

Farm Financial Structure Decisions
Under Different Intertemporal Risk Behavioral Constructs

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Introduction

The unconstrained, expected utility maximization model of farm capital structure developed by Barry, Baker, and Sanint (BBS) and by Collins has yielded important theoretical and empirical insights in agricultural finance. The model produces clear, intuitively reasonable solutions and behavioral predictions. For instance, its comparative static properties provide the basis for the risk-balancing concept while its extended versions have been used to evaluate the effects of several policy and farm-level factors on farm financial structure decisions.

The BBS model, however, contains one important exception to these reasonable behavioral predictions. The model predicts that optimal debt levels will decrease if initial wealth increases. This is a result of the underlying behavioral assumption of constant absolute risk aversion, which implies that investment in a risky asset is a constant function of wealth. When such investments are financed by initial equity and debt, this implies that increases in initial equity will cause one for one decreases in debt.

It appears that the constant absolute risk aversion behavioral hypothesis has caused several agricultural economics researchers to look for modifications of the BBS model that replace the coefficient of absolute risk aversion with the coefficient of relative risk aversion. For example, Collins presents a model where the argument of the expected utility function is rate of return on wealth, and the risk aversion parameter is interpreted as the coefficient of relative risk aversion. Although it is not clear, one reason for such a formulation may be the perceived need to replace the behavioral hypothesis of constant absolute risk aversion.

This paper presents an alternative formulation of the optimal debt model that replaces the behavioral hypothesis of constant absolute risk aversion, with the behavioral hypotheses of decreasing absolute risk aversion and constant relative risk aversion. The alternative behavioral specifications of the proposed model will be formulated using the results of Meyer on the general location-scale model of decision making under uncertainty. Our new formulation retains the positive attributes of the BBS model, while eliminating the behavioral implications of constant absolute risk aversion. This study provides a theoretical justification for a constant relative risk aversion optimal debt model based on its greater empirical strength and more reasonable predictions of optimal farm debt decisions. It is also an example of a direct empirical application of Meyer's general location-scale results.

2. The Analytic Value of the BBS-Collins Model of Optimal Farm Leverage

As subsequent studies have shown, the comparative static properties of both the basic and extended versions of the BBS-Collins model of optimal farm leverage exhibit expected logically deduced relationships between optimal debt and exogenous variables that are empirically verified.

2.1 Applications to Farm Finance Issues

The BBS model, developed as an extension of portfolio theory formulated in a mean-variance framework, was actually designed to establish links between farmers' credit risks and their level of borrowing. The model derives and compares expressions for optimal debt obtained under deterministic and risky borrowing conditions using the first order conditions (FOC) of an expected utility maximization problem. By expanding the concept of total risk to accommodate credit risks that arise from unexpected

variations in the cost and availability of credit, the BBS study describes the influence of the lender's credit rationing policies on the farmer's leverage decisions. As credit risks are introduced in the model, covariance relationships between asset returns and borrowing rates provide important insights into changes in optimal leverage levels under the two borrowing scenarios.

Collins' model used deterministic borrowing rates to present an alternative theoretical validation of the hypothesized balancing of business and financial risks introduced in separate works of Gabriel and Baker, and Barry and Robison. By totally differentiating the expression for optimal leverage, he showed that upward adjustments in the farm's financial leverage position may result when external shocks increase the margin between asset returns and borrowing rates by a greater percentage than the variance of asset returns. Barry and Robison, applying an equilibrium analysis under risk to the capital structure decision problem of the firm, used the same deterministic, static model and the risk balancing concept to decompose possible responses of adjustments in the financial structure to changes in the determining variables into two components. The income effect is determined by the decision-maker's risk attitude while changes in other parameter values determine the substitution effect.

Subsequent extensions of the BBS-Collins model were used to analyze a number of farm finance issues. A theoretical construct developed by Featherstone, *et al.* shows that risk reducing and income-augmenting farm policies actually induce farmers to make optimal leverage adjustments that eventually affect the cumulative probability of earning very low rates of return on equity. Farmers, instead of benefitting from increased welfare, are then faced with the prospect of losing part of their equity investments and

ultimately going bankrupt. Other applications of the model include the analyses of changes in optimal capital structure decisions as a result of capital gains deductions (Moss, Ford, Boggess), depreciation allowances, tax policy, investment tax credits and a partial-adjustment specification (Ahrendsen, Collender and Dixon; Jensen and Langemeier).

2.2 Comparative Static Properties

Optimal capital structure is defined by the solution to the model's FOC. The optimal value of the choice variable, debt, is a function of the mean and variance of asset return, cost of borrowing, equity level and a risk aversion parameter. A comparative static analysis of the model's results supports logically deduced relationships between optimal debt and its determining factors. Higher mean asset returns and optimal debt are positively related, thus, suggesting that higher profitability can induce the farmer to incur additional debt. A greater amount of risk, represented by the variance measure, is associated with a lower optimal debt level. The same inverse relationship is sustained in the case of the other variables. Higher borrowing costs result in greater financial stress that discourages additional debt; and a more risk averse farmer avoids entering into financial contracts that could dilute his own claims on his farm's assets. The inverse relationships in the case of the risk and risk aversion measures require the empirically reasonable condition that mean asset returns always exceed borrowing costs.

The partial derivative that establishes the effect of changes in equity levels on the firm's borrowing activity is consistent with the model's underlying behavioral assumption of constant absolute risk aversion. This risk behavior implies that the decision-maker maintains a constant amount of investments in risky assets even as the

wealth position improves (Barry and Baker). This yields a negative partial derivative of debt with respect to increases in the farm equity position because higher equity must be offset by downward adjustments in the liabilities side of the farm's balance sheet.

3. Behavioral Restrictions of the BBS Model

This section examines the model's assumptions about preferences and the probability distribution of the random variable. Specifically, the assumptions of constant absolute risk aversion and normally distributed returns will be discussed. Consistent with Freund's results, the BBS-Collins capital structure model (and its later versions) is formulated as a static mean variance representation of an expected utility maximization problem with normally distributed returns and a risk behavior exhibiting constant absolute and increasing relative risk aversion (CARA-IRRA). The model's objective function, as Freund suggested, is:

$$E(U(W)) = \mu_w - 0.5\rho\sigma_w^2$$

where $E(U(W))$ is the expected utility of final wealth, μ_w is the expected (or the mean) final wealth, σ_w^2 is the variance of final wealth and ρ is the coefficient of absolute risk aversion.

Applying results of Meyer and Sinn concerning the mean-standard deviation objective function derived from a general economic decision model satisfying the location and scale parameter condition, the above objective function has the following properties, given that $E(U(W)) = V(F, :)$:

- i) Increasing in mean ($V_{:} = 1$)
- ii) Decreasing in standard deviation ($V_F = -\rho$)
- iii) Increasing slope ($S(F, :) = -V_F/V_{:} = \rho$)
- iv) Concave objective function ($V_{:} = 0$, $V_{FF} = -\rho$ and $V_{:F} = 0$)

Following Meyer, the partial derivatives of the slope characterize the function's representation of risk preferences:

$$\frac{\partial S(\sigma, \mu)}{\partial \mu} = 0 \quad (3.1)$$

$$\frac{\partial S(t, \sigma, t\mu)}{\partial t} = \rho\mu \quad (3.2)$$

The derivative with respect to μ , equation 3.1, shows that the model exhibits constant absolute risk aversion. And the derivative with respect to t , equation 3.2, shows that the model exhibits increasing relative risk aversion.

On both theoretical and empirical planes, decreasing absolute risk aversion enjoys greater support than constant absolute risk aversion. The intuitive appeal of the DARA behavior was presented in the original works of Arrow and Pratt, who introduced these intertemporal risk behavioral concepts. Since then, DARA behavior has been regarded as the normative concept. Pratt argues that decision-makers would tend to “pay less for insurance against a given risk the greater their assets (p. 123)” while Arrow contends that a decision maker faced with increasing wealth would find it absurd to be more unwilling to take a fair bet involving a fixed amount.

In agriculture and other applied areas of economics, DARA behavior has gained stronger empirical support than alternatives. Saha, Shumay, and Talpaz (1994) and Chavas and Holt (1996) are recent studies that find DARA behavior by farmers. The econometric analyses employed by other studies also indicate that measures of risk aversion vary inversely with land under control, off-farm income (Moscardi and de Janvry, Young, et al), and net worth (Patrick, Whitaker and Blake), thus, lending more support to DARA behavior.

Furthermore, the assumption that the argument of the utility function is normally distributed may not be supported by the data used in applications of the optimal debt model. The argument of the objective function in the decision model is a linear transformation of the random variable, rate of return on assets or equity. Since linear transformations preserve normality and are invertible, normality of the argument of the objective function can be tested by testing normality of the rate of return. If the rate of return is not normally distributed, then the argument of the utility function is not normally distributed, violating the assumptions of the Freund derivation.

To examine this assumption we conducted a test of normality of the rate of return on equity in agriculture. The data on rate of return is drawn from a sample of 94 Illinois farms for the years 1987 to 1996. The rate of return on equity is calculated by dividing net farm income from operations (net of unpaid labor charges for the farm operator and family) by average farm equity. Summary statistics for the rate of return are presented in Table 1. The mean rate of return ranges between 0.016 and 0.16, and the standard deviation of the rate of return ranges between 0.09 and 0.30.

The test of normality is conducted on each of the annual cross sections of rate of return. This assumes that each year's distribution of the rate of return can be estimated from the realized cross section of rates of return in that year. The cross sectional variation is caused by inherent individual differences and differential responses to shocks in the given year. The mean rate of return across individuals is used as an estimate of the expected rate of return in a given year. And the sample standard deviation is used as an estimate of the standard deviation. This is consistent with some empirical applications of the optimal debt model (Parcell, Featherstone, and Barton).

Table 1 reveals that 1988, 1991, and 1995 were years of low average rates of return for the sample. And 1987, 1989, and 1992 were the years with the three highest

expected rates of return. The standard deviation is between one and two times the expected returns in each year, yielding a coefficient of variation that ranges from one to two.

The rate of return for each year was tested for normality with the test statistic:

$$(6n)^{-\frac{1}{2}} \sum_{i=1}^n e_i^3 + (24n)^{-\frac{1}{2}} \sum_{i=1}^n (e_i^4 - 3) \sim \chi^2(2) \quad (3.3)$$

where e_i is the centered and scaled value of the rate of return and n is 94 – the number of farms in the sample. Equation 3.3 is a slightly modified form of the Jarque-Bera test for normality (Davidson and MacKinnon, p. 596). The computed test statistics and their associated p-values are presented in Table 2. In every case the null hypothesis of normality is strongly rejected. The weakest rejection occurs in 1991 where the p-value is 0.027. This is strong evidence that cross sectional rates of return are not distributed as normal random variables.

4. A DARA-CRRA Alternative for General Distribution of Returns

This section describes the development of an alternative mean, standard deviation objective function that exhibits constant relative risk aversion (and therefore, decreasing absolute risk aversion), and is consistent with expected utility maximization under general probability distributions for the rate of return on assets. The alternative functional form is derived by applying the location-scale condition identified by Meyer and Sinn. The location-scale condition applies to the model of optimal debt because of the way the choice of debt and the random rate of return on assets interact to determine final equity, the argument of the expected utility function.

4.1 Application of the Location-Scale Condition

The problem of choosing an optimal level of debt given a known interest rate on debt and an uncertain rate of return on assets is analyzed here. The random argument of the utility function is:

$$E + \tilde{\pi} = E + \tilde{r}A - iD \quad (4.1)$$

where w is final wealth, E is initial wealth or equity, r is the random rate of return on assets, A is total assets, i is the known interest rate on debt, and D is the chosen level of debt. Assets are financed with initial equity and debt, so:

$$A = E + D.$$

Substituting, the random component of the argument of the utility function can be written as:

$$\tilde{w} = (1 + \tilde{r})E + (\tilde{r} - i)D. \quad (4.2)$$

Given the functional interaction between the underlying source of randomness, r , and the choice variable, D , this argument of the utility function satisfies the location-scale condition.

The location-scale condition can be verified by writing w , the random component of the argument of the utility function, as: $\mu + \sigma \tilde{x}$, where:

$$\tilde{x} = \frac{\tilde{w} - \mu}{\sigma}$$

In order to reparameterize the argument of the utility function, first define the mean and standard deviation:

$$\mu = (\mu_r - i)D + \mu_r E; \quad (4.3)$$

$$\sigma = \sigma_r D + \sigma_r E \quad (4.4)$$

where μ_r is the mean rate of return on assets and σ_r is the standard deviation of the rate of return on assets. This implies:

$$\tilde{x} = \frac{(1 + \tilde{r})E + (\tilde{r} - i)D - ((1 + \mu_r)E + (\mu_r - i)D)}{\sigma_r D + \sigma_r E} = \frac{(\tilde{r} - \mu_r)(D + E)}{\sigma_r (D + E)} = \frac{\tilde{r} - \mu_r}{\sigma_r}$$

which is the centered and scaled rate of return on assets. This implies:

$$\mu + \sigma \tilde{x} = (1 + \mu_r)E + (\mu_r - i)D + \sigma_r (D + E) \frac{(\tilde{r} - \mu_r)}{\sigma_r} = (1 + \tilde{r})E + (\tilde{r} - i)D = \tilde{w} \quad (4.5)$$

Thus, the optimal debt decision problem satisfies the location-scale condition. It can be shown that the location-scale condition also applies to the decision problem that treats the interest rate on debt as a random variable.

Results of Meyer and Sinn can be applied to write the decision problem as:

choose the level of debt, D , to maximize $V(\mathbf{F}; \cdot)$, where \mathbf{F} and \cdot are defined as above, and V satisfies conditions derived by Meyer and Sinn. The properties of V will be discussed in more detail in the following sections that derive an acceptable functional form for V .

4.2 The Behavioral Hypotheses of Decreasing Absolute Risk Aversion and Constant Relative Risk Aversion

The original derivation of the BBS optimal debt model used results of Freund to justify the mean-variance functional form. As described above, this justification carries with it the implicit assumptions of constant absolute risk aversion and normal returns. Constant absolute risk aversion might be an acceptable behavioral hypothesis for a small class of decision-makers who have common wealth levels. In this case, it would serve as an

approximation. But anytime the model is applied to decision-makers with significant differences in initial wealth, constant absolute risk aversion should be replaced by decreasing absolute risk aversion. Differences in initial wealth should cause significant differences in the willingness of decision-makers to undertake risky investments of a given size. Frequently, decreasing absolute risk aversion is introduced by assuming constant relative risk aversion. The implication of decreasing absolute risk aversion is one of the values of the behavioral hypothesis of constant relative risk aversion.

Another reason the hypothesis of constant relative risk aversion is desirable is that such functions exhibit risk vulnerability. The idea of risk vulnerability is: "... that adding an unfair background risk to wealth makes risk-averse individuals behave in a more risk-averse way with respect to another independent risk." (Gollier and Pratt, p.1110) An unfair risk is a risk with a non-positive expected value. One important implication of risk vulnerability is that opening new insurance or contingent claim markets should stimulate economic activity in other risky markets. Risk vulnerability is a desirable behavioral hypothesis. And utility functions with the property that absolute risk aversion is decreasing and convex are risk vulnerable (Gollier and Pratt, p.1117). Thus, constant relative risk aversion utility functions have the desirable properties of risk vulnerability, and decreasing absolute risk aversion.

4.3 A Constant Relative Risk Aversion Mean-Variance Objective Function

This research presents a functional form for V that satisfies the properties defined by Meyer and Sinn, exhibits constant relative risk aversion, and produces a unique optimal level of debt in the optimal leverage problem. The functional form was found by

searching over candidate homogeneous functions and examining the behavior of the optimal solution to the farm leverage problem.

An acceptable functional form must exhibit the following four properties of $V(F, \cdot)$ defined by Meyer and Sinn: 1) $V: \mathbb{R}^+ \rightarrow \mathbb{R}^+$; 2) $VF \neq 0$; 3) $V(F, \cdot)$ concave in (F, \cdot) ; and 4) $V(F, \cdot)$ is homogenous in (F, \cdot) .

Concavity is used as a search criterion, even though the weaker property of quasi-concavity is what the function must satisfy. The stronger property is used because it is easier to verify. This is consistent with Meyer, who wrote: “Since $V(F, \cdot)$ is the representation of ordinal preferences, it is obvious that its quasi-concavity is the relevant property to discuss in order to establish convexity of preferences in (F, \cdot) space. On the other hand, there are those who have addressed the question of which $V(F, \cdot)$ functions can arise, ... That is, which $V(F, \cdot)$ arise directly from an EU calculation. For this latter question, the fact that $V(F, \cdot)$ is concave under appropriate conditions is an important one to be cognizant of.” (Meyer, 1987, pp.424-425)

The homogeneity property is imposed to create a function that exhibits constant relative risk aversion. The homogeneity can be of any degree because the aspect that is needed for constant relative risk aversion is a common factorization of t from V_F and V_μ : as Meyer’s results have shown:

$$\frac{\partial S(t\sigma, t\mu)}{\partial t} = 0 \text{ for all } \mu + \sigma x \quad (4.6)$$

where,

$$S(\sigma, \mu) = \frac{-V_\sigma(\sigma, \mu)}{V_\mu(\sigma, \mu)}$$

The combination of these properties imposes limits on the search. Degrees of homogeneity greater than 1 are ruled out because they are inconsistent with concavity. That is, if doubling (F, \cdot) more than doubles the value of V , then V is not concave in (F, \cdot) . This means that candidate functions should be homogeneous of degree one or less. Further, if $V(F, \cdot)$ is homogeneous of degree one, then the Hessian of V is singular, implying weak concavity of V , which further implies that preferences in (F, \cdot) space are weakly convex. This severely restricts the representation of aversion toward risk. Also, homogeneity of degree zero of $V(F, \cdot)$ should be ruled out because it implies that $S(F, \cdot) = -V_F/V = \cdot / F$ by Euler's theorem. This slope can not be consistent with expected utility maximization because it implies increasing absolute risk aversion and constant relative risk aversion by the results of Meyer. Thus, search should be focused on functions that are negatively homogeneous.

By the definition of $V(F, \cdot)$ in equation (1) of Meyer (1978, p.423) the function should be defined by:

$$V(\sigma, \mu) = \int_a^b \frac{1}{1-\rho} (\mu + \sigma x)^{1-\rho} dF(x) \tag{4.7}$$

where D is the coefficient of relative risk aversion, and $F(x)$ is the cumulative distribution function (cdf) of a scaled and centered random variable. Direct solution for V with candidate cumulative distribution functions does not yield functional forms that are sufficiently tractable. For example, if you assume that F is the cdf of a uniform random variable with mean 0 and variance 1, then the solution for V is:

(4.8)

$$V(\sigma, \mu) = \frac{(\mu - \sqrt{3}\sigma)^{2-\rho} - (\mu + \sqrt{3}\sigma)^{2-\rho}}{2\sqrt{3}(\rho - 2)(\rho - 1)\sigma}$$

This is a rather unrealistic example because it requires that the support of the rate of return on assets be $[-\sqrt{3}, \sqrt{3}]$, but it provides one of the simpler closed form solutions for V.

An alternative approach is to choose a functional form for V and solve for a corresponding F. This requires solution of a complicated functional equation which may have no solution, one solution, or multiple solutions. Given the dependence of the resulting V function on the corresponding cdf F, neither of these approaches would yield a general solution that represents constant relative risk aversion preferences over a range of asset rate of return cdf's.

Therefore, a tractable approach to the problem is to choose a functional form for V with the required properties, and treat it as an approximation to the true but unknown V that is derived from a particular cdf F. A functional form for V that satisfies these criteria is given by:

$$V(\sigma, \mu) = -(\mu^2 + \lambda\sigma^2)^{-1} \quad (4.9)$$

where λ is a constant that is related to the coefficient of relative risk aversion. This function satisfies the properties: 1) $V > 0$; 2) $V_{FF} < 0$; 3) V concave in (F, \cdot) ; and 4) V homogeneous of degree negative 2 in (F, \cdot) . Further verification of the usefulness of this function requires an examination of the optimal leverage solutions that it produces, which will be presented in section 5.

5. Analysis of a Representative Grain Farm

Further evidence of the analytic value of the CRRA mean-standard deviation function is provided by a numerical optimization analysis of a representative grain farm. The analysis uses both the CRRA mean-standard deviation function and the CARA mean-standard deviation function to illustrate different responses of key farm decision variables to a change in the farmer's intertemporal risk behavior.

A representative farm is created using average resource endowments, operating levels and financial conditions of about 1,004 grain farms in the North Central Region that participate under Illinois' Farm Business Farm Management (FBFM) Program. This farm, managed by a 49 year-old farmer belonging to a family of three, produces corn and soybeans in a 50/50 rotation, operates a total of 862 tillable acres of farmland, of which 250 are owned and the remainder covered by a leasing contract.

The fair market value of the farm's assets is \$1.02 Million as of year-end of 1998, including \$209,253 worth of farm equipment and farm real estate property valued at \$376,496. These assets were financed by 28% debt and 72% farm equity. The family's annual withdrawals from equity for family living expenses amount to \$31,729, excluding income taxes.

Using the financial and structural attributes of the representative farm as a starting point, this analysis assumes a two-year planning horizon and considers final wealth as the outcome variable that is determined by the value of farmland, farm equipment and ending cash balance, net of deductions for outstanding loan balances, at the end of the planning horizon. This analytical framework is based on previous multi-period programming models (Barry and Willmann 1976) (Gwinn, Barry and Ellinger 1992). A large matrix of

activities and constraints is developed where sub-matrices along the main diagonal elements correspond to time periods and off-diagonal elements represent transfers among the model's activities. The representative farm in this analysis is assumed to engage in production and marketing, land and machinery investments, cash renting of farmland, liquidity management, consumption, taxation and incurring short-, intermediate- and long-term loans. Financing and operating decisions are constrained by limits set on land availability, machinery requirements, consumption, and borrowing levels. Specifically, borrowing decisions are constrained by the level of unused portion of the farm's credit reserves, which is the amount that lenders are still willing to lend to the farm given existing financial obligations and operating conditions. The level of credit reserves are calculated from liquidity changes in the balance sheet, expected income, changes in outstanding loan levels, and changes in asset values (Gwinn, Barry and Ellinger 1992). In order to capture the timing of cash flows within a particular year, the cash transfer equation has two sub-periods.

Historical data on the decision variables of the model are used to construct a variance-covariance matrix that accounts for the sources of risk in the model. The decision variables in this model represent purchases of additional farmland and farm machinery, cash renting of supplemental farmland acreage, and incremental loans incurred under short-, intermediate- and long-term credit facilities. This analysis utilized fourteen years of historical data (1985-1998) for these variables that were obtained from the Agricultural Finance Data Book published by the U. S. Department of Agriculture and from the Illinois Agricultural Statistics Service web site. The signs of the entries in the matrix, except for covariance terms among asset/income-generating activities and

among liabilities/cost-generating activities, were adjusted to correctly specify directional effects of certain pairs of activities on risk reduction. Specifically, this adjustment ensures that lower correlations are more preferred among assets (and among liabilities) which potentially reduce aggregate risk in the model. Moreover, higher correlations between assets and liabilities were assigned the opposite of their estimated signs to account for the reversal of the preferred correlation relationship between these activities.

Table 3 presents the resulting farm production and financing plans under the two alternative formulations of the decision problem statement (CARA versus CRRA) obtained using the General Algebraic Modeling System (GAMS) software. Each version of the model considers scenarios of low and high risk aversion levels. For the CARA model, coefficients of absolute risk aversion of 0.0000004 and 0.0006 were used to represent low and high risk aversion, respectively. The CRRA model was evaluated using coefficients of relative risk aversion of 0.3 and 2.3 to capture low and high risk aversion, respectively.

Under the CARA model, the significant change in absolute risk aversion levels only resulted in slight adjustments in the average size of farm operations, with the less risk-averse farmer operating about 50 acres more than the more risk-averse farmer. The less risk-averse farmer also incurred a higher overall leverage ratio, mainly due to larger short- and medium-term leverage positions relative to those of the more risk-averse counterpart. On the other hand, CRRA results indicate that the more risk-averse farmer reduced the size of operations by more than 60% compared to the less risk-averse farmer. This change produced substantially lower leverage ratios for all types of credit under

higher relative risk aversion. Notably, solutions for the decision variables are highly sensitive to changes in relative risk aversion levels under the CRRA model.

The almost identical leverage solutions for the less risk-averse farmer under the two models provided a convenient opportunity to further investigate the initial equity effects of changes in intertemporal risk behavioral assumptions. In this analysis, initial equity levels for the less risk-averse farm decision-maker in each of the two models were reduced by \$200,000. Notwithstanding this change, the CARA solutions are identical under the two initial equity scenarios. On the other hand, the CRRA solutions indicate a tendency for the farm decision-maker to implement more aggressive production and financing plans (i.e. larger farm size and short-term leverage position) when operating with lower initial equity. However, the farmer with higher initial equity position maintained a slightly larger proportion of aggregate borrowings to his/her asset base, in spite of the fact that the counterpart with lower initial equity had a larger starting short-term loan account, which was beefed up by the amount deducted from the initial equity account. This result confirms the contention of a positive initial equity effect under the CRRA behavioral construct.

6. Conclusion

Just correctly identified that the location-scale, or linear distribution, condition is a relatively unexplored analytic approach to ordering and evaluating risky alternatives facing agricultural producers. The Social Science Citation Index reveals that about 40 of the 125 citations of Meyer's 1987 paper have been in the agricultural economics literature. How is this substantial citation consistent with the claim that the approach is relatively unexplored? A sampling of the 40 citations provides a partial explanation. The

agricultural economics literature seems to be willing to use Meyer's result to acknowledge that a mean-standard deviation analysis can be consistent with expected utility maximization, but unwilling to fully invest in a mean-standard deviation objective function. The unwillingness is due, in part, to the awareness of the restrictive behavioral assumptions that are inherent in the linear mean-variance objective function which plays a prominent role in prior agricultural economics research.

We argue that a mean-standard deviation objective that exhibits the widely accepted behavioral hypothesis of decreasing absolute risk aversion and decreasing absolute prudence can contribute to a fuller exploration of the location-scale analytic approach. Many objective functions used in agricultural economics research satisfy the location-scale, or linear distribution, condition. So the location-scale analytic approach has broad application in agricultural economics research.

We present a constant relative risk aversion mean-standard deviation objective function, which is a simple extension of the linear mean-variance objective function. Application to an optimal leverage model shows that the objective function produces good analytic results that are substantially different than results from the linear mean-variance model and very similar to direct expected utility maximization. The constant relative risk aversion mean-standard deviation objective function can contribute to broader exploration of location-scale analysis in agricultural economics research.

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Table 1. Summary Statistics of Rate of Return

Year	Mean	Standard Deviation
1987	0.15675820	0.30305994
1988	0.01670560	0.16609410
1989	0.14253366	0.14984186
1990	0.13219492	0.13800189
1991	0.04628229	0.11026131
1992	0.16045788	0.20416415
1993	0.11424879	0.12083227
1994	0.07497370	0.13974322
1995	0.04337900	0.16303315
1996	0.07808465	0.09703295

Table 2. Normality Test of Rate of Return Data

Year	Test Statistic	P-Value
1987	113.84	0
1988	8.97	0.01
1989	48.66	2.7 e-11
1990	23.96	6.2 e- 6
1991	7.19	0.003
1992	61.94	3.6 e-14
1993	7.76	0.002
1994	47.49	4.9 e- 11
1995	60.83	6.2 e- 14
1996	209.46	0

Table 3. Programming Results for CARA and CRRA Models,
Different Risk Aversion and Initial Equity Levels

Selected Variables	CARA			CRRA		
	Low Risk Aversion		High Risk Aversion	Low Risk Aversion		High Risk Aversion
	Low Initial Equity	High Initial Equity		Low Initial Equity	High Initial Equity	
Farm Size (Acres, Average)	2,814	2,814	2,760	2,630	2,596	1,028
Land Purchases (Acres)				33.95		14.75
Equipment Purchases (\$)	480,370	480,370	639,010	480,370	480,370	
New Intermediate-Term Loans	432,340	432,340	575,110	432,340	432,340	
New Short-Term Loans	660,019	660,019	667,256	672,530	570,120	17,716
Short-Term Debt-Asset Ratio	0.5528	0.5528	0.4237	0.6978	0.5644	0.1992
Medium-Term Debt-Asset Ratio	0.4391	0.4391	0.3268	0.4391	0.4391	0.1210
Long-Term Debt-Asset Ratio	0.0788	0.0788	0.0788	0.0788	0.0788	0.0788
Total Debt-Asset Ratio, Average	0.2955	0.2955	0.2424	0.2935	0.2952	0.1124