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On the Ergodic Achievable Rates of Spectrum Sharing Networks with Finite Backlogged Primary Users and an Interference Indicator Signal

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Abstract

Spectrum sharing networks are communication setups in which unlicensed secondary users (SUs) are permitted to work within the spectrum resources of licensed primary users (PUs). This paper aims to study the ergodic achievable rates of spectrum sharing networks with finite backlogged primary user and an interference indicator signal. Here, in contrast to the standard interference-avoiding schemes, the secondary user activity is not restricted within the primary user inactive periods. Considering both fading and nonfading channels, the unlicensed user ergodic achievable rate is obtained for different unlicensed user transmission power and licensed user received interference power or signal-to-interference-and-noise (SINR) constraints. In the case of fading channels, the results are obtained for both short- and long-term primary user quality-of-service requirements. Further, the results are generalized to the case of multiple interfering users. In terms of unlicensed user ergodic achievable rate, analytical results indicate that while the standard interference-avoiding approach is the optimal transmission scheme at low secondary user or high primary user transmission powers, higher rates can be achieved via simultaneous transmission at high secondary user SINRs. Moreover, simulation results show that, using an interference indicator signal, there is considerable potential for data transmission of unlicensed users under different licensed users quality-of-service requirements.

I. Introduction

Spectrum sharing networks are initiated by the apparent lack of spectrum under the current spectrum management policies. Currently, most frequency bands useful to wireless communication are under control of primary license holders that have exclusive right to transmit over their spectral bands. This is the point that has created the perception of spectrum shortage, leading to ever-growing complaints about available spectral resources. On the other hand, recent studies such as [1], [2] show that at any given time large portions of the licensed bands remain unused. Therefore, it is expected that we can improve the data transmission strategies by better utilizing the licensed resources. Spectrum sharing network is one of the most promising techniques created for this purpose.

In general, the goal of spectrum sharing methods, normally modeled as interference [3], cooperative [4] or cognitive radio [5]–[8] networks, is to better utilize the radio spectrum by allowing the unlicensed secondary users (SUs) to coexist with the licensed primary users (PUs). Along with the standard interference channel, where two independent transmitters transmit independent messages to two independent receivers, there are other ways to exploit the idea of spectrum sharing. For example, in a method widely referred as the *interference-avoiding paradigm* [8]–[10], the unlicensed users are permitted to work within the PUs inactive resources. That is, provided that the secondary transmitter can sense the spatial, temporal or spectral gaps of the primary resources, it can adjust its transmission parameters to fill these white spaces. Although this approach can theoretically lead to significant spectral efficiency improvement, it is not desirable in online applications, as the SU transmission is decided based on the PU activation status. In another scheme, normally denoted *simultaneous or controlled transmission* [11], [12], a secondary user can simultaneously coexist with a primary user as long as it works under a certain interference level imposed by the primary user quality-of-service requirements. In these methods, limits on the interference level received at the PU receiver, referred to as the interference temperature, can be considered to be long-term average or short-term peak constraints. Finally, there are the *cooperative networks* in which the two users normally know each others messages prior to transmission, and can potentially help each other to improve the overall efficiency.

Assuming different licensed users quality-of-service requirements, several results about the performance limits of spectrum sharing networks have been presented in the last decade. For instance, in [7] and [8], the authors demonstrated some information theoretic models, limits and open problems of spectrum sharing networks. [13] studied the SU-SU channel capacity under different PU outage constraints. Then, assuming perfect SU-PU channel state information (CSI), [14] investigated the effect of optimal power allocation on the capacity of the secondary channel under different power constraints. With both peak and average interference power constraints, [15] studied the capacity of the secondary channel in the case where all channels are fully known by the SU transmitter. In [16], considering a path loss shadow-fading model with multiple primary and secondary users, the system-level capacities of spectrum sharing networks under an average interference power constraint were analyzed.

The amount of channel state information available at the SU transmitter and receiver is one of the most important issues attracting much attention in the recent years; allowing limited interference power at the PU receiver, Ji *et al.*, [17] studied the capacity of multicast spectrum sharing networks. In their work, while the SU-SU link is assumed to be perfectly known, the results are obtained for the cases where the interference information is perfectly or imperfectly available at the SU transmitter. Furthermore, [18] investigated the ergodic, the outage, and the minimum-rate capacity of spectrum sharing network under average and peak interference constraints. This work, which was based on perfect SU-PU CSI, was later extended by [19] where the same results were obtained under imperfect CSI feedback assumption. Considering different imperfect CSI model and interference constraints, [20] presented the same results as [19] and verified the effect of feedback quantization as well.

Reviewing the literature, there are some points that are the main motivators for this paper:

- Interference-avoiding and simultaneous schemes are normally thought as two different kinds of networks, namely, overlay and underlay networks, with little connections. To the best of our knowledge, all developed simultaneous transmission approaches, e.g., [11]–[20], have tackled the problem under infinite backlogged (or sometimes called full buffer) PU assumption. That is, it is assumed that the PU transmitter has an infinite amount of information to be communicated. This, however, is not generally valid [1], [2]. On the other hand, the interference-avoiding methods permit no data transmission within the PU transmission time slots, reducing their practicality in online applications [8]–[10]. Importantly, it is not clear which of these methods is the best in different conditions and they are normally selected based on the designers interests. (Meanwhile, [21] and [22] have recently considered hybrid overlay/underlay schemes respectively in OFDM cognitive radio channels using an iterative dual decomposition-based power allocation algorithm and in multiple relay-assisted cognitive radios when the PU transmitter is far from the SU.)
- The SU data transmission efficiency is normally investigated under a PU received interference power constraint. However, the PU received SINR[1], which plays an important role in different PU quality-of-service requirements, has not been thoroughly studied [23]. However, as shown in the following, although

[1]Signal-to-interference-and-noise-ratio.

these constraints are interchangeable in some conditions, there are cases where different results are obtained under either of these conditions.

As illustrated in Table 1, this paper investigates the secondary channel ergodic achievable rates in the case of finite backlogged primary user and in the presence of an interference indicator signal. To be more specific, we focus on the case where the primary user turns on only for a portion of time slots indicated to the SU transmitter and receiver via an interference indicator signal. Here, in contrast to interference-avoiding schemes, the SU activity is not restricted within the PU inactive periods and is decided only based on the considered quality-of-service requirements. The goal is to determine the channel ergodic achievable rate and evaluate the effect of different quality-of-service requirements on the network data transmission performance. In this way, we can study the conditions in which the interference-avoiding approach outperforms the simultaneous transmission scheme, and vice versa.

The results are first presented for the nonfading AWGN channels (Section III). Then, considering fading channels, the SU-SU channel ergodic achievable rate is obtained under the SU transmission power and the PU received interference power or SINR constraints (Section IV). Here, the achievable rates are determined under perfect CSI assumption. Further, the results are generalized to the case when arbitrary number of users, experiencing different fading conditions, share the same frequency band for data transmission. Analytical results indicate that while the standard interference-avoiding approach is the optimal[2] transmission scheme at low SU or high PU transmission powers, higher rates can be achieved via simultaneous transmission at high secondary user SINRs. Moreover, it is shown that, depending on the fading conditions and the PU quality-of-service requirements, there are cases where increasing the PU transmission power leads to higher SU-SU channel achievable rate. Finally, simulation results show that, using an interference indicator signal, there is considerable potential for data transmission of unlicensed users under different licensed users quality-of-service requirements.

II. System model

We consider a standard block-fading spectrum sharing network where two primary and secondary users share the same narrow-band frequency with bandwidth B. With no loss of generality we set $B = 1$. Let H_{pp}, H_{ps}, H_{sp}

[2]Here, all optimality conditions are in terms of the SU-SU channel ergodic achievable rate under the considered quality-of service requirements.

and H_{ss} be the fading random variables in the PU-PU, PU-SU, SU-PU and SU-SU links, respectively, which are assumed to be mutually independent. Correspondingly, we define $G_{\text{pp}} \doteq |H_{\text{pp}}|^2$, $G_{\text{ps}} \doteq |H_{\text{ps}}|^2$, $G_{\text{sp}} \doteq |H_{\text{sp}}|^2$ and $G_{\text{ss}} \doteq |H_{\text{ss}}|^2$ which are denoted *channel gains* in the following. Moreover, let $f_{G_{\text{pp}}}$, $f_{G_{\text{ps}}}$, $f_{G_{\text{sp}}}$ and $f_{G_{\text{ss}}}$ represent the gains corresponding probability density functions (pdfs). Both fading and nonfading AWGN channels are investigated. In the case of fading channels, simulation results are obtained for Rayleigh-fading channels. However, the theoretical arguments are valid for a fairly general case where the gain pdfs can be combination of different continuous functions taking positive values over the entire range $(0, \infty)$. The gains remain constant for a duration, normally called a fading block, and then change independently according to their corresponding pdfs. The white complex Gaussian noises added at the PU and the SU receivers, which are denoted by Z_{p} and Z_{s}, are supposed to have complex Gaussian distributions $\mathcal{CN}(0, \delta_{\text{p}}^2)$ and $\mathcal{CN}(0, \delta_{\text{s}}^2)$, respectively. Finally, all results are presented in natural logarithm basis, in all simulations the channel achievable rate is presented in nats-per-channel-use (npcu) and, as illustrated in the following, are restricted to Gaussian input pdfs.

We assume that the secondary user has infinitely many information nats for transmission so that the SU-SU communication link is *continuous* [24]–[26]. On the other hand, the PU transmitter is active only for a portion of time, in harmony with practical investigations reported by, e.g., [1], [2][3]. The SU transmitter and receiver are assumed to have knowledge about the PU activation status. This information can be obtained in different ways; either the SU transmitter or the receiver can detect the PU interference signal and then inform the other side via one bit *interference indicator signal*. Another possible approach may be the implementation of a *band manager* mediating between the two parties [27]. The same setup also represents the cases where the PU is active for specific predetermined periods. Finally, as stated in, e.g., [5]–[7], we can assume the presence of a *genie* informing the SU about the PU activation status. Then, as the blocks are long, enough time can be considered such that PU activation status and the channels quality information are perfectly provided at the SU end-points. Let A be the PU status indicator in which $A = 1$ ($A = 0$) represents its activeness (inactiveness). In this way, the SU received signal can be stated as

$$Y_{\text{s}} = \begin{cases} X_{\text{s}}H_{\text{ss}} + X_{\text{p}}H_{\text{ps}} + Z_{\text{s}} & \text{if } A = 1 \\ X_{\text{s}}H_{\text{ss}} + Z_{\text{s}} & \text{if } A = 0 \end{cases} \tag{1}$$

[3]Extension of the results to the case where both the primary and the secondary users may have no transmission signals is straightforward.

in which $X_{\rm p}$ and $X_{\rm s}$ represent the primary and the secondary users input messages, respectively, and $Y_{\rm s}$ denotes the SU output signal. Also, in harmony with the literature, e.g., [15]–[20], we assume that the PU transmitted signal, whenever available, is made of zero-mean Gaussian codewords with power of $E[|X_{\rm p}|^2] = T_{\rm p}$.

Finally, the following procedure is applied throughout the paper to determine the channel ergodic achievable rate under different conditions: 1) Find the SU-SU channel ergodic achievable rate for a fixed SU transmission power, 2) determine the optimal power allocation criteria based on the SU transmission power constraint and 3) refine the SU power allocation strategy based on the PU quality-of-service requirements.

III. SU-SU CHANNEL ERGODIC ACHIEVABLE RATE IN THE PRESENCE OF INTERFERENCE INDICATOR SIGNAL; NONFADING AWGN CHANNEL SCENARIO

This part focuses on the nonfading AWGN channel which, although it is a special case of the more general fading channels, is more analytically tractable and, therefore, more instructive. Without loss of generality, we set the channel gains to $H_{\rm pp} = 1$, $H_{\rm ps} = h_{\rm ps}$, $H_{\rm sp} = h_{\rm sp}$ and $H_{\rm ss} = 1$. Note that, with proper scaling, every other specification of the gains and noise variances can be mapped to this case. Under such condition, the channel ergodic achievable rate in presence of the interference indicator signal is obtained by[4]

$$C_{\rm s} = \max_{f_{X_{\rm s}|A}} I(X_{\rm s};Y_{\rm s}|A) = \alpha \max_{f_{X_{\rm s}|A=0}} I(X_{\rm s};Y_{\rm s}|A=0) + (1-\alpha) \max_{f_{X_{\rm s}|A=1}} I(X_{\rm s};Y_{\rm s}|A=1) \tag{2}$$

in which $\alpha = \Pr\{A = 0\}$ is the PU inactiveness probability, $I(W;Q) = h(W) - h(W|Q)$ denotes the mutual information between two random variables W and Q and $h(W) = -\int_{-\infty}^{\infty} f_W(w)\log(f_W(w))\mathrm{d}w$ is the differential entropy of the variable W having pdf $f_W(w)$ [28]. Here, independent of the PU activeness status, the secondary channel is a nonfading AWGN channel and so, under power-limited condition, the channel ergodic achievable rate is obtained by Gaussian signals at the transmitter and typical-set based decoding at the receiver which, using (1) and (2), yields to

[4]Here, the results are obtained for simple decoders where the PU signal is treated by the SU receiver as an additive interference. However, the results provide some insights for the cases where joint decoders, which increase the achievable rates by decoding the PU signal, are implemented at the SU receiver. For instance, we can assume that α represents the ratio of times that the PU signal is successfully decoded by the SU receiver. Then, for the rest of the time the PU is treated as additive interference, as it can not be decoded by the SU. Further performance analysis in the presence of joint decoders is an interesting extension of the paper. Finally, note that in order to implement joint decoders the PU codebook must be available at the SU receiver which may not be possible in practice.

$$C_{\rm s} = \alpha\left\{h(X_{\rm s}+Z_{\rm s}) - h(X_{\rm s}+Z_{\rm s}|X_{\rm s})\right\}\Big|_{X_{\rm s}\sim\mathcal{CN}(0,T_0)}$$
$$+(1-\alpha)\left\{h(X_{\rm s}+X_{\rm p}h_{\rm ps}+Z_{\rm s}) - h(X_{\rm s}+X_{\rm p}h_{\rm ps}+Z_{\rm s}|X_{\rm s})\right\}\Big|_{X_{\rm s}\sim\mathcal{CN}(0,T_1)} . \quad (3)$$
$$= \alpha\log(1+\tfrac{T_0}{\delta_{\rm s}^2}) + (1-\alpha)\log(1+\tfrac{T_1}{\delta_{\rm s}^2+T_{\rm p}g_{\rm ps}}),\ g_{\rm ps} = |h_{\rm ps}|^2$$

Here, T_0 and T_1 are the SU transmission powers for the PU inactive and active conditions, respectively, and the SU average transmission power is found as

$$\bar{T}_{\rm s} = \alpha T_0 + (1-\alpha)T_1. \quad (4)$$

SU transmission power constraints: There may be different power constraints in a spectrum sharing network; due to, e.g., hardware or complexity limitations, there are cases where, independently of the channels conditions, the power allocated can not exceed a maximum value T_{total}. In this case, as the transmission rate of AWGN channels is an increasing function of the SINR [29], the optimal powers maximizing the SU-SU channel ergodic achievable rate are obtained by $T_0 = T_1 = T_{\text{total}}$, which is normally called *short-term* or *instantaneous* power allocation [24]–[26], [30], [31]. Under the more relaxed *long-term* power constraint, which is normally considered for systems with limited energy resources such as battery-limited systems [24]–[26], [30], [31], the transmitter can adapt the power based on the channels (and the PU activeness) conditions such that $\bar{T}_{\rm s} \le T_{\text{total}}$.[5] In this way, the optimal powers maximizing the channel ergodic achievable rate can be found by (3), (4) and a Lagrange multiplier function $\Upsilon = C_{\rm s} + \rho\bar{T}_{\rm s}$ leading to the following water-filling equations

$$\frac{\partial\Upsilon}{\partial T_0} = \frac{\partial\Upsilon}{\partial T_1} = 0 \Rightarrow \begin{cases} \hat{T}_0 = \left\lceil \frac{-1}{\rho} - \delta_{\rm s}^2 \right\rceil^+ & \text{(I)} \\ \hat{T}_1 = \left\lceil \frac{-1}{\rho} - T_{\rm p}g_{\rm ps} - \delta_{\rm s}^2 \right\rceil^+ & \text{(II)} \end{cases} . \quad (5)$$

Here, ρ is the Lagrange multiplier satisfying $\bar{T}_{\rm s} \le T_{\text{total}}$ constraint and $\lceil x\rceil^+ \doteq \max(0,x)$.

Remark 1: Intuitively, using optimal (long-term) power allocation the power is not wasted on *weak* channel realizations and the saved power is spent on *good* channel conditions. Furthermore, (5) has an interesting intuitive consequence; as we have $\hat{T}_1 \le \hat{T}_0, \forall T_{\text{total}}$, there is a SU average transmission power threshold T_{thr} under which we have $T_1 = 0$ (water-filling). That is, interference-avoiding is the best transmission scheme for $T_{\text{total}} \le T_{\text{thr}}$. On the other hand, under $T_{\text{total}} \ge T_{\text{thr}}$ condition, higher rates are achieved by continuous

[5]Throughout the paper, T_{total} represents both the short- and the long-term power constraints. This is particularly because with short-term power allocation the average power is the same as the instantaneous power equal to T_{total}. Moreover, using the same notation allows us to easily compare the simulation results under these constraints in a single figure (for instance, in Fig.8).

communication of the secondary user (simultaneous transmission scheme), which is of course at the cost of the PU received SINR reduction. Finally, in order to find the threshold, we can write

$$\begin{cases} (1-\alpha)(\frac{-1}{\rho} - T_{\text{p}}g_{\text{ps}} - \delta_{\text{s}}^2) + \alpha(\frac{-1}{\rho} - \delta_{\text{s}}^2) = T_{\text{thr}} \\ \frac{-1}{\rho} - T_{\text{p}}g_{\text{ps}} - \delta_{\text{s}}^2 = 0 \end{cases} \Rightarrow T_{\text{thr}} = \alpha T_{\text{p}}g_{\text{ps}}.$$

PU quality-of-service requirements: Short- and long-term power constraints provide the SU-SU channel ultimate performance when no (or extremely relaxed) constraint is imposed by the PU. However, there are always some limitations imposed by the PU quality-of-service requirements. Particularly, limiting the received PU interference power, which for nonfading channel is $T_1 g_{\text{sp}}$, $g_{\text{sp}} = |h_{\text{sp}}|^2$, we can use (5) to determine the optimal transmission powers as

$$\begin{cases} \hat{T}_0 = \max\left\{\left\lceil\frac{-1}{\rho} - \delta_{\text{s}}^2\right\rceil^+, \frac{T_{\text{total}} - (1-\alpha)\frac{\varphi}{g_{\text{sp}}}}{\alpha}\right\} & \text{(I)} \\ \hat{T}_1 = \min\left\{\left\lceil\frac{-1}{\rho} - T_{\text{p}}g_{\text{ps}} - \delta_{\text{s}}^2\right\rceil^+, \frac{\varphi}{g_{\text{sp}}}\right\} & \text{(II)} \end{cases} \quad (6)$$

where φ is the maximum tolerable received PU interference power[6]. On the other hand, as the PU received SINR is $\text{SINR}_{\text{p}} = \frac{T_{\text{p}}}{\delta_{\text{p}}^2 + T_1 g_{\text{sp}}}$, constraining the received SINR to be higher than a given value θ, i.e., $\text{SINR}_{\text{p}} = \frac{T_{\text{p}}}{\delta_{\text{p}}^2 + T_1 g_{\text{sp}}} \geq \theta$, leads to the following power allocation criteria

$$\begin{cases} \hat{T}_0 = \max\{\left\lceil\frac{-1}{\rho} - \delta_{\text{s}}^2\right\rceil^+, \frac{T_{\text{total}} - \frac{(1-\alpha)}{g_{\text{sp}}}\left\lceil\frac{T_{\text{p}}}{\theta} - \delta_{\text{p}}^2\right\rceil^+}{\alpha}\} & \text{(I)} \\ \hat{T}_1 = \min\{\left\lceil\frac{-1}{\rho} - T_{\text{p}}g_{\text{ps}} - \delta_{\text{s}}^2\right\rceil^+, \frac{1}{g_{\text{sp}}}\left\lceil\frac{T_{\text{p}}}{\theta} - \delta_{\text{p}}^2\right\rceil^+\} & \text{(II)} \end{cases}. \quad (7)$$

Remark 2: Considering, e.g., (6) and (7), the following conclusions are interesting; Although the PU received interference power and SINR constraints are interchangeable in some conditions, there are cases where different results are obtained under either of these conditions. For instance, while we can always allocate some SU transmission power under every nonzero PU average interference constraint, no transmission is permitted to the secondary user under PU average received SINR constraints less than the PU received SNR[7]. Moreover, as discussed in the following, in contrast to interference-limited condition, the PU transmission power is not necessarily something *bad* for the secondary channel under SINR limited conditions. In other words, as stated in, e.g., Fig. 6 and 7, increasing the PU transmission power can lead to higher SU transmission rates under PU received SINR constraints, as more relaxed power allocation can be done at the SU transmitter.

[6]To have unified notations throughput the paper, the same Lagrange multiplier parameter ρ is used for different power allocation criteria. However, in each case the parameter is determined based on the corresponding PU quality-of-service requirement and the SU power constraint.

[7]Signal-to-noise-ratio.

A. Simulation results

Setting $|h_{\text{sp}}|^2 = |h_{\text{ps}}|^2 = 1$ and $\alpha = 0.7$, Fig.1.a verifies the effect of different power allocation strategies on the SU-SU channel ergodic achievable rate. Moreover, the results are compared with the ones obtained by the interference-avoiding scheme, in which we have $T_1 = 0, \forall T_{\text{total}}$. Here, and in all other simulations, the primary and the secondary noise variances are set to $\delta_{\text{p}}^2 = 1$, $\delta_{\text{s}}^2 = 1$. Then, Fig.1.b demonstrates the optimal SU simultaneous transmission power, i.e., T_1, as a function of the SU average transmission power T_{total}. Note that for a given T_1, the power term T_0 is obtained easily such that the average power constraint $\bar{T}_{\text{s}} \leq T_{\text{total}}$ is satisfied with equality. As it can be seen, considering different PU quality-of-service requirements, there is a threshold under which the maximum rates are achieved by the interference-avoiding approach. Further discussions about the simulation results are presented in Section V.

B. Extension to multiple interfering users case

The results can be generalized to the case where there are $M \geq 1$ interfering users utilizing the same spectral resources. Note that the users considered in this part are not necessarily the license holders but are the users that, while sharing the same spectrum, are out of control for the considered secondary user[8]. Again, it is assumed that the interfering users' activeness status is known by the SU transmitter and receiver. Let $\tilde{J} \subset \{1, \ldots, M\}$ be the set of active interfering users and denote the SU transmission power in this case by $T_{\tilde{J}}$. Therefore, representing the gains by $h_{\text{p}_j\text{s}}, j = 1, \ldots, M$, $h_{\text{ss}} = 1$, the SU received SINR is

$$\text{SINR}_{\text{s}_{\tilde{J}}} = \frac{T_{\tilde{J}}}{\delta_{\text{s}}^2 + \sum_{j \in \tilde{J}} T_{\text{p}_j} g_{\text{p}_j\text{s}}},\ g_{\text{p}_j\text{s}} = |h_{\text{p}_j\text{s}}|^2 \tag{8}$$

and the maximum achievable rate of the channel would be

$$C_{\text{s}_{\tilde{J}}} = \max_{f_{X_\text{s}|\tilde{J}}} I(X_\text{s}; X_\text{s} + \sum_{j \in \tilde{J}} X_{\text{p}_j} h_{\text{p}_j\text{s}} + Z_\text{s}|\tilde{J}) = \log(1 + \frac{T_{\tilde{J}}}{\delta_{\text{s}}^2 + \sum_{j \in \tilde{J}} T_{\text{p}_j} g_{\text{p}_j\text{s}}}). \tag{9}$$

Consequently, the channel ergodic achievable rate is found as

$$\begin{aligned} &C_\text{s} = \textstyle\sum_{\forall \tilde{J} \subset \{1,\ldots,M\}} \gamma_{\tilde{J}} \log(1 + \frac{T_{\tilde{J}}}{\delta_{\text{s}}^2 + \sum_{j \in \tilde{J}} T_{\text{p}_j} g_{\text{p}_j\text{s}}}), \\ &\gamma_{\tilde{J}} \doteq (\prod_{\forall j \in \tilde{J}} (1 - \alpha_j))(\prod_{\forall j \in \tilde{J}^c} \alpha_j) \end{aligned} \tag{10}$$

and the SU average transmission power is

[8]In other words, the results can be considered as the achievable rates of a specific link in a network with $M + 1$ spectrum sharing users with no cooperation between the users.

$$\bar{T}_{\rm s} = \sum_{\forall \tilde{J} \subset \{1,\ldots,M\}} \gamma_{\tilde{J}} T_{\tilde{J}}. \tag{11}$$

Here, α_j is the inactiveness probability of the j-th interfering user and $\tilde{J}^c = \{1,\ldots,M\} - \tilde{J}$ is the complement set of $\tilde{J}$. Finally, the optimal SU transmission powers are found by the same objective function as before resulting in

$$\hat{T}_{\tilde{J}} = \left[\frac{-1}{\rho} - \delta_{\rm s}^2 - \sum_{j \in \tilde{J}} T_{{\rm p}_j} g_{{\rm p}_j{\rm s}}\right]^+. \tag{12}$$

Again, the Lagrange multiplier ρ is determined according to the SU transmission power constraint $\bar{T}_{\rm s} \le T_{\rm total}$.

Equations (8)-(12) are particularly simplified if the interfering users have the same characteristics, i.e., the same inactiveness probability α, gains $h_{\rm ps}$ and transmission powers $T_{\rm p}$. In this case, independent of the interfering users indices, the same power T_m is considered if m users interference signals are detected[9]. Therefore, with the same arguments as before, the channel ergodic achievable rate and the optimal transmission powers, i.e., (10) and (12), are respectively rephrased as

$$C_{\rm s} = \sum_{m=0}^{M} \binom{M}{m} \alpha^{M-m} (1-\alpha)^m \log(1 + \frac{\hat{T}_m}{\delta_{\rm s}^2 + m T_{\rm p} g_{\rm ps}}), \tag{13}$$

and

$$\hat{T}_m = \left[\frac{-1}{\rho} - \delta_{\rm s}^2 - m T_{\rm p} g_{\rm ps}\right]^+ \tag{14}$$

where $\binom{M}{m}$ is the "M choose m" operator. Note that setting $M = 1$ the results are simplified to the ones obtained in (3)-(5). Finally, as the extension to other constraints is straightforward, we do not discuss them any further. Also, the simulation results for the case of multiple interfering users are presented in Fig.8.

IV. Channel ergodic achievable rate in the presence of interference indicator signal; fading AWGN channel scenario

Assuming the fading channel with perfect CSI at the transmitters and the receivers, the power-limited secondary channel ergodic achievable rate, i.e., (2), changes to

[9]The results of this part can be easily adopted for other situations such as the case where a primary user transmission power is selected among a finite set of powers. In general, the received SU SINR is a sufficient statistics for the SU power allocation and there is no need to have information about every individual interfering user.

$$
\begin{aligned}
C_{\mathrm{s}} &= \max_{f_{X_{\mathrm{s}}|A,G_{\mathrm{ss}},G_{\mathrm{ps}}}} I(X_{\mathrm{s}};Y_{\mathrm{s}}|A,G_{\mathrm{ss}},G_{\mathrm{ps}}) \\
&= \alpha \max_{f_{X_{\mathrm{s}}|A=0,G_{\mathrm{ss}}}} \underbrace{I(X_{\mathrm{s}};Y_{\mathrm{s}}|A=0,G_{\mathrm{ss}})}_{\Gamma_0} + (1-\alpha) \max_{f_{X_{\mathrm{s}}|A=1,G_{\mathrm{ss}},G_{\mathrm{ps}}}} \underbrace{I(X_{\mathrm{s}};Y_{\mathrm{s}}|A=1,G_{\mathrm{ss}},G_{\mathrm{ps}})}_{\Gamma_1}.
\end{aligned}
\tag{15}
$$

The term Γ_0 in the last equality is simply the ergodic achievable rate of an interference-free SISO channel with perfect CSI at the transmitter and the receiver which using Gaussian input distribution and typical-set based decoding is obtained by

$$\Gamma_0 = E_{G_{\mathrm{ss}}}\{h(Y_{\mathrm{s}}|G_{\mathrm{ss}}=g) - h(Y_{\mathrm{s}}|X_{\mathrm{s}},G_{\mathrm{ss}}=g)\} = \int_0^\infty f_{G_{\mathrm{ss}}}(g)\log(1+\tfrac{gT_0}{\delta_{\mathrm{s}}^2})\mathrm{d}g. \tag{16}$$

Here, $E_{G_{\mathrm{ss}}}(.)$ is the expectation with respect to random variable G_{ss} and again T_0 denotes the SU transmission power considered in the PU inactive condition. On the other hand, as there is no information about the PU Gaussian transmitted signal (except its presence), the channel in the presence of the primary user is an interference-affected SISO channel. Therefore, the SU received SINR is a random variable

$$\mathrm{SINR}_{\mathrm{s}} = T_1\Phi_{\mathrm{s}},\ \Phi_{\mathrm{s}} \doteq \frac{G_{\mathrm{ss}}}{\delta_{\mathrm{s}}^2 + T_{\mathrm{p}}G_{\mathrm{ps}}} \tag{17}$$

in which T_1 is the SU transmission power and Φ_{s} is defined as an auxiliary random variable with cumulative distribution function (cdf) given by

$$
\begin{aligned}
F_{\Phi_{\mathrm{s}}}(x) &= \Pr\{\Phi_{\mathrm{s}} \le x\} = \Pr\{\frac{G_{\mathrm{ss}}}{\delta_{\mathrm{s}}^2 + T_{\mathrm{p}}G_{\mathrm{ps}}} \le x\} \\
&= \int_0^\infty f_{G_{\mathrm{ps}}}(y)F_{G_{\mathrm{ss}}}(x(\delta_{\mathrm{s}}^2 + T_{\mathrm{p}}y))\mathrm{d}y = E_{G_{\mathrm{ps}}}\{F_{G_{\mathrm{ss}}}(x(\delta_{\mathrm{s}}^2 + T_{\mathrm{p}}G_{\mathrm{ps}}))\}.
\end{aligned}
\tag{18}
$$

Therefore, the channel ergodic achievable rate under active PU condition, i.e., Γ_1 in (15), is found as

$$\Gamma_1 = E_{\Phi_{\mathrm{s}}}\{\log(1+T_1\Phi_{\mathrm{s}})\} = \int_0^\infty f_{\Phi_{\mathrm{s}}}(x)\log(1+T_1x)\mathrm{d}x \overset{(a)}{=} \int_0^\infty \tfrac{1-F_{\Phi_{\mathrm{s}}}(x)}{1+T_1x}T_1\mathrm{d}x \tag{19}$$

where $f_{\Phi_{\mathrm{s}}}(x)$ is the auxiliary variable pdf and (a) is obtained by partial integration. For instance, considering Rayleigh-fading channels, e.g., $f_{G_{\mathrm{ss}}}(g) = \lambda_{\mathrm{ss}}e^{-\lambda_{\mathrm{ss}}g}, g \ge 0$, (16), (18) and (19) are respectively determined as

$$\Gamma_0 = e^{\frac{\lambda_{\mathrm{ss}}\delta_{\mathrm{s}}^2}{T_0}}\mathrm{Ei}(-\frac{\lambda_{\mathrm{ss}}\delta_{\mathrm{s}}^2}{T_0}), \tag{20}$$

$$F_{\Phi_{\mathrm{s}}}(x) = 1 - \frac{\lambda_{\mathrm{ps}}e^{-\lambda_{\mathrm{ss}}\delta_{\mathrm{s}}^2x}}{\lambda_{\mathrm{ps}} + \lambda_{\mathrm{ss}}T_{\mathrm{p}}x}, x \ge 0, \tag{21}$$

and

$$\Gamma_1 = \frac{e^{\frac{\lambda_{\mathrm{ss}}\delta_{\mathrm{s}}^2}{T_1}}\mathrm{Ei}(-\frac{\lambda_{\mathrm{ss}}\delta_{\mathrm{s}}^2}{T_1}) - e^{\frac{\lambda_{\mathrm{ps}}\delta_{\mathrm{s}}^2}{T_{\mathrm{p}}}}\mathrm{Ei}(-\frac{\lambda_{\mathrm{ps}}\delta_{\mathrm{s}}^2}{T_{\mathrm{p}}})}{1 - \frac{\lambda_{\mathrm{ss}}T_{\mathrm{p}}}{\lambda_{\mathrm{ps}}T_1}}. \tag{22}$$

Here, λ_{ss} and λ_{ps} are the exponential parameters of the SU-SU and PU-SU fading gains normally determined by the path loss and shadowing between the terminals and Ei(.) is the standard exponential integral function.

Finally, using (15), (16) and (19), the channel ergodic achievable rate is obtained by

$$C_{\text{s}} = \alpha \int_0^{\infty} f_{G_{\text{ss}}}(g) \log(1 + \frac{gT_0}{\delta_{\text{s}}^2})\text{d}g + (1-\alpha)\int_0^{\infty} \frac{1-F_{\Phi_{\text{s}}}(x)}{1+T_1 x}T_1 \text{d}x \tag{23}$$

which for Rayleigh-fading channels and fixed transmission powers is simplified to

$$C_{\text{s}} = \alpha e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_0}}\text{Ei}(-\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_0}) + (1-\alpha)\frac{e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_1}}\text{Ei}(-\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_1}) - e^{\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}}}\text{Ei}(-\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}})}{1-\frac{\lambda_{\text{ss}}T_{\text{p}}}{\lambda_{\text{ps}}T_1}}. \tag{24}$$

SU transmission power constraints: Again, the short-term power constraint implies that $T_0 = T_1 = T_{\text{total}}$. Under the long-term (optimal) power constraint, as the SU average transmission power is

$$\bar{T}_{\text{s}} = \alpha \int_0^{\infty} T_0 f_{G_{\text{ss}}}(x)\text{d}x + (1-\alpha)\int_0^{\infty} T_1 f_{\Phi_{\text{s}}}(u)\text{d}u, \tag{25}$$

we can use the Lagrange objective function $\Upsilon = C_{\text{s}} + \rho\bar{T}_{\text{s}}$ to find the optimal powers as

$$\begin{cases} \hat{T}_0(x) = \left\lceil \frac{-1}{\rho} - \frac{\delta_{\text{s}}^2}{x}\right\rceil^+ \\ \hat{T}_1(u) = \left\lceil \frac{-1}{\rho} - \frac{1}{u}\right\rceil^+ \end{cases}. \tag{26}$$

Letting $\bar{T}_{\text{s}} \leq T_{\text{total}}$, the Lagrange multiplier ρ is found as the solution of the equation $\alpha \int_{-\delta_{\text{s}}^2\rho}^{\infty}(\frac{-1}{\rho} - \frac{\delta_{\text{s}}^2}{x})f_{G_{\text{ss}}}(x)\text{d}x + (1-\alpha)\int_{-\rho}^{\infty}(\frac{-1}{\rho} - \frac{1}{u})f_{\Phi_{\text{s}}}(u)\text{d}u = T_{\text{total}}$ which for Rayleigh-fading channels leads to

$$\begin{aligned} &\rho = \arg_{\rho}\{\alpha \textstyle\int_{-\delta_{\text{s}}^2\rho}^{\infty}(\frac{-1}{\rho} - \frac{\delta_{\text{s}}^2}{x})f_{G_{\text{ss}}}(x)\text{d}x + (1-\alpha)\int_{-\rho}^{\infty}(\frac{-1}{\rho} - \frac{1}{u})f_{\Phi_{\text{s}}}(u)\text{d}u = T_{\text{total}}\} \\ &\overset{(b)}{=} \arg_{\rho}\{\alpha\delta_{\text{s}}^2 \textstyle\int_{-\delta_{\text{s}}^2\rho}^{\infty}\frac{1-F_{G_{\text{ss}}}(x)}{x^2}\text{d}x + (1-\alpha)\int_{-\rho}^{\infty}\frac{1-F_{\Phi_{\text{s}}}(u)}{u^2}\text{d}u = T_{\text{total}}\} \overset{(c)}{=} \arg_{\rho}\{\alpha\delta_{\text{s}}^2\lambda_{\text{ss}}\text{Ei}(\lambda_{\text{ss}}\delta_{\text{s}}^2\rho) + \frac{e^{\lambda_{\text{ss}}\delta_{\text{s}}^2\rho}}{\rho} \\ &+\frac{\lambda_{\text{ss}}(1-\alpha)}{\lambda_{\text{ps}}\rho}(\rho(T_{\text{p}} + \lambda_{\text{ps}}\delta_{\text{s}}^2)\text{Ei}(\lambda_{\text{ss}}\delta_{\text{s}}^2\rho) - T_{\text{p}}\rho e^{\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}}}\text{Ei}(\frac{\delta_{\text{s}}^2(\lambda_{\text{ps}}+\lambda_{\text{ss}}T_{\text{p}}\rho)}{T_{\text{p}}})) = T_{\text{total}}\}. \end{aligned} \tag{27}$$

Here, (b) comes from partial integration and (c) is for Rayleigh-fading channels. Having ρ, the channel ergodic achievable rate with an average transmission power constraint is found as

$$\begin{aligned} &C_{\text{s}} = \alpha \textstyle\int_0^{\infty} f_{G_{\text{ss}}}(x)\log(1+T_0 x)\text{d}x + (1-\alpha)\int_0^{\infty} f_{\Phi_{\text{s}}}(u)\log(1+T_1 u)\text{d}u \\ &\overset{(d)}{=} \alpha \textstyle\int_{-\delta_{\text{s}}^2\rho}^{\infty} f_{G_{\text{ss}}}(x)\log(\frac{x}{-\delta_{\text{s}}^2\rho})\text{d}x + (1-\alpha)\int_{-\rho}^{\infty} f_{\Phi_{\text{s}}}(u)\log(\frac{u}{-\rho})\text{d}u \\ &\overset{(e)}{=} \alpha \textstyle\int_{-\delta_{\text{s}}^2\rho}^{\infty}\frac{1-F_{G_{\text{ss}}}(x)}{x}\text{d}x + (1-\alpha)\int_{-\rho}^{\infty}\frac{1-F_{\Phi_{\text{s}}}(u)}{u}\text{d}u \\ &\overset{(f)}{=} \text{Ei}(\lambda_{\text{ss}}\delta_{\text{s}}^2\rho) - (1-\alpha)e^{\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}}}\text{Ei}(\frac{\delta_{\text{s}}^2(\lambda_{\text{ps}}-\lambda_{\text{ss}}T_{\text{p}}\rho)}{T_{\text{p}}}) \end{aligned} \tag{28}$$

where (d) follows from (26). Then, (e) is obtained by partial integration and (f) is valid for Rayleigh-fading channels.

Theorem 1: With an average SU transmission power constraint, there is a threshold T_{thr} under which interference-avoiding is the optimal transmission scheme in terms of SU ergodic achievable rate. However, higher rates are achieved via simultaneous transmission as the average power constraint exceeds the threshold.

Proof: Using (17) and (26), it is obvious that $T_1 \leq T_0, \forall T_{\text{total}}$. Therefore, with the same arguments as for the nonfading channels (Remark 1) and according to the water-filling properties, the assertion can be proven easily. Finally, note that, as seen for the nonfading channels and in the following, the same argument is valid under many other SU transmission power or PU quality-of-service requirements. ■

PU quality-of-service requirements: As an example of the PU joint quality-of-service requirements, we can consider the case where, while the SU transmission power is limited to T_{total}, the PU instantaneous received interference power is constrained to be less than a given threshold φ. In this case, the SU transmission power in the presence of the PU signal is changed to $T_1 = \min(T_{\text{total}}, \frac{\varphi}{G_{\text{sp}}})$, as the PU instantaneous received power is $\varphi_{\text{p}} = T_1 G_{\text{sp}}$. Therefore, the SU-SU channel ergodic achievable rate, i.e., (24), is rephrased as

$$\begin{aligned}
&C_{\text{s}} = \alpha e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}}\text{Ei}(-\tfrac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}) + (1-\alpha)\Pr\{\tfrac{\varphi}{G_{\text{sp}}} > T_{\text{total}}\}E_{\Phi_{\text{s}}}\{\log(1+T_{\text{total}}\Phi_{\text{s}})\}\\
&+(1-\alpha)\Pr\{\tfrac{\varphi}{G_{\text{sp}}} \leq T_{\text{total}}\}E_{\Phi_{\text{s}},G_{\text{sp}}}\{\log(1+\tfrac{\varphi}{G_{\text{sp}}}\Phi_{\text{s}})|\tfrac{\varphi}{G_{\text{sp}}} \leq T_{\text{total}}\}\\
&= \alpha e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}}\text{Ei}(-\tfrac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}) + (1-\alpha)(1-e^{-\frac{\lambda_{\text{sp}}\varphi}{T_{\text{total}}}})\int_0^\infty \tfrac{1-F_{\Phi_{\text{s}}}(x)}{1+T_{\text{total}}x}T_{\text{total}}\text{d}x\\
&+(1-\alpha)\int_{x=\frac{\varphi}{T_{\text{total}}}}^\infty \int_0^\infty f_{G_{\text{sp}}}(x)f_{\Phi_{\text{s}}}(y)\log(1+\tfrac{\varphi}{x}y)\text{d}x\text{d}y\\
&= \alpha e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}}\text{Ei}(-\tfrac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}) + (1-\alpha)(1-e^{-\frac{\lambda_{\text{sp}}\varphi}{T_{\text{total}}}})\frac{(e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}}}\text{Ei}(-\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2}{T_{\text{total}}})-e^{\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}}}\text{Ei}(-\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}}))}{1-\frac{\lambda_{\text{ss}}T_{\text{p}}}{\lambda_{\text{ps}}T_{\text{total}}}}\\
&+(1-\alpha)\int_{x=\frac{\varphi}{T_{\text{total}}}}^\infty \lambda_{\text{sp}}e^{-\lambda_{\text{sp}}x}(\frac{e^{\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2 x}{\varphi}}\text{Ei}(-\frac{\lambda_{\text{ss}}\delta_{\text{s}}^2 x}{\varphi})-e^{\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}}}\text{Ei}(-\frac{\lambda_{\text{ps}}\delta_{\text{s}}^2}{T_{\text{p}}})}{1-\frac{\lambda_{\text{ss}}T_{\text{p}}}{\lambda_{\text{ps}}\varphi}x})\text{d}x.
\end{aligned} \tag{29}$$

(For simulation results, see Fig.2.)

A. *System performance using a semi-optimal power allocation strategy*

Depending on the fading pdfs and the PU joint quality-of-service requirements, it may be difficult to determine the channel ergodic achievable rate under the long-term power constraint. Therefore, we mainly focus on the case where, while two different SU transmission powers are considered in the PU active and inactive conditions, the transmission power remains fixed in each of these situations. It is worth noting that, although suboptimal, this is a more instructive constraint leading to less complexity, simpler amplifiers and a negligible rate loss, particularly at high SINRs [24]–[26], [30], [31]. Moreover, this power allocation scheme, which we call *semi-optimal*, permits better comparisons between the fading and nonfading cases. Under such a constraint, the optimal fixed powers T_0 and T_1 are determined by the Lagrange multiplier function $\Upsilon = \alpha\Gamma_0(T_0) + (1-\alpha)\Gamma_1(T_1) + \rho(\alpha T_0 + (1-\alpha)T_1)$ leading to

$$\begin{cases} \hat{T}_1 = \left\lceil \breve{T}_1 \right\rceil^+, & \text{(I)} \\ \hat{T}_0 = \frac{1}{\alpha}\{T_{\text{total}} - (1-\alpha)\hat{T}_1\}, & \text{(II)} \end{cases} \tag{30}$$

where $\breve{T}_1$ is the solution of the equation

$$\breve{T}_1 = \arg_{T_1}\left\{ \frac{\partial\Gamma_1}{\partial T_1} = \frac{\partial\Gamma_0}{\partial T_0}\bigg|_{T_0=\frac{1}{\alpha}\{T_{\text{total}}-(1-\alpha)T_1\}} \right\}. \tag{31}$$

Remark 3: In deriving (30) we have used the fact that, as 1) the ergodic achievable rate of an AWGN is an increasing function of the channel SINR [29] and 2) the PU signal is an additive interference deteriorating the SU channel quality, in the optimal case we have $\hat{T}_1 \le \hat{T}_0$, $\forall T_{\text{total}}$.

PU quality-of-service requirements: Adding new constraints, we can make the model more realistic; as the average interference power received at the PU receiver is $\bar{\varphi}_{\text{p}} = T_1 E[G_{\text{sp}}]$, assuming an average interference power constraint $\bar{\varphi}_{\text{p}} \le \varphi$ changes the power allocation criterion (30.I) to

$$\hat{T}_1 = \min\{\left\lceil \breve{T}_1 \right\rceil^+, \frac{\varphi}{E[G_{\text{sp}}]}\}. \tag{32}$$

On the other hand, the PU received SINR is obtained by

$$\text{SINR}_{\text{p}} = T_{\text{p}}\Phi_{\text{p}}, \Phi_{\text{p}} \doteq \frac{G_{\text{pp}}}{\delta_{\text{p}}^2 + T_1 G_{\text{sp}}}. \tag{33}$$

Consequently, constraining the PU average received SINR to be higher than a given value, e.g., $E\{\text{SINR}_{\text{p}}\} \ge \theta$, leads to

$$\begin{aligned} &\hat{T}_1 = \min\{\left\lceil \breve{T}_1 \right\rceil^+, \lceil t \rceil^+\}, \\ &t = \arg_{T_1}\{E[\Phi_{\text{p}}] = \tfrac{\theta}{T_{\text{p}}}\} \end{aligned} \tag{34}$$

which for exponential fading pdfs can be rewritten as

$$t = \arg_{T_1}\{\frac{\lambda_{\text{sp}}}{\lambda_{\text{pp}} T_1} e^{\frac{\delta_{\text{p}}^2 \lambda_{\text{sp}}}{T_1}} \text{Ei}(-\frac{\delta_{\text{p}}^2 \lambda_{\text{sp}}}{T_1}) = \frac{\theta}{T_{\text{p}}}\}. \tag{35}$$

As more realistic constraints, we can consider the PU instantaneous quality-of-service requirements. For instance, using the PU received interference power random variable φ_{p}, we have

$$\Pr\{\varphi_{\rm p} \leq \varphi\} = \Pr\{T_1 G_{\rm sp} \leq \varphi\} = F_{G_{\rm sp}}(\frac{\varphi}{T_1}) \tag{36}$$

in which $F_{G_{\rm sp}}(.)$ is the SU-PU link fading cdf. Therefore, the constraint that the PU instantaneous received interference power is with probability of ξ less than some value φ, i.e., $\Pr\{\varphi_{\rm p} \leq \varphi\} \geq \xi$, modifies (30.I) into

$$\hat{T}_1 = \min\{\left\lceil \breve{T}_1 \right\rceil^+, \frac{\varphi}{F^{-1}_{G_{\rm sp}}(\xi)}\}. \tag{37}$$

Here, $F^{-1}_{G_{\rm sp}}(.)$ is the inverse function of $F_{G_{\rm sp}}(.)$ which under exponentially distributed gain assumption is obtained as $F^{-1}_{G_{\rm sp}}(x) = \frac{-1}{\lambda_{\rm sp}} \log(1-x)$.

Finally, the optimal SU transmission power which guarantees that the PU instantaneous received SINR (when it is active) is with probability of ξ higher than a threshold θ is obtained by

$$\begin{aligned} &\hat{T}_1 = \min\{\left\lceil \breve{T}_1 \right\rceil^+, \lceil t \rceil^+\}, \\ &t = \arg_{T_1}\{F_{\Phi_{\rm p}}(\frac{\theta}{T_{\rm p}}) = 1-\xi\} \end{aligned} \tag{38}$$

which for exponential pdfs, $F_{\Phi_{\rm p}}(x) = 1 - \frac{e^{-\lambda_{\rm pp}\delta^2_{\rm p}x}}{1+\frac{\lambda_{\rm pp}T_1 x}{\lambda_{\rm sp}}}$, is simplified to $t = \frac{\lambda_{\rm sp}T_{\rm p}}{\lambda_{\rm pp}\theta\xi}(e^{-\frac{\lambda_{\rm pp}\delta^2_{\rm p}\theta}{T_{\rm p}}} - \xi)$. The simulation results for this part can be found in Fig. 3-7.

1) Extension to multiple interfering users case: Provided that the set $\tilde{J} \subset \{1,\dots,M\}$ of the M mutually independent interfering users are active, the SU-SU channel SINR changes to $\mathrm{SINR}_{{\rm s}_{\tilde{J}}} = T_{\tilde{J}}\Phi_{\tilde{J}}, \Phi_{\tilde{J}} \doteq \frac{G_{\rm ss}}{\delta^2_{\rm s}+\sum_{j\in\tilde{J}} T_{{\rm p}_j} G_{{\rm p}_j{\rm s}}}$ in which $T_{\tilde{J}}$ is the SU transmission power in this case, $G_{{\rm p}_j{\rm s}}$ is the fading gain in the link between the j-th interfering user transmitter and the SU receiver and $T_{{\rm p}_j}$ is the power of the j-th interfering user input signal. Consequently, the channel ergodic achievable rate, i.e., (10), is changed to

$$\begin{aligned} &C_{\rm s} = \sum_{\forall\tilde{J}\subset\{1,\dots,M\}} \gamma_{\tilde{J}} I_{\tilde{J}}, \\ &\begin{cases} \gamma_{\tilde{J}} \doteq (\prod_{\forall j\in\tilde{J}}(1-\alpha_j))(\prod_{\forall j\in\tilde{J}^c}\alpha_j) \\ I_{\tilde{J}} = I(X_{\rm s}; X_{\rm s}H_{\rm ss} + \sum_{j\in\tilde{J}} X_{{\rm p}_j}H_{{\rm p}_j{\rm s}} + Z_{\rm s})\Big|_{X_{\rm s}\sim\mathcal{CN}(0,T_{\tilde{J}})} = \int_0^\infty \frac{1-F_{\Phi_{\tilde{J}}}(x)}{1+T_{\tilde{J}}x} T_{\tilde{J}} {\rm d}x. \end{cases} \end{aligned} \tag{39}$$

Here, $F_{\Phi_{\tilde{J}}}(x) = E_{G_{{\rm p}_j{\rm s}}, j\in\tilde{J}}\{F_{G_{\rm ss}}(x(\delta^2_{\rm s} + \sum_{j\in\tilde{J}} X_{{\rm p}_j}G_{{\rm p}_j{\rm s}}))\}$ is the received SINR cdf which for Rayleigh-fading channels, $f_{G_{{\rm p}_j{\rm s}}}(x) = \lambda_{{\rm p}_j{\rm s}}e^{-\lambda_{{\rm p}_j{\rm s}}x}, x \geq 0$ and $f_{G_{\rm ss}}(x) = \lambda_{\rm ss}e^{-\lambda_{\rm ss}x}, x \geq 0$, is obtained as

$$F_{\Phi_{\tilde{J}}}(x) = 1 - \frac{e^{-\lambda_{\rm ss}\delta^2_{\rm s}x}}{\prod_{j\in\tilde{J}}(1+\frac{\lambda_{\rm ss}T_{{\rm p}_j}}{\lambda_{{\rm p}_j s}}x)}. \tag{40}$$

B. Simulation results

Setting $T_{\mathrm{p}} = 1$ and $\alpha = 0.7$, Fig.2 shows the ergodic achievable rate of the SU-SU channel in the presence of both SU short-term transmission power and PU instantaneous received interference power constraints, i.e., (29). Then, considering different PU transmission powers and instantaneous received SINR or interference power constraints, Fig. 3 studies the effect of the probability parameter ξ on the secondary channel ergodic achievable rate. Here, we set $T_{\text{total}} = 2$ and $\alpha = 0.7$. Also, Fig.4 shows the same results for different PU instantaneous received interference or SINR constraint parameters. Fig.5 and 6 respectively represent the achievable rates of the SU-SU channel for different SU or PU transmission powers. Further, the achievable rates are compared with the ones obtained by the standard interference-avoiding approach (Fig.5). Then, Fig.7 shows the effect of the PU transmission power on the achievable rates of the channel constrained to have limited PU average received SINR. Note that, with proper scaling, the results of instantaneous interference limited condition can be interpreted as the ones with the average PU received interference power constraint. Finally, assuming both fading and nonfading channel conditions, Fig.8 investigates the secondary channel ergodic achievable rate in the presence of different number of primary users. Here, for simplicity, we consider identical parameters $\lambda_{\mathrm{p}_j\mathrm{s}} = 1$, $T_{\mathrm{p}_j} = 1$ and $\alpha_j = 0.7$.[10]

V. Discussions

Theoretical and simulation results emphasize on a number of interesting points that can be listed as follows:

- *Long-term vs short-term power allocation:* With relaxed PU quality-of-service constraints, the effect of long-term (optimal) power allocation is increased at higher PU or lower SU transmission powers while its influence diminishes at high SU-SU channel SINRs. That is, the long-term power-limited channel ergodic achievable rate converges to the one obtained with short-term power constraint as $T_{\mathrm{s}} \rightarrow \infty$ and $T_{\mathrm{p}} \rightarrow 0$. Moreover, long-term power allocation is more effective when the number of PUs increases (Fig.8).
- *Interference-avoiding vs simultaneous transmission:* Under different primary or secondary user quality-of-service constraints, interference-avoiding is the most optimal data transmission scheme at low SU or

[10]Note that considering users with the same pdfs does not necessarily mean that they are always at the same distance from the SU, as they can have independent random distances at different fading blocks. But, the same pdf indicates that in long-run, i.e., over infinitely many fading blocks, they experience the same behavior.

high PU transmission powers. This is particularly based on the water-filling, e.g., (5) or (26), properties in which the limited powers are not wasted on *weak* channel conditions and the saved power is used at *good* SU-SU channel SINR conditions. Then, with the same channels conditions, the SU transmission power is preferably allocated to the case where there is no interference from the primary transmitter. Therefore, as illustrated before, there is a threshold under which no power is allocated to the SU transmitter at PU working time slots. However, increasing the input power above the threshold simultaneous transmission increases the channel ergodic achievable rate, that is, T_1 becomes positive in, e.g., (5) or (26). This result is valid for both fading and nonfading channels and under both optimal and semi-optimal power allocation strategies (Fig.1.b and 5).

- *SU received interference vs power allocation capability:* Under instantaneous PU received SINR constraint, increasing the PU transmission power may lead to higher SU-SU channel ergodic achievable rate. This is an interesting result which can be interpreted as follows. At low PU transmission powers, no simultaneous transmission is permitted, as the SINR constraint is not satisfied (flat horizontal line in Fig.6). Then, at some PU transmission powers simultaneous transmission is possible where the SU transmitter can gain from optimal power allocation and simultaneous data transmission (the increasing parts of the SINR related curve in Fig.6). In this situation, the gain due to optimal power allocation and simultaneous transmission is such that it can compensate the loss due to SU received interference increment, i.e., PU input power increment[11]. The received SINR constraint is always satisfied at higher PU input powers where the rates reach the ones obtained by long-term power allocation (the reducing parts of the SINR related curve in Fig.6). In this case, the PU only plays the role of an additional interference reducing the SU-SU channel ergodic achievable rate. Finally, at asymptotically high PU input powers water-filling implies no power allocation at PU transmission time slots and so the achievable rates get back to the ones obtained by interference-avoiding scheme. Finally, as illustrated in Fig.7, the same conclusion is valid under PU average received SINR constraint.
- *The effect of PU quality-of-service requirements:* With hard PU received interference (or SINR) constraints,

[11]Note that the channel ergodic achievable rate can never be less than the one obtained by interference-avoiding scheme, as it can always be implemented, independent of the PU quality-of-service requirements.

i.e., small φ (or large θ), no data transmission is allowed within the PU activation time slots (interference-avoiding scheme). However, higher rates are obtained under more relaxed PU quality-of-service constraints which converge to the rates achieved by optimal transmission power limited SU-SU channel ergodic achievable rate (Fig.4).

- *PU tolerability:* The more the PU instantaneous received interference or SINR probability constraint ξ is, the less transmission rates are achievable in the SU-SU channel (Fig.3). In other words, when the probability parameter ξ increases, that is, the PU tolerability decreases, the SU transmission rates reduce from the ones obtained by long-term power-limited channel ergodic achievable rate to the rates obtained in interference-avoiding scheme. Further, the interference-limited SU-SU channel ergodic achievable rate is more sensitive to the interference probability factor at lower PU transmission powers and interference constraints φ (Fig.3 interference limited curves). On the other hand, increment of the SINR constraint, i.e., θ, intensifies the effect of ξ (Fig.3 SINR limited curves).
- *SU received interference vs PU quality-of-service requirements:* For a given SU transmission power and an instantaneous SINR constraint, the interference created by the PU transmission power is more detrimental for the SU-SU channel ergodic achievable rate at small ξ's. Then, increasing the probability parameter ξ, the PU quality-of-service requirement becomes the main rate reduction factor. Finally, at extremely hard PU constraints, i.e., $\xi \to 1$, or high PU transmission powers, i.e., $T_{\text{p}} \to \infty$, the SU-SU achievable rates decrease to the ones obtained by interference-avoiding scheme (Fig.3 SINR limited curves).
- *The effect of multiple interfering users:* For both fading and nonfading channels, increasing the number of interfering users can drastically reduce the SU-SU channel ergodic achievable rate (Fig.8). There is an interesting intuition behind this point; The SU-SU channel ergodic achievable rate gains very much from the cases where the cross channels, and correspondingly the interferences, are weak. However, in a system with a number of users experiencing independent pdfs it is more likely that, at any time instant, some of the cross channels experience high gain realizations. Therefore, the ergodic achievable rate reduces since the received interference increases.

VI. Conclusion

This paper studies the performance of spectrum sharing networks in the presence of an interference indicator signal representing the finite backlogged primary user activation status. The results are presented for both fading and nonfading AWGN channels and under different primary and secondary user quality-of-service constraints. In the case of fading channels, we study the channel achievable rates under perfect channel state information conditions. Theoretical and simulation results indicate the optimality of the standard interference-avoiding approach at low secondary user or high primary user transmission powers. Also, depending on the licensed users' quality-of-service requirements and the fading channels conditions, increasing the licensed users input power may lead to higher transmission rates of the unlicensed users. Finally, studying the channel sum capacity in the presence of (im)perfect interference indicator signal and considering other models, such as Markov models, for the primary user activity are interesting topics for future work.

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