Proton acceleration by circularly polarized traveling electromagnetic wave

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The acceleration of charged particles, producing collimated monoenergetic beams, over short distances holds the promise to offer new tools in medicine and diagnostics. Here, we consider a possible mechanism for accelerating protons to high energies by using a phase modulated circularly polarized electromagnetic wave propagating along a constant magnetic field. It is observed that a plane wave with dimensionless amplitude of 0.1 is capable to accelerate a 1 keV proton to 386 MeV under optimum conditions. Finally, we discuss possible limitations of the acceleration scheme.

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I. INTRODUCTION

Laser induced particle acceleration has drawn considerable interest among researchers all over the world since the pioneering work by Tajima and Dawson [1]. The acceleration gradient of conventional linear accelerators is of the order of 10^5 V/cm; however, today's state of the art lasers are capable to produce the acceleration gradient many orders of what can be achieved using conventional linacs. In general, laser based accelerators can be divided based on the medium in which the acceleration takes place, which can be either vacuum or a plasma. The vacuum as a medium for particle acceleration has some inherent advantages over plasma medium. The problems like instabilities are absent in vacuum, it is easier to inject the preaccelerated particle beam in vacuum as compared to the plasma, collisions of particles with media causing energy loss and beam spreading is ruled out, etc. Thus, we will focus on particle acceleration in vacuum in this paper.

The relativistic motion of the charge particle in large amplitude electromagnetic (EM) fields is studied in detail by many authors. The motion of the charged particle in transverse EM wave and the constant magnetic field along wave propagation is studied by Roberts and Buchsbaum [2], which was further extended analytically and experimentally by Jory and Trivelpiece [3]. More recently the in-depth Hamiltonian analysis of the dynamics of a charge particle in a circularly polarized traveling EM wave is been studied by Bourdier and Gond [4]. Recently, Kong *et al.* [5] has presented analytical treatment of the interaction of the charged particle with circularly polarized electromagnetic waves. Various different schemes have been proposed for the acceleration of charge particle in traveling EM wave [6–9]; some include the homogeneous magnetic field, however some include the two counterpropagating EM waves.

In this paper we will consider an alternative method to accelerate protons in vacuum by circularly polarized electromagnetic waves, where the main new ingredient is a phase modulation of the EM wave. Emphasis of the paper would be on understanding the dynamics of proton motion under the proposed scheme. The next section will briefly describe the proposed scheme followed by the numerical analysis, discussion, and final remarks.

II. MODEL DESCRIPTION

A circularly polarized traveling wave propagating along the z direction is considered. The electric and magnetic fields of the wave are given by

$$E_x = E_0 \sin[\omega(t - z/c) + \phi(z, t)] \tag{1}$$

$$E_y = -E_0 \cos[\omega(t - z/c) + \phi(z, t)] \tag{2}$$

and the magnetic fields are expressed as

$$B_x = (E_0/c) \cos[\omega(t - z/c) + \phi(z, t)] \tag{3}$$

$$B_y = (E_0/c) \sin[\omega(t - z/c) + \phi(z, t)], \tag{4}$$

where $\phi(z, t)$ is the phase modulation function which is given by

$$\phi(z, t) = \pi \sin[\alpha\omega(t - z/c)], \tag{5}$$

where α is the so-called phase modulation factor which controls the extent of the modulation. A constant magnetic field is also applied along the direction of wave propagation given by $B_z = b_0$.

The electric fields denoted by Eqs. (1) and (2) can be generated by introducing an *electro-optic phase modulator*.

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This is an optical device in which an element displaying the electro-optic effect is used to modulate the beam of light. The modulation can be done in phase, frequency, polarization, and amplitude. The simplest kind of modulator consists of a crystal, such as lithium niobate whose refractive index is a function of the applied electric field [10]. An appropriate electric field along a crystal can be applied in such a way that its refractive index modulates, which eventually will introduce the phase lag in the EM wave. The phase modulation will also depend on the length of the crystal and other parameters. Designing an accurate phase modulator for a specific problem may be an engineering concern, but for the purpose of this article we will assume that such a problem can be solved satisfactorily.

The schematic diagram for the proposed scheme is shown in Fig. 1. The EM wave is initially passed through the phase modulator so that the spatial and temporal dependence of the electric and magnetic fields are modified according to Eqs. (1)–(4). This modified pulse is then injected into the accelerating cavity, protons under the influence of this modified EM wave undergo the acceleration along the transverse direction.

An exact analytical treatment of the problem seems to be too involved because of the nature of electric and magnetic field profiles. It would be a nontrivial task to solve the momentum equations corresponding to the field equations (1)–(4). In view of this we have numerically analyzed the dynamics of the particle under the influence of the given field profiles.

As is well known, the motion of the relativistic particle is described by the following equations,

$$\frac{d\mathbf{p}}{dt} = q[\mathbf{E} + \mathbf{v} \times \mathbf{B}] \tag{6}$$

$$\mathbf{v} = \frac{\mathbf{p}/m_0}{\sqrt{1 + |\mathbf{p}|^2/(m_0 c)^2}} \tag{7}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \tag{8}$$

where, $\mathbf{p}$, $\mathbf{v}$, $\mathbf{r}$, and m_0 are relativistic momentum, velocity, coordinate, and mass of the particle. The above equations are solved numerically by a standard Boris leapfrog scheme where particle motion is decomposed into motion in the electric field and the motion in the magnetic field [11]. The particle orbits are calculated by substituting Eqs. (1)–(4) into the equation of motion, and specifying the initial condition for the injection energy, letting the initial velocity be directed along z. In the rest of the paper we have used the dimensionless units for all physical quantities.

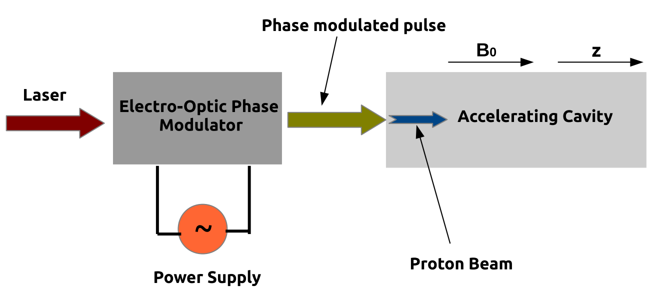

FIG. 1. Schematic diagram of the proposed scheme.

For all the results presented here, the amplitude of the circularly polarized wave is considered to be $a_0 = 0.1$ (unless otherwise stated), where $a_0 = eE/m_e\omega c$ [12]. Similarly magnetic field is denoted by $b_0 = eB/m_e\omega$. Here, e and m_e are the charge and mass of electron, E and B are the amplitude of electric and magnetic field in SI units with ω being the EM wave frequency. The dimensionless space and time are taken in units of the wave number k and the angular frequency ω, respectively.

III. PHASE MODULATION RESONANCE

Before a general numerical study is performed, we will first present the basic properties introduced by the phase modulation. For this purpose we will in this section limit ourselves to modest accelerations with nonrelativistic particle velocities $|\mathbf{v}| \ll c$. Thus, the magnetic field as well as relativistic effects play only a minor role, and we apply the following equation of motion,

$$m_i \frac{d\mathbf{v}}{dt} = q_i \mathbf{E}, \tag{9}$$

where m_i and q are the mass and charge of the particle. Furthermore, only the temporal dependence in Eq. (9) is considered for the purpose of analysis, which is a valid assumption since the dynamics is mostly independent of the spatial coordinates (as have been confirmed numerically). Now using the field profile given by Eq. (1) and phase modulation function given by Eq. (5), this simple equation of motion [Eq. (9)] can be integrated to give

$$\begin{aligned} v_x(\tau) &= \frac{q_i E_0}{m_i} \int_0^\tau \sin[\tau' + \pi \sin(\alpha\tau')] d\tau' + C \\ v_y(\tau) &= -\frac{q_i E_0}{m_i} \int_0^\tau \cos[\tau' + \pi \sin(\alpha\tau')] d\tau' + C, \end{aligned} \tag{10}$$

where τ is time in dimensionless units and C is a constant of integration. For simplification we have chosen $q_i E_0/m_i = 1$ and $C = 0$. The kinetic energy can be calculated by the knowledge of the v_x and v_y as

$$\mathcal{E}(\tau) \equiv \tfrac{1}{2}\sqrt{v_x^2 + v_y^2}. \tag{11}$$

The integrals for v_x and v_y are calculated numerically and corresponding energy [Eq. (11)] is presented in Fig. 2.

The properties of the integrand in Eq. (11) are quite sensitive to the value of α. It is quite clear that if $\alpha = 1/n$, with n being an integer, the integrand is a periodic function. The choice $\alpha = 1/n$ then results in a periodic

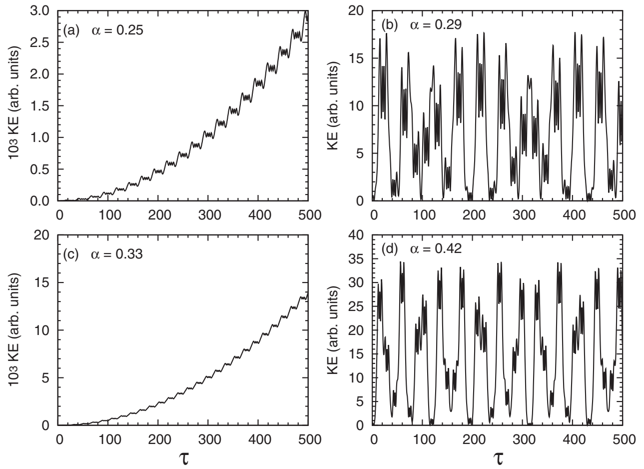

FIG. 2. Temporal evolution of the kinetic energy with different values of α: (a) 0.25, (b) 0.29, (c) 0.33, and (d) 0.42 is presented.

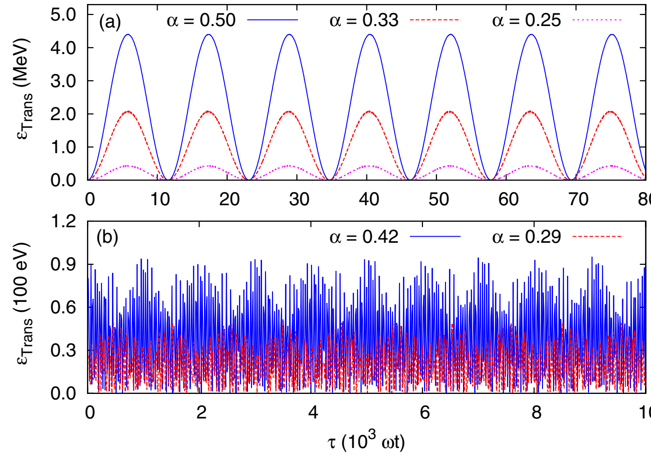

FIG. 3. Temporal evolution of transverse kinetic energy of proton for (a) three different phase modulation factor $\alpha = 1/n$ with $n = 2$, 3, and 4 being an integer; similarly the case when n is not an integer is also presented (b). The amplitude of the EM wave $a_0 = 0.1$ and constant magnetic field along wave propagation $b_0 = 1.0$ is considered.

outcome for $v_x(\tau)$, but the corresponding integral for $v_y(\tau)$ has a nonzero average over a period, and as a result the solution is a linear growth in energy superimposed on periodic oscillations [Figs. 2(a) and 2(c)]. However, for values of α not close to a resonance, the integrand in Eq. (10) is nonperiodic. As a result the solution of the integral is irregular, and on average no energy is gained [Figs. 2(b) and 2(d)]. The observation is that the success and failure of the proposed acceleration scheme firstly depend on the property of the integral given by Eq. (10). Furthermore, long-time acceleration can only be maintained when the frequency of the electromagnetic wave is some harmonics of the phase modulation frequency, i.e. $\alpha = 1/n$ with n being an integer. These properties survive also when the relativistic effects and the magnetic field is added. However, from a practical point of view the above simplified scheme suffers from a poor confinement of the orbits, i.e. for an EM wave with a finite focal spot the particle quickly leaves the region of acceleration. The confinement is helped by a static magnetic field, but this also introduces the need for a small detuning from exact resonance, as will be investigated in the next section.

IV. NUMERICAL ANALYSIS

In this paper, our main focus is to understand the dynamics of a single proton with energy 1 keV (unless otherwise stated), injected along the propagation direction (z) of a phase modulated circularly polarized wave with amplitude $a_0 = 0.1$ and a constant magnetic field with magnitude of $b_0 = 1.0$ (Fig. 1) along wave propagation. Although it will be clear later in the paper that this scheme is equally valid for the proton beam having some energy spread. The central theme of the proposed scheme is the introduction of the so-called *phase modulation factor* (α) which can be expressed as the ratio of the phase modulation frequency (ω_m) to wave frequency ω; i.e., $\alpha = \omega_m/\omega$. Next we will see how the value of α affects the resulting dynamics of the particle.

The time evolution of the transverse kinetic energy of the particle for the phase modulation factor $\alpha = 1/n$ with $n = 2$, 3, and 4 are presented in Fig. 3(a). It is observed that the proposed scheme of acceleration works well only when the wave frequency is some harmonics of the modulation frequency. A large deviation from this condition destroys the acceleration mechanism completely, Fig. 3(b). Furthermore, as can be seen from Fig. 3(a), the efficiency of the scheme deteriorates gradually with higher harmonics of the phase modulation frequency.

Let us focus on the trajectories of the particle under the influence of the phase modulated EM wave [Eqs. (1)–(4)] and external magnetic field ($B_z = b_0$). The time evolution of the particle trajectory along 3 space dimensions is presented in Fig. 4(a). Here we have considered the value of $\alpha = 0.5$ (the same results holds qualitatively for other valid values of α, i.e. $\alpha = 0.20$, 0.25, and 0.33).

The longitudinal displacement along z is mainly governed by the energy at which the particle is being injected in the cavity, however oscillatory motion along y direction is because of the presence of the external magnetic field along z direction.

As can be inferred from the Figs. 4(b) and 4(d), the biggest problem with the acceleration scheme is the excursion along the x direction which is much larger than the displacement along the z direction of propagation. In this scenario, the proposed scheme would be impossible to implement in practice because the excursion along the transverse direction is too large. In order to deal with this problem a small detuning parameter δ can be added to

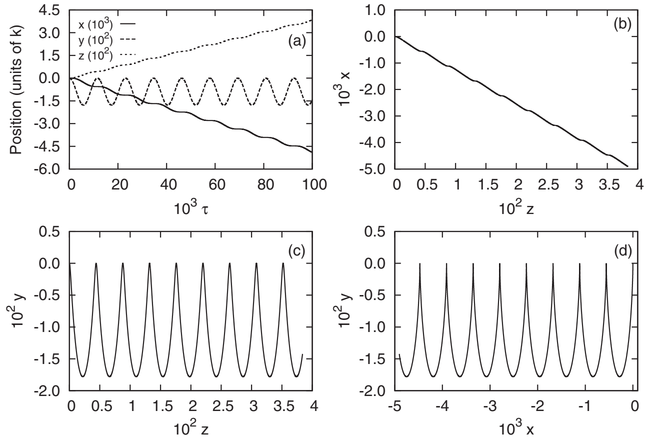

FIG. 4. Temporal evolution of the particle trajectory along three space dimensions (a) is plotted along with the trajectory in the *x*-*z* plane (b), *y*-*z* plane (c), and *x*-*y* plane (d). Here, we have considered the phase modulation factor $\alpha = 0.5$.

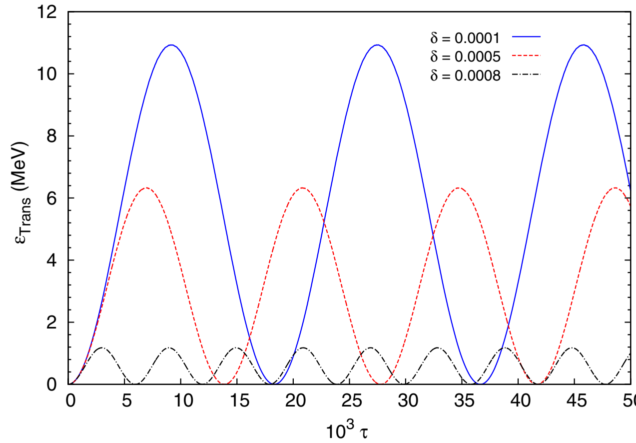

FIG. 6. Temporal evolution of the transverse kinetic energy for various values of detuning parameter δ. The resulting phase modulation factor would be $\alpha + \delta$ with $\alpha = 0.50$.

phase modulation factor α, which would be helpful to confine the particle orbits in the *x*-*y* plane.

Let us examine the trajectory of the particle after the addition of a small detuning parameter $\delta = 10^{-4}$ which modifies the phase modulation factor α from 0.5000 to 0.5001. It can be observed from Fig. 5(a) that the kinetic energy of particle increased by a factor of about 2 as compared to the case when no detuning parameter is present (Fig. 3). Furthermore, the trajectory of the particle is modified significantly after the introduction of the detuning parameter. As can be seen, the particle trajectory in the *x*-*y* plane [Fig. 5(d)] is closed which makes it possible for the acceleration scheme to work in practice.

It should be noted that the selection of the detuning parameter δ is very crucial for the success or failure of the acceleration mechanism. The value of δ must be small

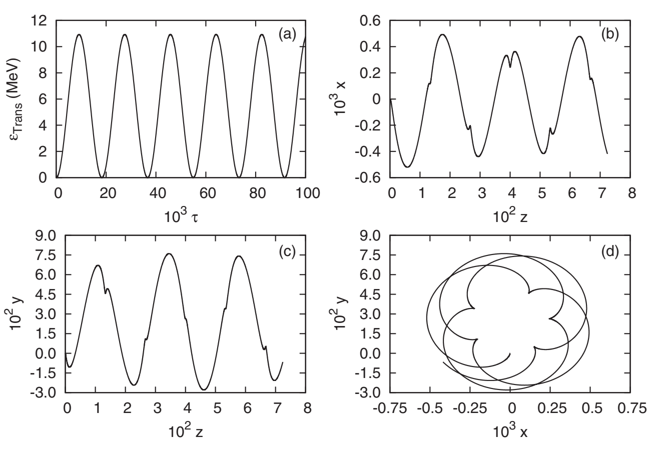

FIG. 5. Temporal evolution of the transverse kinetic energy (a) is plotted along with the trajectory in the *x*-*z* plane (b), *y*-*z* plane (c), and *x*-*y* plane (d) with the detuned phase modulation factor $\alpha = 0.5001$.

enough to make the scheme work in favor of acceleration, i.e., we must still have the wave frequency to approximately be a harmonic of the phase modulation frequency, such that the foundation of the scheme is not destroyed, see Fig. 3(b). Moreover, δ cannot be too small, as it must be large enough to prevent the large excursions [Figs. 4(b) and 4(d)].

The effect of the detuning parameter on the energetics of the particle is presented in Fig. 6. As can be seen from this figure, reducing the detuning parameter from $\delta = 0.0008$ to $\delta = 0.0001$ increases the efficiency of the acceleration significantly. On the other hand, the transverse kinetic energy of the particle is directly related to the transverse excursion of the particle. For lower values of the δ the particle orbits are larger such that the electric field of the wave tends to do more work for each orbital motion of the particle. As the detuning increases the particle orbit becomes shorter and shorter, resulting in lower gain in energy from the wave. Apparently there is an optimum detuning at which one is able to gain maximum energy per orbital cycle of the particle.

So far we have presented all the results with constant magnetic field of amplitude $b_0 = 1$. It can be understood that the magnetic field is also responsible along with the detuning parameter to restrict the excursion of the particle in transverse direction. In view of this, it would be interesting to see how the acceleration efficiency varies with the applied constant magnetic field.

The temporal evolution of the energy varying the constant magnetic field is presented in Fig. 7. The detuned phase modulation parameter in this case is chosen to be 0.5001. It is observed that the proton can now be accelerated to energies of about 386 MeV when the applied magnetic field strength is 0.30. Furthermore, there seems to be an optimum magnetic field for the maximum acceleration of the particle. This behavior can be explained on the

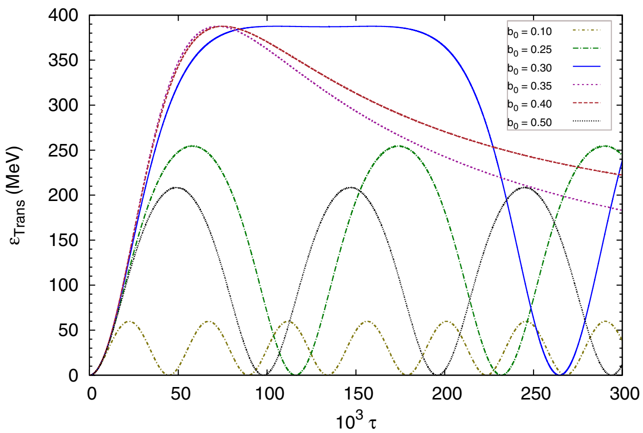

FIG. 7. Temporal evolution of the transverse kinetic energy for various values of applied magnetic fields with detuned phase modulation factor of 0.5001.

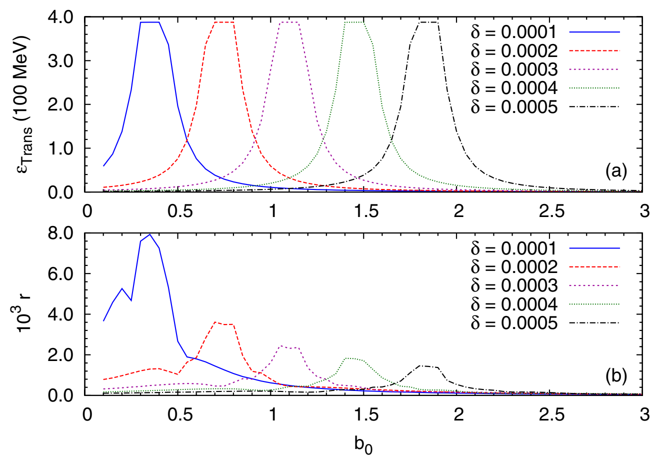

FIG. 9. Maximum energy (a) and radius of the particle orbit in dimensionless units (b) as a function of magnetic field for different values of detuning parameter δ with phase modulation parameter $\alpha = 0.5$.

basis of the Larmor radius of the particle which varies with the applied magnetic field. The magnetic field should be strong enough to bend the particle to avoid large excursion and should not be so large that the particle orbit is very small and the resulting energy gain in one orbital motion is small.

So far we have now established the working principle of the proposed scheme which can accelerate the particles to about 386 MeV of energies. However, it would be also interesting to see the dependence of the maximum energy achievable using the proposed scheme under the given physical parameters. In order to shed some light on the limitations of the proposed acceleration scheme, we have presented the maximum energy as a function of the detuning parameter δ and applied magnetic field b_0 in Fig. 8, with phase modulation factor $\alpha = 0.5$. It can be observed that the maximum energy is independent on the variation of the δ and b_0. The optimum magnetic field for maximum energy increases linearly with the detuning parameter, giving the same maximum energy of the particle. As the optimum magnetic field increases with the increase in the detuning, one can expect that as a consequence particle orbit will reduce with increasing detuning parameter or the magnetic field as can be seen in Fig. 9(b).

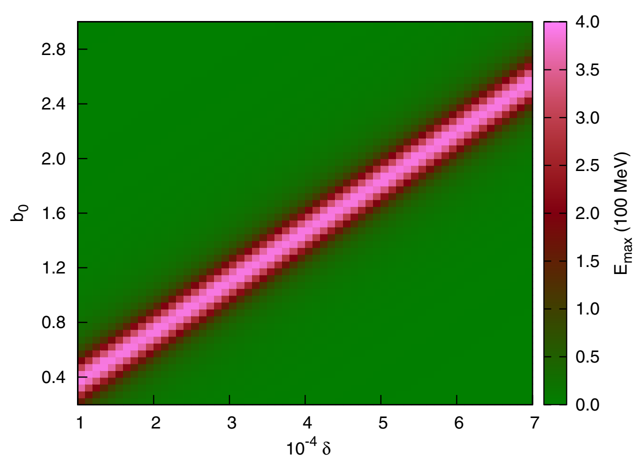

FIG. 8. Maximum energy as a function of magnetic field (b_0) and detuning parameter (δ) for the phase modulation parameter $\alpha = 0.5$.

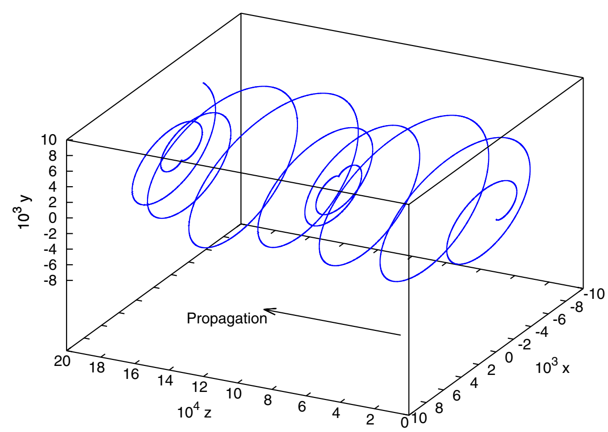

FIG. 10. Particle trajectory under the influence of applied magnetic field of magnitude 0.3. The particle is propagating along the z direction.

The particle orbit for the magnetic field $b_0 = 0.3$ is shown in Fig. 10. As can be seen from Fig. 10 the excursion in the transverse direction is controlled. Thus in this case, particle acceleration using a phase modulated electromagnetic wave propagating along a constant magnetic field seems to be possible in a real scenario.

V. SPECTRAL PROPERTIES

Here we have focused on the understanding of the underlying dynamics of the acceleration. We have established that the phase modulated circularly polarized wave can accelerate particles in the presence of a constant magnetic field. There are optimum conditions on the detuning parameter and the magnitude of the magnetic field, in order to gain maximum energies. However, for the purpose of applications, the spectral properties play a crucial role, in addition to the maximum energy. In view of this it would be important to understand how this scheme works for a particle beam with an energy spread, instead of just a single particle. In order to shed some light on the acceleration dynamics of the particle beam, we have separately simulated the single particle motion when injected with different energies, which can be perceived as the energy spectral spread of the beam.

The time evolution of the transverse kinetic energy of single particle is presented in Fig. 11(a) for different injection energies. The phase modulation factor is considered to be $\alpha = 0.5001$ and the magnitude 0.5 is considered for the constant magnetic field. It can be observed that, even if the injection energies are varied an order of magnitude, the output energy of the beam is not very much affected, keeping the scheme functional. The most important property of the particle beam is its energy distribution function. The energy distribution function of the output beam is calculated in Fig. 11(b) by considering the input beam having a Gaussian energy spectrum with peak energy of 600 keV and FWHM of 166 keV, the energy spread ($\Delta E/E_{\mathrm{peak}}$) of the input beam is about 28%. The peak energy of the output beam is observed to be 167 MeV with FWHM of 5 MeV, with the energy spread about 3%.

The inset in Fig. 11(b) shows the variation of output energy with the injected particle energy. Circles denotes the actual numerical value which is fitted with a 10th order polynomial (solid line) in order to find the energy spectrum. The decline in the output particle energy with input energy can be explained on the basis of the interaction time of the particle with the EM wave. The faster the particle, the lesser is the interaction time with the EM wave, and hence the energy transfer to the particle.

As we have observed in Fig. 11(b), the output particle energy is more or less independent of the injected particle energy, which is apparently visible in the almost monoenergetic energy distribution of the output beam.

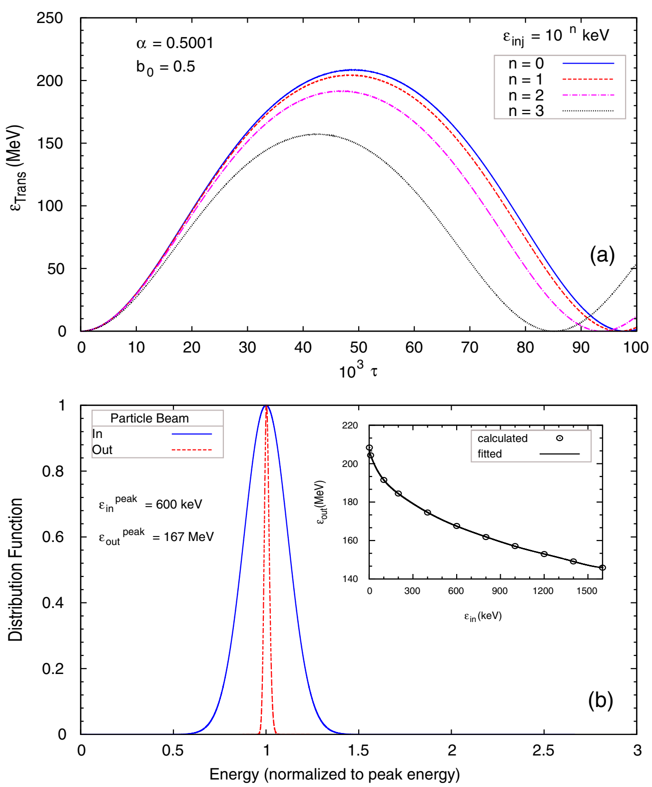

FIG. 11. Temporal evolution of the kinetic energy for different values of injection energies (a). An energy spectrum of the output beam is calculated by injecting a beam with Gaussian energy spectrum (b). The inset in (b) represents the output beam energy as a function of injection energy, circles denote the numerically measured values; however, the solid line is 10th order polynomial fit which is used in calculating the energy distribution function of output beam.

VI. FINAL REMARKS

The dimensionless analysis of the discussed problem seems to fetch prominent results as per as working principle of the acceleration is concerned. It is observed that the maximum proton energy of about 400 MeV can be achieved when driver field (a_0) is 0.1, magnetic field (b_0) is 0.3, and phase modulation factor ($\alpha + \delta$) is 0.5001, radius of the particle trajectory (r_0) in this case found to be 8000 in dimensionless units. The connection between dimensionless units to real world units is as follows: (i) electric field, $E = a_0 m_e \omega c/e$ V/m; (ii) magnetic field, $B = b_0 m_e \omega/e$ Tesla; (iii) radius, $r = r_0 c/\omega$ meters, along with $\omega = 2\pi c/\lambda$. The comparison with different values of the λ is presented in Table I. As can be seen, the requirements of the magnetic field (321 Tesla) are not feasible if we go with the infrared regime. However, in the microwave regime the idea of proton acceleration can be perceived.

A further evaluation of the feasibility of the proposed acceleration scheme needs to go beyond the 1D variations of the electromagnetic fields. In particular, it is clear that picking parameter values corresponding to intense lasers

TABLE I. Parameters comparison for maximum proton energies.

$\lambda(m)$	$\boldsymbol{\omega}$	**E** (V/m)	**B** (Tesla)	**r**(m)
10^{-2}	1.88×10^{11}	3.211×10^{7}	0.321	12.73
10^{-3}	1.88×10^{12}	3.211×10^{8}	3.210	1.273
10^{-4}	1.88×10^{13}	3.211×10^{9}	32.10	0.1273
10^{-5}	1.88×10^{14}	3.211×10^{10}	321.0	0.012 73

gives successful results for the maximum particle energies. However, a potential limitation is that large transverse particle excursions exclude the use of very focused pulses, in which case the available laser intensity drops accordingly. To some extent this may be remedied with a strong value of the static magnetic field, which limits the transverse excursion. In practice, magnetic field strengths well beyond 100 T is needed if the system works in the optical laser regime, which makes this regime less attractive. Decreasing the wave frequency in the scheme reduces the need for extreme magnetic field strengths, since we may allow for somewhat larger particle excursions. The optimal frequency regime may lie in the infrared regime or lower, but a full 3D analysis is needed to optimize the parameters in a realistic scenario. This remains a project for further research.

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