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A Nonlinear Unit Root Test in the Presence of an Unknown Break

Abstract

The Perron test is the most commonly applied procedure to test for a unit root in the presence of a structural break of unknown timing in the trend function. Deriving the Perron-type test regression from an unobserved component model, it is shown that the test regression in fact is nonlinear in coefficient. Taking account of the nonlinearity leads to a test with properties that are exclusively assigned to Schmidt-Phillips LM-type unit root tests.

JEL Classification: C12, C22

Keywords: Unit root tests, nonlinear regression, structural breaks, innovational outliers

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1 Introduction

The procedure proposed by Vogelsang and Perron (Perron 1990, Perron and Vogelsang 1992, Vogelsang and Perron 1998) is the most common procedure to test for unit roots in the presence of an unknown structural break in the trend function. However, as shown by Harvey et al. (2001) and Lee and Strazicich (2001), the innovational outlier unit root test exhibits considerable spurious rejections in finite samples when a break is present under the null hypothesis. Popp (2008) solves this problem by using a Perron-type test regression derived from an unobserved component model (UCM); for another approach starting from a UCM see Lanne et al. (2003). But, as already stated by Gluschenko (2004) for the case of a known level break, the so derived Perron-type test regression is in fact a linear approximation of a nonlinear (in coefficient) one. Exploiting the nonlinearity in the estimation procedure leads to asymptotically more efficient estimates of the autoregressive parameter and consequently to more powerful tests. The importance of restrictions on coefficients was already mentioned by Lee and Amsler (1997).

In the present paper the nonlinear test for a unit root is considered for the cases of a break in level for non-trending (model 0, M0) and trending data (model 1, M1) as well as for the case of a break in level and slope for trending data (model 2, M2). Various favorable properties can be identified. In contrast to the test proposed by Vogelsang and Perron, the linear version derived by Popp (2007) and the nonlinear unit root test do have stable sizes in the presence of a break in finite samples. Furthermore, the power of the nonlinear test is considerably higher compared to the linear versions. Finally, the asymptotic distribution in case of an unknown break date not only equals that for a known break date, but also corresponds to that for the case of no break, i.e. the Dickey-Fuller (DF) distribution. This implies that the nonlinear unit root test is asymptotically independent of the break fraction parameter λ. Commonly, this property is exclusively attributed to LM-type unit root tests (Schmidt and

Phillips 1992, Amsler and Lee 1995, Lee and Strazicich 2003a, 2003b).

The paper is organized as follows. In the next section the models and correspondent linear and nonlinear test regressions are presented. In section 3 the size and power properties are shown by Monte Carlo techniques. The one break minimum LM test by Lee and Strazicich (2003b) is described and analysed in section 4. In section 5 the nonlinear test is applied to the inflation rates of six G7 countries. Section 6 concludes.

2 The models

Consider the first order autoregressive process y_t allowing for a break in level and slope for trending data (M2). Due to Vogelsang and Perron (1998), the null and the alternative hypothesis are formulated as follows:

$$H_0 \quad : \quad y_t = y_{t-1} + b + \theta D(T'_B)_t + \gamma DU'_t + e_t, \tag{2.1}$$

$$H_1 \quad : \quad y_t = a + bt + \theta DU'_t + \gamma DT'_t + e_t \tag{2.2}$$

with $e_t \sim iid(0, \sigma_e^2)$. The parameters θ and γ represent the amount of the level and slope break, respectively. Models M0 and M1 are special cases for M2 and can be derived by assuming $b = \gamma = 0$ and $\gamma = 0$, respectively. The dummy variables DU_t, DT_t and $D(T_B)_t$ are defined as follows, $1(.)$ being the indicator function and T'_B symbolizing the true break date:

$$DT'_t = 1(t > T'_B)(t - T'_B), \tag{2.3}$$

$$DU'_t = \Delta DT'_t = 1(t > T'_B), \tag{2.4}$$

$$D(T'_B)_t = \Delta DU'_t = 1(t = T'_B + 1). \tag{2.5}$$

Both hypotheses are nested in the well-known Dickey-Fuller-type test regression with three dummy variables:

$$y_t = \rho y_{t-1} + a + bt + dD(T_B)_t + \theta DU_t + \gamma DT_t + e_t. \tag{2.6}$$

In order to test the null hypothesis $\rho = 1$ of a unit root against the alternative hypothesis $|\rho| < 1$ the t-statistic of ρ, denoted $t_{\hat{\rho},P}$, is used. Before computing the test statistic, the break date T_B has to be specified in the test regression (2.6). Conventionally, two selection methods are used to estimate the unknown break date:

$$\hat{T}_{B,1} = \arg\min_{T_B} t_{\hat{\rho}}(T_B), \tag{2.7}$$

$$\hat{T}_{B,2} = \begin{cases} \arg\max_{T_B} |t_{\hat{\theta}}(T_B)| & \text{, for M0 and M1} \\ \arg\max_{T_B} |t_{\hat{\gamma}}(T_B)| & \text{, for M2} \end{cases} \tag{2.8}$$

with $T_B \in [\tau T, (1-\tau)T]$, τ: trimming factor. To indicate the used selection method, the test statistic is denoted $t_{\hat{\rho},P}(\hat{T}_{B,1})$ and $t_{\hat{\rho},P}(\hat{T}_{B,2})$, respectively.[1]

The test regression (2.6) can also be derived in a less *ad hoc* manner by representing the DGP of the time series y_t as a UCM consisting of a deterministic component d_t and a stochastic component u_t.[2] Using the same notation as above, the UCM is:

$$y_t = d_t + u_t, \tag{2.9}$$

$$d_t = \alpha + \beta t + \theta DU_t' + \gamma DT_t', \tag{2.10}$$

$$u_t = \rho u_{t-1} + e_t. \tag{2.11}$$

The reduced form of the structural model serves then as test regression and has the following form similar to (2.6):

$$y_t = \rho y_{t-1} + \alpha^* + \beta^* t + \xi^* D(T_B)_t + \kappa^* DU_t + \zeta DT_t + e_t \tag{2.12}$$

with $\alpha^* = \alpha(1-\rho) + \beta\rho$, $\beta^* = \beta(1-\rho)$, $\xi^* = \theta\rho$, $\kappa^* = \rho(\gamma - \theta) + \theta$, $\zeta = -\phi\gamma$ and $\phi = (\rho - 1)$. It has to be noted that the test regression is actually nonlinear

[1] Because both test statistics exhibit similar properties, exclusively $t_{\hat{\rho},P}(\hat{T}_{B,2})$ is considered further.

[2] See Schmidt and Phillips (1992) for advantages of using the UCM representation.

with respect to the coefficients.

For a given break date T_B, the Perron-type test regression (2.6) and the linearized test equation (2.12) yield identical t-values. Also in the context of an unknown break date, using selection method 1 leads for both test regressions to identical results. Things are different when using selection method 2, which focuses on the break coefficients θ and γ. By using test equation (2.6) Perron implicitly assumes that the level break parameter θ is the coefficient of the dummy variable DU_t, and the slope break parameter γ is the coefficient of the dummy variable DT_t. In contrast, according to the test regression (2.12) derived from the UCM, the coefficient of DU_t is $\kappa^* = \rho\,(\gamma - \theta) + \theta$ and of DT_t is $\zeta = -\phi\gamma$.

In order to use selection method 2 analogously it is necessary to reshape equation (2.12) to isolate the break coefficients θ and γ. After reshaping, the test regression is as follows:

$$y_t = \rho y_{t-1} + \alpha^* + \beta^* t + \xi D(T_B)_t + \kappa DU_{t-1} + \zeta DT_{t-1} + e_t \tag{2.13}$$

with $\xi = (\gamma + \theta)$ and $\kappa = (\gamma - \phi\theta)$. For M0 and M1 the parameter ξ corresponds to the level break parameter θ. The selection method can now be formulated as:

$$\hat{T}_{B,3} = \arg\max_{T_B} |t_{\hat{\xi}}(T_B)|. \tag{2.14}$$

The respective test statistic is denoted as $t_{\hat{\rho},L}(\hat{T}_{B,3})$. The restrictions $\xi = (\gamma + \theta)$ and $\kappa = (\gamma - \phi\theta)$ are ignored in the linear version of the test regression leading to asymptotically consistent but inefficient estimates of the autoregressive parameter ρ. It is expected that taking the restrictions into account augment the power of the unit root test even in finite samples. The nonlinear test regression has the following form:

$$y_t = \rho y_{t-1} + \alpha^* + \beta^* t + (\gamma + \theta) D(T_B)_t + (\gamma - \phi\theta) DU_{t-1} - \phi\gamma DT_{t-1} + e_t, \tag{2.15}$$

which is been used in conjunction with $\hat{T}_{B,2}$. Because the break date is identified more accurately using the linear test regression (2.13) with $\hat{T}_{B,3}$ than with the nonlinear version and $\hat{T}_{B,2}$, a two step procedure is recommended. In the first step the break date T_B is estimated with (2.13) and (2.14). In the second step the nonlinear test regression is conducted for the estimated break date. The resulting test statistic is denoted $t_{\hat{\rho},NL}(\hat{T}_{B,3})$.

Due to the nonlinearity of the test regression a closed form of the estimator is not available. Hence, the test distribution is derived by Monte Carlo simulations and displayed for the models M0, M1 and M2 in the next section.

3 Finite sample size, power and break date estimation accuracy

All simulations are based on 10 000 replications and were carried out in GAUSS using the OPTMUM Module. The time series are generated according to equations (2.9), (2.10) and (2.11) with $e_t \sim N(0,1)$. The parameters α and β are set to zero. For M0 and M1 it is further assumed that $\gamma = 0$. For each replication $T + 50$ observations with $T \in \{100, 200\}$ are generated. Afterwards the first 50 observations are discarded to reduce the impact of the initial condition. The trimming factor is $\tau = 0.1$.

One primary goal is to assess the effect of a variation of the break date and the break magnitude on the properties of the test discussed in the previous section. The true break date T'_B is defined as: $T'_B = [\lambda' T]$, [.]: integer. The parameter λ' is called the break fraction, which varies among the following values: $\lambda' \in \{0.1, 0.2, \ldots, 0.9\}$. For M0 and M1 the level break is taken to be equal to the following magnitudes: $\theta \in \{0, 5, 10, 20\}$. For M2 all combinations of various level and slope break magnitudes, $\theta \in \{0, 5, 10\}$ and $\gamma \in \{0, 2, 6, 10\}$, are considered. Under H_0 ρ is equal to 1. The power of the test is assessed for $\rho \in \{0.9, 0.8\}$.

In the case of a known break, the Perron-type unit root test is invariant to a break under the null hypothesis. The same is true for the nonlinear test as shown by Gluschenko (2004) for M0 and for the case without a constant. However, by exploiting the nonlinearity constraint the power of this test is higher. Furthermore, the distribution of the nonlinear test is asymptotically identical to the DF-distribution, a property which implies the independence of the asymptotic distribution to the break fraction λ'. In finite sample indeed the distribution varies with λ', but only slightly. So, the standard deviation of the 5 per cent critical values of $t_{\hat{\rho},NL}(T_B', M0, T = 100)$ calculated for the nine different values of $\lambda' \in \{0.1, 0.2, \ldots, 0.9\}$ is 0.035, which is 3,6-times smaller than the standard deviation of the corresponding Perron-type tests (std = 0.125).

The invariance to the break size holds also for M1 and M2, but is not proved explicitly for the case of a known break date. The critical values of $t_{\hat{\rho},NL}(T_B')$ for M0, M1 and M2 are displayed in Table 1 for various values of λ'. Because of the mentioned invariance to the break size, the critical values are calculated assuming no break, i.e. $\theta = 0$ for M0 and M1 as well as $\theta = \gamma = 0$ for M2.

Now it is assessed if the DF-distribution is a good approximation for the distribution of $t_{\hat{\rho},NL}(T_B')$ as well in finite sample. For that matter the critical values of $t_{\hat{\rho},NL}(T_B')$ are compared with the respective ones of the DF-test. For M0 the distribution of the DF-test with constant is relevant, for M1 that one with constant and trend. That means that the level break and consequently the timing of its occurrence has no impact on the asymptotic distribution. Though, a break in slope does affect the test distribution, otherwise the distribution with constant and trend would be the relevant one. But, the critical values of M2 are absolutely greater and correspond to that of the DF-test with constant, trend and squared trend. More precisely, the critical values for M2 match that with the same number of asymptotically relevant regressors. So, they are identical to that of the DF-test with constant, trend and cubic trend as well. The comparison of $t_{\hat{\rho},NL}(T_B')$ and $t_{\hat{\rho},DF}$ (displayed in Table 2) shows that the critical values of $t_{\hat{\rho},NL}(T_B')$ and of the respective DF-test differ slightly for $T = 100$ and hardly for $T = 200$.

Commonly, the break date is unknown and has to be estimated during the test procedure. Just for the case of an unknown break date, the Perron-type unit root test exhibits considerable problems with spurious rejections in finite samples, when a break is present under the null hypothesis, cf. Lee and Strazicich (2001) and Harvey et al. (2001). As shown by Popp (2007), test statistic $t_{\hat{\rho},L}(\hat{T}_{B,3})$ does not have this problem. In addition, the distributions in the case of an unknown and known break date coincide. Intuitively, this can be explained as follows. If the unit root test accounting for a break of unknown timing is invariant to the break magnitude and additionally identifies the true break date with increasing break size more accurately (e.g. for M0: $\lim_{\theta \to \infty} P(\hat{T}_B = T'_B) = 1$), the test distribution in the case of a known (i.e. in 100 percent of the cases accurately identified) break date and unknown break date must coincide inevitably.

This result also holds for the nonlinear test. Firstly, the application of selection method 3 ensures the accurate estimation of the break date with increasing break magnitude. This can be verified in Tables 4 to 7. There it is shown that the probability of detecting the true break date, $P(\hat{T}_{B,3} = T'_B)$, tends to 1 with increasing break size under the null and under the alternative hypothesis. And secondly, the stable empirical size of $t_{\hat{\rho},NL}(\hat{T}_{B,3},\text{CV}_{\text{endo}})$ shown in Tables 4 to 7 documents the invariance to a break.

This result is in accordance with the critical values of $t_{\hat{\rho},NL}(T'_B)$ and $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ tabulated in Tables 1 and 2. Thus, the distribution of the DF-test $t_{\hat{\rho},DF}$, the nonlinear test in the case of an unknown break date $t_{\hat{\rho},NL}(T'_B)$ and of a known break date $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ are approximately equal in finite samples. In the following, these critical values are denoted by CV_{DF}, CV_{exo} and CV_{endo}, respectively. The correspondence of CV_{DF}, CV_{exo} and CV_{endo} can also be seen in Table 3 where the critical values are shown for fixed break fraction $\lambda' = 0.5$ and various samples sizes T of 50, 100, 200, 300, 500, and 1000. This implies that $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ is invariant to the break magnitude and the break fraction. So far, this invariance property size is denied for DF-type unit root tests and is exclusively assigned to the LM-type unit root test, see Chou (2007) and Im et al. (2005).

Tables 4 to 7 display the 5 percent rejection frequency of $t_{\hat{\rho},P}(\hat{T}_{B,2})$, $t_{\hat{\rho},L}(\hat{T}_{B,3})$ and $t_{\hat{\rho},NL}(\hat{T}_{B,3})$, for the case of $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ using all three sets of critical values. Because the critical values hardly vary with λ' and in order to keep the analysis concise, it is assumed in the following that $\lambda' = 0.5$. It has to be mentioned that the difference to the DF-distribution is largest for $\lambda' = 0.5$. So, it can be expected that the results improve for other values of λ'. But even for this case, the DF-distribution is a good approximation as can be inferred from Tables 1 and 2.

The results are similar for all models. The empirical size of $t_{\hat{\rho},P}(\hat{T}_{B,2})$ tends to 1 with increasing break size. This is also true for the power. Thus, interpreting the rejection frequency as power in the case of an extremely oversized test is not recommended. In contrast, the linearized test as well as the nonlinear test exhibit quite stable size equal to its nominal size of 5 percent. For large break $t_{\hat{\rho},L}(\hat{T}_{B,3})$ and $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ using CV_{endo} are slightly conservative, whereas $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ using CV_{exo} and CV_{DF} are a little bit oversized. These tendencies vanish with increasing sample size.

Independent of the chosen set of critical values, the power of the nonlinear test is higher than that of $t_{\hat{\rho},P}(\hat{T}_{B,2})$ and $t_{\hat{\rho},L}(\hat{T}_{B,3})$. The power is highest using CV_{DF}. Furthermore, the power increases with the sample size suggesting the consistency of the tests.

4 One break minimum LM test

Lee and Strazicich (2003b) use an LM approach to test for a unit root in the presence of an unknown break. The DGP is assumed as stated in equations (2.9) to (2.11).

The test regression has the following form:

$$\Delta y_t = \beta + \theta D(T_B)_t + \gamma DU_t + \phi \tilde{S}_{t-1} + e_t \tag{4.1}$$

where $\tilde{S}_t = y_t - \tilde{\alpha}_u - \tilde{\beta}t - \tilde{\theta}DU_t - \tilde{\gamma}DT_t,\ t = 2, \ldots, T$; $\tilde{\beta}$, $\tilde{\theta}$ and $\tilde{\gamma}$ are the coefficients in the regression of Δy_t on a constant, $D(T_B)_t$ and DU_t; and $\tilde{\alpha}_u$ is the restricted Maximum Likelihood estimator of $\alpha_u (\equiv \alpha + u_0)$. The null of $\phi = 0$ is tested using the t-value of $\hat{\phi}$, $t_{\hat{\phi},LM}$. The estimate of the break date is that point in time which minimizes the value of $t_{\hat{\phi},LM}$:

$$\hat{T}_{B,4} = \arg\min_{T_B} t_{\hat{\phi},LM}(T_B). \tag{4.2}$$

Lee and Strazicich state that the test is invariant to the break size and identifies the true break date accurately. Furthermore, the test is independent of the location of the break for M1 and approximately so for M2. They show that the test has the same asymptotic distribution of the Schmidt-Phillips test (test without break).

The finite sample size, power and break date estimation performance of the LM test is evaluated for M1 and M2. The results are displayed in Tables 5, 6 and 7. For M1, it can be seen that the test gets conservative with increasing level break magnitude. The break date is identified, but not as accurate as in the course of the new test. In the case of M2 the empirical size of the LM test differs considerably from its nominal level. Furthermore, the test encounters difficulties in detecting the true break date.

5 Application

The nonlinear test will be applied to the inflation rates of six G7 countries including Canada, Japan, France, Italy, the UK, and the USA. Data on inflation is taken from the International Financial Statistics database published by the International Monetary Fund. We use annual data for the period 1971 to 2006.

The time series of the inflation rates are plotted in Figure 1. A common feature can be noticed for all analyzed countries. The sample period can be divided into a period of high inflation in the seventies and early eighties followed by a low inflation period.

There is an extensive literature on the integrational properties of inflation, see, inter alia, Culver and Papell (1997), Bos et al. (1999), Lee and Wu (2001), Charemza et al. (2005). They find mixed results on the long-run properties of the inflation rate.

As a benchmark case the ADF-test in the specification without trend are applied to the inflation rates. The results are displayed in Table 8. It shows that the null hypothesis of a unit root can only be rejected for the US at a significance level of 10 percent.

The nonlinear test takes account of a single break of unknown timing. Applying the test to the inflation rates one is able to reject the unit root hypothesis for France and Italy at the 1% level and for Canada and the US at the 5% level. The majority of the break dates obtained from the nonlinear test are dated in the mid-eighties and correspond to our visual inspection of Figure 1.

6 Conclusion

The unit root test in the presence of an unknown break proposed by Perron exhibits considerable spurious rejections in finite samples, when a break occurs under the null hypothesis. In contrast, the test put forward by Popp (2007) based on the representation of the DGP using a UCM is invariant to a break in the trend function. It is shown that the corresponding test regression is in fact nonlinear in coefficients. Exploiting the nonlinearity leads to tests with more favorable properties. In the present paper the test properties of the nonlinear test are compared to that of the approaches by Perron and Popp. It is shown that the nonlinear test possesses various favorable properties. The test is able to identify the true break date very accurately. The test distribution corresponds to that one when the break date is known a priori as well as that one of the DF-test. This implies directly the invariance to the break size and break fraction. In contrast to the LM-type unit root test the invariance to the break fraction also holds for M2. Finally, the test is consistent and exhibits high power.

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Figure 1: Annual Inflation rates of six G7 countries

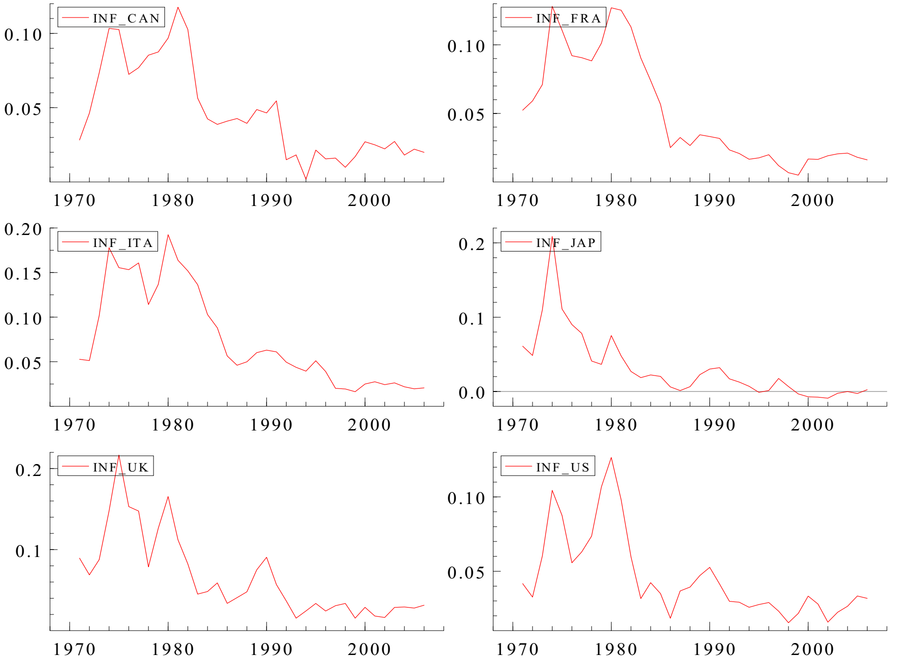

Table 1: Critical Values for $t_{\hat{\rho},NL}(T'_B)$

			λ'									
	T		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	std
$t_{\hat{\rho},NL}(T'_B, M0)$	100	1%	-3.473	-3.518	-3.581	-3.625	-3.642	-3.645	-3.640	-3.613	-3.559	0.058
		5%	-2.875	-2.880	-2.914	-2.945	-2.968	-2.976	-2.968	-2.947	-2.922	0.035
		10%	-2.571	-2.569	-2.589	-2.608	-2.628	-2.636	-2.636	-2.624	-2.605	0.025
	200	1%	-3.452	-3.462	-3.489	-3.513	-3.530	-3.533	-3.536	-3.517	-3.490	0.029
		5%	-2.876	-2.874	-2.884	-2.901	-2.911	-2.915	-2.913	-2.908	-2.899	0.015
		10%	-2.578	-2.575	-2.583	-2.591	-2.600	-2.601	-2.602	-2.599	-2.593	0.010
$t_{\hat{\rho}.NL}(T'_B, M1)$	100	1%	-4.018	-4.114	-4.148	-4.142	-4.094	-4.079	-4.106	-4.125	-4.112	0.037
		5%	-3.429	-3.482	-3.518	-3.485	-3.463	-3.469	-3.483	-3.485	-3.499	0.023
		10%	-3.128	-3.166	-3.190	-3.173	-3.154	-3.164	-3.163	-3.177	-3.184	0.017
	200	1%	-3.983	-4.062	-4.023	-4.037	-4.038	-4.027	-4.030	-4.048	-4.032	0.020
		5%	-3.424	-3.456	-3.450	-3.463	-3.439	-3.440	-3.446	-3.455	-3.455	0.011
		10%	-3.133	-3.146	-3.150	-3.156	-3.139	-3.139	-3.144	-3.148	-3.167	0.010
$t_{\hat{\rho}.NL}(T'_B, M2)$	100	1%	-5.118	-4.378	-4.487	-4.584	-4.625	-4.627	-4.566	-4.468	-4.286	0.222
		5%	-4.148	-3.738	-3.834	-3.928	-3.981	-3.986	-3.935	-3.838	-3.670	0.135
		10%	-3.760	-3.415	-3.505	-3.591	-3.653	-3.670	-3.615	-3.516	-3.352	0.122
	200	1%	-4.950	-4.311	-4.380	-4.461	-4.490	-4.495	-4.469	-4.382	-4.223	0.192
		5%	-4.089	-3.699	-3.760	-3.849	-3.900	-3.912	-3.865	-3.789	-3.644	0.124
		10%	-3.726	-3.392	-3.454	-3.547	-3.602	-3.607	-3.573	-3.488	-3.341	0.112

Critical values for $t_{\hat{\rho},NL}(T'_B, M0)$ are taken from Gluschenko (2004).

Table 2: Critical Values for $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ and DF-Test

	T	$t_{\hat{\rho}.NL}(\hat{T}_{B,3})$ 1%	5%	10%	DF-Test 1%	5%	10%
M0	100	-3.822	-3.122	-2.774	-3.496	-2.890	-2.582
	200	-3.610	-2.989	-2.671	-3.464	-2.876	-2.574
M1	100	-4.356	-3.690	-3.358	-4.052	-3.455	-3.153
	200	-4.170	-3.567	-3.258	-4.007	-3.433	-3.140
M2	100	-4.969	-4.154	-3.789	-4.500	-3.888	-3.587
	200	-4.695	-3.996	-3.655	-4.410	-3.858	-3.571

MacKinnon's (1991) Dickey-Fuller critical values with constant for M0, with constant and trend for M1 and with constant, trend and squared trend for M2 (own simulations with 100 000 replications).

Table 3: 5% Critical Values for $t_{\hat{\rho},NL}(\hat{T}_{B,3})$ (CVendo), $t_{\hat{\rho},NL}(T'_B)$ (CVexo) and DF-test (CVDF), $\lambda' = 0.5$

T	M0 CV_{endo}	M0 CV_{exo}	M0 CV_{DF}	M1 CV_{endo}	M1 CV_{exo}	M1 CV_{DF}	M2 CV_{endo}	M2 CV_{exo}	M2 CV_{DF}
50	-3.334	-3.109	-2.920	-3.935	-3.610	-3.501	-4.379	-4.168	-3.948
100	-3.122	-3.033	-2.890	-3.690	-3.498	-3.455	-4.154	-3.953	-3.851
200	-2.989	-2.888	-2.876	-3.567	-3.448	-3.433	-3.996	-3.900	-3.843
300	-2.933	-2.865	-2.871	-3.557	-3.483	-3.426	-3.904	-3.893	-3.822
500	-2.950	-2.870	-2.868	-3.495	-3.438	-3.421	-3.846	-3.878	-3.859
1000	-2.924	-2.901	-2.865	-3.421	-3.387	-3.417	-3.807	-3.813	-3.809

Table 4: 5% rejection frequency for M0 and probability of detecting the true break date, $\lambda' = 0.5$

$T = 100$						
$\rho = 1$						
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$			
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$
0	0.050	0.050	0.050	0.070	0.081	0.013
5	0.110	0.041	0.036	0.051	0.062	0.977
10	0.434	0.042	0.037	0.054	0.062	1.000
20	0.926	0.044	0.036	0.052	0.062	1.000
$\rho = 0.9$						
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$			
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$
0	0.128	0.217	0.239	0.313	0.348	0.010
5	0.318	0.192	0.246	0.324	0.364	0.979
10	0.875	0.188	0.254	0.329	0.367	1.000
20	1.000	0.198	0.259	0.334	0.374	1.000
$\rho = 0.8$						
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$			
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$
0	0.407	0.676	0.635	0.718	0.753	0.012
5	0.746	0.637	0.765	0.837	0.867	0.973
10	0.997	0.640	0.766	0.844	0.874	1.000
20	1.000	0.637	0.769	0.844	0.874	1.000

$T = 200$						
$\rho = 1$						
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$			
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$
0	0.050	0.050	0.047	0.057	0.060	0.004
5	0.070	0.055	0.039	0.049	0.053	0.969
10	0.229	0.057	0.038	0.048	0.052	1.000
20	0.732	0.060	0.040	0.050	0.055	1.000
$\rho = 0.9$						
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$			
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$
0	0.408	0.726	0.675	0.723	0.742	0.006
5	0.592	0.702	0.793	0.834	0.848	0.967
10	0.947	0.713	0.806	0.850	0.864	1.000
20	1.000	0.713	0.799	0.843	0.861	1.000
$\rho = 0.8$						
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$			
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$
0	0.963	0.999	0.990	0.993	0.994	0.005
5	0.995	0.985	0.985	0.987	0.987	0.961
10	1.000	0.999	1.000	1.000	1.000	1.000
20	1.000	0.999	1.000	1.000	1.000	1.000

Table 5: 5% rejection frequency and probability of detecting the true break date for M1, $\lambda' = 0.5$

$T = 100$								
$\rho = 1$								
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0.050	0.050	0.047	0.081	0.083	0.011	0.050	0.016
5	0.106	0.029	0.025	0.050	0.051	0.972	0.056	0.395
10	0.507	0.033	0.031	0.055	0.056	1.000	0.045	0.486
20	0.953	0.030	0.026	0.053	0.054	1.000	0.019	0.517
$\rho = 0.9$								
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0.130	0.152	0.162	0.259	0.262	0.012	0.247	0.018
5	0.253	0.108	0.115	0.196	0.199	0.968	0.205	0.523
10	0.819	0.102	0.115	0.188	0.192	1.000	0.141	0.720
20	0.999	0.102	0.119	0.196	0.199	1.000	0.102	0.863
$\rho = 0.8$								
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0.366	0.483	0.492	0.640	0.645	0.013	0.723	0.018
5	0.578	0.395	0.474	0.635	0.642	0.970	0.579	0.662
10	0.988	0.414	0.482	0.646	0.651	1.000	0.486	0.894
20	1.000	0.399	0.478	0.637	0.642	1.000	0.453	0.978

$T = 200$								
$\rho = 1$								
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0.050	0.050	0.053	0.072	0.073	0.006	0.050	0.009
5	0.066	0.035	0.037	0.051	0.052	0.967	0.057	0.355
10	0.255	0.035	0.039	0.052	0.053	1.000	0.056	0.474
20	0.803	0.034	0.035	0.049	0.050	1.000	0.034	0.524
$\rho = 0.9$								
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0.340	0.486	0.549	0.625	0.630	0.006	0.706	0.010
5	0.401	0.426	0.543	0.624	0.628	0.966	0.624	0.574
10	0.869	0.428	0.538	0.619	0.623	1.000	0.560	0.831
20	1.000	0.425	0.539	0.616	0.620	1.000	0.480	0.960
$\rho = 0.8$								
			$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	CV_{endo}	CV_{exo}	CV_{DF}	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0.924	0.987	0.983	0.990	0.990	0.005	0.991	0.010
5	0.955	0.978	0.988	0.993	0.993	0.964	0.978	0.740
10	1.000	0.983	0.995	0.998	0.998	1.000	0.970	0.956
20	1.000	0.983	0.995	0.998	0.998	1.000	0.962	0.998

Table 6: 5% rejection frequency and probability of detecting the true break date for M2, $\lambda' = 0.5$

$T = 100$									
$\rho = 1$									
				$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	γ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	$\mathrm{CV}_{\mathrm{endo}}$	$\mathrm{CV}_{\mathrm{exo}}$	$\mathrm{CV}_{\mathrm{DF}}$	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0	0.050	0.050	0.047	0.068	0.084	0.014	0.050	0.018
	2	0.652	0.043	0.015	0.026	0.033	0.152	0.021	0.140
	6	0.991	0.045	0.033	0.049	0.059	0.663	0.023	0.176
	10	0.998	0.046	0.034	0.051	0.061	0.967	0.018	0.204
5	0	0.140	0.048	0.034	0.052	0.063	0.972	0.052	0.140
	2	0.611	0.051	0.035	0.052	0.062	0.999	0.057	0.026
	6	0.998	0.046	0.031	0.049	0.061	1.000	0.096	0.004
	10	1.000	0.050	0.034	0.053	0.064	1.000	0.131	0.011
10	0	0.505	0.043	0.032	0.050	0.061	1.000	0.032	0.269
	2	0.804	0.043	0.032	0.048	0.058	1.000	0.097	0.042
	6	1.000	0.048	0.033	0.050	0.064	1.000	0.318	0.001
	10	1.000	0.048	0.035	0.052	0.062	1.000	0.476	0.000
$\rho = 0.9$									
				$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	γ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	$\mathrm{CV}_{\mathrm{endo}}$	$\mathrm{CV}_{\mathrm{exo}}$	$\mathrm{CV}_{\mathrm{DF}}$	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0	0.091	0.118	0.122	0.168	0.196	0.014	0.143	0.015
	2	0.766	0.071	0.041	0.062	0.075	0.130	0.060	0.163
	6	0.999	0.096	0.080	0.116	0.138	0.609	0.066	0.240
	10	1.000	0.112	0.094	0.137	0.160	0.952	0.048	0.295
5	0	0.143	0.107	0.094	0.137	0.160	0.970	0.107	0.238
	2	0.752	0.103	0.092	0.132	0.157	0.999	0.133	0.036
	6	1.000	0.104	0.089	0.134	0.160	1.000	0.231	0.008
	10	1.000	0.112	0.097	0.140	0.163	1.000	0.250	0.030
10	0	0.459	0.115	0.096	0.140	0.165	1.000	0.052	0.459
	2	0.924	0.104	0.088	0.131	0.158	1.000	0.182	0.070
	6	1.000	0.105	0.089	0.130	0.156	1.000	0.523	0.001
	10	1.000	0.113	0.095	0.135	0.161	1.000	0.706	0.000
$\rho = 0.8$									
				$t_{\hat{\rho},NL}(\hat{T}_{B,3})$					
θ	γ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	$\mathrm{CV}_{\mathrm{endo}}$	$\mathrm{CV}_{\mathrm{exo}}$	$\mathrm{CV}_{\mathrm{DF}}$	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0	0.263	0.373	0.366	0.465	0.515	0.012	0.464	0.016
	2	0.925	0.173	0.151	0.202	0.234	0.106	0.266	0.201
	6	1.000	0.289	0.274	0.348	0.386	0.527	0.254	0.322
	10	1.000	0.349	0.352	0.446	0.493	0.930	0.207	0.394
5	0	0.181	0.349	0.350	0.451	0.506	0.966	0.306	0.391
	2	0.936	0.352	0.350	0.449	0.503	0.996	0.417	0.071
	6	1.000	0.354	0.356	0.455	0.506	1.000	0.562	0.022
	10	1.000	0.354	0.352	0.459	0.514	1.000	0.527	0.103
10	0	0.377	0.350	0.350	0.451	0.504	1.000	0.182	0.695
	2	0.994	0.350	0.347	0.452	0.506	1.000	0.412	0.181
	6	1.000	0.351	0.352	0.448	0.501	1.000	0.845	0.006
	10	1.000	0.355	0.353	0.455	0.506	1.000	0.950	0.000

Table 7: 5% rejection frequency and probability of detecting the true break date for M2, $\lambda' = 0.5$

$T = 200$

$\rho = 1$

θ	γ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{endo}}$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{exo}}$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{DF}}$	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0	0.050	0.050	0.044	0.058	0.066	0.006	0.050	0.009
	2	0.937	0.061	0.019	0.024	0.027	0.103	0.015	0.129
	6	0.965	0.044	0.029	0.037	0.041	0.512	0.019	0.164
	10	0.988	0.052	0.031	0.046	0.052	0.872	0.016	0.177
5	0	0.079	0.054	0.037	0.048	0.053	0.965	0.047	0.098
	2	0.937	0.057	0.038	0.052	0.058	0.998	0.031	0.011
	6	0.979	0.058	0.039	0.051	0.056	1.000	0.050	0.002
	10	0.995	0.054	0.038	0.051	0.056	1.000	0.066	0.007
10	0	0.304	0.054	0.040	0.055	0.059	1.000	0.039	0.201
	2	0.908	0.054	0.038	0.051	0.056	1.000	0.083	0.106
	6	0.992	0.055	0.038	0.051	0.056	1.000	0.157	0.000
	10	0.997	0.054	0.039	0.054	0.058	1.000	0.237	0.000

$\rho = 0.9$

θ	γ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{endo}}$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{exo}}$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{DF}}$	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0	0.253	0.389	0.393	0.462	0.486	0.006	0.420	0.009
	2	0.999	0.178	0.130	0.162	0.171	0.084	0.221	0.184
	6	0.999	0.271	0.229	0.278	0.295	0.428	0.239	0.274
	10	1.000	0.348	0.317	0.380	0.403	0.808	0.204	0.350
5	0	0.192	0.384	0.385	0.458	0.482	0.966	0.325	0.239
	2	0.999	0.379	0.375	0.452	0.480	0.996	0.345	0.026
	6	0.999	0.383	0.384	0.457	0.483	1.000	0.416	0.017
	10	1.000	0.380	0.383	0.454	0.480	1.000	0.436	0.062
10	0	0.312	0.384	0.385	0.457	0.484	1.000	0.209	0.538
	2	0.999	0.385	0.383	0.452	0.478	1.000	0.495	0.027
	6	1.000	0.386	0.385	0.454	0.480	1.000	0.719	0.001
	10	1.000	0.382	0.383	0.452	0.479	1.000	0.815	0.000

$\rho = 0.8$

θ	γ	$t_{\hat{\rho},P}(\hat{T}_{B,2})$	$t_{\hat{\rho},L}(\hat{T}_{B,3})$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{endo}}$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{exo}}$	$t_{\hat{\rho},NL}(\hat{T}_{B,3})$ CV$_{\text{DF}}$	$P(\hat{T}_{B,3} = T'_B)$	$t_{\hat{\phi},LM}(\hat{T}_{B,4})$	$P(\hat{T}_{B,4} = T'_B)$
0	0	0.851	0.960	0.945	0.964	0.969	0.007	0.975	0.007
	2	1.000	0.542	0.450	0.478	0.487	0.057	0.880	0.234
	6	1.000	0.766	0.648	0.677	0.688	0.332	0.870	0.364
	10	1.000	0.901	0.769	0.786	0.791	0.723	0.844	0.444
5	0	0.406	0.951	0.961	0.975	0.979	0.961	0.901	0.444
	2	1.000	0.959	0.966	0.980	0.983	0.992	0.953	0.080
	6	1.000	0.961	0.969	0.983	0.985	0.999	0.959	0.061
	10	1.000	0.963	0.972	0.983	0.986	1.000	0.945	0.255
10	0	0.271	0.963	0.971	0.983	0.987	1.000	0.835	0.820
	2	1.000	0.962	0.969	0.983	0.986	1.000	0.971	0.133
	6	1.000	0.963	0.973	0.982	0.986	1.000	0.995	0.005
	10	1.000	0.960	0.970	0.981	0.984	1.000	0.998	0.001

Table 8: Results of unit root tests for annual inflation rates

Series	Sample	T	ADF τ_α	k	M0	$\hat{T}_B$	$\hat{\lambda}$	k
Canada	1971-2006	36	-1.421	0	-3.688**	1982	0.33	0
France	1971-2006	36	-1.219	1	-5.238***	1984	0.39	5
Italy	1971-2006	36	-1.107	0	-4.815***	1984	0.39	4
Japan	1971-2006	36	-1.991	0	-1.548	1979	0.25	0
UK	1971-2006	36	-1.774	0	-1.108	1979	0.25	3
US	1971-2006	36	-2.679*	1	-3.446**	1984	0.39	4