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The Variable-Rate Input Application Decision for Multiple Inputs with Interactions

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Research has evaluated the relative profitability of variable-rate (VRT) versus uniform-rate (URT) application of a single input in fields with multiple management zones. This study addresses map-based VRT decisions for multiple inputs in fields with multiple management zones. The decision-making framework is illustrated for nitrogen and water applied to irrigated cotton in fields with three management zones. Results suggest traditional methods of determining VRT application of a single input may be suboptimal if interactions exist among VRT inputs and URT inputs. Implications are that a systems approach to multiple-input VRT decisions can produce increased net returns to VRT.

Key words: breakeven analysis, cotton, economic feasibility, multiple inputs, precision farming, variable-rate technology

Introduction

Economic analyses of the decision to use variable-rate technology (VRT) versus uniform-rate technology (URT) to apply inputs within a farm field have concentrated on application of a single input (e.g., Babcock and Pautsch, 1998; English, Roberts, and Mahajanashetti, 2001; Lambert and Lowenberg-DeBoer, 2000; Swinton and Lowenberg-DeBoer, 1998). Unless inputs are independent of one another, a change in the quantity of one input affects the marginal products of other inputs as they interact in producing output. Thus, for the multiple-input VRT decision, optimal quantities of inputs must be determined jointly. For example, Larson et al. (2004) investigated the profitability of a systems approach for cotton production using the Normalized Difference Vegetative Index (NDVI) to delineate management zones. Seed, in-furrow fungicide, insecticide, growth regulator, and harvest aid inputs were then applied at variable rates across management zones. Input levels were determined by implicit yield-response functions (i.e., yield-response decision rules developed by experts), which subjectively accounted for interactions among inputs.

This article deals with the profit-maximizing decision concerning VRT and/or URT application of multiple inputs within a field and evaluates this decision for cases where nitrogen and water are applied to cotton fields with different proportions of their areas

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in three management zones. Results show that interactions among inputs produce suboptimal returns to single-input VRT unless URT and/or VRT application rates of other inputs are also adjusted.

Two basic methods of implementing site-specific information for VRT application of inputs within a farm field include map-based and sensor-based methods (Ess, Morgan, and Parsons, 2001; Swinton, 2005; Zhang et al., 2002). Both methods use yield-response decision rules in the form of implicit or explicit yield-response functions to determine input application rates from site-specific information. Once site-specific information is collected, yield-response decision rules convert the site-specific information into input application rates that are expected to achieve desired yield responses.

The map-based VRT method uses information-gathering technologies such as remote sensing, yield monitoring, topographical mapping, and electrical conductivity mapping, along with implicit or explicit yield-response functions, to create a site-specific input application map for a field, which is then placed in a variable-rate controller on an implement or tractor to guide variable-rate application of the input (Ess, Morgan, and Parsons, 2001; Swinton, 2005; Zhang et al., 2002). With map-based VRT, farmers can gather site-specific information from sensors and other site-specific information-gathering technologies over a period of time and later use the accumulated information to make decisions about applying the inputs (Khanna, 2001; Roberts et al., 2004). In a 2005 precision farming survey of southern U.S. cotton farmers, Roberts et al. (2006) found that 93% of VRT adopters reported using map-based methods. Regardless of its apparent popularity among farmers who have adopted VRT, map-based VRT typically has been used and evaluated for a single input without considering interactions among inputs.

The sensor-based VRT method employs sensors to measure the site-specific characteristics of a field, and uses that information immediately through a set of decision rules (or algorithms) to control a variable-rate input applicator on-the-go (Lowenberg-DeBoer, 2004; Zhang, Wang, and Wang, 2002). Decisions to gather site-specific information and apply inputs are made jointly when using the sensor-based VRT method because the intermediate step of creating an application map is by-passed (Lowenberg-DeBoer, 2004; Swinton, 2005). Sensor-based VRT methods have been developed for application of inputs such as herbicides (e.g., Bennett and Pannell, 1998; Biller, 1998; Gerhards and Christensen, 2003; Lamm, Slaughter, and Giles, 2002; Swinton, 2005; Tian, 2002; Tian, Reid, and Hummel, 1999; Wartenberg and Dammer, 2001; Zhang et al., 2002), fertilizers (e.g., Alchanatis and Schmilovitch, 2005; Auernhammer et al., 1999; Colburn, 1999; Ehlert, Schmerler, and Voelker, 2004; Link, Panitzki, and Reusch, 2002; Lowenberg-DeBoer, 2004; Noh et al., 2005; Raun et al., 2002; Solie et al., 2002; Stone et al., 1996), and irrigation water (e.g., Camp et al., 1998; Evans et al., 2000; Omary, Camp, and Sadler, 1997).

Farmers are increasingly interested in sensor-based VRT. Roberts et al. (2006) found that 11% of VRT adopters reported using sensor-based VRT methods. Four percent of those who used sensor-based methods also reported using map-based VRT methods. This interest stems from the high cost of gathering and mapping site-specific information, interpreting the maps to create accurate site-specific prescriptions, and time delays in decision making (Noh et al., 2005; Swinton, 2005; Tian, 2002; Zang et al., 2002). As with map-based VRT, despite their increasing popularity, sensor-based VRT methods have typically been used and evaluated for single-input VRT without considering interactions with other inputs.

The sequential pattern of decision making used with map-based VRT is assumed in the theoretical framework and illustrative example presented below. Swinton (2005, p. 260) asserts, "Although far from commercially viable, the potential to cut labor costs and timeliness problems associated with map-based SSHWM [site-specific herbicide-based weed management] combined with the possibility of sharply declining capital costs ... make sensor-based on-the-go herbicide application the most promising future for SSHWM." On the other hand, Adamchuk and Jasa (2002, p. 2) suggest that using soil-sensor information in a map-based VRT system "may be more desirable ... than using real-time, on-the-go sensors with controllers" because the map-based method allows the farmer to make prescription decisions in multiple steps using multiple layers of site-specific information. Zhang, Wang, and Wang (2002, p. 116) state, "Most experimental precision-agriculture systems are map-based systems, because on-the-go sensors for monitoring the field, soil, and field variability are too expensive, not sufficiently accurate, or not available."

Although sensor-based, real-time VRT is gaining in popularity, Lowenberg-DeBoer (2004, p. 1) reports several constraints to widespread adoption and notes, "If the algorithms driving the controller are not well understood by the user this is referred to as a 'black box' approach." Thus, decisions about input application rates may be transferred from the farmer to unknown yield-response decision rules that automatically determine input prescriptions.

Adamchuk and Jasa (2002, p. 2) point out that most sensor-based VRT systems require time for measurement, integration, and/or adjustment, which slows operator speed; moreover, applicators require additional information like yield potential to augment or calibrate algorithms used in developing prescriptions. They go on to state, "Currently, there is no site-specific management prescription algorithm proven to be the most favorable for all variables involved in crop production."

As concluded by Ess, Morgan, and Parsons (2001, p. 8), "It is important to match the application system with the objectives of the overall site-specific management program in which it will be used. Producers should expect an increasing number of options for both map-based and sensor-based site-specific operations as research and development efforts continue." Although map-based VRT is assumed in the theoretical framework and example, the conclusions that are drawn apply to both map-based and sensor-based VRT when one input is VRT applied without considering interactions with other VRT or URT inputs.

Farmers who are contemplating the use of map-based VRT are interested in knowing whether VRT is economically viable for their fields. Within-field variability in soil physical and chemical properties is a necessary condition for the economic viability of VRT (English, Roberts, and Mahajanashetti, 2001; Forcella, 1992; Hayes, Overton, and Price, 1994; Roberts, English, and Mahajanashetti, 2000; Snyder, 1996). Profitability of VRT varies among fields with differences in spatial variability, where spatial variability is defined as the distribution across a field of management zones with different crop yield responses to inputs (Roberts, English, and Mahajanashetti, 2000). In this context, a management zone is not necessarily a contiguous area within a field, but can be a set of smaller areas dispersed throughout the field that have similar yield responses to inputs.

Relationships among crop yields, input levels, and soil properties help farmers identify management zones and determine spatial variability within the field. These relationships also determine yield-response variability, where yield-response variability is defined by differences in the magnitudes of yield responses among management zones (English, Roberts, and Mahajanashetti, 2001; Forcella, 1992; Roberts, English, and Mahajanashetti, 2000). Spatial and yield-response variability, along with the crop price, the input prices, and the additional cost of using VRT versus URT, in concert with farm and farmer characteristics, factor into a farmer's decision to adopt VRT (Roberts et al., 2004). In the end, however, no general rule exists for declaring VRT profitable because each field presents a different case (Roberts et al., 2002).

The specific objectives of this research were (*a*) to develop a conceptual framework for the map-based VRT-versus-URT decision for applying multiple inputs in fields with multiple management zones, (*b*) to illustrate the use of the decision-making framework using explicit yield-response functions for irrigated cotton fields with nitrogen and water applied to three management zones, and (*c*) to use the conceptual framework to demonstrate that single-input VRT produces suboptimal returns when rates of other URT or VRT inputs are not adjusted to account for interactions. The illustrative example addresses the map-based, multiple-input VRT decision; however, as will be discussed later, this framework can be adapted to sensor-based VRT methods. The example assumes quadratic yield-response functions but can be modified for other explicit response functions, such as linear-plateau and semi-log forms, or implicit response functions in the form of fertilizer recommendations from a soil-test laboratory, rules of thumb, or sensor readings that are correlated with application of inputs to achieve a desired yield response.

Analytical Framework

Assume farmers are profit maximizers who have used information-gathering technologies to classify their fields into m management zones and have knowledge of the management-zone-specific yield-response functions for a given crop and set of n inputs. Suppose further that yield responses in the management zones can be represented by strictly concave functions and fields can include any of these m management zones in any proportions. Let the response functions be represented by:

$$(1) \qquad Y_i = Y_i(\mathbf{A}_i, X_{i1}, ..., X_{in}), \quad i = 1, 2, ..., m,$$

where Y_i is crop yield/acre for management zone i; $\mathbf{A}_i$ is a vector of yield-response function parameters for management zone i; and X_{ij} is the amount/acre of input j (j = 1, ..., n) applied to management zone i.

Consider an alternative where r inputs are applied with VRT and the remaining $n - r$ inputs are applied with URT. Assuming the order of the inputs is nonspecific and the amount/acre of uniform-rate input j ($j = r + 1, ..., n$) applied to management zone i ($i = 1, ..., m$) is constrained by $X_{ij} = X_{uj}$, the yield-response functions become:

$$(2) \qquad Y_i = Y_i(\mathbf{A}_i, X_{i1}, ..., X_{ir}, X_{u(r+1)}, ..., X_{un}), \quad i = 1, 2, ..., m.$$

When $r = n$, all inputs are applied with VRT, and equations (1) and (2) are the same. Conversely, when $r = 0$, all inputs are applied with URT, and equations (2) become:

$$(3) \qquad Y_i = Y_i(\mathbf{A}_i, X_{u1}, ..., X_{un}), \quad i = 1, 2, ..., m.$$

Thus, the farmer's optimization problem can be stated for any alternative k as:

$$\text{(4)} \qquad \text{Max}\, R_k = \sum_{i=1}^{m} \lambda_i \left[P_Y Y_i(\mathbf{A}_i, X_{i1}, ..., X_{ir}, X_{u(r+1)}, ..., X_{un}) - \sum_{j=1}^{r} P_j X_{ij} \right] - \sum_{j=r+1}^{n} P_j X_{uj},$$

where R_k is the return above input cost/acre for alternative k, which is one of the 2^n alternative ways to apply n inputs using VRT or URT (Larson, 1982); P_Y is the crop price; P_j is the price of input j ($j = 1, ..., n$); X_{ij} is the amount/acre of variable-rate input j ($j = 1, ..., r$) applied to management zone i; X_{uj} is the amount/acre of uniform-rate input j ($j = r + 1, ..., n$) applied to all management zones in the field; and λ_i is the proportion of the field in management zone i such that

$$\sum_{i=1}^{m} \lambda_i = 1.$$

The first-order conditions for profit maximization are:

$$\text{(5)} \qquad \frac{\partial R_k}{\partial X_{ij}} = \lambda_i \left(P_Y \frac{\partial Y_i}{\partial X_{ij}} - P_j \right) = 0,$$

$$\text{which gives} \quad \frac{\partial Y_i}{\partial X_{ij}} = \frac{P_j}{P_Y}, \quad i = 1, ..., m; \; j = 1, ..., r,$$

and

$$\text{(6)} \qquad \frac{\partial R_k}{\partial X_{uj}} = \sum_{i=1}^{m} \lambda_i \left(P_Y \frac{\partial Y_i}{\partial X_{uj}} \right) - P_j = 0,$$

$$\text{which gives} \quad \sum_{i=1}^{m} \lambda_i \frac{\partial Y_i}{\partial X_{uj}} = \frac{P_j}{P_Y}, \quad j = r + 1, ..., n.$$

When $r = 0$, the optimal amounts/acre of the n uniform-rate inputs are found from the simultaneous solution of equations (6) because equations (5) do not exist. When $r = n$, the marginal products of the n variable-rate inputs ($\partial Y_i / \partial X_{ij}$) are independent across management zones and, because equations (6) do not exist, optimal rates of the n variable-rate inputs for management zone i are found from the simultaneous solution of the n first-order conditions for that management zone. When r is between 0 and n, equations (5) and (6) are solved simultaneously to find optimal amounts/acre of the variable-rate and uniform-rate inputs because the marginal products of the variable-rate inputs in each management zone i ($\partial Y_i / \partial X_{ij}, j = 1, ..., r$) are dependent on the amounts/acre of the uniform-rate inputs applied, and the field-average marginal product of each uniform-rate input,

$$\sum_{i=1}^{m} \lambda_i \frac{\partial Y_i}{\partial X_{uj}}, \quad j = r + 1, ..., n,$$

is dependent on the amounts of the variable-rate inputs applied in each management zone.

As indicated by equations (5), optimality requires that the marginal products for each variable-rate input j ($j = 1, ..., r$) be equal across management zones and equal to the respective input-output price ratio, and equations (6) show that for each uniform-rate input j ($j = r + 1, ..., n$) the field-average marginal product must equal the respective input-output price ratio. Thus, optimality is required for each variable-rate input in each management zone, but only field-average optimality is required for each uniform-rate input.

Solving these first-order conditions and substituting the optimal levels of the inputs into the objective function in equation (4) gives the following profit function (Nicholson, 2004) for alternative k:

$$(7) \qquad R_k^* = \sum_{i=1}^{m} \lambda_i \left[P_Y Y_i^*(\mathbf{A}_i, X_{i1}^*, ..., X_{ir}^*, X_{u(r+1)}^*, ..., X_{un}^*) - \sum_{j=1}^{r} P_j X_{ij}^* \right] - \sum_{j=r+1}^{n} P_j X_{uj}^* = R_k^*(\mathbf{A}_1, ..., \mathbf{A}_m, \lambda_1, ..., \lambda_{m-1}, P_Y, P_1, ..., P_n),$$

where the variables and parameters are as defined in equation (4), and an asterisk indicates optimality. The proportion of the field in management zone m (λ_m) is not an argument in the profit function because

$$\lambda_m = 1 - \sum_{i=1}^{m-1} \lambda_i .$$

If alternative k has one or more additional inputs that are applied using VRT compared to another alternative o, R_k^* is expected to be greater than R_o^*, other things equal, because inputs applied with VRT are applied optimally in each management zone, while URT typically under- or over-applies inputs in the management zones. Even when alternative k has fewer inputs applied using VRT, R_k^* can be greater than R_o^* if the set of VRT inputs for alternative k provides a greater increase in return above input cost/acre than the set of VRT inputs for alternative o in going from all inputs being applied using URT to the respective VRT alternative. Regardless, the difference between R_k^* and R_o^* is a profit function that identifies the optimal return to the VRT application of inputs in alternative k compared to the VRT application of inputs in alternative o:

$$(8) \qquad RVRT_{ko}^* = R_k^* - R_o^* = RVRT_{ko}^*(\mathbf{A}_1, ..., \mathbf{A}_m, \lambda_1, ..., \lambda_{m-1}, P_Y, P_1, ..., P_n), \quad k \neq o \text{ and } R_k^* > R_o^*,$$

where $RVRT_{ko}^*$ is the return to VRT, which is the increased optimal return above input cost/acre from the application of the variable-rate inputs in alternative k compared to the application of a different set of variable-rate inputs in alternative o. The profit functions in equations (7) and (8) are important because they allow direct calculation of $RVRT_{ko}^*$ without re-solving the first-order conditions for alternatives k and o every time a model parameter ($\mathbf{A}_i$, λ_i, P_Y, or P_j) changes.

With $RVRT_{ko}^*$ calculated, the question remains whether a profit-maximizing farmer should switch from alternative o to alternative k. Alternative k is more profitable than alternative o if $RVRT^*_{ko} - C1_{ko} - C2_{ko} > 0$, where $C1_{ko}$ is the input application cost/acre for alternative k minus the input application cost/acre for alternative o, and $C2_{ko}$ is the difference in the cost/acre of gathering spatial information and using it to identify management zones and their yield-response functions for alternative k compared to alternative o. If the additional spatial information required to make the decision has already been gathered and processed, $C2_{ko}$ is a sunk cost in making this decision, and the profit-maximizing farmer will undertake alternative k if $RVRT^*_{ko} > C1_{ko}$. That $C2_{ko}$ is a sunk cost is the initial assumption of our analysis.

If, on the other hand, the additional spatial information has not been gathered and processed, the farmer can use conservative, educated guesses about the λ_is, the corresponding yield-response functions, and $C1_{ko}$ to estimate $RVRT^*_{ko} - C1_{ko}$, which can be thought of as an educated guess about the maximum amount/acre a farmer can invest in gathering and processing the additional spatial information required to improve identification of the field's management zones and their yield-response functions. If $RVRT^*_{ko} - C1_{ko} < C2_{ko}$, the farmer would not gather the additional spatial information and would continue to apply inputs under alternative o. The farmer would undertake to gather and process the additional spatial information if $RVRT^*_{ko} - C1_{ko} > C2_{ko}$. Once the additional spatial information has been gathered and processed, the farmer would have better estimates of the management zones and their yield-response functions with which to make the decision between alternatives k and o.

The choice faced by many farmers is whether to switch from applying all inputs with URT to applying some or all inputs with VRT. When $r = 0$, the profit function is:

$$\text{(9)} \qquad \begin{aligned} R^*_1 &= P_Y \sum_{i=1}^{m} \lambda_i Y_i(\mathbf{A}_i, X^*_{u1}, \ldots, X^*_{un}) - \sum_{j=1}^{n} P_j X^*_{uj} \\ &= R^*_1(\mathbf{A}_1, \ldots, \mathbf{A}_m, \lambda_1, \ldots, \lambda_{m-1}, P_Y, P_1, \ldots, P_n), \end{aligned}$$

where R^*_1 is optimal return above input cost/acre when all inputs are applied with URT;

$$\sum_{i=1}^{m} \lambda_i Y_i(\mathbf{A}_i, X_{u1}, \ldots, X_{un})$$

is the field-average yield-response function, which correlates well with common methods used to develop fertilizer recommendations (English, Roberts, and Mahajanashetti, 2001); Y_i is yield obtained from management zone i when optimal uniform rates of the inputs are applied; and X^*_{uj} is the optimal uniform application rate for input j obtained from the simultaneous solution of the n first-order conditions. For convenience and to simplify notation, the subscript on R^*_1 assigns this alternative as the first of 2^n possible alternatives. Assuming $C2_{k1}$ is a sunk cost for a particular field, the profit-maximizing farmer chooses a VRT alternative k ($k = 2, \ldots, 2^n$) that has some or all of its inputs applied using VRT so long as a solution to the following optimization problem exists:

$$\text{(10)} \qquad \begin{aligned} &\text{Max } RVRT^*_{k1} - C1_{k1}, \\ &\text{s.t.: } RVRT^*_{k1} - C1_{k1} > 0, \quad k = 2, \ldots, 2^n. \end{aligned}$$

If the constraint is not satisfied for any VRT alternative, the profit-maximizing farmer chooses to continue applying all inputs with URT.

Spatial break-even variability proportions (SBVPs) (English, Roberts, and Mahajanashetti, 2001; Mahajanashetti, 1998; Roberts, English, and Mahajanashetti, 2000) are defined as the lower and upper limits of λ_{m-1} and λ_m for given levels of $\mathbf{A}_1, ..., \mathbf{A}_m$, λ_1, $\lambda_2, ..., \lambda_{m-2}$, P_Y, P_j, and $C1_{k1}$ such that:

$$(11) \qquad RVRT_{k1}^* = RVRT_{k1}^*(\lambda_{m-1} | \mathbf{A}_1, ..., \mathbf{A}_m, \lambda_1, \lambda_2, ..., \lambda_{m-2}, P_Y, P_1, ..., P_n) = C1_{k1},$$

where

$$\lambda_m = 1 - \lambda_{m-1} - \sum_{i=1}^{m-2} \lambda_i .$$

Identifying the SBVPs can facilitate decision making by classifying fields into those with spatial characteristics that produce positive net returns to VRT ($RVRT_{k1}^* - C1_{k1}$) and those that do not.

More specifically, assume three management zones and express equations (1) as quadratic yield-response functions containing two inputs with interactions between the inputs. Given these assumptions, the maximization problem in equation (10) can be solved and the SBVPs identified using equation (11). Let the respective management-zone proportions be λ_1, λ_2, and λ_3, and let equations (1) be represented by:

$$(12) \qquad Y_1 = a_1 + b_1 X_{11} + c_1 X_{11}^2 + d_1 X_{12} + e_1 X_{12}^2 + f_1 X_{11} X_{12},$$

$$(13) \qquad Y_2 = a_2 + b_2 X_{21} + c_2 X_{21}^2 + d_2 X_{22} + e_2 X_{22}^2 + f_2 X_{21} X_{22},$$

and

$$(14) \qquad Y_3 = a_3 + b_3 X_{31} + c_3 X_{31}^2 + d_3 X_{32} + e_3 X_{32}^2 + f_3 X_{31} X_{32},$$

where Y_i and X_{ij} are defined in equations (1) for $m = 3$ management zones ($i = 1, 2,$ and 3) and $n = 2$ inputs ($j = 1$ and 2). Let the four (2^2) alternatives be defined as follows: alternative 1 = use URT for both X_1 and X_2, alternative 2 = use VRT for X_1 and URT for X_2, alternative 3 = use URT for X_1 and VRT for X_2, and alternative 4 = use VRT for both X_1 and X_2.

For alternative 1, the field-average yield-response function is expressed as:

$$(15) \qquad Y_u = g_1 + g_2 X_{u1} + g_3 X_{u1}^2 + g_4 X_{u2} + g_5 X_{u2}^2 + g_6 X_{u1} X_{u2},$$

where

$$g_1 = \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3;\ g_2 = \lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3;$$
$$g_3 = \lambda_1 c_1 + \lambda_2 c_2 + \lambda_3 c_3;\ g_4 = \lambda_1 d_1 + \lambda_2 d_2 + \lambda_3 d_3;$$
$$g_5 = \lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3;\ \text{and}\ g_6 = \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3.$$

Profit-maximizing levels of the inputs are obtained from the first-order conditions as (Heady and Dillon, 1972):

$$(16) \qquad X_{u1}^* = \left[(P_2 g_6 - 2 g_5 P_1)/P_y + (2 g_2 g_5 - g_4 g_6)\right] / (g_6^2 - 4 g_3 g_5);$$
$$X_{u2}^* = \left[(P_1 g_6 - 2 g_3 P_2)/P_y + (2 g_4 g_3 - g_2 g_6)\right] / (g_6^2 - 4 g_3 g_5),$$

where P_1 is the price of input X_1, P_2 is the price of input X_2, and P_Y is the price of output. Substitute these optimal input rates into equation (9) to obtain R_1^*.

For alternative 4, solve the first-order conditions for management zone i to get optimal levels of X_{i1} and X_{i2} as (Heady and Dillon, 1972):

$$(17) \qquad X_{i1}^* = \left[(P_2 f_i - 2e_i P_1)/P_y + (2b_i e_i - d_i f_i)\right]/(f_i^2 - 4c_i e_i), \quad i = 1, 2, 3,$$

and

$$(18) \qquad X_{i2}^* = \left[(P_1 f_i - 2c_i P_2)/P_y + (2d_i c_i - b_i f_i)\right]/(f_i^2 - 4c_i e_i), \quad i = 1, 2, 3.$$

Substitute these optimal input rates into equation (7) to obtain R_4^*.

For alternative 2, the four first-order conditions for profit maximization are:

$$(19) \qquad \frac{\partial R_2}{\partial X_{i1}} = b_i + 2c_i X_{i1} + f_i X_{u2} = \frac{P_1}{P_Y}, \quad i = 1, 2, 3,$$

and

$$(20) \qquad \frac{\partial R_2}{\partial X_{u2}} = \sum_{i=1}^{3} \lambda_i (d_i + 2e_i X_{u2} + f_i X_{i1}) = \frac{P_2}{P_Y}.$$

Solving equations (19) for X_{i1} ($i = 1, 2, 3$) and substituting them into equation (20) gives the optimal level of X_{u2}^* as:

$$(21) \qquad X_{u2}^* = \frac{P_2 P_Y^{-1} - \sum_{i=1}^{3} \lambda_i (d_i + 0.5c_i^{-1} f_i P_1 P_Y^{-1} - 0.5c_i^{-1} f_i b_i)}{\sum_{i=1}^{3} \lambda_i (2e_i - 0.5c_i^{-1} f_i^2)}.$$

The optimal levels of X_{i1}^* ($i = 1, 2, 3$) are then found by substituting X_{u2}^* into:

$$(22) \qquad X_{i1}^* = (P_1 P_Y^{-1} - b_i - f_i X_{u2}^*)/2c_i, \quad i = 1, 2, 3.$$

Similarly, the optimal levels of the inputs for alternative 3 are given by:

$$(23) \qquad X_{u1}^* = \frac{P_1 P_Y^{-1} - \sum_{i=1}^{3} \lambda_i (b_i + 0.5e_i^{-1} f_i P_2 P_Y^{-1} - 0.5e_i^{-1} f_i d_i)}{\sum_{i=1}^{3} \lambda_i (2c_i - 0.5e_i^{-1} f_i^2)},$$

$$X_{i2}^* = (P_2 P_Y^{-1} - d_i - f_i X_{u1}^*)/2e_i, \quad i = 1, 2, 3.$$

Substitute the optimal input rates from equations (21) and (22) into equation (7) to obtain R_2^*, and do the same with the optimal input rates from equations (23) to obtain R_3^*.

Assuming spatial information about the field has been gathered, the management zones have been delineated, their yield-response functions have been specified (i.e., $C2_{k1}$ is a sunk cost), and the differences in the costs of VRT and URT application of the inputs are available ($C1_{k1}$), then the alternative that meets the criterion in equation (10) can be determined if it exists for a field with a given combination of λ_is, and the SBVPs can be estimated from equation (11) for fields with the specified management zones and yield-response functions.

Example

Assume hypothetical fields suited to cotton production can be classified into three management zones, and let the following quadratic functions represent cotton yield response to nitrogen fertilizer (X_1) and irrigation water (X_2) for the management zones:

$$(24) \quad Y_1 = 233.72 + 0.439X_{11} - 0.003X_{11}^2 + 23.65X_{12} - 0.182X_{12}^2 + 0.021X_{11}X_{12},$$

$$(25) \quad Y_2 = -1{,}103.6 + 2.85X_{21} - 0.004X_{21}^2 + 118.35X_{22} - 1.63X_{22}^2 - 0.046X_{21}X_{22},$$

and

$$(26) \quad Y_3 = -170.93 + 3.74X_{31} - 0.011X_{31}^2 + 32.45X_{32} - 0.022X_{32}^2 + 0.022X_{31}X_{32},$$

where Y_i is cotton lint yield (lbs./acre) for management zone i (i = 1, 2, and 3); X_{i1} is the amount of nitrogen applied (lbs./acre) in management zone i (i = 1, 2, and 3); and X_{i2} is the amount of irrigation water applied in management zone i (i = 1, 2, and 3) plus 5 inches of available preplant moisture plus 1 inch of rainfall (acre-inches). These functions were estimated by Hexem and Heady (1978) using field data from Arizona and California, and are similar to the quadratic yield-response functions in Arce-Diaz et al. (1993); Agrawal and Heady (1972); Mjelde et al. (1991); Schlegel and Havlin (1995); and Vanotti and Bundy (1994). Although they were chosen only for illustrative purposes to serve as examples in this article, they are plausible irrigated cotton yield-response functions for three soil types that could define management zones.

Application of the analytical framework does not require statistically estimated, strictly concave, continuous yield-response functions, although work is currently under way to statistically estimate site-specific yield-response functions for a field (e.g., Anselin, Bongiovanni, and Lowenberg-DeBoer, 2004; Griffin, Brown, and Lowenberg-DeBoer, 2005; Velandia, Rejesus, and Segarra, 2004) and response functions estimated from data generated by crop-growth simulation models have been used (e.g., Larson, English, and Roberts, 2002; Roberts et al., 2002; Watkins, Lu, and Huang, 1998).

Use of the response functions in this example is not meant to imply statistically estimated, strictly concave, continuous yield-response functions are the only ones that can be used in this decision-making framework. Nonetheless, such response functions are many times the basis for rules of thumb (or other algorithms) used to determine input application rates based on soil properties or plant characteristics to achieve a desired yield response. For example, many soil-test laboratories, or others who make fertilizer recommendations, use statistically estimated yield-response functions gleaned from the literature or from their own experiments to develop fertilizer recommendations (e.g., Iowa State University, Agronomy Extension, 2006). Also, a cursory review of research presented at the 7th International Conference on Precision Agriculture (Mulla, 2004) produced at least 23 papers that discussed the use of yield-response functions for determining input application rates, many of which were statistically estimated; 21 were map-based applications and two were sensor-based applications.

Table 1. Optimal Yields, Input Application Rates, and Returns Above Input Costs for Management Zones 1, 2, and 3

Management Zone	Yield (lbs./acre)	Nitrogen (lbs./acre)	Water (acre-inches)	Return Above Input Cost ($/acre)
Zone 1	1,140	160	53	339.08
Zone 2	1,120	107	32	424.96
Zone 3	1,628	214	67	522.75

Quadratic yield-response functions facilitate the example because solutions to the mathematical problem are relatively straightforward compared to other functional forms. For example, the linear-plateau function is linear and discontinuous, precluding the use of calculus and, although the semi-logarithmic functional form is continuous, analytical solutions for the SBVPs cannot be found. Solutions to the optimization problem and SBVPs can be found with iterative algorithms for any continuous or discontinuous functional form or for any rule of thumb based on an implicit yield-response function (e.g., label rates for chemical application) so long as yield response to inputs is concave, but not necessarily strictly concave, and interactions among inputs can be assumed through statistical estimation, review of literature, or judgment of experts. As the example will show, if interactions exist among inputs, VRT application of one input without adjusting URT and/or VRT application rates of other inputs will yield suboptimal returns.

An average cotton lint price received by farmers (P_Y = $0.52/lb.) and an average nitrogen price (P_1 = $0.26/lb.) over the 2000–2003 period (U.S. Department of Agriculture, 2004), combined with an irrigation water price of $4/acre-inch ($P_2$), were used in the analysis. Hughes (2005) assumed a water price of $58.33/acre-foot ($4.86/acre-inch) in his irrigation-water-cost analysis. Our price of $4/acre-inch was chosen to be conservative. Optimal yields, input application rates, and net returns above input costs/acre were determined for each management zone (table 1). Assuming the same alternatives as those associated with equations (12)–(14), equation (7) was used to determine R_4^* as a weighted average of the last column in table 1, given the assumptions about the λ_is presented below. R_2^* and R_3^* were also calculated using equation (7), and R_1^* was calculated from equation (9) using the field-average yield-response function. Expressing the λ_is as percentages in this example, equation (8) was calculated for hypothetical cotton fields for all combinations of the λ_is when each λ_i varied between 0% and 90% in increments of 10%, such that the sum of the λ_is equaled 100% (e.g., λ_1 = 20%, λ_2 = 30%, and λ_3 = 50%; or λ_1 = 0%, λ_2 = 40%, and λ_3 = 60%).

The additional custom charge for VRT nitrogen application compared to URT application was assumed to be $3/acre. This additional charge was close to the mean of $3.08/acre obtained from personal telephone interviews with firms providing precision farming services to Tennessee farmers (Roberts, English, and Sleigh, 2000). Based on information developed in Georgia (Fairchild, 2003), a center pivot irrigation system can be retrofitted with VRT for somewhere between $5,000 and $10,000 depending on the number of sprinklers controlled. Assuming a five-year useful life, no salvage value, and a 150-acre irrigation system, the additional cost is between $9/acre and $18/acre. As a conservative estimate, we assumed $18/acre. Thus, a farmer would have to receive

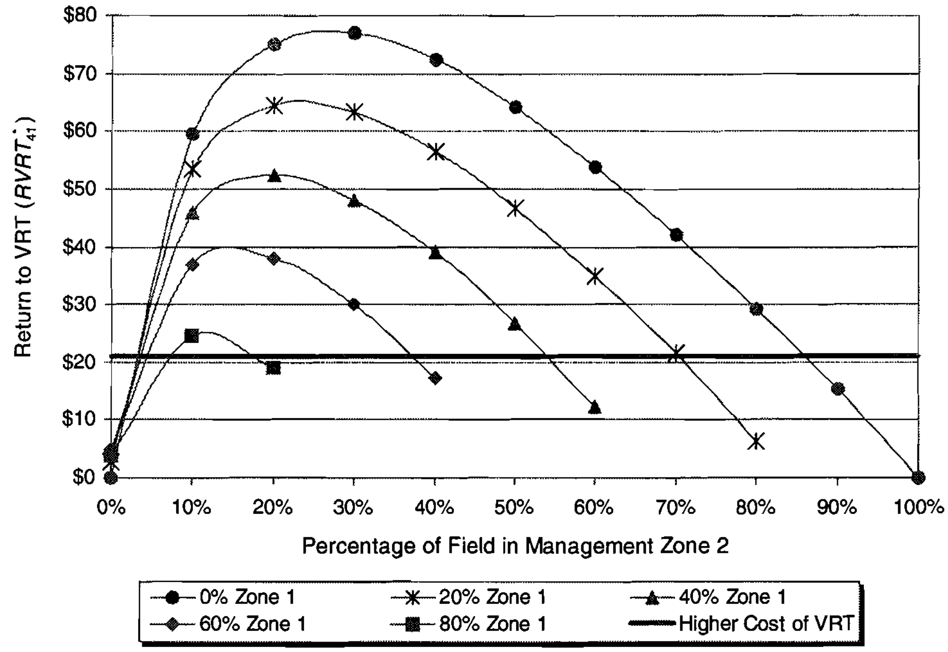

Figure 1. Return to variable-rate technology for VRT nitrogen and VRT water compared with URT nitrogen and URT water for various percentages of a field in management zone 2, given various percentages of the field in management zone 1

an $RVRT^*_{ko}$ of \$21/acre ($C1_{41}$), \$18/acre ($C1_{31}$), \$3/acre ($C1_{21}$), \$18/acre ($C1_{42}$), \$15/acre ($C1_{32}$), and \$3/acre ($C1_{43}$) to break even when going from alternative *o* to alternative *k*. This increased return above input cost/acre would come from increased yield and/or decreased input usage for VRT application of nitrogen and/or water compared to URT application of these inputs.

Figure 1 shows $RVRT^*_{41}$ and $C1_{41}$ as they relate to λ_2 (λ_3 = 100% - λ_2) for fields with different levels of λ_1 between 0% and 80%. If a field has no area in management zone 1 (λ_1 = 0%), the percentage of the field in management zone 2 (λ_2) must be between 2% and 86%, and the percentage in management zone 3 (λ_3) must be between 98% (100% - 2%) and 14% (100% - 86%) for VRT application of both inputs to at least break even with URT application of the inputs. As the percentage of a field in management zone 1 becomes larger, the SBVPs in the other management zones become narrower. For example, if the proportion of a field in management zone 1 is 60%, the SBVPs for management zone 2 are 3% and 37%, and for management zone 3 they are 37% (100% - 60% - 3%) and 3% (100% - 60% - 37%). Within these ranges of λ_2 and λ_3 (given λ_1 = 60%), $RVRT^*_{41} - C1_{41}$ is greater than or equal to zero, and the farmer at least breaks even by using VRT instead of URT to apply nitrogen and water.

As a specific example of a field assumed to have spatial variability within the ranges of its SBVPs, $RVRT^*_{41}$ equals \$63.36/acre for a field with 20% of its area in management zone 1 (λ_1 = 20%), 30% in management zone 2 (λ_2 = 30%), and 50% in management zone 3 (λ_3 = 100% - 20% - 30%). Subtracting \$21/acre ($C1_{41}$) from $RVRT^*_{41}$ gives a positive net return to VRT ($RVRT^*_{41} - C1_{41}$) of \$42.36/acre, suggesting the farmer would be \$42.36/acre better off using VRT instead of URT to apply both inputs in this field.

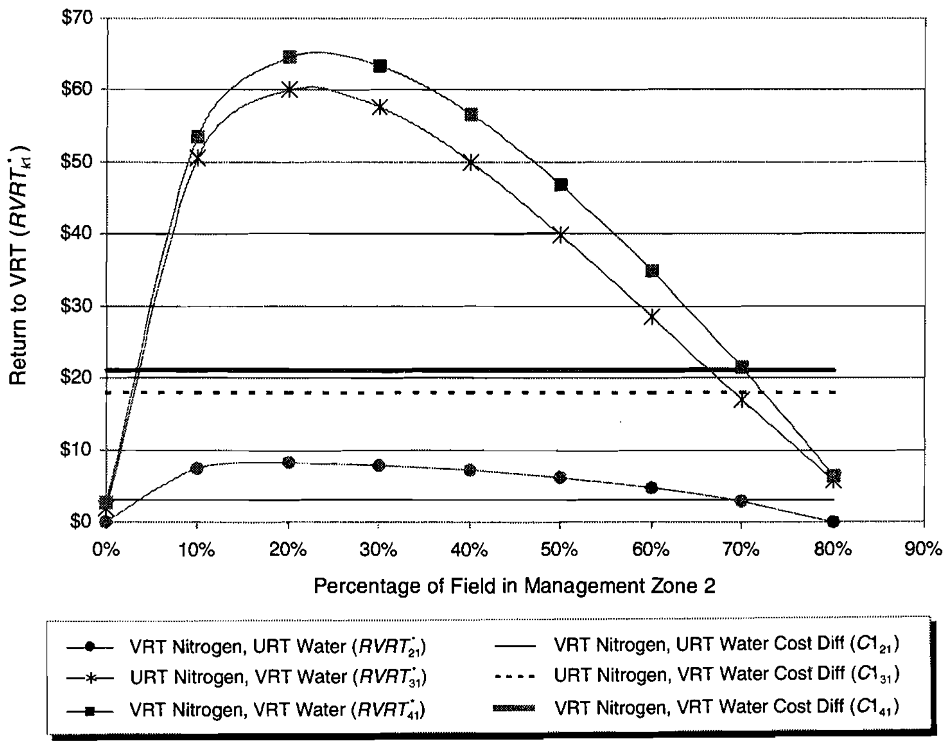

Figure 2. Return to variable-rate technology for three VRT alternatives compared with URT nitrogen and URT water for various percentages of a field in management zone 2, given 20% of the field in management zone 1

Alternatively, if the λ_is and the yield-response functions in equations (24)–(26) were considered educated guesses, this \$42.36/acre could be thought of as an estimate of the maximum amount/acre the farmer can invest in gathering and processing additional spatial information to improve management-zone delineation and increase yield-response function accuracy.

Figure 2 shows $RVRT^*_{k1}$ $(k = 4, 3, 2)$ and $C1_{k1}$ $(k = 4, 3, 2)$ as they relate to λ_2 ($\lambda_3 = 100\% - 20\% - \lambda_2$) for fields with 20% of their area in management zone 1 ($\lambda_1 = 20\%$). Three observations with regard to figure 2 are worth noting.

- First, $RVRT^*_{41}$ is higher than either $RVRT^*_{31}$ or $RVRT^*_{21}$ because alternative 4 has both nitrogen and water applied optimally to the three management zones compared to alternative 2, which only has nitrogen applied optimally to the management zones, and alternative 3, which only has water applied optimally to the management zones.

- Second, the increase in return above input cost/acre in going from alternative 1 (URT both inputs) to alternative 3 (URT nitrogen and VRT water) is greater than the increase in going from alternative 1 to alternative 2 (VRT nitrogen and URT water), indicating that the increase in return above input cost/acre from VRT water application is considerably greater than the increase in return above input cost/ acre from VRT nitrogen application.

Table 2. Net Returns to Variable-Rate Technology ($RVRT^*_{ko} - C1_{ko}$) for Fields with 20% of Their Area in Management Zone 1 (λ_1 = 20%)

% in Zone 2 (λ_2): →	0	10	20	30	40	50	60	70	80
% in Zone 3 (λ_3): →	80	70	60	50	40	30	20	10	0
				—	($/acre)	—			
Compare VRT and URT Alternatives:[a]									
$RVRT^*_{21} - C1_{21}$	-2.93	4.36	5.22	4.92	4.20	3.18	1.78	-0.16	-2.98
$RVRT^*_{31} - C1_{31}$	-16.17	32.64	42.10	39.54	31.89	21.78	10.51	-1.09	-12.25
$RVRT^*_{41} - C1_{41}$	-18.38	32.39	43.55	42.36	35.63	25.82	13.97	0.50	-14.59
Compare 2-Input and 1-Input VRT:									
$RVRT^*_{42} - C1_{42}$	-15.45	28.02	38.33	37.44	31.42	22.64	12.19	0.66	-11.61
$RVRT^*_{43} - C1_{43}$	-2.21	-0.25	1.45	2.82	3.73	4.04	3.46	1.59	-2.34

[a] $RVRT^*_{ko} - C1_{ko}$ is optimal return above input cost/acre for alternative k minus optimal return above input cost/acre for alternative o ($RVRT^*_{ko}$) minus the difference in input application costs/acre between alternatives k and o ($C1_{ko}$). Alternative 1 is URT application of both water and nitrogen; alternative 2 is VRT application of nitrogen and URT application of water; alternative 3 is URT application of nitrogen and VRT application of water; and alternative 4 is VRT application of both nitrogen and water.

- Third, the SBVPs for λ_2 are almost indistinguishable from one another when alternatives 2, 3, and 4 are compared with alternative 1. The lower (upper) SBVPs for λ_2 are 2.1% (69%), 1.9% (69.3%), and 2.3% (70.3%) for alternatives 2, 3, and 4, respectively. Thus, a profit-maximizing farmer would switch from URT application of both inputs to VRT application of one or both inputs in fields that have 20% of their area in management zone 1 and between about 2% and 70% of their area in management zone 2 (between 78% and 10% in management zone 3).

Although figure 2 gives a good visual approximation of the solution to the problem in equation (10) for fields with 20% of their area in management zone 1, the results in the upper portion of table 2 present a more accurate estimate of the net return to VRT ($RVRT^*_{k1} - C1_{k1}$; k = 2, 3, 4) for the fields evaluated. Among the alternatives meeting the constraint in equation (10), alternative 4 has the highest net return to VRT for most fields. Nevertheless, this example illustrates that one alternative does not provide the solution to the optimization problem in equation (10) for all fields. Alternative 3 (URT nitrogen and VRT water) provides the optimal solution for fields with 20% of their area in management zone 1 and between 11.4% and 1.9% of their area in management zone 2. The latter λ_2 is the lower SBVP for alternative 3, and the former is the λ_2 that satisfies $RVRT^*_{41} - C1_{41} = RVRT^*_{31} - C1_{31}$.

The lower portion of table 2 gives the net return to VRT received by switching from URT to VRT water, given VRT nitrogen ($RVRT^*_{42} - C1_{42}$), and the net return to VRT received by switching from URT to VRT nitrogen, given VRT water ($RVRT^*_{43} - C1_{43}$), for fields with 20% of their area in management zone 1 (λ_1 = 20%). The lower and upper SBVPs for λ_2 are 2.2% and 71% for switching from URT to VRT water, given VRT nitrogen, and these values are 11.5% and 75% for switching from URT to VRT nitrogen, given VRT water.

A question arises as to why a farmer might want to make this decision about switching from a single variable-rate input to two variable-rate inputs. If the multiple-input response functions and management zones were known, the farmer would have

Table 3. Sensitivity of Results for Alternatives 1 and 2 to Changes in Model Parameters for a Hypothetical Cotton Field with 20% (λ_1), 30% (λ_2), and 50% (λ_3) of Its Area in Management Zones 1, 2, and 3

Item	Net $RVRT^*_{21}$ [b] ($/acre)	Alternative 1[a] per Acre Means Yield (lbs./acre)	URT Nitrogen (lbs./acre)	URT Water (acre-in.)	Alternative 2[a] per Acre Means Yield (lbs./acre)	VRT Nitrogen (lbs./acre)	URT Water (acre-in.)
Base Levels:	4.92	1,133.4	160.7	38.6	1,145.1	136.4	39.7
Parameter[c] × 1.25:			**— Changes from Base Levels —**				
P_1	1.68	-4.8	-8.5	0.0	-5.9	-11.4	0.1
P_2	-1.05	-13.1	-0.1	-1.5	-13.8	1.0	-1.6
P_Y	1.60	11.6	6.9	1.2	12.5	8.3	1.3
b_1	-0.41	4.3	1.5	0.0	4.9	3.3	0.1
c_1	1.23	-5.9	-3.5	0.0	-4.8	-4.6	-0.1
d_1	0.72	53.4	0.1	0.9	55.0	-0.6	1.0
e_1	-0.40	-17.5	0.0	-0.6	-18.6	0.4	-0.6
f_1	-0.52	9.1	2.8	0.1	10.6	6.2	0.2

[a] Alternative 1 is URT application of both water and nitrogen; alternative 2 is VRT application of nitrogen and URT application of water.
[b] Net $RVRT^*_{21}$ is optimal return above input cost/acre for alternative 2 minus optimal return above input cost/acre for alternative 1 ($RVRT^*_{21}$) minus the difference in input application costs/acre between alternatives 2 and 1 ($C1_{21}$).
[c] P_1 is the nitrogen price; P_2 is the water price; P_Y is the cotton lint price; and b_1, c_1, d_1, e_1, and f_1 are yield-response function parameters for management zone 1.

chosen alternative 4 for most fields, as indicated in the upper portion of table 2. Therefore, comparisons between alternatives 2 and 4 or between alternatives 3 and 4 are most suited to situations where a farmer has already made the VRT decision for one input based on single-input spatial information, and wants to know whether adding another variable-rate input will improve optimal net return to VRT. For these cases, the multiple-input yield-response functions and the λ_is would be considered educated guesses, and the farmer would gather the additional spatial information required to delineate multiple-input management zones and specify their yield-response functions if $RVRT^*_{4o} - C1_{4o}$ were greater than the cost of gathering and processing the additional multiple-input spatial information ($C2_{4o}$; $o = 2$ or 3).

The rows headed "base levels" in tables 3 and 4 present net $RVRT^*_{ko}$s, yields, and per acre input rates for alternatives 1–4 for a field with $\lambda_1 = 20\%$, $\lambda_2 = 30\%$, and $\lambda_3 = 50\%$. Results show that input use/acre does not necessarily decline with VRT application. In going from alternative 1 to alternative 2, nitrogen use declines from 160.7 lbs./acre to 136.4 lbs./acre, but water use increases slightly from 38.6 acre-inches to 39.7 acre-inches (table 3). The more efficient use of nitrogen in each management zone increases the field-average marginal productivity of water, leading to an increase in the optimal uniform water rate. The revenue from increased yield and reduced fertilization cost outweighs the increased water cost to give a net $RVRT^*_{21}$ of $4.92/acre. This net return contrasts sharply with net $RVRT^*_{31}$ and net $RVRT^*_{41}$, which are $39.54/acre and $42.36/acre (table 4), respectively. When VRT is used to apply water, field-average water use increases from 38.6 acre-inches in alternative 1 (table 3) to 53.3 acre-inches in alternative 3 (table 4). The increased efficiency of irrigation water in each management

Table 4. Sensitivity of Results for Alternatives 3 and 4 to Changes in Model Parameters for a Hypothetical Cotton Field with 20% (λ_1), 30% (λ_2), and 50% (λ_3) of Its Area in Management Zones 1, 2, and 3

		Alternative 3[a] Field Means				Alternative 4[a] Field Means		
Item	Net $RVRT^*_{31}$[b] ($/acre)	Yield (lbs./acre)	URT Nitrogen (lbs./acre)	VRT Water (acre-in.)	Net $RVRT^*_{41}$[c] ($/acre)	Yield (lbs./acre)	VRT Nitrogen (lbs./acre)	VRT Water (acre-in.)
Base Levels:	39.54	1,373.5	192.7	53.3	42.36	1,377.9	171.0	53.8
Parameter[d] × 1.25:			— **Changes from Base Levels** —					
P_1	-2.06	-7.4	-9.1	-0.3	-0.53	-9.8	-12.7	-0.3
P_2	-13.72	-33.1	-4.5	-3.6	-14.13	-35.4	-5.3	-3.8
P_Y	32.27	26.3	10.9	3.1	33.07	29.4	14.4	3.3
b_1	0.37	5.4	1.6	0.1	0.09	7.6	4.1	0.2
c_1	-0.94	-9.4	-4.5	-0.1	0.50	-10.3	-7.5	-0.4
d_1	14.87	103.5	5.0	3.4	14.73	111.4	12.7	4.0
e_1	-4.45	-41.3	-3.3	-2.3	-3.25	-42.7	-7.9	-2.5
f_1	2.61	20.0	5.3	0.7	2.57	30.2	14.4	1.5

[a] Alternative 3 is URT application of nitrogen and VRT application of water; alternative 4 is VRT application of both nitrogen and water.

[b] Net $RVRT^*_{31}$ is optimal return above input cost/acre for alternative 3 minus optimal return above input cost/acre for alternative 1 ($RVRT^*_{31}$) minus the difference in input application costs/acre between alternatives 3 and 1 ($C1_{31}$).

[c] Net $RVRT^*_{41}$ is optimal return above input cost/acre for alternative 4 minus optimal return above input cost/acre for alternative 1 ($RVRT^*_{41}$) minus the difference in input application costs/acre between alternatives 4 and 1 ($C1_{41}$).

[d] P_1 is the nitrogen price; P_2 is the water price; P_Y is the cotton lint price; and b_1, c_1, d_1, e_1, and f_1 are yield-response function parameters for management zone 1.

zone increases the field-average marginal productivity of nitrogen, giving an optimal uniform nitrogen rate of 192.7 lbs./acre, up from 160.7 lbs./acre for alternative 1. Although input costs/acre increase for alternative 3 compared to alternative 1, the added revenue from the yield increase of 240.1 lbs./acre (1,373.5 lbs./acre - 1,133.4 lbs./acre) greatly outweighs the increased cost. When both nitrogen and water are applied with VRT (alternative 4) instead of VRT application of water alone (alternative 3), the field-average nitrogen rate declines and water use increases slightly as the increased efficiency of nitrogen in each management zone increases the marginal productivity of water in those management zones.

Results reported in the upper portion of table 5 provide a more detailed view of VRT versus URT water application given URT nitrogen application—a comparison between alternatives 1 and 3. With URT, water is economically under-applied in management zones 1 and 3 by 16.4 and 27.3 acre-inches, respectively, while it is over-applied in management zone 2 by the smaller amount of 7.4 acre-inches. Increased irrigation directly produces most of the large increases in yields of 177.6 lbs./acre and 397 lbs./acre for management zones 1 and 3, respectively. Another portion of the increased yields results from an increase of 32 lbs./acre in the URT nitrogen fertilization rate. This increase in nitrogen fertilization results from an increase in the field-average marginal product of nitrogen. The field-average marginal product of nitrogen increases because of the large increases in irrigation rates in management zones 1 and 3 relative to the smaller decrease in management zone 2. In management zone 2, the lower irrigation rate

Table 5. Management-Zone Results for Alternatives 1 and 3, and Sensitivity of Results for Alternative 3 to Changes in Model Parameters for a Hypothetical Cotton Field with 20% (λ_1), 30% (λ_2), and 50% (λ_3) of Its Area in Management Zones 1, 2, and 3

	Yield (lbs./acre)			URT	VRT Water (acre-inches)		
Item	Zone 1	Zone 2	Zone 3	Nitrogen (lbs./acre)	Zone 1	Zone 2	Zone 3
				— **Base Levels** —			
Alternative 3	1,168.2	1,126.7	1,603.7	192.7	55.0	31.2	65.9
Alternative 1	990.6	1,106.5	1,206.7	160.7	38.6[a]	38.6	38.6
Difference	177.6	20.2	397.0	32.0	16.4	-7.4	27.3
Parameter[b] **× 1.25:**		— **Changes from Base Levels for Alternative 3** —					
P_1	-7.2	1.9	-13.3	-9.1	-0.5	0.1	-0.5
P_2	-49.2	-4.1	-44.1	-4.5	-5.5	-0.5	-4.6
P_Y	37.3	0.2	37.5	10.9	4.9	0.3	4.0
b_1	22.5	-0.4	2.1	1.6	0.1	0.0	0.1
c_1	-32.7	1.0	-6.3	-4.5	-0.3	0.1	-0.2
d_1	503.3	-1.3	6.4	5.0	16.5	-0.1	0.2
e_1	-196.0	0.8	-4.6	-3.3	-11.1	0.0	-0.2
f_1	84.9	-1.4	6.8	5.3	3.2	-0.1	0.3

[a] The URT water rate is 38.6 for alternative 1.

[b] P_1 is the nitrogen price; P_2 is the water price; P_Y is the cotton lint price; and b_1, c_1, d_1, e_1, and f_1 are yield-response function parameters for management zone 1.

is offset by increased nitrogen fertilization to produce 20.2 lbs./acre more lint. The revenue from the field-average increase in yield of 240.1 lbs./acre minus the costs from increased URT nitrogen fertilization of 32 lbs./acre and increased field-average water use of 14.7 acre-inches produces an $RVRT^*_{31}$ of $57.54/acre and a net $RVRT^*_{31}$ of $39.54/acre (table 4) after subtracting $18/acre for the higher cost of VRT water application.

Tables 3 and 4 present sensitivity results for 25% changes in parameter values. Several findings are of interest with regard to alternative 1 (table 3). First, when both inputs are applied with URT, yield and input use are insensitive to 25% changes in parameter values. Second, when the parameters that directly affect nitrogen use change (P_1, b_1, or c_1), URT water use does not change, and when the parameters that directly affect water use change (P_2, d_1, or e_1), URT nitrogen use does not change appreciably. This phenomenon occurs because the weighted sum of the interaction terms in equations (24)–(26), g_6 in equation (15), is only 0.001; essentially no interaction exists between URT nitrogen use and URT water use. As f_1 increases by 25%, from 0.021 to 0.026, g_6 doubles to 0.002, which gives small positive increases in URT nitrogen and water use. Third, nitrogen use is more sensitive to changes in the parameters that directly affect nitrogen use (P_1, b_1, c_1, and f_1) than water use is to changes in the parameters that directly affect water use (P_2, d_1, e_1, and f_1). Nevertheless, as reflected in the assumed yield-response functions, yield is more responsive to changes in the water application rate than it is to changes in the nitrogen application rate.

Discussion of the sensitivity results for alternatives 2–4 concentrates first on alternative 3 (tables 4 and 5) and then more briefly addresses alternatives 2 and 4. The effects of the negative interaction between nitrogen and water in equation (25) can easily be seen in the results in the lower portion of table 5. The effects on the water rate for management zone 2 are always smaller than the effects for management zones 1 and 3, and in most instances the signs of the effects for management zone 2 are opposite the signs of the effects for the other management zones. For example, a 25% increase in the price of nitrogen (P_1) decreases the URT nitrogen rate by 9.1 lbs./acre, which in turn decreases the optimal VRT water rates by 0.5 acre-inches in management zones 1 and 3 because f_1 and f_3 are positive, while the negative interaction produces a slight increase in the water rate in management zone 2. Reduced URT nitrogen and VRT water in management zones 1 and 3 cause yield to decline in those management zones, while the increased optimal water rate overwhelms the reduction in the URT nitrogen rate to increase yield slightly in management zone 2.

The sensitivity of the results for alternative 3 (table 4) is greater than the sensitivity of the results for alternative 1 (table 3) for changes in every parameter; i.e., when both inputs and yield decline (increase), they decline (increase) more for alternative 3 than alternative 1. Nevertheless, the value of the increased (decreased) lint yield for alternative 3 compared with the value of the increased (decreased) lint yield for alternative 1 outweighs the difference in increased (decreased) costs between the alternatives; thus, net $RVRT_{31}^*$ (table 4) moves in the same direction as the change in input rates and yield. Results for net $RVRT_{31}^*$ ($RVRT_{31}^* - C1_{31}$) in table 4 show that, for this particular field, increases in the prices of nitrogen and water (P_1 and P_2) decrease the profitability of VRT alternative 3 compared to alternative 1 by \$2.06/acre and \$13.72/acre, respectively. In addition, the greater the yield-response variability, the more profitable alternative 3 is relative to alternative 1, as indicated by net $RVRT_{31}^*$ increases of \$0.37/acre and \$14.87/acre resulting from 25% increases in b_1 and d_1, respectively. In contrast, the smaller the yield-response variability, as indicated by 25% changes in c_1 and e_1, the less profitable alternative 3 is relative to alternative 1. A 25% increase in the interaction term (f_1) increases the relative profitability of alternative 3 compared to alternative 1 by \$2.61/acre.

Sensitivity results for alternative 4 (table 4) are similar to those for alternative 3, suggesting that adding nitrogen as a VRT input has a relatively small effect on profitability compared to adding water as a VRT input. Changes in net $RVRT_{31}^*$ and net $RVRT_{41}^*$ (table 4) are considerably larger than changes in net $RVRT_{21}^*$ (table 3) for 25% changes in model parameters that directly affect yield response to water (P_2, P_y, d_1, e_1), and changes in the parameters that directly affect yield response to nitrogen (P_1, P_y, b_1, c_1) have smaller impacts on net returns to VRT in table 4 than in table 3. These results suggest when water is applied with VRT it is the dominant input, having the greatest impact on relative profitability, whether it be alternative 3 or alternative 4 compared with alternative 1. Based on the sensitivity results, the relative profitability of VRT nitrogen is lower than the relative profitability of VRT water but more stable, in that the relative profitability of VRT nitrogen changes much less than the relative profitability of VRT water when model parameters change.

Conclusions

The theoretical decision-making framework presented in this study suggests that traditional methods of determining economically optimal map-based VRT application rates for a single input may be suboptimal if interactions exist among VRT inputs and other inputs applied with URT. The amount of inaccuracy depends on the inputs and the yield-response decision rules used. In our illustrative example of nitrogen and water applied to cotton fields with three management zones, we found that the optimal uniform rate of irrigation water increased 2.8% when nitrogen was applied using VRT. A greater increase of 20% was found in the uniform rate of nitrogen when irrigation water was applied using VRT. Thus, if interactions exist among inputs, single-input VRT may provide suboptimal returns above input costs/acre unless uniform rates for other inputs are also adjusted.

Many potential crop-production applications exist for this multiple-input VRT framework. Two other examples include decisions about plant population and nitrogen fertilization in corn production and application of nitrogen, plant growth regulator, and harvest aid inputs in cotton production. Results from the theoretical decision-making framework and the illustrative example indicate increased profit could be obtained by evaluating potential interactions among inputs through a systems approach to the VRT or URT application of multiple inputs. Work is currently under way to develop decision aids using a systems approach to help cotton farmers make decisions about using precision farming technologies.

The approach taken in this analysis could be adapted to decisions about sensor-based VRT. The yield-response decision rules (algorithms) used with sensor-based VRT could be modified to account for potential interactions among inputs. In addition, the "black-box" nature of sensor-based VRT suggests that farmers might want to reduce the risk of attaining disappointing economic returns by developing a preliminary set of site-specific information from yield monitoring, aerial photography, soil survey maps, and/or other information-gathering technologies before making the sensor-based VRT decision. This information could be used to anticipate whether sufficient spatial and yield-response variability exist within a field to cover the increased cost of sensor-based VRT relative to URT application of inputs.

For the sensor-based VRT decision, the decision-making framework presented above could be modified in three ways. First, sunk information-gathering costs (C_2) and the difference in input application costs (C_1) would be different for the sensor-based and map-based VRT decisions. With the sensor-based VRT decision, C_2 still might exist, but some information-gathering costs would become part of C_1 as variable costs in making this VRT decision. For example, suppose a farmer making the sensor-based VRT decision could use yield-monitoring information to help determine whether using electrical conductivity-based, on-the-go VRT would be more profitable than URT. The cost of collecting and mapping site-specific yield information would be C_2, and the cost of collecting electrical conductivity readings would be included in C_1 as part of VRT input application costs. Alternatively, decisions about map-based VRT would include the costs of collecting site-specific yield and electrical conductivity information as sunk costs in C_2, and C_1 would exclude the cost of collecting electrical conductivity readings. Second, C_1 could be different for sensor- and map-based methods depending on differences in machinery, equipment, and labor costs for VRT application. Third, the return to VRT (RVRT) for sensor-based and map-based VRT could be different if input costs differed, but would be the same if both methods used soil electrical conductivity readings and the same yield-response decision rule to apply inputs at variable rates across the field.

Farmers are interested in knowing whether VRT is economically viable on their fields. Fields generally exhibit yield-response and spatial variability; however, as demonstrated here, not all fields warrant VRT from an economic standpoint. The economic viability of single- or multiple-input VRT varies from field to field depending on yield-response interactions among inputs, as well as spatial variability and yield-response variability among management zones. As shown in the example, no general rule exists for determining whether single- or multiple-input VRT is more profitable than URT application of all inputs because each field presents a different case. Nevertheless, for the case presented in this paper, a wide range of spatial and yield-response variability would provide increased net returns to map-based VRT application of nitrogen and/or water relative to URT application of both inputs.

To utilize this map-based, multiple-input VRT framework in a theoretically ideal world, farmers, consultants, researchers, and/or extension personnel would need knowledge of the field-specific management zones for a particular crop and inputs, including the parameters of the corresponding yield-response functions. Unfortunately, this knowledge is difficult to obtain with certainty, but agricultural practitioners are currently employing information-gathering technologies (e.g., yield monitoring, remote sensing, field mapping) that can be used to identify management zones and their yield-response potentials (e.g., Anselin, Bongiovanni, and Lowenberg-DeBoer, 2004; Larson et al., 2004; Roberts et al., 2004). As in Larson et al. (2004), yield-response decision rules that account for interactions among inputs based on expert knowledge of farmers, consultants, researchers, and extension personnel about underlying implicit yield-response functions can be used in a systems approach to VRT application of multiple inputs. Even when information about the management zones and their multiple-input yield-response functions is far from perfect, these methods can be used to develop rough estimates about whether investment in obtaining additional multiple-input spatial information to more precisely identify management zones and improve knowledge of their implicit yield-response functions is potentially worthwhile.

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