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The Impact of Polarization-Dependent Loss on the Constant Modulus Algorithm for Varying Number of Fiber Spans Based on an Outage Criterion

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***Abstract*—We investigate the effect of polarization-dependent loss (PDL) on the constant modulus algorithm (CMA) in 4-QAM and 16-QAM polarization multiplexed (PolMux) systems with varying number of fiber spans and different PDL values. To quantify this effect, outage probability is introduced as the probability of having a signal-to-noise ratio (SNR) degradation smaller than 1 dB 99% of the time. We observe that by increasing the number of fiber spans, the SNR penalty for CMA reaches a limiting value. Moreover, in the 16-QAM multi-span case, the effect of PDL on CMA depends on the amount of PDL, while for 4-QAM, the effect of PDL on CMA is insensitive to the amount of PDL. These results will provide guidelines for designing systems and adjusting transmission power.**

***Index Terms*—Coherent optical communication, blind equalization, constant modulus algorithm, polarization-dependent loss.**

I. Introduction

Much research has been conducted in the area of coherent optical communication, since it enables the combination of polarization multiplexing (PolMux) with higher order modulation formats to increase spectral efficiency for next-generation optical networks. In coherent systems, digital signal processing (DSP) is used to design equalizers in the receiver to compensate for linear effects in the fiber and components [1], [2]. These effects include polarization-mode dispersion (PMD), chromatic dispersion (CD), and polarization-dependent loss (PDL). CD, PMD, and polarization mixing are unitary phenomena and as a result, their effect can be undone at the receiver. On the other hand, PDL is not a unitary impairment, so its effect can not be compensated for completely. In particular, PDL causes signal degradation and affects the statistical properties of the noise. Moreover, in PolMux systems, PDL destroys the orthogonality between polarizations. Hence, it is important to understand the impact of PDL on equalizers in order to properly design systems and adjust transmission power. Many methods have been introduced to perform equalization. One of the most efficient equalization strategies is blind equalization, as it does not require a training sequence for the initialization step. Due to this benefit, many coherent PolMux systems use blind equalizers. In particular, the constant modulus algorithm (CMA) [3], [4] is a commonly used blind equalization method.

The effect of PDL on coherent PolMux systems was previously investigated in [5], where limits on the performance of the coherent receivers were found. The penalty on channel capacity induced by PDL was evaluated in [6], [7]. In [8], the PDL effect on a lumped channel model was determined, while [9] studied the best and worst cases of this model. This was extended in [10], for both lumped and distributed channel models, for worst, best, and average alignment between the signal and PDL. In [8], [10], it was shown that the lumped model is not precise since it does not consider the effect of PDL on noise. Finally, in [11], the effect of PDL combined with nonlinear effects was studied. While investigating the impact of worst, best, and average alignments of the polarizations with respect to the PDL are useful, they do not accurately represent the typical system performance. Moreover, the effect of the number of fiber spans on PDL has not been investigated.

In this paper, we will investigate the effect of PDL on the performance of CMA for two modulation formats: 4-QAM and 16-QAM. We will evaluate the effect of PDL on CMA, as well as two other linear equalizers: the zero-forcing (ZF) equalizer and the minimum-mean squared error (MMSE) equalizer [12]. ZF and MMSE have been chosen as a reference, since they have been shown to yield near-optimal performance [13]. Contrary to the works listed above, we consider (i) the effect of a varying number of fiber spans; and (ii) a performance criterion based on outage probability (i.e., such that the SNR degradation after equalization can be guaranteed 99% of the time to be below 1 dB), rather than worst, best, or average case. Our main findings are that (i) the loss due to PDL in CMA is not sensitive to the number of fiber spans; (ii) for multi-span 16-QAM, the effect of PDL on CMA depends on the amount of PDL; and (iii) for 4-QAM and single-span 16-QAM, the effect of PDL on CMA does not significantly depend on the amount of PDL.

The remainder of the paper is organized as follows. Section II presents the system model, where we describe the observation model, and how CMA works. In section III, we introduce the performance measure. Section IV analyzes the performance of the system considering different scenarios, corresponding to different modulations, number of fiber spans, amount of PDL per span, and the type of equalizer, where conclusions are drawn for the effect of PDL and the number

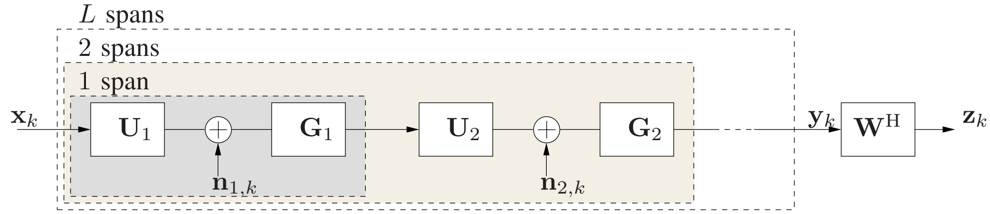

Fig. 1: System model, including polarization mixing, ASE noise, and PDL, for a system with L fiber spans. Here $\mathbf{U}_i$ refers to the polarization mixing in the i-th fiber span, $\mathbf{n}_{i,k}$ is ASE noise in optical amplifiers, and $\mathbf{G}_i$ refers to the PDL and rotation/derotation in optical components, and $\mathbf{W}$ is the equalizer.

of fiber spans on CMA.

II. System Model

A. Observation Model

Figure 1 represents the system model, where $\mathbf{x}_k$ is a vector of 2 transmitted complex independently and identically distributed M-QAM symbols (one per polarization) at k-th symbol interval. The i-th fiber span consists of (i) a fiber with polarization mixing (modeled by a random unitary matrix $\mathbf{U}_i$), followed by (ii) the optical amplifier which adds amplified spontaneous emission (ASE) noise, $\mathbf{n}_{i,k}$, (modeled as zero-mean complex additive white Gaussian noise with variance $\mathbf{\Lambda}_i = (N_0/L)\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, L is the number of fiber spans, and N_0 is the power spectral density of the noise in the whole system when there is no PDL), and finally (iii) PDL due to optical components, such as optical amplifiers, add-drops, couplers [14] (modeled by $\mathbf{G}_i = \mathbf{\Phi}_i^{\mathrm{T}}\mathbf{\Gamma}_i\mathbf{\Phi}_i$, where[1] $\mathbf{\Phi}_i$ is a random unitary polarization rotation matrix, due to the random orientation between the signal and the PDL element, and $\mathbf{\Gamma}_i = \mathrm{diag}(1, \gamma_i)$, where γ_i is the polarization-dependent attenuation on the i-th span). As in [15], the effects of CD, PMD, self-phase modulation, and synchronization are neglected.

The discrete-time observation after L fiber spans is given by $\mathbf{y}_k = \mathbf{H}_{\mathrm{total}}\mathbf{x}_k + \mathbf{m}_k$, where $\mathbf{H}_{\mathrm{total}}$ is the total equivalent channel, as seen by the receiver, and $\mathbf{m}_k$ is the total equivalent noise. It is easily verified that[2]

$$\mathbf{H}_{\mathrm{total}} = \left(\prod_{i=0}^{L-1} \mathbf{G}_{L-i}\mathbf{U}_{L-i}\right), \tag{1}$$

and that $\mathbf{m}_k$ is Gaussian, zero-mean and has covariance matrix

$$\mathbf{\Upsilon} = \mathbf{G}_L\mathbf{\Lambda}_L\mathbf{G}_L^{\mathrm{H}} + \sum_{i=1}^{L-1} \mathbf{Q}_i\mathbf{G}_i\mathbf{\Lambda}_i\mathbf{G}_i^{\mathrm{H}}\mathbf{Q}_i^{\mathrm{H}}, \tag{2}$$

where $\mathbf{Q}_i = \prod_{m=0}^{L-(i+1)} \mathbf{H}_{L-m}$, $\mathbf{H}_i = \mathbf{G}_i\mathbf{U}_i$. It must be noted that the signal passes through the whole channel whereas each element of noise experiences that part of channel that exists between where the noise is added to the signal and receiver. The channel $\mathbf{H}_{\mathrm{total}}$ and covariance matrix $\mathbf{\Upsilon}$ may be slowly varying, and are assumed to be constant during a short sequence of observations. According to (2), due to the existence of PDL effect (i.e., $\mathbf{G}_i\mathbf{G}_i^H \neq \mathbf{I}$), the noise contributions of different polarizations are correlated.

The observation $\mathbf{y}_k$ is fed to a 2×2 complex-valued equalizer matrix $\mathbf{W}$, leading to the discrete-time equalizer output

$$\mathbf{z}_k = \mathbf{W}^{\mathrm{H}}\mathbf{H}_{\mathrm{total}}\mathbf{x}_k + \mathbf{W}^{\mathrm{H}}\mathbf{m}_k \doteq \mathbf{B}\mathbf{x}_k + \mathbf{v}_k, \tag{3}$$

where $\mathbf{v}_k$ is the noise at the output of the equalizer with zero mean and covariance matrix $\mathbf{\Psi} = \mathbf{W}^{\mathrm{H}}\mathbf{\Upsilon}\mathbf{W}$.

We will consider three equalizers: the ZF equalizer $\mathbf{W}_{\mathrm{ZF}} = \mathbf{H}^{-\mathrm{H}}$, the MMSE equalizer $\mathbf{W}_{\mathrm{MMSE}} = \mathbf{H}(\mathbf{H}^{\mathrm{H}}\mathbf{H}+\mathbf{\Upsilon})^{-\mathrm{H}}$, and the CMA equalizer. Note that CMA is a blind equalizer, while ZF and MMSE require a channel estimate.

B. Constant-Modulus Algorithm

In blind equalization, a popular method uses the constant modulus criterion. The aim of this method is to minimize the following function

$$J(\mathbf{W}) = \mathbb{E}\left\{(|\mathbf{z}_k(\mathbf{W})|^2 - R_2)^2\right\} \tag{4}$$

where $|.|$ is the modulus of the complex variables, $\mathbb{E}\{.\}$ is the expectation operator, and $R_2 = \mathbb{E}\left\{|\mathbf{x}_k|^4\right\}/\mathbb{E}\left\{|\mathbf{x}_k|^2\right\}$. This minimization problem can be solved using stochastic gradient descent method given by

$$\mathbf{W}_{r+1} = \mathbf{W}_r - \mu\nabla J(\mathbf{W}_r) \tag{5}$$

where $\mathbf{W}_{r+1}$ is the estimated matrix for equalizer on the $(r+1)$-th iteration of (5), and $\mu > 0$ is the step size. The update in (5) can be performed one symbol at a time (in this case $r = k$), or based on a block of observations. The gradient $\nabla J(\mathbf{W})$ is the derivative of (4) with respect to $\mathbf{W}^{\mathrm{H}}$ given by

$$\nabla J(\mathbf{W}) = \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}^H} = \mathbb{E}\left\{\begin{bmatrix} \left(|z_{1,k}^2| - R_2\right) y_{1,k} \\ \left(|z_{2,k}^2| - R_2\right) y_{2,k} \end{bmatrix} \mathbf{z}_k^H\right\}$$

where $z_{j,k}$ (respectively $y_{j,k}$) refers to the k-th output (respectively input) of equalizer on the j-th polarization ($j \in \{1, 2\}$), and the expectation occurs over all symbol indices k in the block.

[1]The superscript T, H, -H denotes the transpose, Hermitian transpose, and inverse Hermitian transpose, respectively.

[2]$\prod_{i=1}^{L} \mathbf{A}_i$ is a shorthand for $\mathbf{A}_1\mathbf{A}_2 \ldots \mathbf{A}_L$.

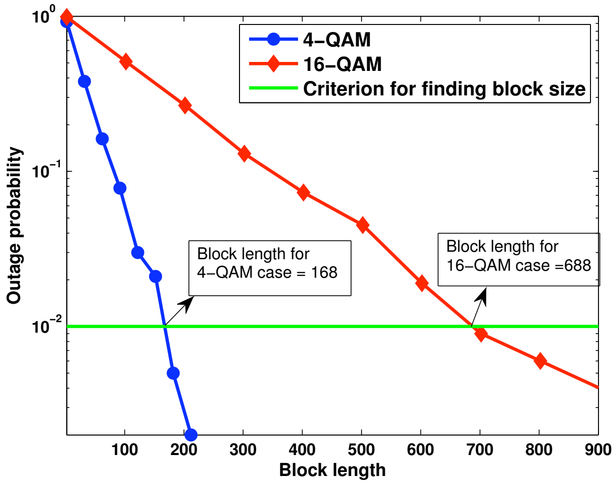

Fig. 2: The outage probability versus the length of the block used for computation of the CMA equalizer (the outage probability is defined as having SNR degradation more than 1 dB).

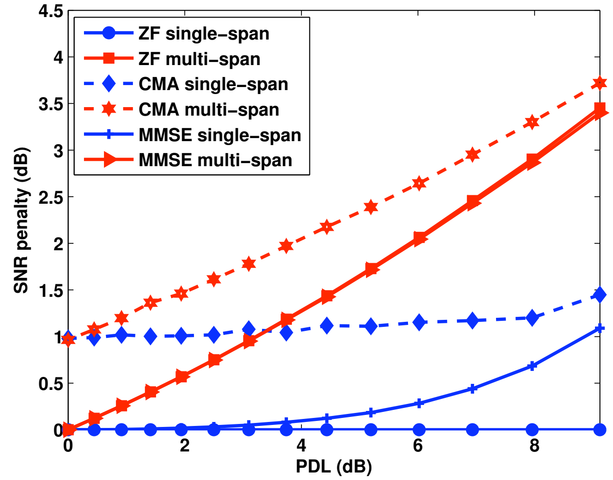

Fig. 4: PDL-induced SNR penalty versus PDL at BER 10^{-3} for single and multi-span case with ZF, MMSE, and CMA based equalizers for 16-QAM modulation.

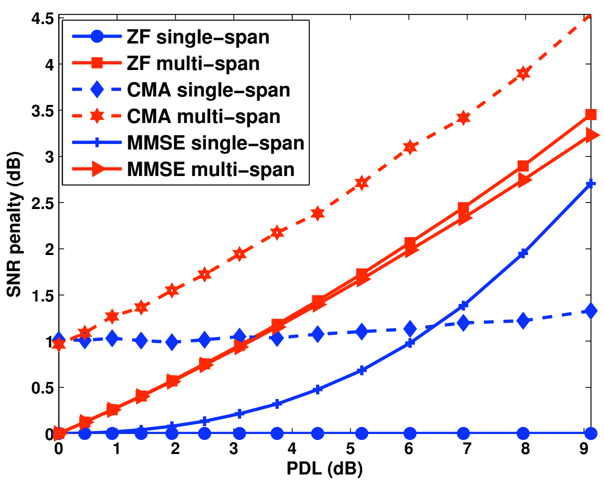

Fig. 3: PDL-induced SNR penalty versus PDL at BER 10^{-3} for single and multi-span case with ZF, MMSE, and CMA based equalizers for 4-QAM modulation.

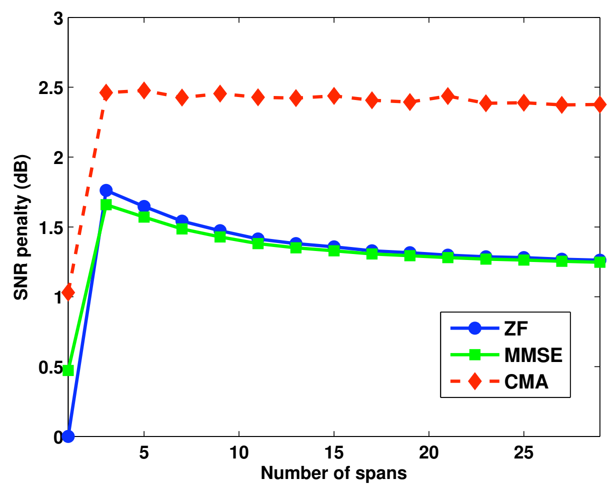

Fig. 5: PDL-induced SNR penalty versus number of spans with ZF, MMSE, and CMA based equalizers for 4-QAM modulation and PDL=4.5 dB.

III. Performance Measure

Assuming that the data symbols have energy E_s per polarization, we set E_s/N_0 such that the bit-error-rate (BER)[3] under no PDL (i.e., $\gamma_i = 1, \forall i$) is 10^{-3}. This target SNR depends on the constellation and is denoted by $\mathrm{SNR}_{\mathrm{target}}$ and corresponds to the largest SNR that can be achieved after equalization. The target SNR for 4-QAM is 9.807 dB, and for 16-QAM is equal to 16.541 dB. In order to investigate the performance degradation for an equalizer due to PDL, we determine the SNR after equalization, denoted by $\mathrm{SNR}_{\mathrm{out}}$, so that the SNR degradation is $\mathrm{SNR}_{\mathrm{target}} - \mathrm{SNR}_{\mathrm{out}}$, expressed in dB. It must be noted that $\mathrm{SNR}_{\mathrm{out}}$ depends on the amount of PDL, number of spans, and constellation type.

For a given equalizer, $\mathrm{SNR}_{\mathrm{out}}$ is computed as follows. The first output of the equalizer, i.e., the first row in (3), can be written as $z_1 = b_{11}x_1 + b_{12}x_2 + v_1$, where b_{ij} is the element on row i, column j of $\mathbf{B}$ defined in (3). When $|b_{11}| > |b_{12}|$, we can consider the interference from the second polarization $b_{12}x_2$ as noise, and determine the SNR on the first polarization as

$$\mathrm{SNR}_1 = \frac{|b_{11}|^2 E_s}{|b_{12}|^2 E_s + \psi_{11}}, \tag{6}$$

where ψ_{11} is the first diagonal element of $\mathbf{\Psi}$. In case $|b_{11}| < |b_{12}|$, the role of b_{11} and b_{12} in (6) should be reversed. A

[3]A BER of 10^{-3} is typical value prior to error-correction.

similar procedure is applied to the second polarization, i.e., the second row in (3), giving us SNR_2. The total SNR is then $\mathrm{SNR}_{\mathrm{out}} = (\mathrm{SNR}_1 + \mathrm{SNR}_2)/2$.

IV. Simulations

A. Setup

We will consider different scenarios, corresponding to different constellations (4-QAM and 16-QAM), the number of fiber spans (L), the amount of PDL per span (γ_i), and the type of equalizer (CMA, ZF, MMSE). For every scenario, we have generated 100000 channels $\mathbf{H}_{\mathrm{tot}}$ and covariance matrices $\mathbf{\Upsilon}$, according to the model in Figure 1. The PDL rotations and polarization mixing matrices per span were drawn uniformly from the set of unitary 2×2 matrices. For a system with L spans, we write the total amount of PDL as $\mathrm{PDL(dB)} = 10\log_{10}(1/\gamma_{\mathrm{tot}}^2)$, where $\gamma_{\mathrm{tot}} = \gamma_i^L$, and $\gamma_i = \gamma_j,\ \forall i, j \in \{1, 2, ..., L\}$.

For CMA, the SNR degradation also depends on the block size used to compute the CMA equalizer coefficients. We have chosen the block size such that the outage probability is 0.01 under $\gamma_i = 1, \forall i$ [15], where the outage event is defined as having SNR degradation more than 1 dB. Hence, we do not account for worst, best, or average SNR after equalization, but rather the SNR that can be guaranteed most of the time. Figure 2 shows the outage probability for 4-QAM and 16-QAM cases versus the block size. According to this figure, for the 4-QAM case, the required block size is 168, and for the 16-QAM case, the required block size is 688. It must be noted that in contrast to [15] in our case CMA is block-based and in all iterations of (5), we use the same block, whereas in the [15], an online method is used.

B. Discussion

Figure 3 shows the SNR degradation for 4-QAM, considering both $L = 10$ and $L = 1$. ZF for a single span is not affected by PDL, as both signal and noise are affected by the same channel, up to a unitary matrix. MMSE for a single span incurs a degradation with increasing PDL, as it is suboptimal for the single-span system. CMA incurs a roughly 1 dB penalty compared to ZF, as long as PDL is below 9 dB. This penalty is due to the block size on which CMA operates, and can be reduced by increasing the block size. For very large amounts of PDL, the degradation increases. For 10 spans, all equalizers degrade with increasing PDL, but the 1 dB gap between ZF and CMA remains. This indicates that CMA incurs no additional penalty due to PDL. Again, the 1 dB gap can be reduced by considering longer observations. The degradation of ZF and MMSE is due to the non-unitary effect of PDL, which in the multi-span case affects the signal and noise differently, hence it cannot be fully compensated for through DSP.

Figure 4 shows corresponding results for 16-QAM. We observe that the same conclusions hold for single-span as in the case of 4-QAM. However, for multi-span, the gap between CMA and ZF varies as a function of the PDL. This indicates that PDL is the dominating source of degradation, while the additional impact due to CMA is negligible for PDL below 9 dB.

A different view is offered in Figure 5, showing the effect of the number of spans on the output SNR for 4-QAM and a PDL of 4.5 dB. For ZF, we again see no penalty when $L = 1$, then a rapidly increasing penalty for $L = 3$, as the performance is dominated by the worst span. For large L, the degradation then reduces again, due to the averaging effect of the different spans, and reaches a limiting value. For a large number of spans, both $\mathbf{H}_{\mathrm{tot}}$ and $\mathbf{\Upsilon}$ turn out to be near-constant (i.e., irrespective of the realization of the unitary matrices), explaining the existence of a limiting value of the SNR degradation. MMSE has similar behavior, but incurs a larger degradation for $L = 1$, as discussed above. Finally, CMA has a very different behavior: for $L = 1$, we see the expected 1 dB degradation, while for $L > 1$, CMA immediately reaches a limiting value of the degradation around 1.13 dB above the limiting value of ZF. Hence, CMA is less sensitive to the number of fiber spans than ZF or MMSE.

As an aside, we mention that we have also performed similar simulations for an outage probability of 0.001. Our findings regarding the impact of PDL on the SNR penalty remained qualitatively unchanged.

V. Conclusion

We have studied the effect of PDL on CMA for 4-QAM and 16-QAM PolMux systems, as a function of the number of fiber spans and amount of PDL. We introduced a new performance criterion for CMA, based on the maximum SNR penalty that can be guaranteed 99% of the time. A comparison was made with ZF and MMSE equalization. We found that in 4-QAM, for PDL below 9 dB CMA incurs no additional PDL-induced penalty in both single-span and multi-span scenarios. The same result holds for the single-span 16-QAM case. However, for 16-QAM multi-span, the performance is limited due to PDL, not CMA. Furthermore, we noticed that by increasing the number of fiber spans, the PDL-induced penalty reaches a limiting value. For CMA, this limiting value is reached for 2 or 3 spans, while for ZF and MMSE, more spans (depending on total amount of PDL) are required to observe the limiting value.

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