

A Derivative Security Approach to Setting Crop Revenue Coverage Insurance Premiums

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The nature of indemnities and reliance on futures price averaging during two distinct time intervals throughout the production year imply Crop Revenue Coverage (CRC) insurance behaves like an exotic put option. Treating this type of insurance as a derivative security, an analytical model is developed and an algorithm for solving the model to place a lower bound on insurance premiums is presented. Monte Carlo simulation, taking into account the path-dependent nature of an Asian-type option, is then used to determine lower-bound estimates for insurance premiums on corn gross revenue under specified price and yield distributions.

Key words: Asian option, CRC insurance, derivative, discrete arithmetic averaging, Monte Carlo

Introduction

Production agriculture has always been inherently risky. Price and yield uncertainty at the farm level have, in the past, seemed to justify government involvement in agriculture. This is so even in the long-standing presence of commodity markets which can (and in many cases do) offer price risk protection for commodity producers. Numerous arguments have been made to justify such continual direct government involvement in agriculture as managing low farm incomes, stabilizing farm prices and supplies, and maintaining an adequate and safe food supply (Knutson, Penn, and Boehm). Emerging trends toward diminishing the role of the federal government in agriculture and the new Federal Agriculture Improvement and Reform (FAIR) Act have made the government's future role in the production agriculture sector uncertain.

Traditional price support programs and commodity markets have offered producers a mechanism to manage price risk. However, with the exceptions of Multiple-Peril Crop Insurance (MPCI) and catastrophic coverage, little or no yield risk protection has been available through these sources. In response to this deficiency and recent government attitudes regarding farm policy, commodity markets and private insurance companies have become innovative in both conceiving and offering products designed to better provide risk protection for agricultural producers. For example, in June 1995, the Chicago Board of Trade began trading yield futures contracts to complement its numerous commodity price-oriented products.

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Private insurance companies have also become more creative in terms of the types of products they now offer agricultural producers. Companies like American Agrisure have been providing revenue-based insurance products designed to offer both price and yield protection concurrently. Gross revenue guarantees are now available in several states for a number of commodities. Income Protection, Revenue Assurance, Crop Revenue Coverage (CRC), and Crop Revenue Coverage Plus are four such revenue-based insurance policies offered by private insurance companies.

The CRC product is especially noteworthy as it appears to have gained widespread acceptance from producers. However, current procedures to determine premium prices for CRC (for example, those suggested by Barnaby) are confusing and ad hoc in nature. Although it is not altogether clear why, crops that can be covered with MPCII have CRC premium rates that are closely linked to MPCII rates. Such approaches to setting CRC rates can have serious consequences if the inaccuracy or mispricing induced by these current procedures is large. Additionally, sellers of income-based insurance products may have limited understanding of the complexity of the derivative securities they are marketing.

Given the potential for mispricing, the consequences could be large for taxpayers as a whole because CRC insurance providers apply for reinsurance through the federal government. Underpricing and/or inadequate hedging of the risk the insurance company assumes may have the undesirable effect of forcing the government back into a more proactive agricultural role if a farm crisis ensues and a crop insurance industry bailout becomes necessary. Similarly, the threat of overpricing may result in even more government regulation of the private crop insurance market. In either case, there is the potential for increased government involvement in agriculture, albeit indirectly, but nonetheless in opposition to the current retreat.

The objective of this research is to present an equilibrium framework from which CRC insurance premiums can be approximated that is more closely aligned with contemporary economic and financial theory. To accomplish this objective, CRC insurance is discussed in detail and a derivative security pricing methodology is suggested as a means of setting a lower bound on CRC insurance premiums. In this discussion, it becomes apparent that CRC insurance behaves like an Asian put option. An analytic solution to the model is shown to be extremely complex because of the dependency of the policy on at least two path-dependent state variables. As a simple empirical example and practical alternative, CRC insurance premium approximations are generated for Iowa corn using Monte Carlo simulation under some basic assumptions regarding the underlying diffusion equations for price and yield of corn. Finally, these results are summarized and conclusions are drawn.

Crop Revenue Coverage

Crop Revenue Coverage is a Federal Crop Insurance Corporation (FCIC) approved alternative to Multiple-Peril Crop Insurance. Piloted in the spring of 1996, more than one-third of Iowa and Nebraska corn and soybean producers transferred MPCII policies to CRC. In the fall of 1996, the FCIC [now the Risk Management Agency (RMA)] approved expansion of CRC for wheat producers with plans to further expand into other crops and geographic areas. In 1997, a total of 166,896 policies were sold in 18 states for corn, cotton, grain sorghum, soybeans, and wheat (FCIC).

CRC policies insure that the gross revenue per acre associated with a particular commodity grown by a producer will fall within some range of possible outcomes. The insurance essentially works in the following manner.¹ On or before the sales closing date, producers determine the crops for which they desire insurance, and for what level of coverage.² Coverage levels range from 50% to 75% in 5% increments. The producer's gross revenue is determined in all cases by the product of prices and yields. However, two types of yields, and prices from two different time periods, determine (a) whether an indemnity is necessary and, if so, (b) how much indemnity is necessary.

Both a base price and a harvest price are used to determine indemnities. For example, the base price for corn is calculated as 95% of the average daily settlement price on the December corn futures contract traded at the Chicago Board of Trade during February of the production year for which coverage is desired. Similarly, the harvest price for corn is calculated as 95% of the average daily settlement price on the December corn futures contract traded at the Chicago Board of Trade during November of the production year for which coverage is desired.

The two types of crop yield on which CRC policies are dependent are actual production history (APH) and actual realized yields. APH is an average of the producer's own personal yield history, while actual realized yields are for the cropping year covered by the insurance. The existence of an indemnity is based on whether or not actual realized gross revenue is in excess of guaranteed gross revenues. Actual realized gross revenue is determined by multiplying the calculated harvest price by actual harvest time yields. Guaranteed gross revenue is determined by multiplying the greater of base price and harvest price by the coverage level and APH yield. In this sense, the indemnity offers a three-tiered payoff because the producer receives nothing if actual gross revenue exceeds guaranteed gross revenue or, alternatively, if actual gross revenue is less than guaranteed gross revenue, the producer receives a payoff equal to the difference between actual and guaranteed gross revenue. In this last case, guaranteed gross revenue will be determined with the greater of base and harvest prices.

Given the preceding discussion regarding the key variables and events that trigger a CRC insurance indemnity, at the end of the production year (or more generally, at termination time T), the following mathematical expression describes the setting:

$$(1) \quad V(T) = \max\{0, \gamma b \bar{y} - h y(T), \min\{\gamma(b + \zeta) \bar{y} - h y(T), h[\gamma \bar{y} - y(T)]\}\}.$$

Here, V represents the time T value of the CRC insurance contract in dollars per acre, \max is the maximization operator, γ denotes the guarantee (in percentage terms), b and h are the base and harvest prices in dollars per bushel, \bar{y} represents the APH yield per acre, $y(T)$ is the producer's actual realized yield per acre, and ζ denotes the price limit in dollars per bushel. Thus equation (1) describes the conditions under which an indemnity will not be paid (in which case $V = 0$), or the conditions under which an indemnity will be paid ($V > 0$).

¹ Interested readers desiring more detailed information should consult Harwood et al., or (for example) American Agriculture's online web site (<http://www.amag.com>).

² The sales closing date varies by geographic area and refers to the date at which coverage must be obtained to be eligible for indemnities.

In mathematical terms, equation (1) is a boundary condition for a partial differential equation (PDE), the solution of which is the value of the CRC insurance contract.³ This interpretation presupposes that CRC insurance can be treated as a derivative security. To confirm this, consider that producers buying the insurance for a given commodity are in effect buying a European put option on the gross revenue associated with their production of that commodity. In the case of many commodities covered by CRC, this feature—in conjunction with the policy's fundamental reliance on futures contracts publicly traded at regional exchanges—implies the insurance derives its value from the value of other (underlying) assets.

In fact, all income protection policies operate very much like options or, more generally, derivative securities (Stokes, Nayda, and English; Turvey 1992a, b; Turvey and Amanor-Boadu). In the majority of the cases, a threshold level of income per acre, for which the insurance is purchased, is specified. Failure to achieve this threshold level usually results in an indemnity equal to the difference between actual and insured income. Given these types of insurance products can be viewed as put-like derivative securities, then a logical first approach to premium approximation is the application of contemporary derivative security pricing techniques.

However, as noted above, CRC imposes some rather complex conditions that determine whether an indemnity will be paid. Because a producer could only purchase CRC before yields and the harvest price were known with certainty, there are multiple sources of uncertainty that must be priced to accurately set premiums.⁴ Additionally, CRC is also classified as an exotic or path-dependent derivative because of the nature of the construction of base and harvest prices. That is, the value of the CRC contract depends on the history of futures prices (through the constructions of base and harvest prices) rather than just on their value at T . These ideas are developed more fully in the next section.

Asian Options and Implications for Crop Revenue Coverage

Numerous research efforts have been directed at developing pricing models for options. The majority of this research has centered on analytic and numerical solutions to European- and American-type options. (Interested readers should consult Rogers and Talay, or Dempster and Pliska for contemporary treatises on the current state of development of modern option pricing theory and practice.) Exotic or path-dependent derivative securities typically pose substantial valuation problems stemming from their complexity. An analytic solution to a PDE derived to characterize the value of an exotic derivative is a rare occurrence, with most valuation attempts ending in numerical solutions to the PDE or simulation.

Of critical interest for the valuation of CRC insurance is the recognition that this type of insurance is a complex form of an Asian option, a special kind of exotic option. The term "Asian" in this context denotes that such options originated for stocks trading on

³ Two examples of PDEs for CRC insurance are presented in the appendix to demonstrate the derivations of the PDEs and, by way of example, to highlight the complexity of the equations.

⁴ Furthermore, a producer may not know the base price in part or in full, depending on the time that CRC is purchased. For example, in a given production year, if corn CRC is purchased before the last day of February or, alternatively, if corn CRC premiums are set before the last day of February, then the base price is also stochastic.

Asian exchanges (Campbell, Lo, and MacKinlay). All exotic options have payoffs that depend in some nontrivial way on the past history of the underlying asset price as well as its spot price at exercise or expiry (Wilmott, Howison, and Dewynne).

Asian options are fully path-dependent and typically depend on some form of average of the underlying asset or exercise prices over one or more time intervals while the option is alive. Averaging can be discrete or continuous, as well as geometric or arithmetic. The link to CRC insurance for exchange-traded commodities should be clear. The value of the insurance is critically dependent on base and harvest prices which are averages determined by the evolution of the futures price. However, harvest price is not known at the sales closing date or any time prior to the purchase of the insurance. Similarly, while base price may be known by the sales closing date, it may not be known (in full or in part) at the time the producer buys the coverage or the time the rates are set. Because futures prices are an ongoing revelation, and base and harvest prices are averages of futures prices, the path-dependent nature of CRC is established.

As mentioned previously, different types of averaging can take place. In the context of CRC insurance, discrete arithmetic averaging is used to build the base and harvest prices by using the settlement prices for each day during the appropriate intervals. This type of averaging is typical of many derivative securities, but is atypical in that the underlying asset price portion of the payoff (harvest price) in addition to the exercise portion (base price) is being averaged.⁵ Usually, either the asset price or the exercise is averaged, but not both. Nonetheless, this type of averaging does allow one to get close to a solution when using an analytic approach to valuation (under some special assumptions), and offers an algorithm to employ if a numerical solution is satisfactory.

An Algorithm for Determining Crop Revenue Coverage Premiums

An analytic derivative security modeling approach to approximating CRC insurance premiums commences as would any other approach to valuing a derivative security. Namely, the underlying state variables influencing the value of the derivative need to be identified and their diffusion equations specified. Depending on the nature of the state variables modeled (e.g., whether their risk can be hedged), equilibrium and/or arbitrage approaches can be employed to arrive at a PDE, the solution of which describes the value of the derivative or, alternatively, the premium for the insurance. In most cases, path dependency precludes a closed-form solution for the value of exotic options. Numerical methods are, however, fully capable of enumerating option values by numerically solving the PDE given appropriate boundary data.

CRC insurance for exchange-traded commodities is fundamentally dependent on five stochastic state variables: the futures price of the underlying commodity (f), the yield of the underlying commodity (y), the base price (b), the harvest price (h), and time (t). Time dependence exists for all the state variables, but is intentionally suppressed to facilitate notational convenience. For example, we write f to mean $f(t)$. The insurance

⁵Technically, further averaging is at work because APH yield is also an (annual) average and is part of the exercise portion of the payoff. For simplicity, we ignore this averaging and treat APH as a parameter—recognizing a more complex model would account for annual sampling of producer yields.

functional is denoted $V = V(f, y, b, h, t)$, and is taken to mean the value or lower-bound premium estimate a crop producer should be willing to pay for CRC-type insurance coverage. Alternatively, V represents a lower-bound premium estimate that the insurance company should charge for a given level of coverage. If the assumptions of the model hold, this is also the actuarially fair premium.

The difficulty in applying contemporary derivative security pricing methods is readily apparent because the PDE characterizing the insurance is multidimensional. Such is always the case when dealing with PDEs, but usually only one state variable and time make up the multidimensional nature of the equation. In the present case, however, four state variables in addition to time influence the value of the insurance policy.

One method of simplifying the problem is to recognize that b and h are arithmetic averages where the sampling takes place at discrete points in time, i.e., at daily settlement during the months when base price and harvest price are being determined. In between these sampling points, the value of the insurance is influenced by price and yield evolution and time passage, but cannot be impacted by a changing base or harvest price because these state variables change only when updating the running sum of prices during the specified intervals.

Define B and H to be the discretely sampled running sums of base and harvest prices, respectively, during the time interval when base and harvest prices are determined. Then we can write

$$(2) \quad B(t) = \sum_{i=1}^{j(t)} f(t_i) \quad \text{and} \quad H(t) = \sum_{i=1}^{k(t)} f(t_i),$$

where the t_i are the sampling dates, and $j(t)$ and $k(t)$ are the largest integers such that $t_{j(t)}, t_{k(t)} < t$. Time dependence has been reintroduced explicitly in the equations in (2) for improved clarity. Thus the discretely sampled arithmetic averages are given by $B(t)/j(t)$ and $H(t)/k(t)$. The equations in (2) merely define the two time intervals when running sums of futures prices are needed to determine base and harvest prices.

One additional piece of information is needed to complete the simplification—namely, a jump condition. Across a sampling date, a running sum is necessarily discontinuous. For example, the running sum of harvest price is $H(t_i^-)$ just prior to a given sampling date (t_i), and is $H(t_i^-) + f(t_i)$ just after the sampling date. It is desirable from an analytic standpoint to have the value of the insurance continuous across sampling dates, implying the following relations must hold during the base and harvest price estimation intervals:

$$(3) \quad V(f, y, B, H, t_i^-) = V(f, y, B + f, H, t_i^+),$$

and

$$V(f, y, B, H, t_i^-) = V(f, y, B, H + f, t_i^+).$$

The arithmetic averaging procedure influences the valuation process to the extent that the B and H state variables are effectively reduced to parameters on either side of a sampling date. Provided the conditions in equation (3) are applied when necessary to jump the sampling date, finding a solution is greatly facilitated because the dimensionality of the problem has been reduced twofold.

With these thoughts in mind, an algorithm for determining the value of CRC insurance for any exchange-traded commodity is summarized in the six steps below.

- STEP 1. Beginning at expiry when the value of the insurance is known and equal to the boundary condition in equation (1), solve the PDE characterizing the value of the derivative from expiry back to the moment in time just after the last sampling date using equation (1) as boundary or final data.⁶
- STEP 2. Apply equation (3) to allow for the jump across the sampling date.
- STEP 3. Return to step 1, and solve the PDE characterizing the value of the derivative using the equation resulting from the application of the jump condition as final data. The solution gives the value of the insurance over the interval between the sampling dates when the running sum (H) is constant.
- STEP 4. Repeat step 2 to jump the sampling date. Continue the process, stepping back through the harvest price estimation interval by solving the PDE characterizing the value of the derivative in an iterative fashion subject to the final data provided by the subsequent sampling date.
- STEP 5. Over the time interval between the last sampling date for the base price and the first sampling date for the harvest price, solve the PDE characterizing the value of the derivative using as final data the value of the insurance just prior to the first sampling date for the determination of the harvest price.
- STEP 6. If necessary, progress through the base price time interval in an analogous manner to arrive at the time-zero value of the insurance.

Having reduced the dimensionality of the PDE characterizing the value of the derivative from five to three state variables through the use of discrete arithmetic averaging and the use of the algorithm presented, a reasonable analytic solution is probably still elusive for at least two reasons. First, the PDE characterizing the value of the derivative is still quite difficult to solve analytically, especially given the three-tiered nature of the boundary data presented in equation (1) and the potential existence of cross-partial terms induced by price and yield correlation.

More importantly, such analytical solutions only hold for a small portion of the time domain. The nature of the insurance (with two lengthy averaging intervals) and iterative-type solution procedure ensures the existence of a cumbersome analytic expression because the PDE must be solved numerous times. CRC for corn, for example, would potentially involve solving the PDE about 59 times—one time for each day in November (during the harvest price determination period), once more between base and harvest price averaging intervals, and one time for each day in February (during the base price determination period).

Fortunately, two alternative approaches are readily available. The PDE characterizing the value of the derivative can be solved numerically using explicit, implicit, or Crank-Nicolson finite differencing schemes in the iterative fashion described. A second approach, and the one used here and almost exclusively by Wall Street to value exotic

⁶ For an example of such a PDE, see equation (A5) in the appendix.

options, is to conduct a Monte Carlo experiment where the average payoff (across replications) is determined and interpreted as the value of the derivative or, alternatively, the lower-bound premium estimate the insurance company should charge for coverage. Whether using a numerical procedure to solve the PDE characterizing the value of the derivative or to simulate the most likely outcome, the functional form of the diffusion equations for the underlying state variables must be specified. In the next section, a simple specification is described in the context of an empirical example.

Of course, the preceding approach to premium estimation does not account for all costs. Certainly the insurance company bears costs when offering a product like CRC, such as the cost of developing, marketing, and administering such a product/program. In addition, premiums are often grossed up or buffered slightly as an added measure of risk protection. It may be possible to pass on most, if not all, of these costs to the insured. It is also likely that producers bear costs such as the dollar costs or disutility associated with learning about their risk management alternatives and gathering the requisite information needed to document APH yields.

While these costs are not considered in the modeling approach outlined above, at least two approaches are available to accommodate their existence. First, the cost to the insurer and/or insured could be expressed as a fixed fraction of the CRC premium. In such a case, an additional term enters the PDE characterizing the value of the derivative. This approach is indicative of a cost structure that can be fully passed on to the producer and increases linearly with a producer's CRC premium. An alternative exogenous approach would be to add a fixed amount onto premiums to account for administration and buffering costs. Such loading is often practiced by insurance companies and, again, is reflective of the ability to pass these costs on to the insured. In this case, however, the cost is equally divided among those demanding insurance. Both of these approaches are beyond the scope of the present research, but the preceding discussion demonstrates at least two ways whereby the issue of transactions costs could be accommodated in the present framework.

Additional considerations include the existence of adverse selection and moral hazard, as these risks are very real problems in the insurance industry and are not well addressed in the present approach to premium approximation. This consequence is due more to the design of CRC than to the pricing methodology itself. The effects of moral hazard are tempered somewhat through the use of APH yields because the decision to exhibit moral hazard this year carries with it implications for the subsequent years' APH yield estimation (APH is a moving average). At best, however, producers probably are forced into making a conscientious effort to evaluate the potential monetary gains associated with exhibiting moral hazard rather than the more desirable alternative of eliminating moral hazard altogether.

The problem of adverse selection is a double-edged sword. If premiums were based on county- or state-level yield averages, the potential for a separating equilibrium exists (Rasmusen). Those producers who can always obtain regionalized yield averages would not see any benefit from coverage and would elect to forego purchasing the insurance. In contrast, those producers incapable of achieving the regionalized yield averages would opt to purchase the insurance. However, the occurrence of adverse selection can be mitigated somewhat through the use of a producer's own yields when setting premiums.

Iowa Corn CRC Premiums via Monte Carlo Simulation: An Empirical Example

Boyle first demonstrated that Monte Carlo simulation could be used to price options, and the technique works well on both American and European options (Gemmill). Many options without an early exercise feature are complex enough that the technique is currently the only reasonable solution method available in practical settings (Campbell, Lo, and MacKinlay). Such is the case for many exotic derivatives, including CRC insurance.

The technique is straightforward to apply and relies on the notion that if enough sample paths (replications) for the stochastic state variables are generated, the eventual payoff can be determined under a variety of settings that may actually occur in the real world. The average payoff is then discounted (at a risk-neutral rate) back to the beginning of the life of the option and represents the value of the derivative or, in this case, the minimum premium per acre the insurance company should charge for a given level of coverage.

To employ the technique in the context of corn CRC insurance, one needs to specify diffusion equations for corn prices and yields, and simulate a collection of sample paths. The present value of the average payoff (indemnity) across the sample paths is then the value of the insurance or, alternatively, the premium. For analytic convenience, many Monte Carlo simulations conducted to price derivative securities rely on the assumption that the prices of the underlying assets evolve according to geometric Brownian motion (GBM). Such a specification probably is based less on empirical grounds and more on analytic convenience and intuition. The diffusion is particularly easy to work with analytically, which facilitates the solution of complex PDEs. Also, if the price of an asset follows GBM, the distribution of prices is lognormal. Hence, the intuition that negative prices should not result and that zero should be an absorbing barrier for price (to preclude infinite arbitrage opportunities) is facilitated by the specification.

While GBM and the ensuing imposition of lognormality for prices and yields may not be the most palatable assumption, it is justifiable in the present context. First, the empirical example to follow is merely an example that demonstrates the behavior of the model. The interpretation of CRC as a complex derivative and the valuation technique itself are unencumbered by distributional assumptions. Consequently, other diffusion equations that could more accurately characterize the evolution of prices and yields can be specified. Such specifications are, however, beyond the scope of the present research. Further, as pointed out by an anonymous reviewer, county-level yield data tend to be left-skewed in direct contrast to the right-skewness associated with the lognormal distribution. However, the use of futures contracts to some extent mitigates this concern because yields are inferred from yield futures contract prices at a high level of aggregation.

Assuming that the prices of corn futures and corn yield futures contracts traded at the Chicago Board of Trade can be modeled as geometric Brownian motion implies the following diffusion equations hold:

$$(4) \quad \frac{df}{f} = \mu_f dt + \sigma_f dZ_f \quad \text{and} \quad \frac{dy}{y} = \mu_y dt + \sigma_y dZ_y,$$

where the drift and volatility components are constants.

To account for correlation between price and yield, expected gross revenue is specified as $R = f \times y$, and Ito's lemma is applied to determine the dynamics of R , giving

$$(5) \quad dR = ydf + fdy + dfdy.$$

Substituting for df and dy in equation (5) using the equations in (4) results in the following stochastic differential equation describing the dynamics of gross revenue per acre:

$$(6) \quad \frac{dR}{R} = \mu dt + \sigma_f dZ_f + \sigma_y dZ_y.$$

Equation (6) is a two-dimensional Brownian motion with $\mu = \mu_f + \mu_y + \rho_{fy} \sigma_f \sigma_y$.

The solution to the risk-neutralized representation of equation (6) is used in the Monte Carlo simulations and is presented below.⁷ This equation has the added advantage of accounting for price and yield correlation explicitly through the drift term. In addition, the solution to the risk-neutralized futures price diffusion equation adapted from equation (4) also must be simulated to be able to account for the determination of base and harvest prices. The expression $R(T)/f(T)$ is then used to determine $y(T)$ as needed in the boundary condition.

Assuming the no-arbitrage condition holds, and carrying forward the assumption of risk-neutral dynamics [as conjectured in Cox and Ross, and detailed in Campbell, Lo, and MacKinlay (p. 355)] implies the following two discrete solution equations are to be simulated:

$$(7) \quad R(t + \Delta t) = R(t) e^{\left(r - \frac{\sigma^2}{2}\right) \Delta t + \sigma_f Z_f \sqrt{\Delta t} + \sigma_y Z_y \sqrt{\Delta t}}$$

and

$$(8) \quad f(t + \Delta t) = f(t) e^{\left(r - \frac{\sigma_f^2}{2}\right) \Delta t + \sigma_f Z_f \sqrt{\Delta t}},$$

where $\sigma^2 = \sigma_f^2 + \sigma_y^2 + 2\sigma_f \sigma_y \rho_{fy}$, and r have been substituted for μ in accordance with the assumption of risk neutrality. Z_f and Z_y are normally distributed random variables with mean zero and unit variance, and determine the direction of travel for the simulated Brownian motion. Additionally, daily time steps are assumed; thus Δt is equal to 1/365. Equations (7) and (8) are merely the discretized versions of the continuous form of the solutions to one- and two-dimensional geometric Brownian motion found by employing the well-known transformation—i.e., $x_R = \ln(R)$, and $x_f = \ln(f)$.

⁷ Wholesale substitutions of the risk-free rate of return for the drift coefficient can occur when no arbitrage prevails, implying all the risk associated with the derivative can be hedged away. Technically, one does not make these substitutions literally, but rather would proceed by finding an equivalent probability measure under which the state variables are martingales. As outlined in Cox and Ross, the solution equations (7) and (8) would then result. If yield futures are an imperfect means of hedging the yield risk of CRC insurance, an alternative is to factor basis into the analysis. If no yield and/or price futures are available, or the basis is inadequate, the equilibrium-based PDE presented at the end of the appendix [equation (A7)] or a variant can be used.

Price and Yield Data

Futures price data for the 1996 December corn futures contract and the 1996 December Iowa corn yield futures contract were used to estimate price and yield volatility and the correlation coefficient. The procedure for estimating the parameters of the diffusion equations is based on that suggested by Nowman under the special case of GBM (a nested specification).

The estimation results in annualized volatility estimates of 25.2327% for the futures price of corn, 19.5961% for the yield futures, and -0.0829 for the correlation between price and yield. Using the yield futures prices gives the market's perception of Iowa corn yields (after adjusting for the dollar denomination of the contract) on a daily basis for the year 1996, and implies hedging yield risk can be adequately accomplished using these yield futures. Thinly traded yield futures that are more aggregate in scope may be problematic for the hedging approach to valuation illustrated here. Longer price and yield series and a more localized yield series could also be used to reflect wider swings and more realistic prices and yields, but for demonstration purposes, it is felt the series chosen is adequate. The risk-free rate of return was estimated using daily Center for Research in Security Prices (CRSP) data on U.S. Treasury bills for the year 1996, resulting in a 5.84% annualized return. Additionally, a limit upward price movement for corn is given as \$1.50 per bushel.

Empirical Results

The Monte Carlo experiment consisted of 20,000 replications of equations (7) and (8). Equations (7) and (8) are simulated on a daily basis, and equation (8) is used to determine the b and h for each replication over the appropriate time intervals. Equation (7) is used to determine $y(T)$ via the expression $R(T)/f(T) = y(T)$. Subsequently, $V(T)$ in equation (1) is estimated for each replication using the simulated b , h , and $y(T)$ data. Then, each simulated $V(T)$ is discounted back to time zero (assumed for simplicity to be January 1), and all the $V(0)$ are averaged to give the value of the CRC insurance policy on that date.

A cross-section of simulated premiums (in dollars per acre) under alternative guarantee and APH yield levels is presented in table 1 to help convey information about the nature of the value of CRC insurance under the present modeling approach. As noted above, the premiums listed have been estimated for January 1, reflecting a sales closing date, premium setting date, or CRC purchase date prior to partial or full revelation of the base price.⁸ As shown, premiums are generally an increasing function of the guarantee percentage and APH yield. This is reasonable because both the guarantee

⁸ To clarify, the assumption that time zero is January 1 implies that the mean premiums reported are for a date when neither the base price nor the harvest price has been revealed partially or in full. Although January 1 is chosen as a matter of simplicity, the selection does highlight the flexibility of the modeling approach by allowing for premiums to be calculated when there are two averaging periods (i.e., before base price or harvest price are determined). Presumably, premiums could be set or a producer could elect to purchase CRC insurance later than January 1, possibly during the month of February (after the base price is partially revealed but before the harvest price is at all revealed), or after the month of February (after the base price is fully revealed but before the harvest price is at all revealed). There is no universal advantage to doing so, however. Of course, some of the uncertainty (base price) associated with the problem is resolved after February, but unanticipated events in February could make purchase in January less costly in retrospect.

Table 1. Mean Simulated and Actual CRC Premiums, in Dollars per Acre, for Alternative APH Yields and Guarantee Percentages for Iowa Corn

APH Yield (bushels/acre)	Guarantee Percentage					
	50%	55%	60%	65%	70%	75%
	----- (\$ per acre) -----					
60 \bar{p} simulated	0.00	0.01	0.02	0.04	0.08	0.14
actual	6.25	7.40	8.79	10.69	13.72	18.10
70 \bar{p} simulated	0.01	0.03	0.08	0.16	0.29	0.51
actual	6.32	7.48	8.90	10.88	13.92	18.34
80 \bar{p} simulated	0.05	0.12	0.24	0.47	0.84	1.39
actual	5.84	6.91	8.28	10.11	12.98	17.21
90 \bar{p} simulated	0.14	0.31	0.64	1.16	1.97	3.12
actual	6.57	7.78	9.31	11.38	14.60	19.36
100 \bar{p} simulated	0.34	0.73	1.39	2.43	3.93	6.00
actual	6.36	7.33	8.92	10.91	13.79	18.53
110 \bar{p} simulated	0.73	1.48	2.69	4.49	6.99	10.24
actual	6.99	8.06	9.81	12.01	15.17	20.38
120 \bar{p} simulated	1.39	2.69	4.69	7.52	11.26	16.02
actual	6.58	7.84	9.46	11.55	14.88	19.73
130 \bar{p} simulated	2.43	4.49	7.52	11.62	16.91	23.43
actual	7.13	8.49	10.25	12.51	16.12	21.38
140 \bar{p} simulated	3.93	6.99	11.26	16.91	23.98	32.49
actual	6.89	8.29	9.88	12.24	15.80	21.03
150 \bar{p} simulated	6.00	10.24	16.02	23.43	32.49	43.08
actual	7.04	8.03	9.93	12.40	15.86	21.44
160 \bar{p} simulated	8.66	14.32	21.82	31.18	42.33	55.09
actual	7.51	8.57	10.59	13.22	16.92	22.87
170 \bar{p} simulated	11.98	19.28	28.66	40.11	53.41	68.37
actual	7.36	8.70	10.83	12.95	16.80	22.70
180 \bar{p} simulated	16.02	25.11	36.55	50.13	65.62	82.80
actual	7.79	9.21	11.47	13.71	17.79	24.04

Notes: For each APH yield and guarantee percentage combination, the top entry (\bar{p} simulated) is the mean simulated premium price per acre; the lower entry (actual) is the actual premium price per acre for Boone County, Iowa, which was randomly selected as a representative Iowa county. Simulated premiums are for an assumed date of January 1, implying the base price is unknown and must also be simulated. A base price of \$2.59/bushel, a low price factor of \$0.217/bushel, and a high price factor of \$0.212/bushel were used to estimate the actual premiums reported.

percentage and APH yield have the effect of increasing the “exercise price” of the insurance. More concisely, increasing the guarantee percentage and/or the APH yield increases the probability of an indemnity, *ceteris paribus*. Consequently, the insurance is offered for a higher price.

While not directly comparable owing to differences in modeling technique, distributional assumptions, and aggregation, corresponding actual CRC premiums for Boone County, Iowa, are also reported in table 1. (Boone County was randomly selected as a representative Iowa county.) These rates were determined by applying procedures currently in use where price, yield, and revenue risks are all priced individually and summed to determine the CRC premium in dollars per acre.⁹ As shown, actual CRC premiums are also a strictly increasing function of the guarantee percentage, but they are only generally an increasing function of APH. Further, premiums are always positive and considerably lower than those predicted by the model developed in this study for high APH-guarantee combinations.

Actual CRC premiums, then, appear to be less sensitive to APH than simulated rates. This finding is further substantiated by the results reported in table 2. Presented in table 2 are simulated CRC rates assuming time zero is March 1, the first full day when base price is known with certainty. The base price for corn in 1997 was \$2.59 per bushel, and this value was used in the simulations. For comparative purposes, table 2 also provides the actual CRC rates for Boone County, Iowa, under the same assumptions. As shown, simulated and actual CRC rates are an increasing function of the percentage guarantee, although it appears that actual CRC rates are much larger but less sensitive to changes in the guarantee percentage (for a fixed APH) than are the simulated CRC rates. Additionally, actual CRC rates are a decreasing function of APH, while simulated rates are an increasing function of APH.

Arguably, the differences in distributional assumptions and levels of aggregation cause at least a portion of the differences in empirical results presented in tables 1 and 2. However, the most likely reason for these differences was cited above as one of the motivations for the development of alternative premium-setting models. Current procedures used to determine CRC premiums are closely linked to MPCIR rates. When constructing CRC premiums, both yield and price risk are determined using MPCIR base premium rates (revenue risk is determined by CRC rate factors). MPCIR base premium rates are a decreasing function of APH consistent with the notion that at the county level, differences in the production practices, soil fertility, rainfall, etc., associated with individual farms in that county are relatively smaller than at higher levels of aggregation. Findings suggest that producers with higher APHs demonstrate superior managerial ability and consequently less risk than producers in the same county with lower APHs. This result is less likely to hold at higher levels of aggregation because differences in APH at the state level are more likely to be influenced by those very factors cited above that tend to exhibit relatively small differences at the county level.

It should be noted that these results do not imply that under current rate-setting procedures, producers with high APHs pay less than producers with low APHs for a given guarantee level. Recall that premiums in dollars per acre are generally an increasing function of APH under current rate-setting procedures. However, it does appear that

⁹ Data used to construct these premiums were taken from the FCIC-RMA Actuarial Document online web site at <http://act.fcic.usda.gov/actdoc/>.

Table 2. Mean Simulated and Actual CRC Rates (Premium/Liability) for Alternative APH Yields and Guarantee Percentages for Iowa Corn

APH Yield (bushels/acre)	Guarantee Percentage					
	50%	55%	60%	65%	70%	75%
	----- (\$Premium/\$Liability) -----					
60 \bar{w} simulated	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012
actual	0.0804	0.0866	0.0943	0.1058	0.1261	0.1553
70 \bar{w} simulated	0.0001	0.0003	0.0007	0.0013	0.0023	0.0038
actual	0.0698	0.0750	0.0818	0.0923	0.1097	0.1349
80 \bar{w} simulated	0.0004	0.0010	0.0019	0.0035	0.0060	0.0093
actual	0.0563	0.0607	0.0666	0.0751	0.0895	0.1107
90 \bar{w} simulated	0.0012	0.0024	0.0046	0.0079	0.0126	0.0187
actual	0.0563	0.0607	0.0666	0.0751	0.0895	0.1107
100 \bar{w} simulated	0.0026	0.0053	0.0093	0.0151	0.0229	0.0328
actual	0.0491	0.0514	0.0574	0.0648	0.0761	0.0954
110 \bar{w} simulated	0.0053	0.0098	0.0165	0.0257	0.0373	0.0514
actual	0.0491	0.0514	0.0574	0.0648	0.0761	0.0954
120 \bar{w} simulated	0.0093	0.0165	0.0266	0.0397	0.0556	0.0743
actual	0.0423	0.0458	0.0508	0.0572	0.0684	0.0847
130 \bar{w} simulated	0.0151	0.0257	0.0397	0.0571	0.0776	0.1007
actual	0.0423	0.0458	0.0508	0.0572	0.0684	0.0847
140 \bar{w} simulated	0.0229	0.0373	0.0556	0.0776	0.1026	0.1300
actual	0.0380	0.0416	0.0454	0.0519	0.0622	0.0773
150 \bar{w} simulated	0.0328	0.0514	0.0743	0.1007	0.1300	0.1613
actual	0.0363	0.0376	0.0426	0.0491	0.0583	0.0736
160 \bar{w} simulated	0.0447	0.0678	0.0951	0.1260	0.1592	0.1935
actual	0.0363	0.0376	0.0426	0.0491	0.0583	0.0736
170 \bar{w} simulated	0.0585	0.0862	0.1180	0.1528	0.1891	0.2260
actual	0.0334	0.0359	0.0410	0.0453	0.0545	0.0687
180 \bar{w} simulated	0.0743	0.1063	0.1423	0.1805	0.2195	0.2582
actual	0.0334	0.0359	0.0410	0.0453	0.0545	0.0687

Notes: For each APH yield and guarantee percentage combination, the top entry (\bar{w} simulated) is the mean simulated rate; the lower entry (actual) is the actual rate for Boone County, Iowa, which was randomly selected as a representative Iowa county. Assuming a March 1 date for premium price estimation, the base price is known and set equal to \$2.59/bushel, with a low price factor of \$0.217/bushel, and a high price factor of \$0.212/bushel. All rates were determined by dividing the premium (in dollars) by dollars of liability (APH times guarantee percentage times CRC base price).

in a given county, producers with higher APHs pay less in relative terms (premium dollars per unit of liability) than their counterparts with lower APHs. One explanation for this finding is that another government farm program (MPCI) imposes such a structure.

In addition, low APH and/or low guarantees typically imply “free” coverage, while high APH and/or high guarantees imply excessively high premiums for coverage under the Monte Carlo valuation technique. If accurate, low levels of coverage should never be purchased by producers because the value of the policy is, for all intents and purposes, zero. Similarly, most producers probably would be reluctant to pay the excessively high prices associated with high levels of coverage in the absence of a subsidy or cost-sharing arrangement to induce them to do so. It is likely that at least two potentially related issues cause these phenomena—volatility and the GBM assumption. An understatement of the volatility of yields can cause low or no value for low exercise prices because the random draws have too little variance to make these extremes a reality. Likewise, if the distributions of prices and/or yields are conditionally heteroskedastic or follow some other fat-tailed distribution, underpricing at the extremes can occur again because of the relatively lower probability in the tail of the lognormal distribution. Finally, no administrative costs have been accounted for which would have the effect of raising the simulated premiums.

Summary and Conclusions

Governmental retreat from the agricultural sector has been met with innovative and timely products by exchanges and insurance companies to help producers manage risk. The newness and complexity of many of these products make evaluation by sellers and buyers of the products potentially ad hoc. The purpose of this study was to present one approach that can be used to set premiums on Crop Revenue Coverage, one of the more complex insurance products.

After noting the similarities between this product and Asian options, an analytic approach to premium approximation was described. The Monte Carlo simulation method of valuation was determined to offer the practitioner an approach toward valuation of this type of insurance. Numerical estimates of premiums for corn Crop Revenue Coverage were presented as an empirical example under alternative guarantee percentages and actual production history yields.

Future research should proceed along three lines. First, CRC premiums and/or rates should be analyzed under alternative diffusion equations after a thorough empirical analysis of state- and/or county-level yields and prices (depending on the level of aggregation desired for rate setting). While the theoretical model and methodological approach outlined here are independent of any particular distributional assumption, it is clear that CRC insurance premiums are critically influenced by distributional choice. Therefore, in practice, attention should be paid to the proper selection of diffusion rather than the simplifying assumption used in this analysis.

If yield futures contracts are inadequate for properly hedging the yield risk that CRC insurance is designed to protect against, and/or the valuation of CRC for commodities without a price and/or yield futures contract is desired, then alternatives to the hedging arguments in this research need to be considered. Equilibrium arguments centered around returns associated with other derivatives whose value is fundamentally dependent on the same set or subset of state variables as CRC may be of great benefit in this

regard. An alternative approach for CRC on commodities with exchange-traded prices and yields is to incorporate basis as the link between the individual's yield expectations and the market's perception of yield at a more aggregate level. Methods for incorporating administrative and/or transactions type costs into the valuation framework and more explicit consideration of the impacts of moral hazard and adverse selection are also desirable.

Last, the consequences of costly reinsurance if underpricing or price gouging occurs need to be examined in detail. Distinct differences in premium prices between those reported here and those charged by insurance companies are most likely attributable to methodological differences in distributional assumptions, the level of price and yield correlation, and linkages between existing programs like MPCI. The emergence of products such as CRC in recent years is in direct response to government retreat from production agriculture. However, because of reinsurance, the government potentially can be forced back into a proactive role in agriculture if mispricing occurs at a level where insurance companies are inadequately hedged against the risk they are assuming or if they excessively overprice these products.

[Received March 1999; final revision received July 1999.]

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Appendix: An Example Partial Differential Equation (PDE) for CRC Insurance

Here, an example PDE characterizing the value of CRC insurance is derived assuming the risk associated with the derivative can be fully hedged. The derivation is expository in nature with its purpose meant to convey a sense of the complexity of the equation characterizing the value of the CRC derivative as well as the complexity of the derivative itself.

Assuming the price (f) and yield (y), stochastic state variable dynamics are described by diffusion equations of the form

$$(A1) \quad df = \mu_f dt + \sigma_f dZ_f \quad \text{and} \quad dy = \mu_y dt + \sigma_y dZ_y,$$

where $\mu_f = \mu_f(f, t)$ and $\mu_y = \mu_y(y, t)$ are drift coefficients, and $\sigma_f = \sigma_f(f, t)$ and $\sigma_y = \sigma_y(y, t)$ are volatility coefficients. Let f_y be the price of a yield futures contract, and define $y = f_y/c$ to be the market’s expectation of yield implied by such a price when the conversion factor, c , is a constant. Applying Ito’s lemma to y gives $dy = df_y/c$. Assuming f_y follows a diffusion equation similar to that of f in (A1) then implies the second equation in (A1) must hold. Notice also that the diffusion specifications are distribution free, as is the derivation that follows.

The dynamics of V , the value functional describing the price of the insurance, can be determined by applying Ito’s lemma to V . The result is

$$(A2) \quad dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial f} df + \frac{\partial V}{\partial y} dy + \frac{1}{2} \left(\frac{\partial^2 V}{\partial f^2} df^2 + 2 \frac{\partial^2 V}{\partial f \partial y} df dy + \frac{\partial^2 V}{\partial y^2} dy^2 \right).$$

Notice the exclusion of base and harvest prices as stochastic state variables because of the assumption of discrete arithmetic averaging.

A hedge portfolio, Π —consisting of one CRC insurance policy, δ_f sold price futures contracts, and δ_y sold yield futures contracts—moves according to the following equation:

$$(A3) \quad d\Pi = dV - \delta_f df - \delta_y dy.$$

To construct the hedge portfolio, it is assumed the underlying assets can be shorted in the amounts specified. Substituting (A2) into (A3) and forcing the hedge ratios to equal the partial effect on V from changes in the price and yield state variables (i.e., $\partial V/\partial f = \delta_f$ and $\partial V/\partial y = \delta_y$) to eliminate randomness results in

$$(A4) \quad d\Pi = \frac{\partial V}{\partial t} dt + \frac{1}{2} \left(\frac{\partial^2 V}{\partial f^2} df^2 + 2 \frac{\partial^2 V}{\partial f \partial y} df dy + \frac{\partial^2 V}{\partial y^2} dy^2 \right).$$

The return on an amount Π invested in riskless assets over a time step dt would be $r\Pi dt$, where r is the risk-free rate of return. If the right-hand side of (A4) were larger than $r\Pi dt$, then arbitrage would take place by individuals borrowing to invest in the portfolio. Conversely, if the right-hand side of (A4) were lower than $r\Pi dt$, arbitrageurs would short the portfolio, placing the proceeds in an interest-bearing account. Either way, the right-hand side of (A4) must equal $r\Pi dt$ in equilibrium. Applying this result, substituting using the definition of Π , and employing the diffusion equations in (A1) results in the following:

$$(A5) \quad \frac{\partial V}{\partial t} + \frac{1}{2} \left[\sigma_f^2 \left(\frac{\partial^2 V}{\partial f^2} \right) + \sigma_y^2 \left(\frac{\partial^2 V}{\partial y^2} \right) + 2\sigma_f^2 \sigma_y^2 \rho_{fy} \left(\frac{\partial^2 V}{\partial f \partial y} \right) \right] + r \left[f \left(\frac{\partial V}{\partial f} \right) + y \left(\frac{\partial V}{\partial y} \right) \right] - rV = 0,$$

where $\rho_{fy} = \rho_{fy}(f, y, t)$ measures the correlation between price and yield. Again, notice the PDE in equation (A5) relies on the assumption that prices and yields can be specified as diffusions, but it is free from any particular distributional assumptions for each diffusion.

The boundary conditions for the PDE in (A5) are as follows:

$$(A6) \quad V(f, y, T; b, h) = \max\{0, \gamma b \bar{y} - hy, \min[\gamma(b + \zeta)\bar{y} - hy, h(\gamma \bar{y} - y)]\},$$

$$V(f, 0, t; b, h) = \max\{\gamma b \bar{y}, \min[\gamma(b + \zeta)\bar{y}, h\gamma \bar{y}]\}e^{-r(T-t)},$$

and

$$V(f, y, t; b, h) \rightarrow 0, \text{ as } f, y, \rightarrow +\infty,$$

where γ represents the guarantee percentage, ζ denotes the limit futures price move, \bar{y} is the APH yield, and all other variables and parameters are as defined previously.

The first boundary condition is merely the time T indemnity presented in text equation (1), while the second represents the value of the insurance when $y = 0$ for any time t . Such a condition ensures that the value of the insurance is the discounted value of the indemnity when yield expectations fall to zero. The remaining condition simply ensures that the value of the insurance or, alternatively, the premium that the insurance company could charge, must approach zero as price and/or yield expectation get arbitrarily high.

Two additional boundary conditions for a zero futures price are necessary and depend on the time space. $V(0, y, t; b, h) = \gamma b \bar{y} e^{-r(T-t)}$ if the futures price hits zero any time prior to the first harvest price sampling date. This is because the price of any asset should have an absorbing barrier at a price of zero to preclude arbitrage, indicating the harvest price will be zero in this case. In the event the futures price hits zero any time on or after the first harvest price sampling date, the first boundary condition above is valid because the harvest price still can be nonzero in this case.

Not all commodities covered by CRC have exchange-traded prices, let alone yields. If the price and/or yield of the commodity covered by CRC are not traded on an exchange, the PDE describing the dynamics of the policy still can be specified provided price and yield are adequately described by diffusion equations. The expected value of equation (A2) is the expected capital gain on the insurance policy. The policy pays no intermittent cash flows to the producer or insurer, implying the expected cash flow equals zero. The sum of expected capital gain and cash flow equated to an equilibrium return implies that

$$(A7) \quad \frac{\partial V}{\partial t} + \frac{1}{2} \left[\sigma_f^2 \left(\frac{\partial^2 V}{\partial f^2} \right) + \sigma_y^2 \left(\frac{\partial^2 V}{\partial y^2} \right) + 2\sigma_f^2 \sigma_y^2 \rho_{fy} \left(\frac{\partial^2 V}{\partial f \partial y} \right) \right] + \mu_f f \left(\frac{\partial V}{\partial f} \right) + \mu_y \left(\frac{\partial V}{\partial y} \right) - \theta V = 0$$

is the PDE which must be solved to value the coverage. Here, θ is an equilibrium rate of return, an unknown parameter or coefficient function. The boundary conditions listed in equation (A5) also apply to equation (A7) with the exception that the parameter θ should be substituted for the risk-free rate of return. Additional equilibrium and/or hedging arguments must be made to endogenize or risk-neutralize the equilibrium rate of return, θ . Such arguments are beyond the scope of the present research.