

Consistent height transformations between geodetic and meteorologic reference systems

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IAG ICCT Study Group 12

Numerical weather models (NWM's) contain valuable information relevant for removing the environmental signal from geodetic data. Currently no clear documentation exists regarding how to deal with the height systems when carrying out the calculations in a geodetic reference frame. A "conventional" transformation model (available also as source code) would enable geodesists to handle such data easily and allow them to use data from different meteorologic datasets. In addition, geodetic products such as GNSS-derived zenith total delays are being assimilated into NWMs. Thus, the transformations that convert the meteorological data into a geodetic reference frame should also support the use of geodetic data in meteorological models. The IAG ICCT Special Study Group 12 "Coordinate systems in numerical weather models" has been set up to 1) deal with the differences between geodetic and meteorologic reference systems and 2) provide consistent models for transforming between the two systems.

Ellips. Heights ↔ geop. heights

Ellipsoidal heights (h) can be obtained from orthometric heights (H) when the geoid undulation (N) is known.

$$h = H + N \quad (1)$$

Furthermore, orthometric heights relate to geopotential heights (Z) by

$$H = Z \cdot g_n / \bar{g}_0 \quad (2)$$

where g_n denotes the conventional gravity constant used throughout the numerical weather model and \bar{g}_0 is the mean gravity, defined as

$$\bar{g}_0 = 1/\zeta \int g dz \quad (3)$$

where (vertical) integration has to be performed from the geoid surface to height ζ .

Error sources

Although the transformation between numerical weather model heights and geodetic (ellipsoidal) heights can be described in a mathematically unique sense (equations 1-3) the choice of geophysical models, the selection of constants or the definition of the origin can lead to uncertainties of the transformation which can reach several meters. Thus, in the next sections the following effects on ellipsoidal heights are being studied:

- Impact of the gravity model and the way how the mean gravity \bar{g}_0 is calculated
- Impact of the vertical direction w.r.t. the ellipsoid instead of the vertical w.r.t. a sphere (as used for numerical weather models)
- Uncertainty of the geoid (undulation)
- Using a different value for the conventional gravity constant.

Choice of the gravity model

In order to choose the mean gravity for the height transformation the study group has investigated how and to what extent the choice of the gravity model changes the obtained ellipsoidal height. In doing so, geopotential heights from a numerical weather model ($g_n=9.80665 \text{ m/s}^2$) had to be transformed to ellipsoidal heights (assuming a constant geoid undulation of $N=20 \text{ m}$). Calculations were performed on global 1×1 grids and it was assumed that geodetic latitude/longitude is identical to the one used in the numerical weather models. In total 8 contributions (from GFZ/Germany, GRGS/France, NICT/Japan, UNB/Canada(5 solutions) and VUT/Austria) were submitted. Figure 1 shows the RMS w.r.t. an average map (over all solutions) for selected height levels.

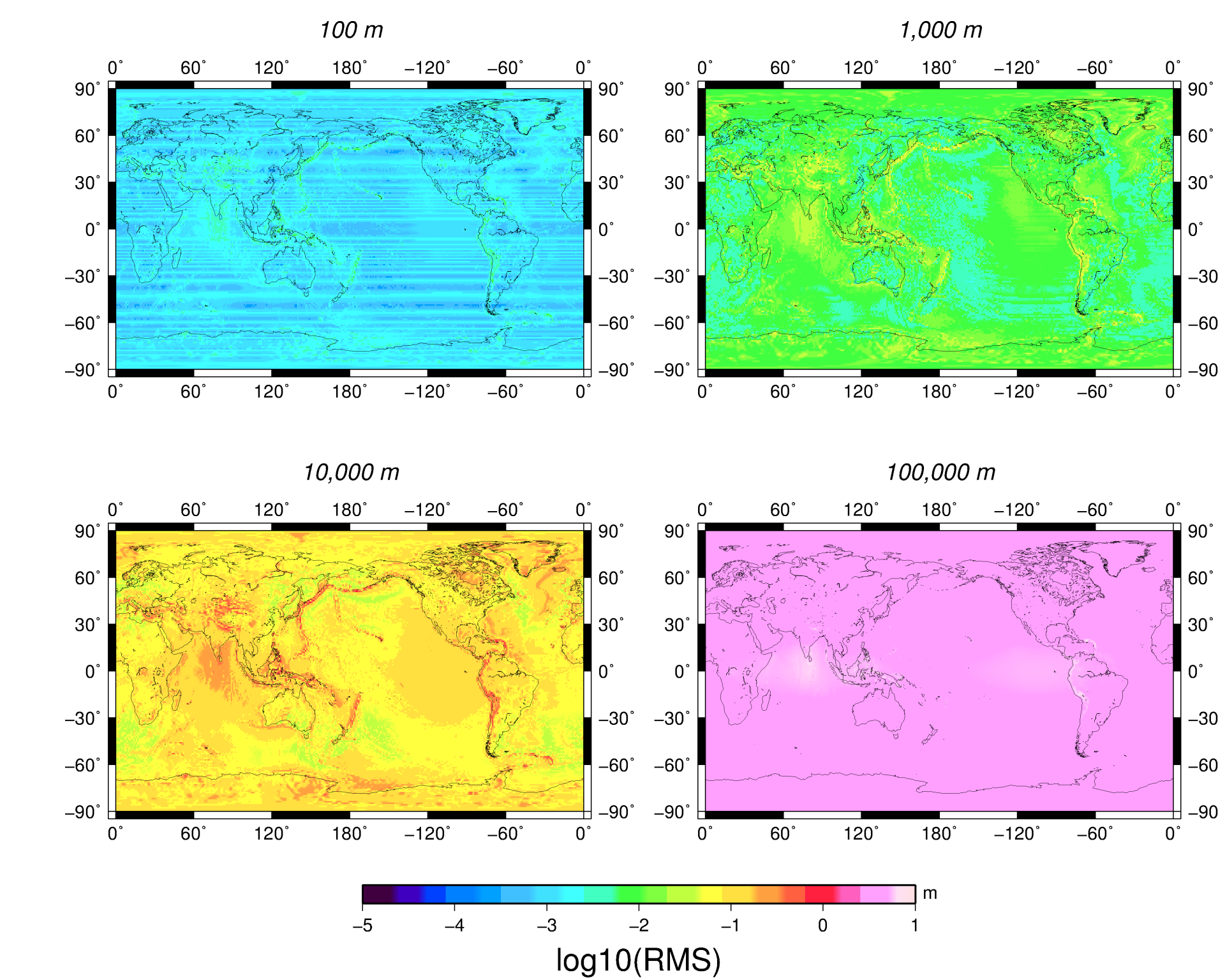


Figure 1: RMS w.r.t. an average map at (geopotential) heights of 100, 1000, 10000 and 100000 m.

Figure 3 depicts the root mean square (RMS) w.r.t. a median map for each solution over all heights. Fitting a linear function over all results, allows to derive a simple estimate for the uncertainty due to the choice of the mean gravity

$$\sigma(\bar{g}_0) = 2.53 \cdot 10^{-5} z \quad (4)$$

where z is the geopotential height in meter.

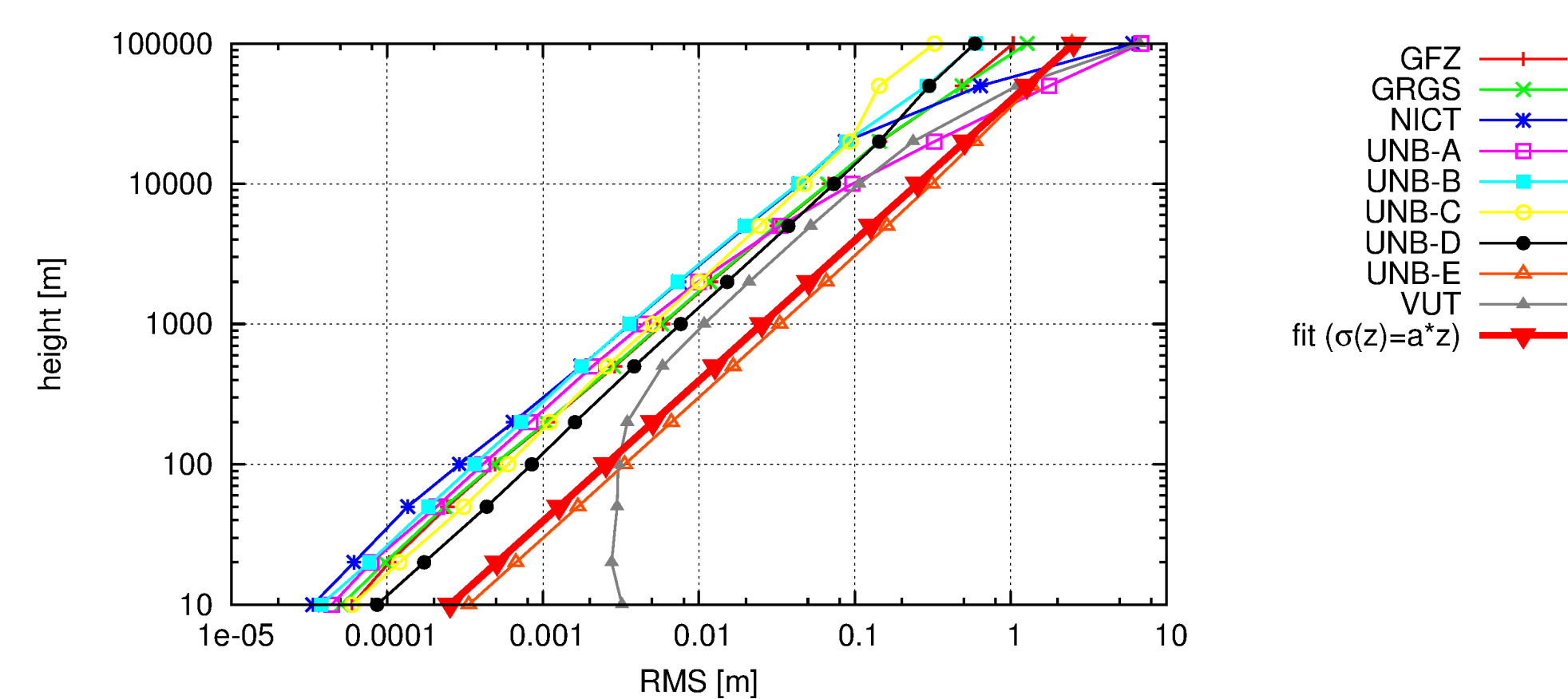


Figure 2: RMS w.r.t. a median (from all eight submitted solutions) for each solution over all heights.

Direction of the normal

When the normal to the sphere is used instead of the normal to the ellipsoid, transformed heights are expected to be changed slightly as well. Similar to the study about the mean gravity model, GRGS evaluated data at various heights and grid points and computed the difference between two transformations, one using the normal to the ellipsoid and one using the normal w.r.t. a mean sphere. Results of selected height levels are displayed in figure 3 and figure 4 shows the RMS in height dependence together with a linear fit for the expected error.

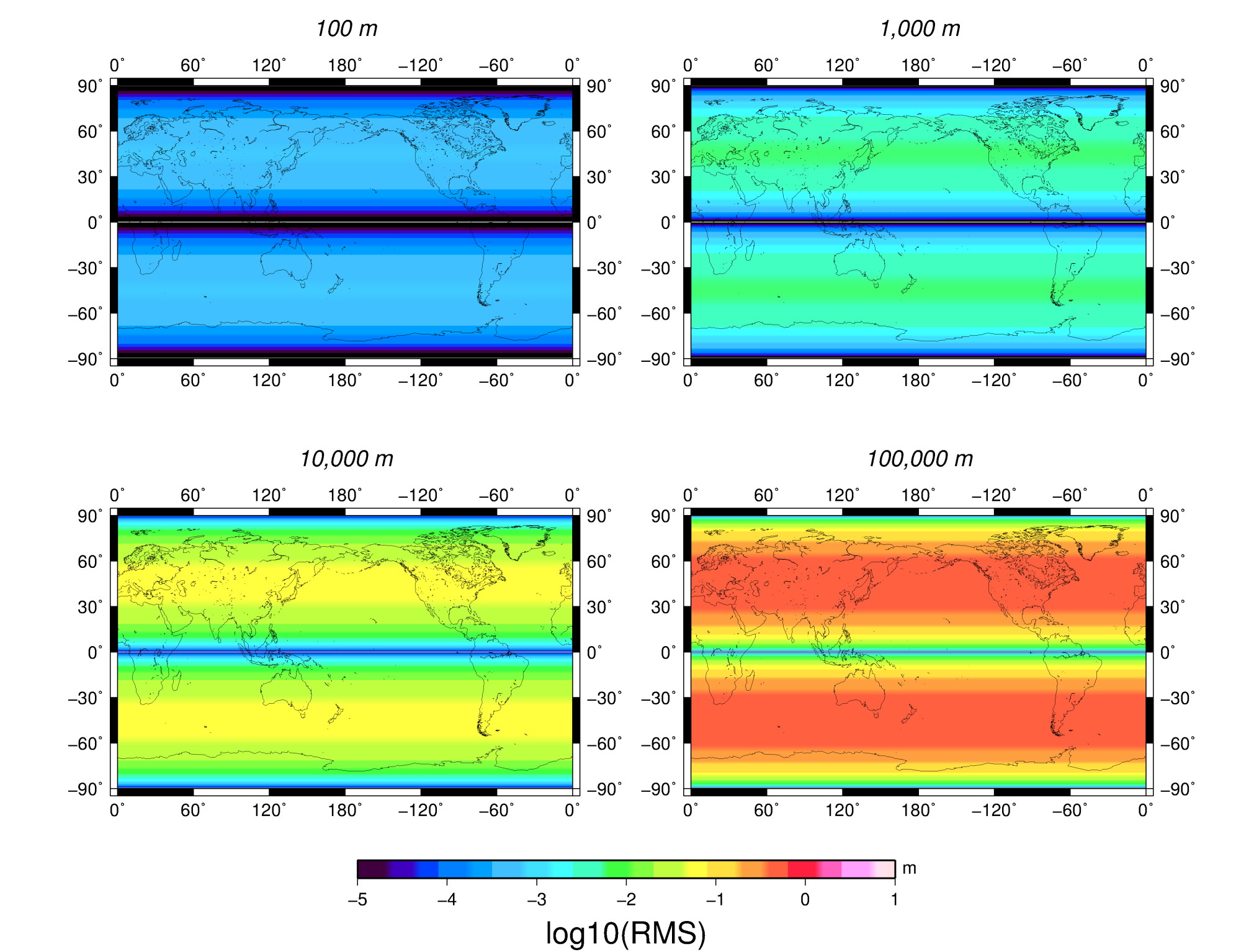


Figure 3: Absolute differences between heights transformed w.r.t. a normal to the ellipsoid and those transformed using a mean sphere.

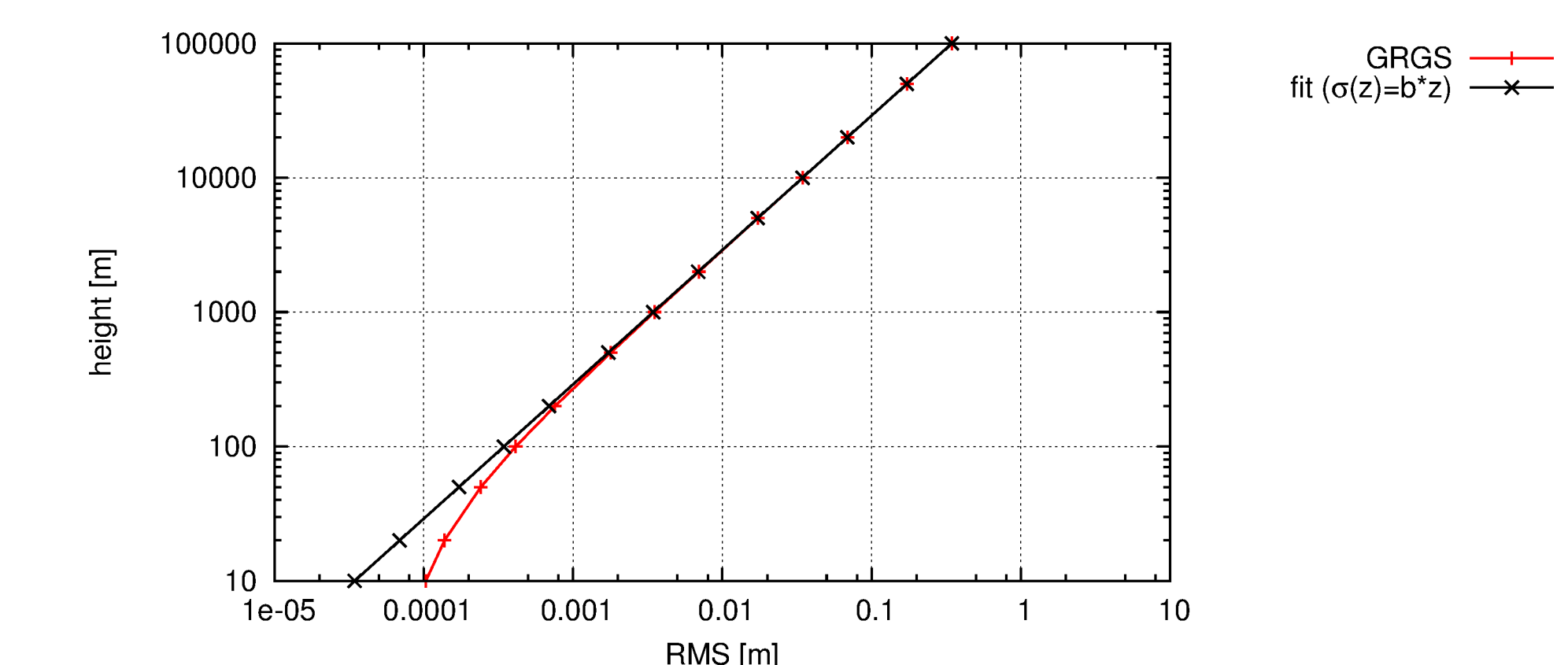


Figure 4: RMS over each map in dependence of the height. The uncertainty due to this effect can be expressed as $\sigma(norm) = 3.45 \cdot 10^{-6} z$.

Uncertainty of the geoid undulation

Geoid undulations N need to be obtained from regional or global geoid models and applied to all grid points of the numerical weather model before obtaining ellipsoidal heights from orthometric heights (equation 1). Thus, any error/uncertainty of these models directly propagates into the obtained ellipsoidal heights. Although regional geoid solution scan provide mm-accuracy such models don't cover the whole area of the numerical weather model. Thus,

$$\sigma(N) = 0.01 \text{ [m]} \quad (5)$$

can be taken as a (conservative) value for the uncertainty of geoid undulations on a global scale.

Conventional gravity constant for NWMs

In case the gravity constant is inaccurate and not properly considered for the transformation (2), an additional error source for obtaining ellipsoidal heights results. However, most of the NWMs rely upon a value of $g_n=9.80665 \text{ m/s}^2$ or explicitly document the usage of another value. Thus, the impact from this error source can be assumed to be zero, i.e.

$$\sigma(g_n) = 0 \text{ [m]} \quad (6)$$

Error budget

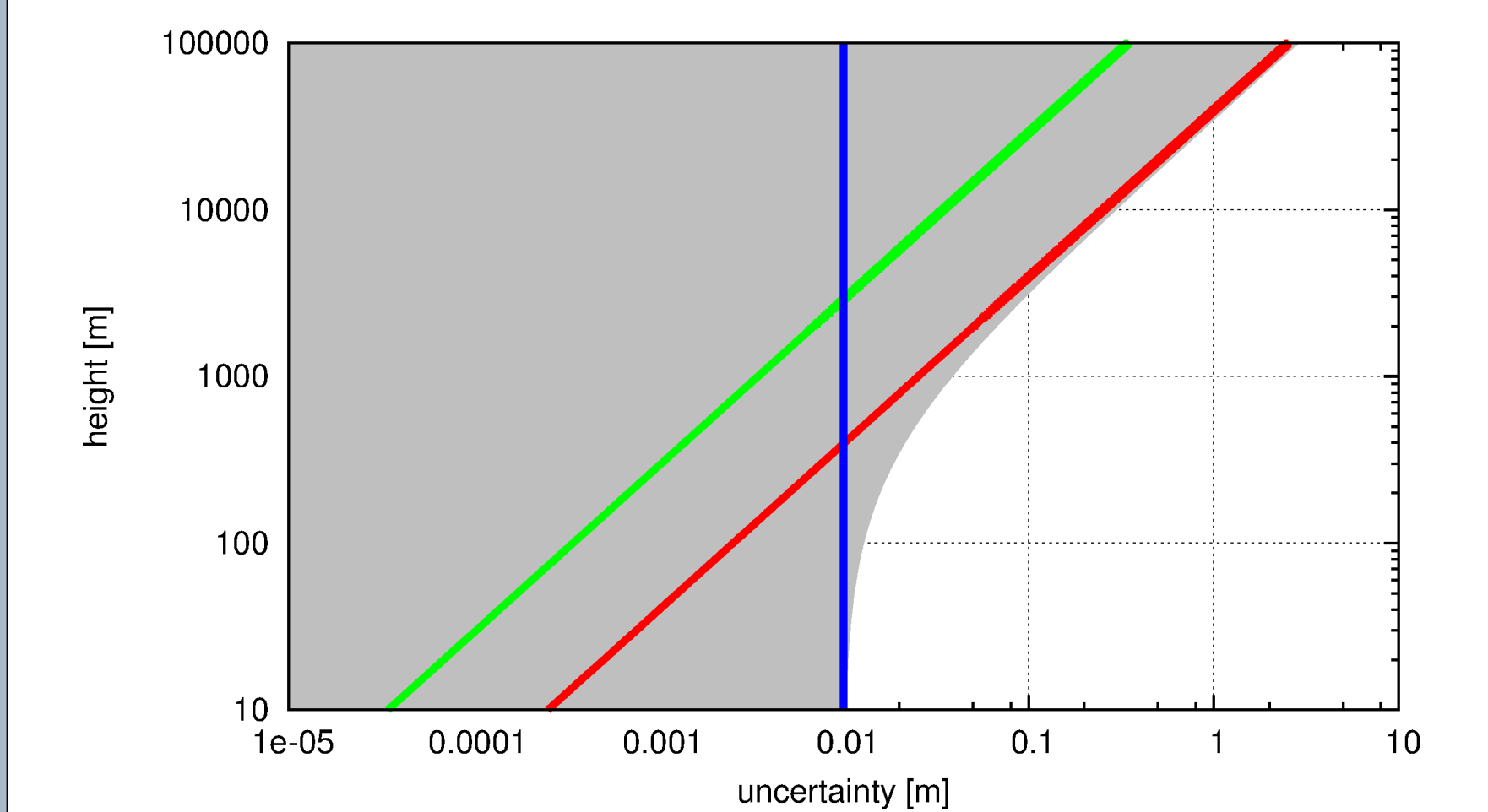


Figure 5: Uncertainties of $\sigma(\bar{g}_0)$, $\sigma(norm)$, $\sigma(N)$ and the total error budget of the height transformation.

Discussion

As shown in figure (5), the uncertainty of the geoid model, resulting mostly from the geoid undulation (N) dominates the overall error budget in the lower height domains, i.e. $<500 \text{ m}$. Above that height, the choice of the gravity model and the way in which the mean gravity acceleration is computed becomes more important, and this error source starts to reduce the accuracy of the transformation. Thus, for a consistent and conventional height transformation between geopotential heights from a numerical weather model and ellipsoidal heights it is important that,

- geoid undulations are known with mm-accuracy on a global scale
- the gravity model provides both geoid undulations and gravity acceleration at a given location
- the proper direction of the normal w.r.t. the reference figure is properly considered for the highest accuracy.

Fortunately, most of the atmospheric parameters relevant for geodesy (mainly pressure) decrease exponentially with height, which reduces the impact of an imperfect height transformation when performing an integration/summation in vertical direction.

Further information

All plots of the comparison campaign, details of the models utilized, as well as software routines for a suggested conventional height transformation can be found on:

<http://hobiger.org/blog/iag-ic-sgg12/>