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An LLR-based Cognitive Transmission Strategy for Higher Spectrum Reutilization

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Abstract—Reutilization of the spectrum licensed to services with low occupancy is of great interest for cognitive radios (CRs). To achieve this goal, we introduce a simple hidden Markov model which captures the primary users activity, signal uncertainties, and noise. For evaluating the performance of any CR, two new criteria are presented entitled spectrum utilization ratio (UR) and interference ratio (IR). Based on this model and new measures, a new a-posterior log-likelihood-ratio based CR is designed and implemented. Its performance is compared with standard energy-detection based spectrum-sensing CR. We demonstrate more than 300% increase in UR for up to 1% allowed interference at the SNR of -5 dB.

Index Terms—Cognitive radio, hidden Markov model, interference ratio, spectrum sensing and spectrum utilization.

I. INTRODUCTION

In spite of all attractive features of cognitive radio (CR), there exist many basic questions that limit the usage of CR in real-world applications. First and foremost, especially for CRs utilizing energy detection spectrum sensing, the capability of CR is limited by the so-called SNR wall [1]. This is due to the low received power of the PU signal at the CR receiver and uncertainties in signals, noise, and channels. There are several attempts in overcoming this problem, e.g., by using sequential spectrum sensing methods [2] [3]. The other impediment on the way to fully reutilizing the spectrum is knowledge about the future activities of the PU in the same band. Hence, since CRs must be cautious in transmitting over the bands with possible future PU activities, spectrum reuse is limited by the causality of spectrum information. On the other hand, the performance measures normally used for characterization of CRs performance are dedicated to the detection efficiency of their spectrum sensing, e.g., receiver operating characteristic (ROC) curves. These measures do not consider the PU transmission model, channel uncertainties or the interaction between CR and PU. Thus, any CR design based on such criteria will not be ideal.

In this contribution, we deploy a hidden Markov model (HMM) to form a framework for modeling the behavior of CRs in the presence of PUs and all the uncertainties. Additionally, a benchmark for evaluation of CR performance is introduced. Then, using this foundation and these measures, a new CR

transmission strategy is designed and implemented. This new design ensures that the vacant spectrum is optimally used conditioned on the level of interference for the PU, due to all uncertainties in the model, is not exceeding a certain level.

HMMs are long in use for modeling different phenomena ranging from speech signals [4] to the complex behavior of computer networks. In the context of cognitive radio, many researchers modeled the spectrum white space with Markov models and spectrum sensing using HMMs [5]–[8]. In our paper, HMMs are used not only for spectrum sensing but also as a tool for CR transmission strategy making. The closest published approach to our method is presented in [9], which employs a partially observed Markov decision process (POMDP). They introduced the POMDP for optimal policy making for multiple channel sensing and access. The approach is similar to ours due to Markovian assumption for the PU transmission model and in the presence of sensing error. However, the sensing model, performance metric and constraints are different from ours.

This paper will be continued by introducing the system model in the next section, which will cover the signal and noise models and the HMM representation of the CR perception of spectrum activities. In Section III, the new performance measures utilization ratio (UR) and interference ratio (IR) will be introduced. Section IV computes the UR and IR for standard CR based on spectrum sensing using energy detection. Section V is devoted to the design of a new CR strategy, which improves reutilization of the spectrum. It will be followed in Section VI by the simulation scenario, some results, and a comparison between the baseline and new CR. The paper will be concluded in Section VII by some final remarks.

II. SYSTEM MODEL

A cognitive radio system is interested in utilizing the spectrum vacancies. To take advantage of time-frequency slots which are not used by the PU, the CR must be aware of the PU activities. In this research, it is assumed that the CR has a full buffer to reuse the spectrum whenever it is available.

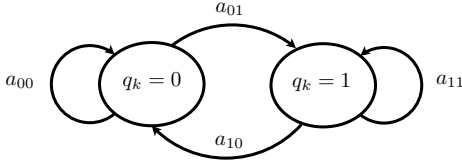


Figure 1. PU transmission model

A. PU transmission model

Here, the PU transmissions are slotted with a slot length T . The existence of a PU transmission in slot k is denoted by hypothesis the $H_1 \triangleq \{q_k = 1\}$ and its absence is denoted by $H_0 \triangleq \{q_k = 0\}$. A simple model which represents the PU transmission is the two-state on-off Markov process depicted in Fig. 1, where the Markov chain is represented by the transition probabilities $a_{i,j} = \Pr\{q_{k+1} = j | q_k = i\}$ for $i, j \in \{0, 1\}$ and q_k stands for the PU state at time slot k . This model is represented by the transition matrix

$$\mathbf{A} \triangleq \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \quad (1)$$

where $a_{00} + a_{01} = a_{10} + a_{11} = 1$.

Due to the noise and other channel impairments, CR is unable to directly observe q_k . In the following sections, the uncertainties in signal and noise are modeled as Gaussian random processes. The initial distribution of the states is assumed to be in steady state [4] and defined as

$$\begin{aligned} \boldsymbol{\pi} &\triangleq [\pi_0 \quad \pi_1] \triangleq [\Pr\{q_k = 0\} \quad \Pr\{q_k = 1\}] \\ &= \left[\frac{a_{10}}{a_{01} + a_{10}} \quad \frac{a_{01}}{a_{01} + a_{10}} \right], \quad k = 0, 1, 2, \dots \end{aligned} \quad (2)$$

B. Signal and noise models

The receiver front end is an energy detector whose its output x_k is

$$x_k = \sum_{i=0}^{K-1} |r(kT + iT_s)|^2, \quad (3)$$

where $r(\cdot)$ is the complex envelope of received signal low-pass filtered to the PU signal bandwidth W , T is the period in which energy is collected, which happens to be the same as the period for the PU to change its state (for simplicity), T_s is the sampling time, and K is the total number of samples in each period. The channel is assumed to be an additive white Gaussian noise channel. We also assume that the PU signal can be modeled as a Gaussian signal (which is a reasonable model for many PU signals [10] [11]). If we select T_s such that $T_s \gg 1/W$, then the samples $r(iT_s)$ are approximately statistically independent. We note that K is constrained as $K \leq T/T_s$. The following is a review of the assumptions that the model is based on.

1) *Noise only*: This model presumes that the noise $n(iT_s) \sim \mathcal{CN}(0, \sigma_n^2)$ is a zero-mean complex circular Gaussian sample with variance σ_n^2 , and the received signal will be $r(iT_s) = n(iT_s)$. Thus, x_k is chi-square distributed with $2K$ degrees of freedom and Gaussian variance σ_n^2 .

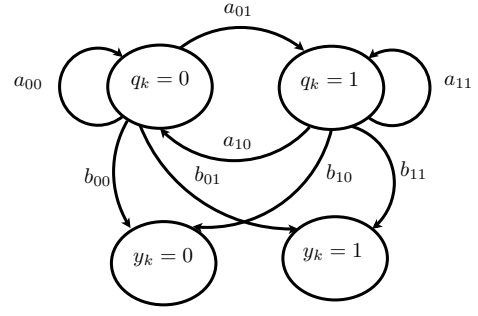


Figure 2. HMM model for the energy detector.

2) *Signal plus noise*: This model assumes that the noise is a zero-mean complex circular Gaussian sample with variance σ_n^2 , the signal is also zero-mean complex circular Gaussian with variance σ_s^2 , and $r(iT_s) = s(iT_s) + n(iT_s)$, $r(iT_s) \sim \mathcal{CN}(0, \sigma_r^2)$, where $\sigma_r^2 = \sigma_s^2 + \sigma_n^2$. Thus, x_k is chi-square distributed with $2K$ degrees of freedom and Gaussian variance σ_r^2 .

C. Hidden Markov model representation of energy detector

Energy detection, which is one of the most widely deployed spectrum sensing methods due to its simplicity, compares the estimated received energy (x_k) with a threshold to detect the existence or absence of the PU signal. Using this threshold at a certain received PU signal power to CR noise ratio (SNR) will result in certain probabilities of mis-detection and false alarm. This procedure introduces the HMM presented in Fig. 2. In this model, $y_k = 0$ and $y_k = 1$ denote the detected state to be H_0 and H_1 , respectively, and is decided by

$$y_k = \begin{cases} 0, & \text{if } x_k \leq \theta_e; \\ 1, & \text{if } x_k > \theta_e, \end{cases} \quad (4)$$

where θ_e is detection threshold. Thus, the elements of the emission matrix \mathbf{B} are

$$\begin{aligned} \mathbf{B} &\triangleq \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix}, \\ b_{00} &\triangleq \Pr\{y_k = 0 | q_k = 0\} = 1 - b_{01} = 1 - P_{\text{FA}} \\ &= 1 - P_0(\theta_e) = 1 - \frac{\gamma(K, \theta_e/2\sigma_n^2)}{\Gamma(K)}, \end{aligned} \quad (5)$$

$$\begin{aligned} b_{11} &\triangleq \Pr\{y_k = 1 | q_k = 1\} = 1 - b_{10} = 1 - P_{\text{M}} \\ &= P_1(\theta_e) = \frac{\gamma(K, \theta_e/2\sigma_r^2)}{\Gamma(K)}, \end{aligned} \quad (6)$$

where Γ is the Gamma function, γ is the lower incomplete Gamma function, P_0 and P_1 are the CDF of a chi-square distribution with $2K$ degrees of freedom and Gaussian variance σ_n^2 and σ_r^2 , respectively, and P_{M} and P_{FA} are the probabilities of mis-detection and false-alarm, respectively. This model, fully specified by $\lambda \triangleq (\mathbf{A}, \mathbf{B}, \boldsymbol{\pi})$ [4], captures the whole behavior of the standard energy detection based CR in the presence of the simplified PU model.

III. SPECTRUM UTILIZATION RATIO AND INTERFERENCE RATIO

The performance of a cognitive radio is usually assessed based on its spectrum-sensing algorithm. Spectrum sensing is judged based on its P_{FA} and P_M , which are normally presented in receiver operating characteristic plots. However, the ultimate goal of CRs is to reutilize the idle spectrum slots while keeping the level of interference for PUs below a certain level. The two aforementioned measures are not taking the PU behavior into account. Besides, utilization and interference are defined by the presence or absence of PU transmission. Therefore, it is necessary to define new criteria which consider the full picture including PUs, CRs, and even the channel.

A. Definitions

Let the CR transmission strategy at time k be denoted by u_{k+1} , where $u_{k+1} = 0$ and $u_{k+1} = 1$ represents no transmission and transmission, respectively in slot $k + 1$. Interference will happen whenever the CR transmits at the same time as the PU. Thus, the interference ratio (IR) ρ is defined as

$$\rho \triangleq \Pr\{u_{k+1} = 1 | q_{k+1} = 1\}. \quad (7)$$

Utilization of spectrum occurs whenever the CR transmits in a vacant time–frequency slot. Thus, we define spectral utilization ratio as

$$\eta \triangleq \Pr\{u_{k+1} = 1 | q_{k+1} = 0\}. \quad (8)$$

The intention of any CR is to design a strategy that keeps ρ below a specified level, say ρ_{\max} , and then maximize the utilization ratio η . Hence, we call a transmission scheme that maximizes η while $\rho \leq \rho_{\max}$ an optimal transmission scheme.

B. Relation of UR and IR to transmission rate

It is noticeable that UR can be directly translated into cognitive transmission rate. However, in this paper there is no assumption on any particular modulation and coding; thus, we will not demonstrate any exact rate for the CR. Still, an average rate might be calculated based on UR and IR as

$$R = R_b(\eta\pi_0 + \rho\pi_1) = R_b(\pi_0(\eta - \rho) + \rho), \quad (9)$$

where R and R_b is average CR transmission rate in bit/s and data rate for continuous CR transmission in bit/s, respectively. Thus, for small $\rho\pi_1/\pi_0$ (9) is approximated by $R \approx R_b\pi_0\eta$. In the same way, the probability of error can be derived for the CR as

$$\Pr\{\text{error}\} = (\Pr\{\text{error} | q_{k+1} = 0, u_{k+1} = 1\}\eta\pi_0 + \Pr\{\text{error} | q_{k+1} = 1, u_{k+1} = 1\}\rho\pi_1) / (\eta\pi_0 + \rho\pi_1). \quad (10)$$

The first term in (10) includes the probability of error in the absence of PU and the second term includes the probability of error in the presence of PU. In both expressions (9) and (10), the first term represents the rate or error due to reutilizing slots and the second term represents the rate and error during interference with the PU. Thus, the UR and IR are also useful in evaluating actual transmission performance of cognitive communication links.

IV. UR AND IR FOR THE BASELINE CR TRANSMISSION STRATEGY

The CR strategy for transmission, in the presence of an active PU, can be adapted and optimized according to these new measures. One might consider different methods in which UR is maximized while IR is limited. An ideal rule for transmission would be $u_{k+1} = \overline{q_{k+1}}$, where $\bar{\cdot}$ denotes the negation. Obviously, this rule is unrealistic because it is non-causal. Thus, the next PU state might be predicted and used instead of its real state, i.e., $u_{k+1} = \hat{q}_{k+1}$, where \hat{q}_{k+1} denotes a prediction of the next PU state. In an even simpler scheme, the CR may simply transmit in next slot whenever the current slot is estimated to be vacant, i.e., $u_{k+1} = \overline{\hat{q}_k}$. The strategy which simply takes current detection y_k for deciding whether to transmit or not ($u_{k+1} = \overline{y_k}$) is widely used due to its simplicity [11] and is herein referred to as the baseline system.

Theorem 1: In the HMM presented in Section II-C with the transmission strategy of $u_{k+1} = \overline{y_k}$, UR and IR are

$$\eta_e = a_{01}b_{10} + a_{00}b_{00} = a_{01}(b_{10} - b_{00}) + b_{00}, \quad (11)$$

$$\rho_e = a_{10}b_{00} + a_{11}b_{10} = a_{10}(b_{00} - b_{10}) + b_{10}. \quad (12)$$

Proof: The UR and IR for $u_{k+1} = \overline{y_k}$ can be computed from (7) and (8) as

$$\eta_e = \Pr\{u_{k+1} = 1 | q_{k+1} = 0\} = \frac{\Pr\{y_k = 0, q_{k+1} = 0\}}{\Pr\{q_{k+1} = 0\}}, \quad (13)$$

$$\rho_e = \Pr\{u_{k+1} = 1 | q_{k+1} = 1\} = \frac{\Pr\{y_k = 0, q_{k+1} = 1\}}{\Pr\{q_{k+1} = 1\}}. \quad (14)$$

From the HMM it follows that y_k and q_{k+1} are independent conditioned on q_k . Thus, the numerators can be written as

$$\begin{aligned} & \Pr\{y_k = 0, q_{k+1} = j\} \\ &= \sum_{i=0}^1 \Pr\{q_{k+1} = j | q_k = i\} \Pr\{y_k = 0 | q_k = i\} \Pr\{q_k = i\} \\ &= \sum_{i=0}^1 a_{ij} b_{i0} \pi_i. \end{aligned} \quad (15)$$

Hence, by substituting (15) into (13) and (14) and using (2) the theorem follows. ■

Expressions (11) and (12) capture the impact of the PU and the CR on spectrum reutilization and collisions. They depend on P_{FA} and P_M , which are properties of the CR front end as well as the matrix \mathbf{A} , which is the PU's property.

V. LLR-BASED TRANSMISSION STRATEGY

In Section IV, a simple strategy which is widely used for CR was presented. In this section a new HMM-based transmission strategy, which depends on observations until time k $\mathbf{y}_k \triangleq [y_1, y_2, \dots, y_k]^T$ is introduced.

As described in Section IV, a generic transmission scenario for a full buffer CR can be phrased as $u_{k+1} = \overline{\hat{q}_{k+1}}$. Thus, the better the PU state prediction performed by the CR, the higher the spectrum reutilization. One reasonable way to incorporate

both the model and whole observations is to form the a posterior probability of $\Pr\{q_{k+1} = 1 | \mathbf{y}_k; \lambda\}$. This probability will be used in the decision rule as

$$u_{k+1} = \begin{cases} 1, & \text{if } z_k \leq \theta_l \\ 0, & \text{if } z_k > \theta_l \end{cases}, \quad (16)$$

$$z_k \triangleq \log \frac{\Pr\{q_{k+1} = 1 | \mathbf{y}_k\}}{\Pr\{q_{k+1} = 0 | \mathbf{y}_k\}}, \quad (17)$$

where z_k and θ_l are the *a posteriori log-likelihood ratio* and the threshold for z_k , respectively. The z_k , which is based on the future state of PU, hereafter will be addressed as the LLR. To calculate $\Pr\{q_{k+1} = i | \mathbf{y}_k\}$, $i \in \{0, 1\}$, the fact that q_{k+1} and \mathbf{y}_k , conditioned on q_k , are independent, is used. Thus, this probability might be expressed as

$$\begin{aligned} & \Pr\{q_{k+1} = i | \mathbf{y}_k\} \\ &= \sum_{j \in \{0,1\}} \Pr\{q_{k+1} = i | q_k = j, \mathbf{y}_k\} \Pr\{q_k = j | \mathbf{y}_k\} \\ &= \sum_{j \in \{0,1\}} \Pr\{q_{k+1} = i | q_k = j\} \Pr\{q_k = j | \mathbf{y}_k\}. \end{aligned} \quad (18)$$

In (18), $\Pr\{q_{k+1} = i | q_k = j\} = a_{ji}$ is given in the matrix \mathbf{A} and $\Pr\{q_k | \mathbf{y}_k\}$ can be calculated using the forward-backward method [4]. Since only the information about the past is available, $\Pr\{q_k | \mathbf{y}_k\}$ is the forward variable $\alpha_k(i) = \Pr\{q_k = i | \mathbf{y}_k\}$, $i \in \{0, 1\}$ which is computed recursively [4, eqs. 19–21] with moderate complexity. Thus, the LLR is

$$z_k = \log \frac{a_{01}\alpha_k(0) + a_{11}\alpha_k(1)}{a_{00}\alpha_k(0) + a_{10}\alpha_k(1)}. \quad (19)$$

For the cognitive transmission scheme in (16), a threshold for the LLRs is needed. This threshold firstly should fulfill a required IR $\rho \leq \rho_{\max}$. To achieve this, one must derive the expression for the IR for this new transmission strategy. By substituting (16) in (7) and (18), we can write the IR as a function of θ_l as

$$\rho_l(\theta_l) = \Pr\left\{z_k \leq \theta_l \mid q_{k+1} = 1\right\} = \mathcal{F}_{z_k | q_{k+1}=1}(\theta_l) \quad (20)$$

and the UR as

$$\eta_l(\theta_l) = \Pr\left\{z_k \leq \theta_l \mid q_{k+1} = 0\right\} = \mathcal{F}_{z_k | q_{k+1}=0}(\theta_l), \quad (21)$$

where $\mathcal{F}_{z_k | q_{k+1}=i}$ is the CDF of z_k conditioned on $q_{k+1} = i$. Since both $\rho_l(\theta_l)$ and $\eta_l(\theta_l)$ are nondecreasing functions of θ_l , it follows that the optimum threshold, which does not cause more interference than the allowed ρ_{\max} and maximizes the UR, is

$$\theta_l = \mathcal{F}_{z_k | q_{k+1}=1}^{-1}(\rho_{\max}), \quad (22)$$

where $\mathcal{F}_{z_k | q_{k+1}=1}^{-1}$ is the inverse CDF of z_k conditioned on $q_{k+1} = 1$.

VI. PERFORMANCE EVALUATION AND RESULTS

To assess the performance of the CR specified in expressions (16)–(19), and to have a fair comparison with the classical energy detection based spectrum sensing modeled in Section IV, a setup in which both introduce the same level of interference ρ_{\max} for the PU must be used. According to (12), energy detection can attain a certain IR if for a given \mathbf{A} , the threshold θ_e is selected to be

$$\begin{aligned} \theta_e &= \rho_e^{-1}(\rho_{\max}), \\ \rho_e(\theta) &= a_{10}(1 - P_0(\theta)) + a_{11}(1 - P_1(\theta)). \end{aligned} \quad (23)$$

By the same reason as in Section V, this threshold will give the maximum achievable UR for a given interference. The threshold given in (23) through expressions (5)–(6) gives the matrix \mathbf{B} . The HMM model specified by $\lambda = (\mathbf{A}, \mathbf{B}, \boldsymbol{\pi})$ will be used as the front-end model.

The same model is used to evaluate the new CR transmission strategy presented in Section V. For this new model to work, a new threshold θ_l is needed. To compute this threshold, the CDF of z_k conditioned on $q_{k+1} = 1$ is estimated empirically.

The rest of this section discusses the evaluation setup by which these CRs are assessed. Then, some results and a comparison are presented.

A. Evaluation setup

In simulating the performance of a CR transmission strategy, the ratio of received primary signal power (at the CR receiver) to the CR receiver noise power is important. Here, for simplicity, we assume one PU link and one CR link. Of course it can be extended to a case with multiple coordinated PUs and multiple coordinated CRs. Hence, we define the SNR as $\text{SNR} \triangleq \sigma_s^2 / \sigma_n^2$ (in dB). In this simulation, K is selected to be 10. This parameter plays a role for the SNR scaling. The other factor which is important in evaluating CRs is the maximum allowable IR ρ_{\max} . This parameter is normally decided by regulatory bodies like FCC. The intention is to keep it low and we assume that $\rho_{\max} = 1\%$.

B. Results

First, the UR for LLR-based CR and baseline CR are compared for different PU parameters and SNRs. Figure 3 depicts η vs. a_{01} . In this figure, it is apparent that UR increases with the SNR. This is expected due to the simplicity of PU detection for the CR in higher SNRs. There exists an obvious gain in UR in LLR-based CR over baseline CR. This gain is due to the inclusion of model and observations in the CR transmission decisions. The smaller the a_{01} , the higher the chance that the PU remains in the zero state when $q_k = 0$. Thus, the LLR-based CR presents much higher UR gains over the baseline CR for lower values of a_{01} . The gain disappears when a_{01} increases because the prediction capability of the LLR-based CR decreases when there is higher chance of a PU transition to the transmission state when $q_k = 0$.

Figure 4 illustrates that both CRs have higher sensitivity to a_{10} than a_{01} . This sensitivity is caused by the limitation

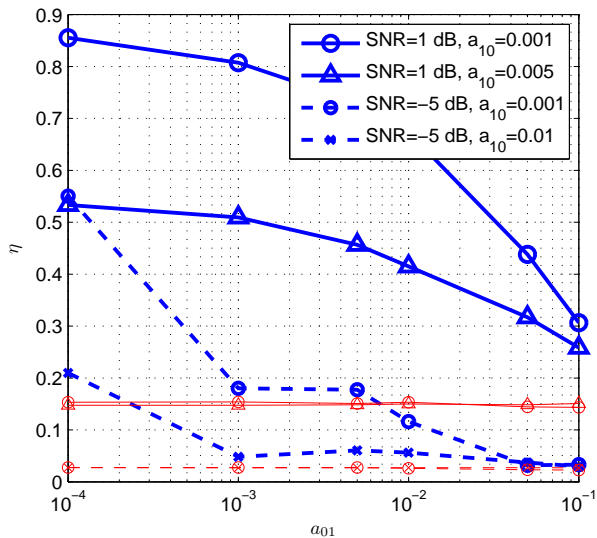


Figure 3. UR vs. a_{01} for the baseline CR (thin red lines) and corresponding LLR-based CR (thick blue lines) at $\rho_{\max} = 1\%$ over different a_{10} and SNRs

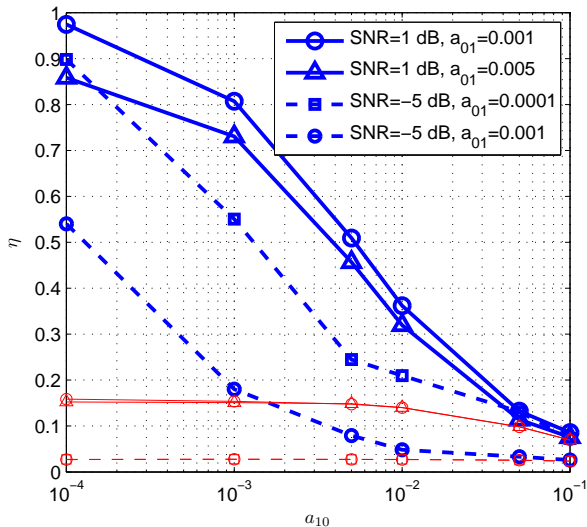


Figure 4. UR vs. a_{10} for the baseline CR (thin red lines) and corresponding LLR-based CR (thick blue lines) at $\rho_{\max} = 1\%$ over different a_{01} and SNRs

introduced due to ρ_{\max} . A smaller a_{10} means a higher chance of the PU staying in transmission state. For keeping the interference below ρ_{\max} , the CR should back off in the threshold θ_e more. This will eventually reduce the UR for both schemes.

VII. CONCLUSIONS

In this paper, we have proposed a new framework for the design and evaluation of cognitive radios. In this new model, the behavior of both PU and CR is captured in a single hidden Markov model. Moreover, two new performance measures, UR and IR, for evaluating the performance of CRs were introduced. In short, the UR is the fraction of the PU-unused slots that the CR transmits in, and the IR is the fraction of the PU-used slots that the CR transmits in. Hence, the UR shows

how much of the vacant spectrum is reutilized by the CR and the IR indicates how much interference the CR causes to the PU. The HMM was introduced to model the standard energy-detection based CR.

The same HMM was used to define a new CR which considers CR and PU models and the history of observation. For the scenarios with low PU spectral occupancy, this new CR shows significant improvement in UR for a given IR in comparison with the baseline system. For a maximum IR of 1%, the LLR-based CR yields more than 300% increase in UR over the baseline CR at the SNR of -5 dB for $a_{01} = a_{10} = 0.01$.

The ideas presented herein are dependent on perfect knowledge of the model, which is not always reasonable. This knowledge might only be available partially or erroneously, which will have direct impact on the performance of the CR. These issues are subject to future research by the authors and will be addressed in forthcoming papers.

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