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AN ANISOTROPIC BUBBLE MODEL FOR STRUCTURED CLAYS

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ABSTRACT: This paper describes the mathematical formulation of the new bubble model for natural clays which exhibit structure and anisotropy. The formulation of a bubble model namely B-SCLAY1S is based on the previously developed S-CLAY1S model using critical state theory and bounding surface plasticity. The kinematic yield surface of the S-CLAY1S model is treated as a bounding surface and a bubble surface is introduced within the bounding surface. The bubble surface is similar in shape to the S-CLAY1S yield surface, and assumes an isotropic elastic behaviour and an associated flow rule. A translation rule of the bubble is adopted to control the movement of the bubble. The influence of the parameters related to the bubble surface in predicting cyclic behaviour were highlighted in a simple constant q slow cyclic triaxial test.

1 Introduction

During past few decades several modifications have been proposed to enhance elasto-plastic models developed within the framework of kinematic hardening plasticity. One of the most successful approaches is to introduce one or two kinematic surfaces within a conventionally defined yield surface (Mróz et al. 1978, Mróz et al. 1979). Models of this type are often termed kinematic hardening ”bubble” models (Al-Tabbaa 1987, Al-Tabbaa and Wood 1989). This paper presents a new constitutive model that is capable of representing structured anisotropic cyclic behaviour of clay. The proposed constitutive model is developed within the framework of the critical state theory and bounding surface plasticity. The model is an extension of the S-CLAY1S model (Koskinen et al. 2002, Karstunen et al. 2005). The kinematic yield surface of S-CLAY1S is treated as bounding surface, and a bubble surface (kinematic yield surface) is introduced within bounding surface to enclose a truly elastic region. The bounding surface can describe the effect of initial anisotropy caused by one-dimensional deposition and K_0 -consolidation process, and the subsequent evolution of anisotropy due to plastic strains is described by a kinematic hardening law of the S-CLAY1S model. The effect of the bonding (destructuration) is introduced by the intrinsic and natural yield surfaces (Gens & Nova 1993). The intrinsic yield surface is of smaller size but same orientation as the bounding surface of the natural soil. With the introduction of the bubble based on the idea of Al-Tabbaa (1987), the model allows the simulation of important features of soil behaviour not reproduced by the S-CLAY1S model, such as non-linearity and plasticity from early stages of loading, and hysteretic behaviour during cyclic loading. B-SCLAY1S model would be ideal for simulating the behaviour of overconsolidated soils and/or the cyclic response of soils.

2 Description of the B-SCLAY1S model

For the sake of simplicity, the mathematical formulation of B-SCLAY1S is presented in triaxial stress space, which can be only used when cross-anisotropic samples (cut vertically from the soil deposit) are tested in laboratory in oedometer or triaxial apparatus. Stress quantities $p' = (\sigma_a + 2\sigma_r)/3$ and $q = (\sigma_a - \sigma_r)$ and strain quantities $\epsilon_v = \epsilon_a + 2\epsilon_r$ and $\epsilon_q = 2(\epsilon_a - \epsilon_r)/3$ are used where subscripts a and r denote the axial and the radial directions, respectively, of a triaxial stress space.

The bounding surface of the model in triaxial stress space is the same as the S-CLAY1S model yield surface, given as follows:

$$f_y = \frac{(q-\alpha p')^2}{M^2-\alpha^2} + \left(p' - \frac{p'_m}{2}\right)^2 - \left(\frac{p'_m}{2}\right)^2 \tag{1}$$

where M is the slope of the critical state line, p'_m defines the size of the yield curve and α defines the orientation of the yield curve, see Fig. 1. The scalar parameter α is a measure of the degree of plastic anisotropy of the soil. The intrinsic yield surface is linked to the size of the bounding surface is given below where p'_{mi} defines the size of the intrinsic yield surface and χ defines the amount of bonding.

$$p'_m = (1+\chi)p'_{mi} \tag{2}$$

The kinematic bubble surface, enclosing the truly elastic region, has a similar shape to the bounding surface, but smaller in size is formulated as follows:

$$f_b = \frac{[(q-p'\alpha)-(q_b-p'_b\alpha)]^2}{M^2-\alpha^2} + (p'-p'_b)^2 - R^2\left(\frac{p'_m}{2}\right)^2 \tag{3}$$

where p'_b and q_b are centre of bubble surface and R is the ratio of the size of the kinematic bubble surface to that of the bounding surface, see Fig. 1.

The model assumes the associated flow rule, hence the plastic potential is same as the yield surface. The B-SCLAY1S model incorporates four hardening rules namely isotropic hardening rule, rotational hardening rule, destructuration rule and translation rule. Isotropic hardening rule relates to volumetric hardening which is the same formulation as the S-CLAY1S model is given below.

$$dp'_{mi} = \frac{(1+e)}{\lambda_i - \kappa} p'_{mi} d\epsilon_v^p \tag{4}$$

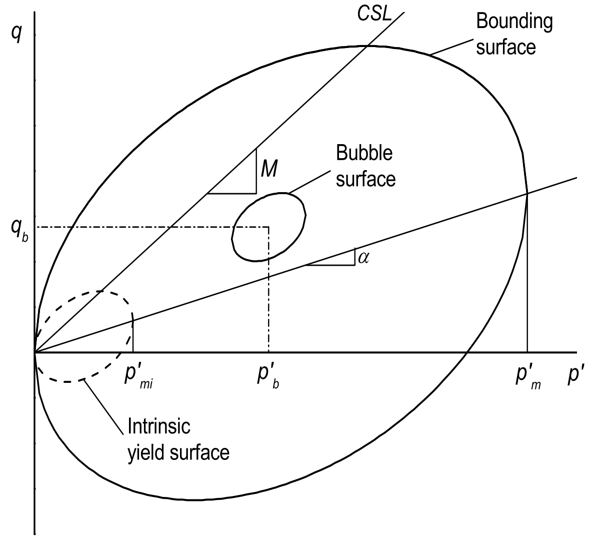

Figure 1: The B-SCLAY1S model yield surface in triaxial stress space

In the equation above the change in size of the bounding surface is controlled by the plastic volumetric strain increment. λ_i and κ are slopes of the intrinsic normal compression line and swelling line in the $e - \ln p'$ space, where e is void ratio.

Rotational hardening rule is to control the rotation of the bubble surface due to the change of anisotropy, and it is defined in the same way as S-CLAY1S model.

$$d\alpha = \mu\Big[\Big(\frac{3}{4}\frac{q}{p'} - \alpha\Big)\langle d\epsilon_v^p\rangle + \beta\Big(\frac{1}{3}\frac{q}{p'} - \alpha\Big)d\epsilon_d^p\Big] \quad (5)$$

where $d\epsilon_v^p$ is the plastic volumetric strain increment and $d\epsilon_d^p$ is plastic deviatoric strain increment. Parameter β controls the relative influence of $d\epsilon_d^p$ and μ the absolute rate of the plastic strain increments on the rotation of the bubble surface.

The effect of the bonding is introduced by using the concept of an intrinsic yield surface given by Eq. 2. The destructuration law describes the degradation of bonding with plastic straining where the plastic volumetric strains and plastic deviatoric strains tend to reduce the bonding parameter χ towards a target value of zero as follows:

$$d\chi = -a\chi\left(|d\epsilon_v^p| + b\,|d\epsilon_d^p|\right) \quad (6)$$

where a and b are two additional soil constants; parameter a controls the absolute rate of destructuration and parameter b controls the relative effectiveness of plastic deviatoric strains and plastic volumetric strains in destroying the bonding.

The translation rules of the bubble surface in B-SCLAY1S are formulated based on (Al-Tabbaa 1987) translation rules. Two different translation rules are adopted, one for when bubble surface moves inside the bounding surface and one for when two surfaces are in contact. The first translation rule describes the bubble surface movement within the bounding surface in such a way the bubble surface and bounding surface can come in contact at common normal but never intersect as follows:

$$\left\{\begin{array}{c} dp'_b \\ dq_b \end{array}\right\} = \frac{dp'_m}{p'_m}\left\{\begin{array}{c} p'_b \\ q_b \end{array}\right\} + S\left\{\begin{array}{c} \frac{p'-p'_b}{R} - (p' - p'_m) \\ \frac{(q-p'\alpha)-(q_b-p'_b\alpha)}{R} - (q - p'\alpha) \end{array}\right\} \quad (7)$$

where S scalar quantity can be derived from the consistency condition of the bubble surface.

The second translation rule describes the movement of bubble when two surfaces are in contact at the current stress state, Eq. 7 is reduced to:

$$\left\{\begin{array}{c} dp'_b \\ dq_b \end{array}\right\} = \frac{dp'_m}{p'_m}\left\{\begin{array}{c} p'_b \\ q_b \end{array}\right\} \quad (8)$$

The hardening modulus of B-SCLAY1S is defined in such a way that when the two surfaces are in touch, and the yielding is continuous, the model predicts the same behaviour as the S-CLAY1S model. It is initially formulated for special case when two surfaces are in contact, and then modified for the general case when two surfaces are not in contact and the stress state is within the bounding surface. When the two surfaces are in contact the hardening function $\mathcal{H}_0$ is given by the following equation:

$$\mathcal{H}_0 = \frac{4(1+e)}{\lambda_i - \kappa}\Big[(p' - \frac{p'_m}{2}) - \frac{(q - \alpha p')}{M^2 - \alpha^2}(\alpha)\Big] * \Big[\frac{(q - \alpha p')^2}{M^2 - \alpha^2} + (p' - \frac{p'_m}{2})p'\Big] \quad (9)$$

When the bubble lies inside the bounding surface, hardening modulus is defined based on (Al-Tabbaa 1987) description, and $\mathcal{H}_0$ is replaced with a more general expression as follows:

$$\mathcal{H}_0 = \mathcal{H}_{0b} + \mathcal{H}_b \quad (10)$$

Al-Tabbaa (1987) assumed, after Hashiguchi (1985), that $\mathcal{H}_b$ is a function of a measure of the proximity of the bubble surface to the bounding surface, for further details of formulation see Sivasithamparam (2012).

3 Model parameters

The proposed formulation of the model in general stress space requires values for 10 soil constants and 3 state variables. They are summarized in Table 1.

The soil constants of the B-SCLAY1S model include four parameters from the MCC model (κ, λ_i, M and Poisson's ratio ν') that can be determined from conventional laboratory tests. Two additional parameters (R, ψ) are required for introduction of the bubble surface into the S-CLAY1 model. Al-Tabbaa (1987) explains how these additional model parameters can be obtained from simple standard tests or multi-stage test using the triaxial apparatus. Two additional soil constants (μ and β) and additional state variable (α_0) govern the evolution of anisotropy and the initial anisotropy, respectively. (Wheeler et al. 1999, Wheeler et al. 2003) discussed the determination of these three parameters in detail and generally no non-standard tests are needed to get reasonable estimates for these values. Two additional soil constants (a and b) and additional state variable (χ_0) relates to initial bonding control the destructuration process. The initial value of the state variable χ_0 can be estimated by the sensitivity (S_t) of the clay. Parameters a and b have to be determined from model simulations by optimization of the destructuration process.

The model is hierarchical, so it is possible to reduce the model to the S-CLAY1S model, by setting R equal to one. The S-CLAY1S can be switched to S-CLAY1 by setting the relevant structure parameters to zero. Furthermore, if initial anisotropy is switched off, by setting α_0 and μ equal to zero, the model simplifies to the isotropic MCC model.

Table 1: Parameters required for the B-SCLAY1S model

Soil constants:

κ	Initial slope of swelling/recompression line in $e - \ln p'$ space
ν'	Poisson's ratio
λ_i	Slope of post yield compression line in e-lnp'-space for reconstituted sample
λ	Slope of post yield compression line in e-lnp'-space
M	Stress ratio at critical state (in triaxial compression)
μ	Absolute effectiveness of rotational hardening
β	Relative effectiveness of rotational hardening
a	Absolute rate of destructuration
b	Relative rate of destructuration
R	Ratio of the size of the bubble surface to that of the bounding surface
ψ	Exponent in the hardening function $\mathcal{H}$

State variables:

e_0	Initial void ratio
α_0	Initial inclination of the yield surface
χ_0	Initial bonding

Table 2: Bothkennar clay parameters

Soil constants:							
κ	ν'	λ_i	M	μ	β	a	b
0.02	0.2	0.21	1.4	30	0.94	9.0	0.4
State variables:							
e_0	α_0	χ_0					
2.0	0.31	10.0					

4 Numerical simulations

Two set of numerical examples are shown in this section to highlight the influence of parameters (R and ψ) related to bubble surface. Simple constant q slow cyclic triaxial simulations were performed to represent cyclic behaviour of Bothkennar clay. Bothkennar clay is a soft normally consolidated marine clay deposit in Scotland (Symposium 1992). Table 2 summarizes the model parameters used in these simulations. The simulations were initially started from slightly over-consolidated (OCR=1.1) state with an initial stress of p'= 66.7 kPa and q = 50 kPa. The deviatoric stress was kept constant while cell pressure was increased and then decreased (one way cyclic loading). Ten load cycles were applied. First four cycles, p' varies from 91.6 kPa to 41.6 kPa; second three cycles, p' varies from 111.6 kPa to 41.6 kPa and final three cycles, p' varies from 121.6 kPa to 41.6 kPa. One out of two parameters was kept constant while second parameter was varied so that its effect on the model's performance could be seen.

Firstly, the influence of the parameter R which relates to the size of the bubble surface is investigated. The permanent strains (both axial and volumetric) increase as the value of R decreases, see Figures. 2 - 4 for R = 0.10,0.15 and 0.20. This is because a smaller value of R causes the soil to be softer during loading/reloading stages and strains increase as a result. Secondly, the parameter ψ (exponent in the hardening function) is examined in cyclic behaviour of clay. Figures. 5 - 7 shows the results when = 1.0, 1.5 and 2.0, respectively. It can be seen that a bigger value of ψ will make the soil become softer during reloading, and as a result more permanent strains occur. It should be noted that the both parameters show significant influence in predicting cyclic behaviour of clay.

5 Conclusions

This paper provides mathematical formulation of bubble surface, hardening law, hardening modulus and kinematic translation of bubble with required parameters. Simulations on constant q slow cyclic triaxial have been performed using B-SCLAY1S model with varying R and ψ values. From these simulations, it can be found that B-SCLAY1S is very flexible in predicting cyclic soil behaviour. For future work, the implicit integration scheme for the B-SCLAY1S model will be studied and implemented into finite element codes.

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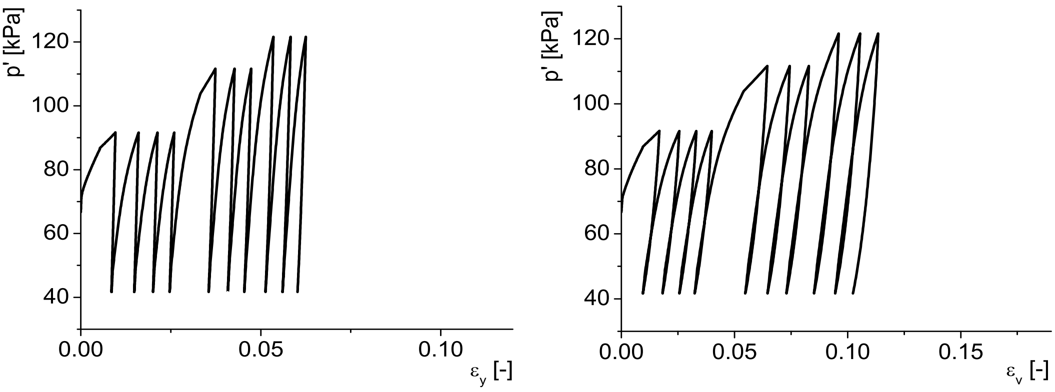
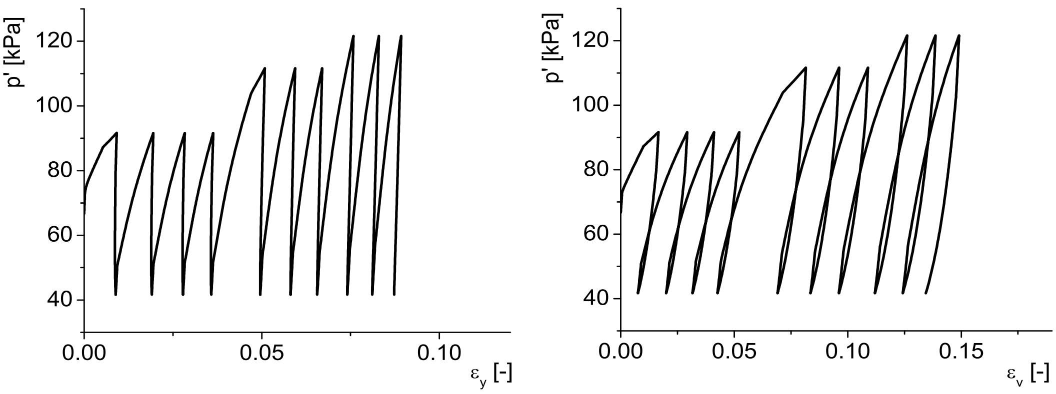


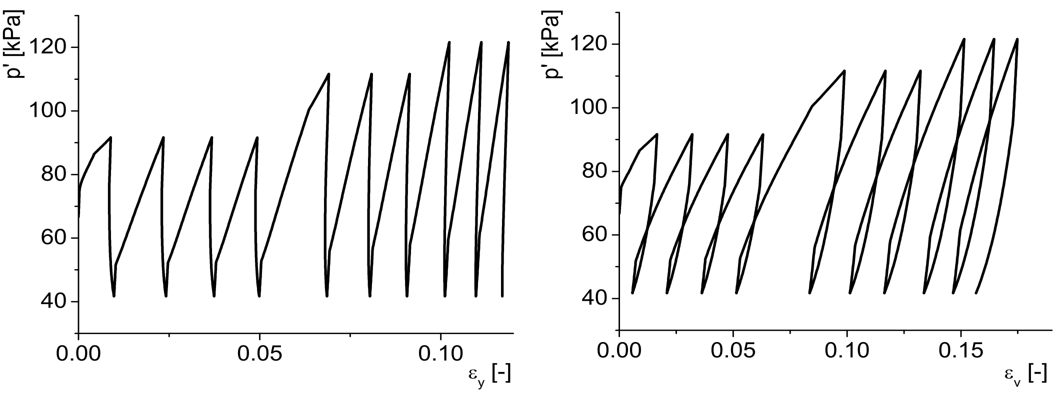
(a) axial strain (b) volumetric strain

Figure 2: Influence of $R = 0.10$



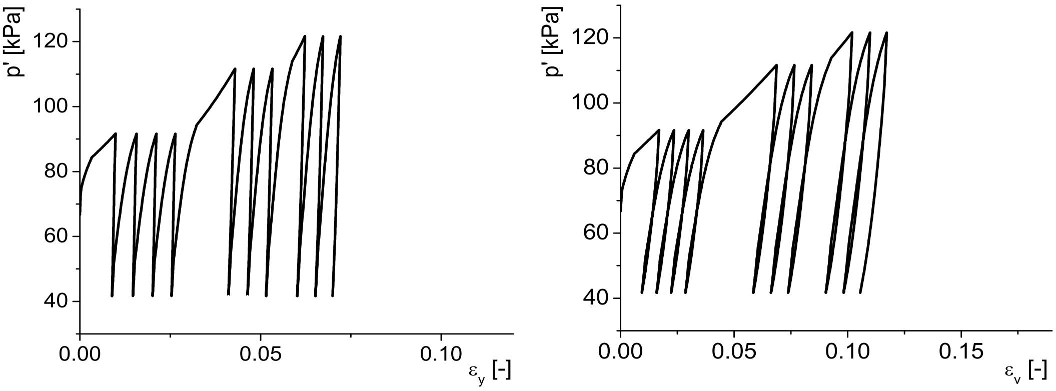
(a) axial strain (b) volumetric strain

Figure 3: Influence of $R = 0.15$



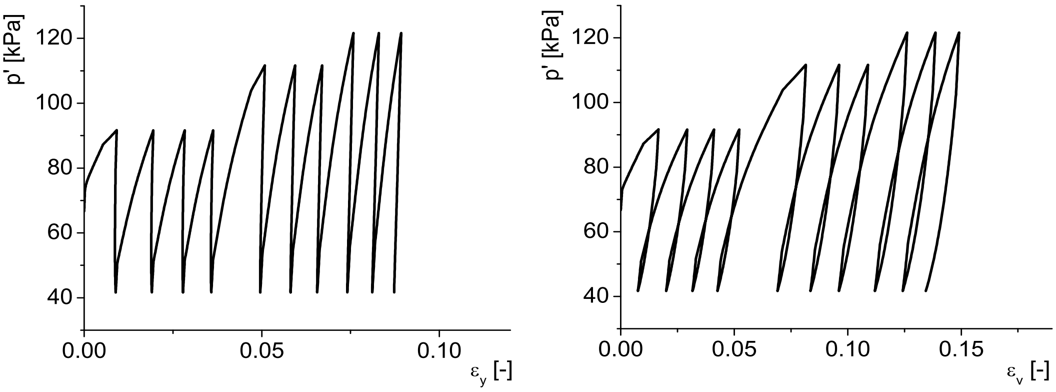
(a) axial strain (b) volumetric strain

Figure 4: Influence of $R = 0.20$



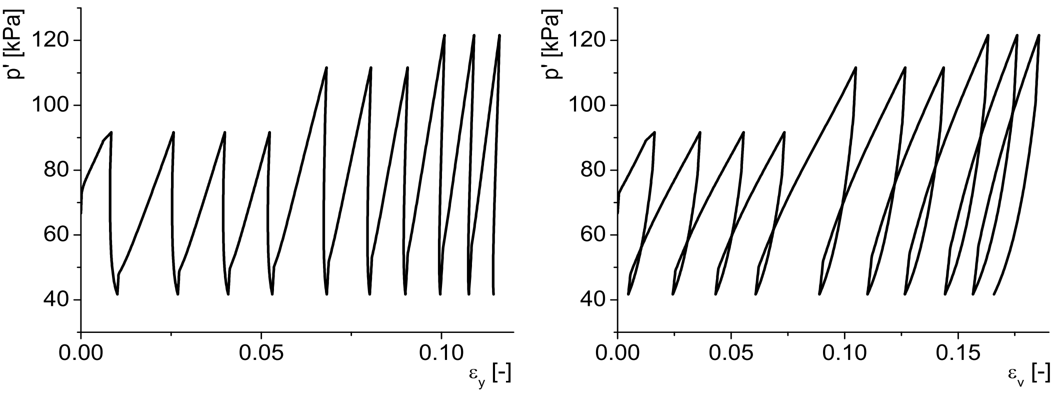
(a) axial strain (b) volumetric strain

Figure 5: Influence of $\psi = 1.0$



(a) axial strain (b) volumetric strain

Figure 6: Influence of $\psi = 1.5$



(a) axial strain (b) volumetric strain

Figure 7: Influence of $\psi = 2.0$