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TOOLS: A NEW EMPIRICAL MODEL**

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**Short Title: Strategic Interaction**

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## **Strategic Interaction with Multiple Tools: A New Empirical Model**

### **Abstract**

The Lanchester model of strategic interaction typically considers only two-firm rivalry and one strategic tool. This paper presents an alternative that considers rivalry among several firms using multiple tools. Marketing decisions are dynamically optimal and use equations of motion for market share that are consistent with optimal consumer choice. Using a single-market case study that consists of five years of monthly data on ready to eat cereal sales, advertising, product development investments and new product introductions, we test our model against a similar Lanchester specification. Non-nested specification tests fail to reject the proposed model, but reject the Lanchester alternative.

**keywords:** advertising, brands, cereal, dynamic, Lanchester, oligopoly, strategic interaction.

## **Introduction**

Although researchers agree that marketing activities such as advertising, promotion or product development have persistent sales effects, there is considerable debate over their fundamental cause and how to model them (Feichtinger, Hartl, and Sethi, 1994). In fact, there are two broad model types that appear in the marketing and industrial organization literatures: (1) Nerlove-Arrow, or “goodwill” models, and (2) Lanchester, or “market share” models. Among the first studies to recognize the dynamic impact of advertising, Nerlove and Arrow (1962) maintain that advertising expenditures are in fact investments in long-lived capital assets they termed “goodwill.” Because goodwill is both slow to develop and depreciates slowly over time once established, the impact of an investment made in one period can be felt for many periods into the future so economic models that describe the effect of advertising on sales must be inherently dynamic. On the other hand, Vidale and Wolfe (1957) and Kimball (1957) develop an alternative characterization, based on Lanchester’s models of battlefield strategy, that assumes advertising instead acts directly on the rate of change of sales or, more precisely, on the evolution of a firm’s market share. Theoretical and empirical advances based on these two different approaches tend to follow independent paths through the marketing literature (Deal 1979; Sorger 1989; Erickson 1992; Chintagunta and Vilcassim 1992, 1994) and in the context of empirical dynamic oligopoly games in the economics literature (Roberts and Samuelson 1988; Gasmi, Laffont, and Vuong 1992; Slade 1995). Each of the disparate strains of research attempt to achieve similar objectives, but rarely do they address the appropriateness or value of the others’ approach. However, there appears to be great value in achieving a synthesis of these two approaches.

Whereas the state variable in Nerlove and Arrow-type models is the stock of goodwill itself, the state variable in Lanchester-type models is instead market share. Feichtinger, Hartl, and Sethi (1994) review the considerable literature on advertising dynamics. Despite the rather compelling logic of either approach, each has its conceptual and empirical strengths and weaknesses. Whereas the notion of advertising as contributing to a capital asset that has a lasting effect on sales is intuitively plausible, Nerlove - Arrow models are difficult to apply because goodwill is inherently latent or unobservable and its rate of depreciation is typically underidentified in most empirical applications. On the other hand, although the market-share state variable in a Lanchester model is readily observable, this approach is highly restrictive in that it allows only a limited number of rivals, it implies no aggregate sales impact, and the equations of motion for market share are not based in a rigorous model of firm optimization but are rather conjectures that seem logically sound. Despite the potentially contrary implications that may emerge from each of these approaches, no previous studies directly test one against the other as an appropriate explanation for sales dynamics. Nor do they attempt to address some of the limitations of existing models in order to develop one that combines the best features of each.

Beyond the methodological question of which empirical approach is most appropriate, very little research considers the strategic impact of a firm's simultaneous use of many different marketing tools, namely price, product development, new product introduction and advertising.<sup>2</sup> While several consider either advertising alone (Deal, 1979; Roberts and Samuelson, 1988;

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<sup>2</sup> We include both product development expenditures and new product introductions in order to capture the fact that not all product investment results in new brands, but often is used to conduct basic research in innovative new products or to improve taste or packaging attributes of existing products. In this sense, product development expenditures are a proxy measure of a firm's investment in quality and reputation.

Sorger, 1989; Erickson, 1992,1997, and many others) or price and advertising together (Liang, 1986; Slade, 1995), none consider product development product line extensions as either substitutes for, or more likely complements to, advertising and price strategies. With some 25,000 new products introduced to supermarkets alone in 1999 (Food Institute Report) and 95% of those failing to exist beyond the required six month proving period, product development and new product introduction represent some of the most significant and risky investments undertaken by a firm. This is particularly true for the firms that constitute the empirical example used in this paper -- ready-to-eat cereal manufacturers. Few industries are more active in developing new products and in using strategic new product introductions as a competitive tool (Liang, 1986; Hausman, 1997; Erickson, 1997; Nevo, 2001). Further, with slotting fees for new products now estimated at more than \$250,000 for some categories (FTC Hearings) for a single retail grocery chain in addition to significant investment in developing and promoting a new product, maximizing the probability of success is now paramount. While pricing strategies for new products (Dockner and Jorgensen, 1988; Kadiyali, Vilcassim, and Chintagunta, 1999) and the strategic motives for product line changes (Brander and Eaton, 1984; Bayus and Putsis, 1999; Kadiyali, Vilcassim, and Chintagunta, 1996, 1999) are relatively well understood, the strategic interplay of these decisions with investments in product research and the use of various advertising and promotion techniques remains to be thoroughly studied. Particularly in the case of cereal manufacturers, it is also not clear whether such non-price tools obviate the need for strict price competition.

Despite the central role that price-rivalry plays in much of the literature (Liang, 1986; Slade, 1995; Cotterill and Putsis, 2000) competition in prices is typically a maintained

assumption and authors do not test against any alternatives. Rather, rivalry in quantities may be more appropriate if manufacturers have market share goals that are achieved through non-price tools. Arguing that price is a strategic variable, Liang (1986) and Cotterill and Putsis (2000) estimate direct (price-independent) demand systems simultaneously with price-response equations. However, if quantities are truly endogenous, then a more correct approach is to estimate an indirect demand system simultaneously with quantity-response equations. In this study, we conduct specification tests to determine which provides a better fit to the data.

The objectives of this paper are, therefore, both substantive and methodological. Our primary objective is to obtain a better understanding of the strategic roles played by product development, new product introduction, advertising, and market share rivalry in the ready-to-eat cereal industry. In order to achieve this goal, however, it is necessary to examine the adequacy of the existing methodological orthodoxy in this area and to rigorously test against equally viable alternatives. Consequently, our second objective is to develop a model of strategic rivalry that represents a synthesis of existing models and to determine whether this model provides a better fit to the data than models currently in popular use. In order to achieve these objectives, we follow Slade (1995) and Kadiyali, Vilcassim and Chintagunta (1996) by adopting a case-study empirical approach. Specifically, we estimate our model with market-level scanner data from a single metropolitan market (Baltimore / Washington, D.C.) for one product category (ready-to-eat cereals).

The paper begins by developing two alternative conceptual models of strategic rivalry. Next, we present econometric models that follow from each. We then explain the techniques necessary to estimate these models and provide a detailed description of the ready-to-eat cereal

data. A discussion of the estimation results follows, both in terms of specification tests and tests of specific hypothesis generated from the conceptual models. The final section presents our empirical results, provides a discussion of some implications and suggests avenues for future research.

### **Conceptual Model of Market Share Rivalry**

Although valuable in terms of their parsimony and ability to explain complex dynamic interrelationships between firms, Lanchester-type models have several weaknesses that limit their generality. First, in order to retain the mathematical tractability of dealing with only one state variable, the basic Lanchester framework is only able to analyze duopoly situations (Sorger, 1989; Chintagunta and Jain, 1995; and many others). Erickson (1997) recognizes this weakness by extending the model to include dynamic conjectural variation terms wherein several competitors respond to changes in market share with contingent advertising strategies. Structural game theoretic models, however, do not face the same restrictions on the number of potential rivals because they do not purport to solve for equilibrium control paths. For example, Slade (1995) employs a differential game approach similar to Karp and Perloff (1993) in analyzing both price and advertising competition among rival brands of crackers in a local oligopolistic market. Similarly, Roberts and Samuelson (1988) and Gasmi, Laffont, and Vuong (1992) are of this same broad class of model, but rather focus on the problem of estimating strategic responses among duopolistic rivals using multiple tools.

Despite their application to duopolistic market structures, these studies address a second limitation of Lanchester-type models, namely their analysis of only one strategic tool. Although notable exceptions exist -- Chintagunta and Vilcassim (1994), for example, consider the roles of



both advertising and detailing in a duopoly framework -- these studies typically estimate dynamic models and simulate optimal dynamic solutions, so consideration of multiple tools is usually infeasible.

A third potential weakness of Lanchester-type models has yet to be addressed in the literature. While differential equations describing the evolution of duopoly market share are mathematically plausible and admit a simple yet powerful solution technique, they are *ad hoc* as they are not grounded in any theoretical model of consumer optimization. Rather, market share should evolve according to consumers' responses to firms' various marketing activities. Chingtagunta and Rao (1996) develop a model of optimal strategic pricing based on a random utility model of consumer choice, but their use of a single-control steady-state framework prevents them from considering the interaction of several possible marketing tools. By grounding our strategic-marketing model in consumer theory, we are able to estimate a system of equations and matrix of response elasticities that better describe the likely outcome of market share rivalry.

The model we propose addresses each of these weaknesses by developing a synthesis of Nerlove - Arrow and Lanchester-type models. First, by treating each firm within an oligopoly as being in an "us versus the rest of them" battle for market share, we are able to condense a potentially intractable oligopoly problem into one that is mathematically manageable. This is a realistic approach in that oligopolistic firms rarely single-out particular rivals for a targeted price cut or advertising campaign. Second, we explicitly account for the fact that marketing strategies can, and do, include choices over several marketing variables and that these choices are endogenous to market performance and rival strategies. Consequently, we specify a fully

simultaneous model of product demand and marketing variable choice. Chintagunta and Jain (1995), Liang (1986) and Cotterill and Putsis (2000) demonstrate the importance of simultaneously estimating market-share response equations with equations representing the optimal choice of each strategic marketing tool. This approach has been applied in the duopoly-game theory literature by Roberts and Samuelson (1988) in the cigarette industry, Gasmi and Vuong (1991), and Gasmi, Laffont, and Vuong (1992) in the soft drink market, and Liang (1986) in the ready-to-eat cereal industry. It has not, however, been used to estimate an oligopoly model with multiple decision variables. Third, we base the dynamics governing market share in a model of consumer optimization, so strategic interaction impacts firm performance not directly, but through demand for their product. This is a more realistic and plausible motivation for the evolution of market share over time in a rivalrous environment. By incorporating these three features into a model of strategic rivalry, we hope to create a synthesis that performs better than existing models.

We assess the relative performance of these models using two non-nested testing methods: (1) the J-test of Davidson and Mackinnon , and (2) the likelihood dominance criterion (LDC) of Pollak and Wales (1991). With these tests, we are able to determine which of the market share rivalry specifications is more appropriate to our particular data set.<sup>3</sup> Although the J-test is popular due to its compelling logic and ease of calculation, it frequently provides indecisive results. Therefore, we also use the LDC test to either refute or corroborate the J-test

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<sup>3</sup> If the explanatory variables of the two models were the same, Barten's method of constructing a composite model that nests the two approaches would be the preferred way to select the better specification. In our case, however, the dependent variables are the same and the explanatory variables differ so the models are not nested in any way and construction of a composite model would be unnecessarily difficult.

result. With these tests, the data determine which model represents a better description of the strategic interactions that are occurring in the industry, rather than the modeler alone. With a formal comparison of the performance of our new model with one that represents the current orthodoxy, we intend to demonstrate the potential value of the synthetic model in improving the quality of information available on the relative effectiveness of different types of marketing activity.

In this section, we develop two structural models of strategic interaction -- the first based in Lanchester's model of market share rivalry, and the second based on a system of consumer demand equations. We begin with an extension of the well-understood Lanchester approach. With this model, firms are assumed to maximize the present value of future profits subject to the dynamic evolution of their market share, which is in turn determined by the nature of the strategic response of their rivals. In a duopoly model, the single state variable is defined in terms of firm  $i$ 's market share ( $M_i$ ) so that rival market share is the simple complement of this:  $M_j = 1 - M_i$ . From the perspective of firm  $i$ , however, they are more likely to be in an oligopoly than a true duopoly, so are more concerned with the share of the market served by *all others* rather than a specific rival firm. This represents a very general and simple extension of the Lanchester duopoly model to the oligopoly case where the equation of motion for the market share of firm  $i$  is written as:

$$\dot{M}_i = M_{i,t} - \Phi_i M_{i,t-1} = \left( \sum_j \beta_{ij} A_{ij}^{\alpha_{ij}} \right) (1 - M_i) - \left( \sum_j \beta_{-ij} A_{-ij}^{\alpha_{-ij}} \right) M_i + \epsilon_i, \quad (1)$$

for all  $i$  firms, where  $M_{-i} = 1 - M_i$  is the market share of all other firms,  $\dot{M}_i$  is the change in market share of the  $i^{\text{th}}$  firm, and  $A_{ij}, A_{-ij}$  are a set of  $j$  marketing tools available to firm  $i$  and all

other firms, respectively. In the empirical application to follow, this set of tools consists of firm-level advertising, product development expenditure, number of distinct brands (product line length) and average shelf price. In this general form for firm  $i$ 's market share dynamics,  $\beta$  measures the effectiveness of the particular strategic tool, whereas  $\alpha$  provides an estimate of its curvature. Usually,  $\alpha$  is assumed to be 0.5, so each element of the marketing mix demonstrates diminishing marginal returns typical of the square root function. To maintain comparability with existing research, we adopt this convention as well. Further, we include the parameter  $\Phi$  to account for the possibility that market share adjustment from one period to the next is costly, so it is not instantaneous if firms behave optimally. Equation (1) forms the basis of the Lanchester model of market share rivalry. Despite its simplicity, intuitive appeal, and considerable empirical support, it nonetheless rests on an *ad hoc* specification for the evolution of market share.

Rather, a firm's market share does not respond directly in response to the actions of rivals, but through the consequence of consumers' responses to those actions. The Lanchester model ignores this fact, so any dynamic solution is necessarily *ad hoc*, even if it is fully optimal with respect to its own equations of motion. On the other hand, directly specified consumer demand functions describe the direct impact of marketing tools on aggregate consumer budget, and hence, market shares. By selecting the appropriate demand specification, the equations of motion for market share can describe optimal consumer behavior. From among the large class of functional forms that are consistent with utility maximization, we choose the linear version of Deaton and Muellbauer's (1980) Almost Ideal Demand System (AIDS) model. Originally developed as a direct demand system, in which prices are assumed to be exogenous, there is some question in this case as to whether prices or quantities are more plausibly assumed to be

exogenous. For example, Liang (1986) and Cotterill and Putsis (2000) specify simultaneous systems of demand and price-response equations, arguing that prices are endogenous choice variables. Indeed, Cotterill and Putsis (2000) use a direct linear approximate AIDS (LA / AIDS) model to estimate both the demand for ready to eat cereals and price-response equations in simultaneous, three-stage-least squares framework. While this approach is consistent in an econometric sense, if prices are indeed endogenous as the authors argue, then the demand system itself should be written in inverse, or *price-dependent* form with firm-level quantities as explanatory variables. Evidence of high cereal prices (Cotterill, 1999) does not necessarily mean that price is a competitive variable, but rather suggests the opposite – that firms cooperate in holding prices relatively high and compete for market share using other means. Ultimately, therefore, the appropriate form of the demand-and-response system is an empirical question.

We resolve this question using a series of Wu-Hausman specification tests (Hausman, 1978), which is a common method of testing for price or quantity endogeneity (Eales and Unnevehr, 1993). Comparing an estimator that is consistent under both the null and alternative hypotheses with one that is efficient under the null hypothesis yields a chi-square statistic of 64.733 when prices are specified as exogenous and 1,340.241 when quantities are considered exogenous. This result provides strong evidence that prices are exogenous, but quantities are not. Because it is not appropriate to specify price as the primary competitive variable in these data, we use an inverse AIDS (IAIDS) model and estimate with fully simultaneous methods in a manner similar to Moschini and Vissa (1992) or Eales and Unnevehr (1993). Consequently, we describe this model in some detail next.

Derivation of the IAIDS model begins from a distance function analogous to Deaton and

Muellbauer's (1980) PIGLOG expenditure function:

$$\ln D(u, \mathbf{q}) = (1 - u) \ln a(\mathbf{q}) + u \ln b(\mathbf{q}), \quad (2)$$

where  $\mathbf{q}$  is a vector of quantities. Eales and Unnevehr (1993) define  $a(\mathbf{q})$  and  $b(\mathbf{q})$  as:<sup>4</sup>

$$\begin{aligned} \ln a(\mathbf{q}) &= \sum_i \gamma_i \ln q_i + 0.5 \sum_i \sum_{-i} \gamma_{-ii} \ln q_{-i} \ln q_i \\ \ln b(\mathbf{q}) &= \eta_o \prod_i q_i^{-\eta_i} + \ln a(\mathbf{q}). \end{aligned} \quad (3)$$

Substituting these expressions into the distance function (2), differentiating with respect to quantities, and solving for each of the  $i$  budget shares leads to an estimable system:

$$M_i = \gamma_{i0} + \gamma_{i1} \ln q_i + \gamma_{-i2} \ln q_{-i} + \eta_i \ln Q, \quad (4)$$

where  $M_i$  is the budget share of firm  $i$ , and  $Q$  is a total quantity index, commonly approximated with Stone's quantity index:  $\ln Q = \sum_i w_i \ln q_i$ . Theoretical conditions of homogeneity, adding up, and symmetry can be imposed on the system, and thereby tested through simple likelihood ratio tests with the following set of restrictions:

$$\sum_i \gamma_i = 1, \sum_i \gamma_{ij} = 0, \sum_i \eta_i = 0, \sum_j \gamma_{ij} = 0, \quad (5)$$

Although it is more usual to specify a full system of separable goods, or brands in this case, we model oligopolistic rivalry among cereal manufacturers with a set of equations that represent each firm's share as a function of own quantity and all others. This not only makes the model

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<sup>4</sup> This distance function is assumed to be linearly homogenous, concave, non-decreasing in  $Q$  and decreasing in  $u$  (Eales and Unnevehr 1993).

more parsimonious, but is consistent with our objective of condensing an oligopolistic problem into a set of single-state variable duopoly models, each of which is comparable to the more traditional Lanchester specification. To aid in interpreting the structural parameters, which are of little meaning themselves, the price and scale flexibilities in the linear-approximate version are given by:

$$\begin{aligned} F_{-ii} &= -\delta_{-ii} + (\gamma_{-ii} + \eta_{-i}M_i)/M_{-i} \\ F_i &= -1 + \eta_i/M_i, \end{aligned} \tag{6}$$

respectively, where  $\delta_{ij}$  is Kronecker's delta. In the inverse model, negative cross-flexibilities indicate gross quantity-substitutes, whereas positive cross-flexibilities suggest that the firms' products are quantity-complements. Similarly, products with scale flexibilities below -1.0 are termed necessities, while a scale flexibility above -1.0 indicates the firm's output is a luxury. In order to utilize (4) as an equation of motion for market share, however, we need to specify a more general version of this model that allows for dynamics in consumer choice, and their response to advertising and new product introduction.

To incorporate consumer response, Pollak and Wales (1980) describe two methods of modifying the underlying utility structure to be consistent with either shifts in demand (translating) or pivoting the demand curve (scaling). Because our interest lies in the effect of marketing tools on market share, we adopt the translating method and incorporate the log of advertising and new product count into the intercept of (4). Second, as in the Lanchester model case, we develop a dynamic version of (4) by recognizing the fact that budget-share adjustment is a costly process as consumers seek out and try new varieties before changing their current

consumption patterns. In high frequency data such as that used below, these short-run costs of adjustment imply that budget shares adjust only partially toward their desired levels each period. This partial adjustment assumption is incorporated into (4) by writing each budget share equation in geometric lag form, with the coefficient on the lagged share representing the rate of adjustment per period (Heien and Wessels, 1990). As a result, market share evolves according to the equations of motion described by the system of inverse demand equations:

$$M_{it} = \theta_i M_{i,t-1} + \sum_j \phi_{ij} \ln A_{ij} + \sum_j \phi_{-ij} \ln A_{-ij} + \gamma_i \ln q_i + \gamma_{-i} \ln q_{-i} + \eta_i \ln Q + \epsilon_i, \quad (7)$$

for all  $i$  firms using  $j$  marketing tools, where  $\theta$  is the rate of market-share adjustment,  $q_i$  is the unit volume sold by firm  $i$ ,  $Q$  is the total quantity index,  $\epsilon$  is a random error term and the other variables are as defined above. Notice that average price levels for each rival firm are implicitly endogenous with this specification, so they are strategic variables in the IAIDS model, but not the Lanchester model. This is an important point of departure between the two models as industry observers note that intense non-price competition has led to tacit collusion in price among industry members (Cotterill, 1999). With both models, therefore, we focus on the full set of strategic variables introduced above -- advertising, product development expenditures and product line manipulation -- in order to capture the complete strategic arsenal employed since the near-universal market share battles that arose in early 1996 (Corts 1998). Several studies estimate the equations of motion alone and use these parameters to characterize a dynamic solution, but Chintagunta and Vilcassim (1994) note that this generates policy rules that assume firms behave according to the particular game that is played. Rather, by estimating each firm's actual response, we are better able to differentiate between behaviors under the actual game being



played (Liang, 1986; Roberts and Samuelson, 1988; Erickson, 1992; Chintagunta and Jain, 1995; Slade, 1995). With the competing market-share adjustment equations defined in (1) and (7), we derive equations that define the optimal strategic response in each marketing mix variable from the first order conditions of the firm's dynamic optimization problem.

Assuming each firm maximizes the discounted sum of future profits subject to the market share evolution described by either (1) or (7), the optimization problem becomes:

$$\max_{A_{ij}} V(M_i) = \int_0^{\infty} e^{-rt} [g_i M_i(t) \sum_i q_i - \sum_j A_{ij}(t)] dt, \quad (8)$$

where  $g_i$  is firm  $i$ 's gross margin per unit of sales and we assume each strategic variable is expressed in expenditure or equivalent terms to facilitate comparison. This objective function, together with the dynamic market share equations, describe a differential game that can be solved under assumptions of either a closed- or open-loop policy. Essentially, an open-loop solution results in policy equations for each of advertising, product development, and brand introduction that are functions of time alone. Therefore, the firm's managers are assumed to commit to a particular trajectory for each and adhere to this strategy irrespective of the competitive environment. Alternately, if we assume that the firms play a non-cooperative game in each marketing tool in every time period, a closed-loop solution to (8) constitutes a Nash equilibrium where each decision variable is a function not only of time, but of the current state of the game. The state is, of course, summarized by the market share of firm  $i$ . As commonly recognized, solving this problem in more than two market shares ie. in an oligopoly, is analytically intractable. Therefore, we adopt a new approach by considering the oligopoly solution as simply

a series of “us versus them” duopoly games. This approach, while unique, is valuable in two respects. First, it reduces the number of state variables, thus making what would otherwise be an intractable economic problem easily solvable with analytical methods. Second, perspective is intuitively preferable because firms do not single-out rivals for targeted advertising or pricing strategies as the more traditional approach implies. Rather, they set policies conditional on the strategic environment they face, where their particular environment may consist of any number of rivals. By solving a similar problem for each firm in the industry, we derive share and response functions that capture each firm’s response to all other firms in the industry at once. There are many examples of studies that use the simpler duopoly approach to arrive at analytical solutions for strategic variables such as price or advertising, and use parametric assumptions to simulate optimal control paths (Sorger, 1989; Erickson, 1992, Chintagunta and Vilcassim, 1992, 1994), however, we are concerned with only the first-stage of this problem. Consequently, in the next section we derive the necessary conditions for a closed-loop solution to the game described above and then to derive an empirical model of strategic rivalry.

These necessary conditions describe each firm’s dynamic optimal response for each element of its marketing mix. Specifying optimal response equations along with the equations of motion for market share complete the market-share rivalry model for each firm. First, we derive the optimal-response equations for the Lanchester model and then the IAIDS specification. Because the solution to a Lanchester duopoly game is well understood, however, we simply recognize that our approach represents an extension of existing solutions wherein a firm’s market share is the sole state variable. Given this observation, the Hamiltonian representing the optimal control problem facing each firm is written:

$$H(M_i) = g_i M_i Q - \sum_j A_{ij} + \lambda_i [M_i - \sum_j \beta_{ij} A_{ij}^{\alpha_{ij}} (1 - M_i) + \sum_j \beta_{-ij} A_{-ij}^{\alpha_{-ij}} (M_i)] \quad (9)$$

for each firm  $i$  and marketing tool  $j$  where  $\lambda$  is the costate variable for firm  $i$ . To derive an estimable model, we take the solution derived by Sorger (1989) as given, and derive a corresponding solution to the IAIDS model below. Specifically, the closed loop solution for each oligopolist under curvature conditions that are generally accepted in this literature (Erickson, 1992, for example assumes  $\alpha_{ij} = \alpha_{-ij} = 1/2$ ) yields response functions that are particularly appealing for econometric purposes, because they are linear in own-market share (Chintagunta and Jain, 1995):

$$A_{ij} = \left( \frac{a_{ij}}{\beta_{ij}} \right)^2 (1 - M_i), \quad (10)$$

and, for each set of rival firms:

$$A_{-ij} = \left( \frac{a_{-ij}}{\beta_{-ij}} \right)^2 M_i, \quad (11)$$

where the  $a_{ij}$  parameters solve the set of simultaneous equations given in Sorger (1989). Notice from this expression that the cross-equation restrictions imply that a full-information method will be more efficient than a limited-information, or single-equation estimator. An equivalent method provides a similar set of response equations for each firm and marketing tool in the IAIDS model.

The critical difference between the IAIDS and Lanchester models is, however, that the

optimal response depends both upon a firm's share of the market as well as the absolute quantity sold by each of its rivals due to the inverse demand specification. To see this, we apply the maximum principle to the Hamiltonian defined for the IAIDS problem. In this case, the Hamiltonian is:

$$H(M_i) = g_i M_i \sum_i q_i - \sum_j A_{ij} + \lambda_i [\dot{M}_i - \sum_j \phi_{ij} \ln A_{ij} + \sum_j \phi_{-ij} \ln A_{-ij} + \gamma_i \ln q_i + \gamma_{-i} \ln q_{-i} + \eta_i \ln Q], \quad (12)$$

Applying the Maximum Principle to (12) to find the stationary point with respect to  $A_{ij}$  provides the necessary conditions for a Nash closed-loop solution:

$$A_{ij} = \lambda_i \phi_{ij}, \quad (13)$$

for each firm  $i$  and instrument  $j$ , and the costate equation:

$$\dot{\lambda}_i = -g_i \sum_i q_i - \lambda_i (\eta_i (\ln q_i - \ln q_{-i}) + r). \quad (14)$$

recognizing that  $\ln \sum_i q_i = M_i \ln q_i + (1 - M_i) \ln q_{-i}$ . Differentiating (13) with respect to time, converting all time derivatives to their discrete-time counterparts, using the result to eliminate the costate variable in (14) and solving for the optimal response for instrument  $j$  by firm  $i$  gives an expression for  $A_{ij,t}$  that is a function only of the current state of the system:

$$A_{ij,t} = (1 + \eta_i (\ln q_i - \ln q_{-i})) (A_{ij,t-1} - g_i \phi_{ij} \sum_i q_i). \quad (15)$$

Using a similar procedure to find the optimal, endogenous quantity (ie. market share) response leads to an analogous expression in market share and rival quantities:

$$\dot{q}_i/q_i + \eta_i \ln q_i = (\gamma_i + \eta_i M_i)/M_i + \eta_i \ln q_{-i} - (\gamma_i / ((\gamma_i + \eta_i M_i) M_i)) (M_{i,t} - M_{i,t-1}). \quad (16)$$

By estimating (15) and (16) together with the IAIDS equations of motion (7) with a non-linear three-stage-least-squares method, we account for the maintained endogeneity of output in the marketing response equations, and of marketing expenditure in the market share equations. This approach also allows us to impose cross-equation restrictions between (7) and (15) that follow from the theoretical derivation of the IAIDS model. As a result, the only unique parameter to be identified in the response equations is the firm-specific margin,  $g_i$ . On their own, however, these structural parameters have little intuitive meaning for firm strategy.

Response elasticities, on the other hand, provide unit-free estimates of the percentage change in a marketing tool by one firm in response to a given percentage change in another tool by a rival firm. As such, these elasticities permit a common-size comparison of the strength of reaction among firms, and among the different tools at their disposal. In the IAIDS model, these estimates are similar to the dynamic conjectural variations of Roberts and Samuelson (1988), or the synthetic conjectural parameters of Erickson (1997). For the general case of tool  $j$  used by firm  $i$ , the response elasticity is:

$$\begin{aligned} \epsilon_{A_{ij}, A_{-ij}} &= \left( \frac{\partial A_{ij}}{\partial A_{-ij}} \right) \left( \frac{\bar{A}_{-ij}}{\bar{A}_{ij}} \right) = \\ &= \left( \left( \frac{\gamma_{ij} (A_{ij,t-1} - G_i \phi_{ij} \sum_i q_i)}{(1 + \gamma_{i9} (\ln q_i - \ln q_{-i}))^2} \right) \left( \sum_{-i} \phi_{-ij} \left( \frac{\ln q_i}{A_{-ij}} \right) \right) + \left( \frac{-G_i \phi_{ii}}{(1 + \gamma_{i9} (\ln q_i - \ln q_{-i}))} \right) \right) \left( \frac{\bar{A}_{-ij}}{\bar{A}_{ij}} \right), \end{aligned} \quad (17)$$

recognizing that Stone's quantity index is a weighted average of each firm's quantity, where the

weights are endogenous in the IAIDS model. Using these expressions, we test the null hypothesis that each is equal to zero using Wald chi-square tests. Unlike Erickson (1997), who defines each firm's marketing reaction function in terms of a rival firm's market share, with these elasticities we define the conjecture directly in the marketing-mix space. This is both necessary and convenient because each firm's response to changes in its rivals' market share (the state variable) is subsumed in (17) and the information contained in (17) is of more direct relevance to managers interested in strategic responses to decisions, rather than outcomes, from rival firms. Moreover, the statistical significance of these elasticities determine whether the solution is closed- or open-loop.

Specifically, if these elasticities and the share-response parameters in the Lanchester model are both non-zero, then each model represents a closed-loop solution to the dynamic oligopoly marketing game. However, each embodies a different hypothesis as to how competitive reactions are formed. Whereas firms in the Lanchester model are assumed to respond directly to changes in market share, in the IAIDS model they respond to both share and aggregate market size. Both of these solutions, however, represent closed-loop Nash equilibria in that the responses they describe reflect only the current state of the system. In this sense, they are also sub-game perfect. Open-loop solutions, on the other hand, depend only upon the initial state of the system and time so are, therefore, not sub-game perfect even though they remain Nash in that the oligopolist is doing the best that he can given that his rivals are doing likewise under the assumed rules of the game. Based on our observation of firm behavior in the cereal industry, the only plausible model must describe a closed-loop solution as firms clearly do not commit to long-term marketing strategies independent of the competitive environment in the

industry. The model selection tests described in the next section, therefore, provide evidence as to the nature of the competitive interaction among cereal manufacturers in the US -- whether they respond to changes in market share as hypothesized by the Lanchester model, or to changes in firm sales and total industry size as in the IAIDS model.

### **Data and Methods**

To provide a comparison to existing research, we apply our model of market share rivalry to sales, advertising, product development, and product line data from the ready-to-eat cereal industry. Specifically, we focus on the period following the industry-wide price cuts of early 1996 in order to focus on the methods of competition in use since that time (Cotterill 1999). By choosing this example, we are also able to compare our results to the dynamic conjectural variation model estimated by Erickson (1997). This industry forms a particularly suitable example for our model because it is one of the most concentrated oligopolies in the packaged product industry, and one in which it is widely regarded that new product introductions do indeed form an important method of competitive foreclosure (Scherer 1990; Hausman 1997; Cotterill 1999; Nevo 2001). Our data are from the IRI InfoScan database for the Baltimore / Washington market covering the four-weekly periods from the third quarter of 1996 to the fourth quarter of 1999. Over this period, the top five cereal companies (Kellogg, Post, General Mills, Quaker Oats, and Ralston) sold a total of 224 different cereal brands, some 84 of these were introduced during the sample period. In order to capture the market-level impact of firm-level marketing strategies, it is necessary to use retail scanner data from all stores within a sample market, rather than aggregate out to a national-level data set. This “case study” approach is now common among empirical studies of rivalry among consumer product firms (Slade, 1995; Kadiyali,

Vilcassim, and Chintagunta, 1996). To summarize the scanner data, table 1 provides each firm's average price, unit sales, and market share over the sample period. Among the marketing strategy variables, both advertising and product development expenditure data are provided by firm records obtained from the *Compustat* financial database. Given that these data are available only on a quarterly basis, they are converted to four-weekly series using a cubic-spline smoothing algorithm. Further, both of these variables necessarily reflect firm-wide strategies rather than market-level measures designed to increase local market share. These assumptions are reasonable given that most advertising campaigns target national broadcast and print media, while defining product development spending on a market-level basis would be impossible and meaningless. We calculate the number of brands for each company from the scanner data, adjusting each product line to allow for firm mergers, acquisitions, and divestitures over the sample period. Because there are several periods during which Ralston had a zero market share, we exclude these cereals from all estimates and subsequent model selection tests. With the relatively short time period covered by these data and the low rate of prevailing inflation, all dollar values are in nominal rather than real terms. Further, given that many of the explanatory variables in both models are endogenous, we estimate each using non-linear three-stage-least-squares where the instruments include all exogenous and pre-determined variables in the model. Due to the fact that neither of these models is a special case of the other, and a synthetic model combining them both would be very large and difficult to estimate, so we test between them using non-nested testing methods.

Anticipating the likelihood that one test alone will not provide conclusive evidence either for or against a particular model, we use three such non-nesting testing methods: (1) the J-test,



(2) the likelihood dominance criterion, and (3) tests of prediction accuracy. This section briefly describes these tests and explains how they are able to either determine which model provides a better representation of the data, or, failing a conclusive result, to rank the models according to a dominance criterion. Davidson and McKinnon (1981) develop the J-test to select between two alternative models where one is not a special form of the other. To carry out this test, we create a composite model that artificially nests the two competing hypotheses, written in generic notation as:

$$\begin{aligned} H_1: E(y) &= X_1 \beta_1 \\ H_2: E(y) &= X_2 \beta_2 \end{aligned} \tag{18}$$

where the composite model that includes both specifications is written:

$$y = (1 - \alpha)X_1\beta_1 + \alpha X_2\beta_2. \tag{19}$$

If  $H_1$  is the correct model,  $\alpha = 0$ , so a simple t-test can be used to either reject or fail to reject the hypothesis. The problem with the test as written in (18) above is that  $\alpha$  is not identified in that form. Consequently, Davidson and McKinnon (1981) suggest estimating an alternative where the fitted values of  $X_2\hat{\beta}_2$  are used instead. Conducting a similar test with  $H_2$  as the maintained hypothesis ensures that the conclusions of the test are not due to the particular specification chosen to be represented as fitted values in the other. Although this test is relatively simple to implement, it does not rule out the possibility of an inconclusive result -- one in which either both or none of the models is rejected in favor of the other. In anticipation of this eventuality, therefore, we also use a non-nested test that does not rely on the estimation of a formal composite

model, but only on the likelihood function values of each competing model. Specifically, Pollak and Wales (1991) develop the likelihood dominance criterion (LDC) to test non-nested alternatives when estimation of the composite model is unfeasible. The reasoning behind the LDC lies in comparing the likelihood function values from each competing model to that of a fictitious composite model. Define the “adjusted likelihood value” for alternative  $i$  as:

$$V_i = L_i + C(n_c - n_i)/2, \quad (20)$$

where  $L_i$  is the likelihood value for alternative  $i$ , and  $C$  is the critical chi-square value for degrees of freedom equal to the difference between the parametric size of the composite ( $n_c$ ) and maintained ( $n_i$ ) models. Pollak and Wales (1991) show that if  $V_2 > V_1$  there is no value of the composite likelihood function value that would suggest accepting  $H_1$  and rejecting  $H_2$ . Thus,  $H_2$  is said to dominate  $H_1$ . Clearly, the opposite reasoning holds if the inequality is reversed.

Developing the LDC from this logic in terms of the likelihood function values gives the following set of rules:

- (1) LDC prefers  $H_1$  to  $H_2$  if  $L_2 - L_1 < [C(n_2 + 1) - C(n_1 + 1)] / 2$
- (2) LDC indecisive if  $[C(n_2 - n_1 + 1) - C(1)] / 2 > L_2 - L_1 > [C(n_2 + 1) - C(n_1 + 1)] / 2$
- (3) LDC prefers  $H_2$  to  $H_1$  if  $L_2 - L_1 > [C(n_2 - n_1 + 1) - C(1)] / 2$ .

Although this method still admits the possibility of an inconclusive result, it nonetheless provides a selection method that avoids the same type of ambiguity inherent in the J-test. By using both, we are more likely to be able to determine which of the IAIDS or Lanchester models provides a better description of dynamic rivalry in the cereal industry. If neither of these comparisons provides a conclusive result, however, there remains a third method of model selection that

approaches the problem from an entirely different perspective.

Namely, many argue that the best model is the one that is able to forecast more accurately, whether in sample or out of sample. This is particularly true for models that are intended to be used for managerial guidance rather than testing economic hypotheses. To this end, we compare the two specification's based on two similar measures of forecast accuracy: (1) the root mean square error, and (2) Theil's U (Theil, 1961). Although Theil's U and the RMSE are both based on a measure of quadratic loss, the former normalizes the sum of squared deviations of the forecast from actual by the square of each realized value to produce a unit-free measure that is comparable among different data sets. These measures are calculated using the following formulae for RMSE:

$$\text{RMSE} = \sqrt{\sum_i (w_i - \hat{w}_i)^2 / M}, \quad (21)$$

and Theil's U:

$$U = \sqrt{\frac{\sum_i (w_i - \hat{w}_i)^2 / M}{\sum_i w_i^2 / M}}, \quad (22)$$

where  $M$  is the number of observations in the forecast data set and  $\hat{w}_i$  is the forecasted value of market share. Notice that a perfect forecast implies a U value of zero. With these measures -- the J-test, the LDC, RMSE and Theil's U -- providing three conceptually-different indicators of model performance, any corroboration between them would provide strong evidence in support

of the favored model. One caveat to these results must be noted. Namely, given the limited number of observations for each company's sales available to this study, we do not attempt to compare out-of-sample forecast accuracy, but calculate these measures over the entire sample data set for each model. Based on the results of these model selection tests presented in the next section, we interpret the structural elasticities and conjectural response parameters from the preferred model.

### **Results and Discussion**

Much has been written regarding the marketing strategies of the largest cereal manufacturers prior to and following the industry-wide price reductions of early 1996 (Cotterill, 1999, for example). Consequently, it is important to put the data describing these firm's strategies in context before analyzing the results of estimating the dynamic behavioral models described above. First, note from table 1 that General Mills has a leadership role in the Baltimore / DC market, but their local market share is considerably less than their share of the national market. Therefore, all of the results presented here should be interpreted as the realization of larger corporate strategies in a local market, rather than an indication of the overall performance. Second, note that total advertising and product development expenditure by Kellogg is significantly greater than that of General Mills, despite Kellogg's slightly lower market share. However, the simple snap-shot provided by these summary statistics does not reveal the evolution of each firm's strategic choices over time. In fact, Kellogg has been spending less in all areas of marketing in recent years in order to lower costs -- a strategy that has resulted in sharply declining market share and the loss of their once-dominant market position to General Mills. This table also does not show the general trend toward lower aggregate cereal sales across

the entire sample data period. As U.S. consumers move more toward convenient meal solutions targeted to specific tastes and perceived nutritional requirements, mass-market ready-to-eat cereals are becoming less popular among working-age market segments. Rather, new brands are, in fact, making inroads into niche markets such as woman-specific cereals, healthy snack-food alternatives, or as functional foods. Further, generic and private label cereals, generally produced by Quaker Oats and Ralston, are making gains at the value-end of the market at the expense of the top two. These data are significant in two ways. First, they highlight the importance of controlling for aggregate trends in demand when investigating firm-level changes in marketing strategy. Second, in this competitive environment it is apparent that a better understanding of the effect and effectiveness of the wide array of tools available to cereal marketers is key to helping them design strategy, which places a premium on designing the best possible model of strategic interaction.<sup>5</sup>

To determine which of the IAIDS or Lanchester models provides a better fit to the data, this section summarizes the results from each of the selection tests described above. These results are presented in table 2. First, the critical chi-square value for the J-test at a 5% level of significance and 4 degrees of freedom is 9.488. Based on the Wald chi-square values in table 2, we fail to reject null hypotheses that the IAIDS specification is preferred to the Lanchester, but reject the opposite comparison. Therefore, the J-test provides support for the IAIDS model relative to the Lanchester. Second, although the LDC is designed to address cases in which other

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<sup>5</sup> Although some studies of cereal demand consider the substitutability among specific brands within certain sub-categories such as “children’s cereals,” “adult cereals,” or “family cereals,” the objective of this study is to better understand firm-level market share strategies (Hausman 1997). Consequently, the number of brands a firm sells is a strategic variable of considerable interest, the impact of which is lost by invoking assumptions of separability among cereal categories.

non-nested tests are inconclusive, we consider results from this test to help confirm the results of the J-test. Using the selection criteria defined above, we find that  $L_2 - L_1 = 205.460$ , which is greater than  $C(n_2 - n_1 + 1) - C(1) = 3.615$ . By this test, therefore, we are again led to prefer the IAIDS to the Lanchester model. Finally, table 2 also shows that the IAIDS model performs significantly better than the Lanchester in predicting the market share of each manufacturer. Consequently, we estimate and interpret optimal strategies implied by the structural and response elasticities calculated from the IAIDS model.

Comparing the sign and significance of each response parameter between the IAIDS and Lanchester models suggests situations in which reliance on a Lanchester model may provide misleading guidance for firm strategy. In fact, comparing the parameter estimates between tables 3 and 4 reveals some marked and significant differences. Specifically, the estimates in table 3 suggest that product development is the only effective strategy General Mills may use to increase market share, whereas the Lanchester model parameters in table 4 indicate that advertising is likely to be more effective than either product development or extending the product line.<sup>6</sup> Further, its rivals are able to take market share from General Mills through any of the other tools according to the IAIDS parameter estimates, while the Lanchester model suggests that only rival product development causes General Mills' market share to fall. On the other hand, Kellogg is able to raise market share through any of the three tools, and loses market share only to rival product development according to the IAIDS estimates. Product development is not only less effective in the Lanchester model, but counterproductive as the parameter estimate is significant

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<sup>6</sup> The results from the Lanchester model are very similar to those obtained by Erickson (1997), who estimates a similar model of dynamic rivalry in the ready-to-eat cereal industry. However, he does not restrict the scale parameter to 0.5 as we do, nor does he consider the likely simultaneity of each marketing tool.

and less than zero. The Lanchester model also suggests that Post is only able to increase market share by advertising and loses market share to advertising by its rivals.

Clearly, using these results to guide marketing strategy would sacrifice the potential benefits attributable to both product development and brand introduction that are implied by the IAIDS model results. Given that firms rarely introduce new products without advertising, and never without some prior commitment to significant product development efforts, it is perhaps not surprising to find that advertising is complementary to the other tools. Whereas the IAIDS model captures these complementary effects, the Lanchester model does not. Further, the Lanchester model suggests an entirely different set of competitive threats to Post market share compared to the IAIDS model. While all other firms' advertising causes Post market share to fall in the Lanchester model, only product development and brand introductions are likely to do the same according to the IAIDS estimates. This may be due to the fact that the IAIDS model is able to capture the impact of advertising and product creation on the total potential market size in addition to share, whereas the Lanchester focuses entirely on movements in share. Finally, the IAIDS estimates indicate that Quaker has a relatively limited ability to raise market share by bringing new brands to market, whereas the Lanchester model would recommend both product introductions and product development as part of an optimal dynamic strategy. However, the Lanchester results do not convey the significant negative impact that rival brand introduction may have on Quaker share that would arise if indeed the IAIDS results are more accurate. Although these comparisons provide valuable insight into the confidence that one may place in the strategy implied by one specification, elasticity measures provide better input to quantitative decision making.

Not only do the elasticity estimates in table 5 provide more intuitive, unit free estimates of the impact of each marketing tool, but they are more complete in the sense that their derivation takes into account the total response of one variable with respect to all direct and indirect effects of changes in other variables. Consequently, while the structural response equations in the IAIDS model do not have a direct interpretation, the response elasticities indicate the nature of the response of each firm to strategic decisions taken by their rivals. With these estimates, therefore, we are able to describe the strategies in a manner similar to Fudenberg and Tirole's (1984) "top dog," "fat cat," "lean and hungry," or "puppy dog" depending upon whether a firm overinvests to deter or accommodate a rival, or underinvests to do the same, respectively. Whereas the elasticities with respect to  $A_i$ ,  $R_i$ ,  $B_i$ , and  $q_i$  in this table measure the direct effect of each type of investment, the reaction elasticities indicate the nature of the indirect, or strategic effect of rival investments. For General Mills and Post, note that the strategic response to changes in each tool is negative (the bottom three lines in table 5), which implies that the set of tools considered here are strategic substitutes. If these firms' reaction curves slope downward for each tool, then in the context of the strategies observed since "Grape Nuts Monday" in 1996, we can interpret this behavior as each firm attempting to become "lean and hungry" to appear tough in the face of rival attempts to grab market share.

In terms of the estimates presented in table 5, General Mills' lean and hungry strategy means that it reacts to a 10% cumulative rise in advertising spending by all rivals with a reduction of just under 1.0% of its own, and that they react to rival product development in the same direction, but with about twice the intensity. On the other hand, Kellogg responds to rival product development with a comparatively small increase in its own, but is relatively more



aggressive in attacking any move that would reduce their market share. In contrast, Post is relatively less aggressive in response to new products than they are with respect to rivals' strategies in either of product development, advertising or quantity. Quaker is more similar to Kellogg as they have upward sloping reaction functions in advertising and product development, implying that they alone may opt to be a "puppy dog" in the face of reductions in either of these tools, but are also relatively aggressive in matching any output-based strategies by rivals. For each of the other tools, however, they in effect signal their willingness to roll over and not compete aggressively for share in the future when they reduce their investment along with the other firms. In fact, this explanation appears entirely plausible given Quaker's advertising-to-sales (A/S) and product development-to-sales (B/S) ratios relative to the industry leaders. Whereas General Mills and Kellogg operate under nearly identical 8% A/S and 2% B/S ratios, Quaker's A/S ratio is only 1.5%, while its B/S ratio is far below industry average at approximately 1.6%. This low level of investment may explain why advertising and product development by Quaker are relatively more effective compared to the other companies, simply because of diminishing marginal returns to any strategic variable. Beyond these insights into the usage and effectiveness of each form of marketing rivalry in this particular oligopoly, the approach illustrated here may be valuable to market analysts in a wide variety of similar contexts.

In particular, recognizing that strategic variables affect firm performance only indirectly through consumer demand is more consistent with marketing practice than a simple, mechanistic assessment of the tactical benefits of putting capital toward each marketing tool. Indeed, well-planned marketing decisions are taken with customer-oriented goals in mind so gains on rivals are achieved by reaching the same set of customers in a more effective way. Therefore, strategic

goals are attained only if the primary goals are met first. Further, this analysis shows that these strategic goals need not be phrased quantitatively in terms of a series of one-on-one interactions with rivals as in traditional oligopoly analysis, but rather as if each firm exists in a duopoly -- a duopoly consisting of itself and all other rivals. This perspective not only serves to make dynamic empirical analysis of oligopolistic rivalry mathematically tractable, but also provides more general recommendations as to the optimal policy of any one industry member. These results are also consistent with recent research conducted with similar, but more detailed data sets.

In particular, Nevo (2001) finds that high price-cost margins in the ready-to-eat cereal industry are due more to consumers paying for preferred, differentiated products and manufacturers' multi-product pricing strategies than they are to formal price-collusion. While our results do not address these specific types of behaviors, our general conclusions are the same, namely that cereal manufacturers tend to compete intensely using non-price methods while fully exploiting their abilities to price as the market will bear.

## **Conclusions**

This study develops a new empirical model of dynamic oligopolistic interaction with multiple strategic tools. The primary insight of this model is that strategic decisions are reflected in performance variables (market share or total sales) only through their impact on consumer demand and not directly through market shares as is typically assumed. Further, models of dynamic oligopoly strategy need not require multiple state variables to represent each firm's market share if it is recognized that marketing managers typically take decisions from an "us versus them" perspective. Each firm, therefore, considers its own market share as the only

relevant state variable.

We demonstrate the empirical performance of this model by deriving a multiple-firm market share model in which advertising, product development, new brand introductions (product line extensions) and quantity changes are considered key strategic variables. Recognizing that each of these strategic variables is endogenous when a firm is engaged in strategic rivalry, the model includes equations representing the optimal dynamic response to changes in the state of the system (own and industry sales) by each firm in their use of each strategic tool. The specific functional form for consumer-demand driven changes in market share that we choose to demonstrate this model is an inverse Almost Ideal Demand System (IAIDS), which also dictates the specific form of the marketing-tool response equations that we estimate. By estimating both the demand and strategic response equations together, we are able to provide a much better fit to a sample of ready-to-eat cereal manufacturer data than with a traditional market-share combat model.

In fact, three different non-nested testing procedures show the statistical superiority of the IAIDS model relative to a dynamic Lanchester alternative. All tests provide strong support for a dynamic IAIDS specification. Comparing the strategy recommendations implied by each also reveals a significant difference between the two models. In fact, these results suggest that policy decisions taken based on the results of the statistically inferior model could be seriously flawed. Further, errors may be both of commission by spending money on a tool with a negative impact on sales, or of omission if a potentially lucrative tool is avoided. Finally, future research in this area would consider a wider range of marketing tools, and investigate which specific form of consumer demand model is most appropriate for analyzing dynamic strategic interactions.

## Reference List

- Bayus, B. L. and W. P. Putsis, Jr., 1999, "Product Proliferation: An Empirical Analysis of Product Line Determinants and Market Outcomes," *Marketing Science*, 18, 137-153.
- Brander, J., and J. Eaton, 1984, "Product Line Rivalry," *American Economic Review*, 74, 323-334.
- Chintagunta, P. K. and N. J. Vilcassim, 1992, "An Empirical Investigation of Advertising Strategies in a Dynamic Duopoly," *Management Science*, 39, 1230-1244.
- Chintagunta, P. K. and N. J. Vilcassim, 1994, "Marketing Investment Decisions in a Dynamic Duopoly: A Model and Empirical Analysis," *International Journal of Research in Marketing*, 11, 287-306.
- Chintagunta, P. K. and D. C. Jain, 1995, "Empirical Analysis of a Dynamic Duopoly Model of Competition," *Journal of Economics and Management Strategy*, 4, 109-131.
- Chintagunta, P. K. and V. R. Rao, 1996, "Pricing Strategies in a Dynamic Duopoly: A Differential Game Model," *Management Science*, 42, 1501-1514.
- Corts, K. S., 1998, "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment," *RAND Journal of Economics*, 29, 306-323.
- Cotterill, R. W., 1999, "High Cereal Prices and the Prospects for Relief by Expansion of Private Label and Antitrust Enforcement," *Agribusiness*, 15, 229-245.
- Cotterill, R. W. and W. P. Putsis, Jr., 2000, "Market Share and Price Setting Behavior for Private Labels and National Brands," *Review of Industrial Organization*, 17, 17-39.
- Davidson, R. and MacKinnon, J. G., 1981, "Several Tests for Model Specification in the Presence of Alternative Hypotheses," *Econometrica*, 49, 781-793.
- Deal, K. R., 1979, "Optimizing Advertising Expenditures in a Dynamic Duopoly," *Operations Research*, 27, 682-692.
- Deaton, A. and Muellbauer, 1980, "An Almost Ideal Demand System," *American Economic Review*, 70, 312-326.
- Dockner, E. and S. Jorgensen, 1988, "Optimal Pricing Strategies for New Products in Dynamic Oligopolies," *Marketing Science*, 7, 315-334.
- Eales, J. S. and L. J. Unnevehr, 1993, "Simultaneity and Structural Change in U.S. Meat

Demand,” *American Journal of Agricultural Economics*, 75, 259-268.

Erickson, G. M., 1992, “Empirical Analysis of Closed-Loop Duopoly Advertising Strategies,” *Management Science*, 38, 1732-1749.

Erickson, G. M., 1997, “Note. Dynamic Conjectural Variations in a Lanchester Oligopoly,” *Management Science*, 43, 1603-1608.

Federal Trade Commission Hearings, Washington, D.C.

Feichtinger, G., R. F. Hartl, and S. P. Sethi, 1994, “Dynamic Optimal Control Models in Advertising: Recent Developments,” *Management Science*, 40, 195-226.

Fudenberg, D. and J. Tirole, 1984, “The Fat-Cat Effect, The Puppy-Dog Ploy, and the Lean and Hungry Look,” *American Economic Review: Papers and Proceedings*, 74, 361-366.

Gasmi, F. and Q. H. Vuong, 1991, “An Econometric Analysis of Some Duopolistic Games in Prices and Advertising,” *Advances in Econometrics*, 9, 225-254.

Gasmi, F., J. J. Laffont, and Q. H. Vuong, 1992, “An Econometric Analysis of Collusive Behavior in a Soft-Drink Market,” *Journal of Economics and Management Strategy*, 1, 277-311.

Hausman, J., 1978, “Specification Tests in Econometrics,” *Econometrica*, 46, 1251-1271.

Hausman, J., 1997, “Valuation of New Goods under Perfect and Imperfect Competition.” in *The Economics of New Goods*, eds. T. F. Bresnahan and R. J. Gordon. National Bureau of Economic Research. Chicago, IL: University of Chicago Press.

Heien, D. and C. Wessels, 1990, “Demand System Estimation with Microdata: A Censored Regression Approach.” *Journal of Business and Economic Statistics*, 8, 365-371.

Kadiyali, V., P. Chintagunta, and N. Vilcassim, 1996, “Empirical Analysis of Competitive Product Line Pricing Decisions: Lead, Follow, or Move Together,” *Journal of Business*, 69, 459-487.

Kadiyali, V., P. Chintagunta, and N. Vilcassim, 1999, “Product Line Extensions and Competitive Market Interactions: An Empirical Analysis,” *Journal of Econometrics*, 89, 339-363.

Kalai, E. and W. Stanford, 1985, “Conjectural Variations in Accelerated Cournot Games,” *International Journal of Industrial Organization*, 3, 133-152.

Karp, L., and J. Perloff, 1993, “A Dynamic Model of Oligopoly in the Coffee Export Market,” *American Journal of Agricultural Economics*, 75, 448-457.

- Kimball, G. E., 1957, "Some Industrial Applications of Military Operations Research Methods," *Operations Research*, 5, 201-204.
- Leeftang, D. S. H. and D. R. Wittink, 1992, "Diagnosing Competitive Reactions Using (Aggregated) Scanner Data," *International Journal of Research in Marketing*, 9, 39-57.
- Liang, J. N., 1986, "Price Reaction Functions and Conjectural Variations: An Application to the Breakfast Cereal Industry," *Review of Industrial Organization*, 4, 31-58.
- Maskin, E. and J. Tirole, 1988, "A Theory of Dynamic Oligopoly: I. Overview and Quantity Competition with Large Fixed Costs," *Econometrica*, 56, 549-570.
- Moschini, G., and A. Vissa, 1992, "A Linear Inverse Demand System," *Journal of Agricultural and Resource Economics*, 17, 294-302.
- Nerlove, M., and K. Arrow, 1962, "Optimal Advertising Policy Under Dynamic Conditions," *Economica*, 29, 129-142.
- Nevo, A., 2001, "Measuring Market Power in the Ready-to-Eat Cereal Industry," *Econometrica*, 69, 307-342.
- Pollak, R. A., and T. J. Wales, 1980, "Comparison of the Quadratic Expenditure System and Translog Demand System with Alternative Specifications of Demographic Effects," *Econometrica*, 48, 595-612.
- Pollak, R. A. and T. J. Wales, 1991, "The Likelihood Dominance Criterion: A New Approach to Model Selection," *Journal of Econometrics*, 47, 227-42.
- Roberts, M. J. and L. Samuelson, 1988, "An Empirical Analysis of Dynamic, Non-Price Competition in an Oligopolistic Industry," *Rand Journal of Economics*, 19, 200-220.
- Scherer, F. M., 1990, "The Breakfast Cereal Industry," in W. Adams, ed. *The Structure of American Industry*, New York: MacMillan.
- Slade, M. E., 1995, "Product Rivalry with Multiple Strategic Weapons: An Analysis of Price and Advertising Competition," *Journal of Economics and Management Strategy*, 4, 445-476.
- Sorger, G., 1989, "Competitive Dynamic Advertising: A Modification of the Case Game," *Journal of Economic Dynamics and Control*, 13, 55-80.
- Theil, H., 1961, *Economic Forecasts and Policy*, Amsterdam: North-Holland.
- Vidale, M. L., and H. B. Wolfe, 1957, "An Operations Research Study of Sales Response to

Advertising,” *Operations Research*, 5, 370-381.

**Table 1. Summary Statistics for Top Four Cereal Companies**

<b>Variable</b>	<b>N</b>	<b>Mean</b>	<b>Standard Deviation</b>	<b>Minimum</b>	<b>Maximum</b>
$M_1^1$	65	0.392	0.031	0.339	0.459
$M_2$	65	0.337	0.030	0.274	0.395
$M_3$	65	0.150	0.016	0.114	0.184
$M_4$	65	0.074	0.012	0.052	0.103
$P_1$	65	3.603	0.226	2.999	4.033
$P_2$	65	3.099	0.246	2.517	3.725
$P_3$	65	3.054	0.285	2.584	3.744
$P_4$	65	3.013	0.340	2.339	3.760
$Q_1$	65	52.396	5.157	40.920	62.576
$Q_2$	65	52.811	8.530	35.336	72.093
$Q_3$	65	23.775	3.843	15.759	33.581
$Q_4$	65	12.021	3.016	7.586	21.811
$A_1$	65	453.630	89.327	258.120	610.230
$A_2$	65	518.380	145.450	280.230	725.090
$A_3$	65	303.890	64.937	211.560	439.990
$A_4$	65	79.968	16.103	55.090	104.860
$R_1$	65	123.460	10.032	63.569	134.670
$R_2$	65	164.700	44.667	41.681	242.570
$R_3$	65	68.746	8.112	30.137	76.186
$R_4$	65	87.020	20.239	52.535	113.940
$B_1$	65	36.123	2.503	32.000	41.000
$B_2$	65	40.738	2.857	35.000	46.000
$B_3$	65	26.077	1.915	23.000	31.000
$B_4$	65	18.354	0.975	16.000	20.000

<sup>1</sup> In this table, the firm indices are defined as follows: General Mills = 1, Kellogg = 2, Post = 3, Quaker Oats = 4. The total share does not sum to 1.0 because firm 5, Ralston, is excluded from this table and from the system estimation to avoid singularity of the design matrix. Volume and expenditure data are in millions of current dollars.



**Table 2. Results of Model Selection Tests: IAIDS v Lanchester Rivalry Models**

<b>Criterion:</b>	<b>Strategic Response Model</b>	
	<b>IAIDS</b>	<b>Lanchester</b>
<b>RMSE:<sup>2</sup></b>		
$M_1$	0.006	0.029
$M_2$	0.005	0.427
$M_3$	0.003	0.008
$M_4$	0.004	0.073
<b>Theil's U:</b>		
$M_1$	0.135	0.341
$M_2$	0.091	0.498
$M_3$	0.106	0.216
$M_4$	0.141	0.511
<b>J-Test<sup>t</sup></b>	4.128	273.408
<b>LDC (LLF Values)</b>	1125.221	919.761

<sup>1</sup> The J-Test statistic is chi-square distributed with 4 degrees of freedom, while the LDC is chi-square distributed with the comparison rule given in the text. The null hypothesis for the J-test is that the parameters on the fitted market share values from the other model are jointly not significantly different from zero when included in the first as regressors.

<sup>2</sup> In this table,  $M_1$  = General Mills market share,  $M_2$  = Kellogg market share,  $M_3$  = Post market share, and  $M_4$  = Quaker Oats market share.

**Table 3. IAIDS Model Parameter Estimates: Ready-to-Eat Cereal - Baltimore / DC**

<b>Market Share Equations:</b>								
<b>Variable</b>	<b>General Mills</b>		<b>Kellogg</b>		<b>Post</b>		<b>Quaker Oats</b>	
	<b>Estimate<sup>1</sup></b>	<b>t-ratio</b>	<b>Estimate</b>	<b>t-ratio</b>	<b>Estimate</b>	<b>t-ratio</b>	<b>Estimate</b>	<b>t-ratio</b>
$M_{i,t-1}$	0.015	1.101	0.002	0.132	-0.040*	-4.047	-0.086*	-3.425
$\ln(A_i)$	-0.004	-1.078	0.040*	32.682	0.016*	12.489	0.001	0.420
$\ln(A_{.i})$	-0.007*	-3.874	0.007*	5.314	0.002*	2.327	-0.007*	-2.261
$\ln(R_i)$	0.240*	14.893	0.048*	14.687	0.088*	16.201	0.009	1.545
$\ln(R_{.i})$	-0.075*	-8.142	-0.189*	-18.491	-0.080*	-14.071	0.014*	2.154
$\ln(B_i)$	-0.063*	-4.738	0.056*	10.025	0.071*	18.144	0.024*	2.641
$\ln(B_{.i})$	-0.084*	-6.171	-0.002	-1.178	-0.067*	-10.905	-0.024*	-2.211
$\ln(Q_i)$	0.056*	7.893	0.025*	5.017	0.043*	13.831	0.043*	15.459
$\ln(Q_{.i})$	-0.029*	-2.993	-0.054*	-8.485	-0.033*	-8.094	-0.017*	-2.192
$\ln(Q)$	-0.047*	-41.958	0.001	1.684	-0.049*	-74.826	0.027*	47.133
$R^2$	0.976		0.982		0.979		0.938	
<b>Marketing Tool Response Functions:<sup>2</sup></b>								
$A_i / M_{.i}$	0.498*	2.094	0.527*	24.481	0.503*	9.556	0.496*	3.669
$R^2$	0.978		0.586		0.948		0.503	
$R_i / M_{.i}$	0.498*	2.094	0.527*	24.481	0.503*	9.556	0.496*	3.669
$R^2$	0.996		0.452		0.998		0.767	
$B_i / M_{.i}$	0.498*	2.094	0.527*	24.481	0.503*	9.556	0.496*	3.669
$R^2$	0.993		0.523		0.989		0.389	
$Q_i / M_{.i}$	0.498*	2.094	0.527*	24.481	0.503*	9.556	0.496*	3.669
$R^2$	0.978		0.291		0.991		0.371	

<sup>1</sup> A single asterisk indicates significance at a 5% level.

<sup>2</sup> Note that each marketing tool response function identifies one unique parameter per firm due to the cross-equation restrictions implied by the optimal dynamic solution.

**Table 4. Lanchester Model Estimates: Ready-to-Eat Cereal - Baltimore / DC**

<b>Market Share Estimates:</b>								
	<b>General Mills</b>		<b>Kellogg</b>		<b>Post</b>		<b>Quaker Oats</b>	
	<b>Estimate<sup>1</sup></b>	<b>t-ratio</b>	<b>Estimate</b>	<b>t-ratio</b>	<b>Estimate</b>	<b>t-ratio</b>	<b>Estimate</b>	<b>t-ratio</b>
$M_{i,t-1}$	1.318*	7.870	0.539	0.315	1.531*	5.428	2.719	1.911
$\sqrt{A_i} \sqrt{M_{-i}}$	-0.001*	-22.356	0.003*	7.829	0.000	-0.854	-0.001*	-5.402
$\sqrt{R_i} \sqrt{M_{-i}}$	0.094*	12.566	-0.162*	-7.339	0.036*	8.270	-0.026*	-3.754
$\sqrt{B_i} \sqrt{M_{-i}}$	-0.049*	-4.903	-0.095	-1.784	0.049*	5.799	0.133*	6.276
$\sqrt{P_i} \sqrt{M_{-i}}$	0.125*	3.027	0.524	1.701	-0.071*	-2.004	0.128*	2.099
$\sqrt{A_{-i}} \sqrt{M_i}$	-0.002*	-7.714	0.032*	11.254	0.001*	5.599	-0.001*	-3.703
$\sqrt{R_{-i}} \sqrt{M_i}$	-0.001	-0.947	-0.314	-1.877	-0.107*	-6.314	-0.067	-1.684
$\sqrt{B_{-i}} \sqrt{M_i}$	-0.192*	-5.312	0.843*	2.088	-0.141*	-3.945	0.280*	2.185
$\sqrt{P_{-i}} \sqrt{M_i}$	-0.264*	-2.197	-1.718	-1.315	0.096	0.675	-0.958*	-2.291
$R^2$	0.956		0.968		0.947		0.824	
<b>Marketing Tool Response Functions:</b>								
$A_i / M_{-i,t-1}$	7.010*	44.095	55.679*	102.460	192.300*	125.390	5.519*	144.360
$R^2$	0.217		0.975		0.203		0.174	
$R_i / M_{-i,t-1}$	7.860*	42.573	62.727*	103.280	261.130*	62.060	4.719*	118.010
$R^2$	0.269		0.285		0.232		0.320	
$B_i / M_{-i,t-1}$	3.570*	57.067	32.113*	132.150	83.275*	127.900	3.592*	116.940
$R^2$	0.284		0.160		0.261		0.272	
$P_i / M_{-i,t-1}$	0.871*	47.901	19.677*	139.450	94.510*	49.363	0.001	0.572
$R^2$	0.400		0.135		0.136		0.196	

<sup>1</sup> A single asterisk indicates significance at a 5% level. All estimates obtained using NL 3SLS estimator.

**Table 5. IAIDS Q-Flexibilities and Response Elasticities**

	Firm							
	GM		K		P		Q	
	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
$Q_i$	-0.791*	-38.795	-0.931*	-66.034	-0.678*	-34.351	-0.483*	-13.322
$Q_{-i}$	0.003	0.115	-0.157*	-8.562	0.058*	2.234	-0.526*	-5.331
$A_i$	-0.012	-1.078	0.116*	32.682	0.104*	12.489	0.015	0.419
$A_{-i}$	-0.021*	-3.875	0.021*	5.341	0.015*	2.327	-0.091*	-2.261
$R_i$	0.688*	14.892	0.137*	14.687	0.556*	16.201	0.111	1.545
$R_{-i}$	-0.215*	-8.142	-0.542*	-18.491	-0.504*	-14.701	0.171*	2.153
$B_i$	-0.180*	-4.738	0.159*	10.025	0.449*	18.441	0.299*	2.641
$B_{-i}$	-0.241*	-6.171	-0.007	-0.237	-0.424*	-10.905	-0.307*	-2.211
$X$	0.865*	269.211	1.003*	667.395	0.687*	163.977	1.337*	187.077
$A_i / A_{-i}$	-0.070*	-6.877	0.005*	2.243	-0.028*	-3.689	0.007*	11.765
$R_i / R_{-i}$	-0.019*	-15.502	0.006*	9.647	-0.021*	-14.579	0.022*	9.563
$B_i / B_{-i}$	-0.007*	-7.143	-0.006*	-3.781	-0.002	-1.859	-0.015*	-2.747
$Q_i / Q_{-i}$	-0.583*	-4.019	-0.640*	-4.383	-0.523*	-7.425	-0.326*	-9.444

<sup>1</sup> The Q estimates are flexibilities rather than elasticities. All remaining parameters reported here are elasticities. Values below the elasticity estimates are t-ratios. A single asterisk indicates significance at a 5% level. The strategic variables are defined as follows: A = advertising, R = product development expenditure, B = product line length (number of brands) and Q = quantity.