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CMA Misconvergence in Coherent Optical Communication for Signals Generated from a Single PRBS

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***Abstract*—In the experimental study of modulators for coherent optical communications, it is common to generate multilevel signals using a single pseudo-random binary sequence. This simple experimental realization leads to symbol correlation. When modulators are studied in combination with adaptive receiver algorithms that rely on independent data, misconvergence may result. In this contribution, we investigate the impact of such correlation on a standard blind equalization method and provide guidelines to avoid misconvergence. Our results indicate that care needs to be taken when using a single PRBS sequence and that decorrelation delays must be chosen appropriately. We present simulation and theoretical results for 16-QAM, and show how our results can be applied to other multi-level constellations.**

***Index Terms*—Coherent optical communication, PRBS, blind equalization, correlation.**

I. Introduction

Encoding information onto the amplitude and phase of the optical carrier has attracted a lot of research interest. In particular, the use of multi-level modulation formats has the potential to boost the transmission rates and spectral efficiencies for long-haul communication. However, such coherent optical communication systems put higher requirements on the digital signal processing (DSP) in order to mitigate the effect of group-velocity and polarization-mode dispersion which could limit the performance of such systems. The latter is typically done simultaneously with polarization demultiplexing and intersymbol-interference suppression using equalization techniques. From the different equalization techniques, blind equalization seems to be the most attractive option compared to the conventional adaptive equalization techniques. This is due to the fact that blind equalization does not require a known training sequence for the filter initialization step which could be costly in bandlimited channels. Of the many blind equalization techniques, the constant modulus algorithm (CMA) [1], [2] is a widely spread algorithm for equalizing two-dimensional signals [3].

In addition to DSP at the receiver side, much work has been done in the design of suitable modulators at the transmitter side. Recently, many experimental studies have been focusing on the 16-ary quadrature amplitude modulation (16-QAM) format to increase the spectral efficiency of the system since each of its symbols carries 4 bits. In order to analyze the performance of a coherent optical system in a laboratory setting, the transmitter can be realized in several different ways, including using i) multi-level driving signals and a single I/Q modulator (IQM) [4], ii) an integrated structure with several optical modulators in parallel and binary electrical signals [5], or iii) a cascade of more than one optical modulator [6]. In the setup for an experimental study, we chose to work according to the first suggestion and, for hardware simplicity, a single pseudorandom binary sequence (PRBS) was used. Combining delayed copies of this signal, a 16-QAM signal could be generated. However, for some chosen values of the delays, we noticed that the CMA failed to converge, which lead us to investigate the phenomenon more closely.

In this paper, we consider the interaction between the signal generated by the modulator and the behavior of CMA at the receiver. A key requirement for the CMA equalizer to converge is that the transmitted symbols are stationary and uncorrelated [1]. However, generating a 16-QAM signal from a single PRBS will invariably cause correlation between symbols. This leads to violation of the CMA assumptions of independence and stationarity [1]. It is therefore known that equalizer failure may occur. For example, misconvergence of CMA in several scenarios was presented in [7], but there is no full analytical understanding of the behavior in the presence of cyclostationary, periodic, and nonwhite inputs.

In this work, we report the observation of CMA misconvergence for a specific choice of the 16-QAM modulator. In this way, we identify a potential pitfall when generating a multi-level modulation format from correlated data streams. By close investigation of the problem and the autocorrelation properties of the generated symbols, we propose a way to avoid misconvergence of the CMA equalizer.

The remainder of the paper is organized as follows. Section II presents the system model where we describe how the multi-level signalling generation is done, the receiver

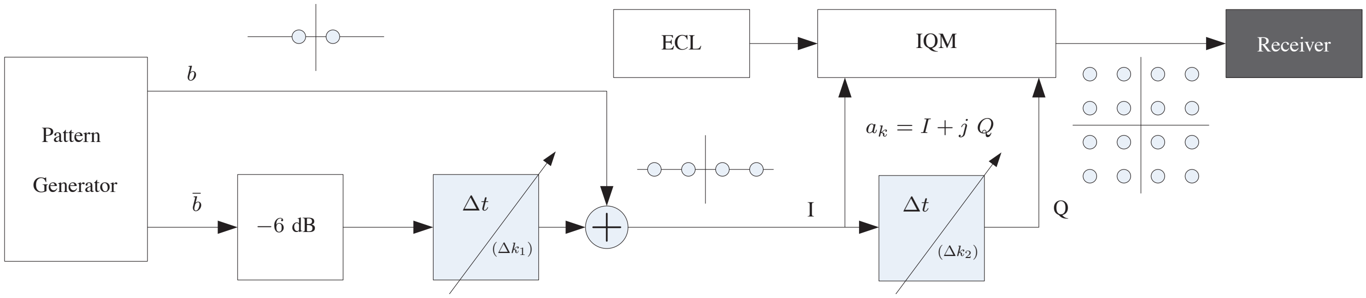

Fig. 1: The 16-QAM modulator setup using a single PRBS. The tunable delays, marked Δt, were set to an integer number of symbol slots. These are denoted by Δk_1 and Δk_2, respectively.

used, and how the CMA algorithm works. In section III, we analyze the performance of the system when a PRBS sequence is used to generate the 16-QAM signal, as well as in the presence of an independent and identically distributed sequence. Then the autocorrelation properties of the sequence generated is presented where conclusions are drawn to avoid the misconvergence of CMA.

II. System Model

A. Multi-level signal generation

The experimental setup of the 16-QAM modulator using a single PRBS is seen in Fig. 1. The output of the pattern generator is a real continuous-time signal, b, driven by the binary PRBS. This signal is added to an inverted, 6 dB attenuated, and Δk_1 symbol slots delayed copy of b. The resulting signal can take on four different values, and this electrical signal is used to generate the real part, I, of the constellation. The imaginary part, Q, is constructed by delaying the I signal Δk_2 symbol slots. These two signals are fed to the IQM, with the optical input connected to an external cavity laser (ECL), resulting in the 16-QAM symbols. The output of the IQM is directly fed to the receiver.

The generated 16-QAM symbols are characterized by the length of the PRBS sequence, the correlation properties of the PRBS sequence, and the parameters Δk_1 and Δk_2. According to [7], increasing the period of the PRBS is one of the factors for the CMA equalizer to converge. Hence, Δk_1 and Δk_2 are the only parameters that can be varied for a certain PRBS to tune the correlation between the output signals a_k.

We note that other modulation formats can be generated in a similar way, using either more delays (e.g., 3 delays for 64-QAM) or fewer delays (e.g., a single delay for QPSK).

B. Receiver

Initially, the experimental investigation focused on the back-to-back performance, from which we identified the overall channel impulse response, which lasted around 2 symbol slots. The received signal was oversampled at 10 samples per symbol and processed offline. Offline processing consisted of CMA equalization, downsampling to the symbol rate, carrier phase compensation, frame synchronization, and data detection. An 80 taps CMA equalizer (corresponding to 8 symbols) was used, while data detection was performed according to the maximum likelihood criterion.

C. The constant modulus algorithm

We consider a transmitted sequence of data symbols $a_k \in \Omega$, where Ω is an M-point constellation. The corresponding electrical signal is given by $s(t) = \sum_k a_k p(t - kT)$, where $1/T$ is the baud rate, and $p(t)$ is a shaping pulse (e.g., non-return-to-zero), which has a support of duration T. The electrical signal is converted to an optical signal, passes through the optical components, and is converted again to an electrical signal at the receiver side. The received electrical signal is filtered and sampled at a rate $1/T_s > 1/T$. The observations can be expressed as

$$r(nT_s) = \sum_{k=-\infty}^{+\infty} a_k h(nT_s - kT) + w(nT_s), \tag{1}$$

where $w(nT_s)$ are i.i.d. complex AWGN samples with variance N_0, and $h(t)$ is the overall equivalent electrical channel.

The goal of the blind equalizer is to recover the data symbols without knowledge of $h(t)$. CMA is a well-known blind equalizer and operates as follows. To recover symbol a_k, we consider a window of observations $\mathbf{r}_k$ of length L, where L is approximately $1/T_s$ times the duration of the support of $h(t)$. The equalizer output at time k is given by

$$z_k = \mathbf{w}_k^{\mathrm{T}} \mathbf{r}_k.$$

The goal of CMA is to determine the equalizer filter taps $\mathbf{w}_k$ so as to minimizing the cost function

$$J_2(\mathbf{w}) \triangleq \mathbb{E}[(|z(\mathbf{w})|^2 - R_2)^2],$$

where the constant $R_2 = \mathbb{E}[|a_k|^4]/\mathbb{E}[|a_k|^2]$. The equalizer taps are updated using a stochastic gradient descent according to

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu \nabla J_2(\mathbf{w}_k),$$

where μ is a step size parameter.

III. Performance analysis

A. Symbol error rate

An important figure of merit in analyzing the system performance is the symbol error rate (SER) after data detection.

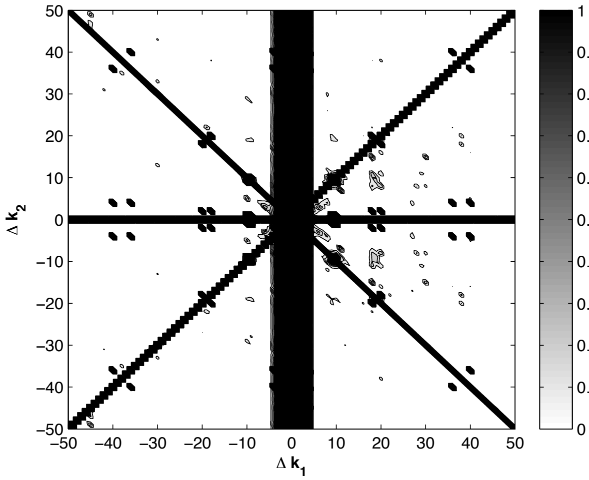

Fig. 2: The symbol error rate when using a single $2^{10}-1$ PRBS with various delays Δk_1 and Δk_2.

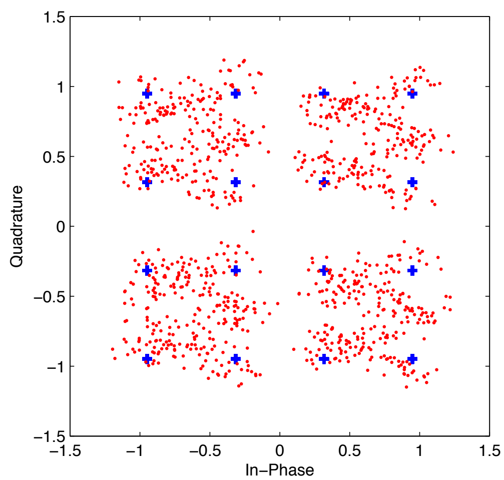

Fig. 3: The equalized received symbols which are the result of a correlated sequence generated by $\Delta k_1 = 3$ and $\Delta k_2 = 16$.

Fig. 2 depicts the SER performance for a 16-QAM signal generated using a PRBS with length of $2^{10}-1$ bits for different Δk_1 and Δk_2 values, where

$$-50 \leq (\Delta k_1, \Delta k_2) \leq 50.$$

(Negative values of the delays are included to show the inherent symmetry in the performance.) We have assigned an SER of 1 for the delay values that fail to generate a 16-QAM constellation. It is clear that there is a high SER for small values of Δk_1, which lead to correlation between neighboring symbols and CMA misconvergence. Fig. 3 shows the symbols after performing the equalization procedure. In this scenario, the generated 16-QAM symbols using $\Delta k_1 = 3$ and $\Delta k_2 = 16$ for example, led to high correlation between the data symbols which in turns led to CMA misconvergence. This could be

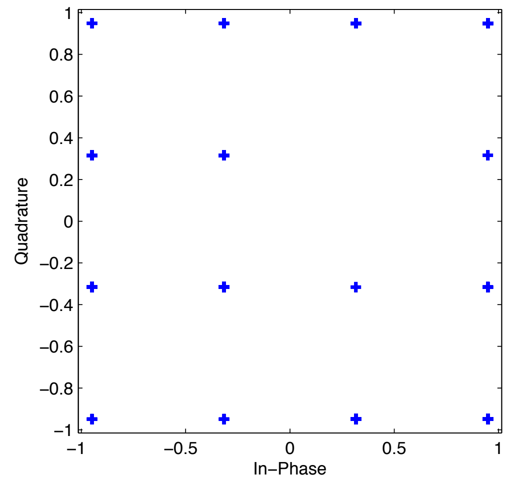

Fig. 4: The constellation generated using the delay parameters $\Delta k_1 = 40$ and $\Delta k_2 = 4$.

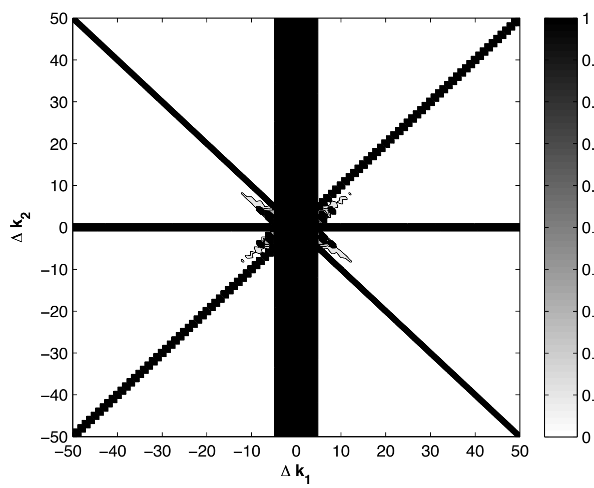

Fig. 5: The symbol error rate when using a $2^{15}-1$ i.i.d. sequence with various delays Δk_1 and Δk_2.

observed by looking at the red dots and comparing with how the original constellation should look like.

On the other hand, larger values of Δk_1 and Δk_2 lead to lower SER, as a result of equalizer convergence. Exceptions to this are when

$$|\Delta k_1| \approx |\Delta k_2|,$$

i.e., the diagonals, and a few noticeable points (e.g., around $\Delta k_1 = 40$ and $\Delta k_2 = 4$) which are highly dependent on the PRBS where the SER is high. In those cases, the high SER is not due to equalizer misconvergence, but rather to the fact that the generated constellation is not 16-QAM. For instance, Fig. 4 shows the constellation generated by the delay parameters $\Delta k_1 = 40$ and $\Delta k_2 = 4$). As we can see, this is not a complete 16-QAM constellation due to one missing symbol which was not generated with these values of the delays.

To rule out the effect of the length of the PRBS and the correlation properties of the PRBS on the SER, we have also evaluated the SER for a 16-QAM signal generated from a very long independent and identically distributed (i.i.d.) binary sequence (length $2^{15}-1$) as shown in Fig. 5. It is clear that the relationship between SER and the choice of Δk_1 and Δk_2 that was mentioned for the specific PRBS still holds. However, we see that the additional points with high SER in Fig. 2 (e.g., around $\Delta k_1 = 40$ and $\Delta k_2 = 4$), which were due to the wrong signal constellation being generated, are not present in Fig. 5. This indicates that those points are due to the specific properties of the PRBS sequence. Preliminary results (not shown) indicate that when the length of the PRBS sequence is increased, these points move towards higher Δk_1 and Δk_2.

Empirically, we could conclude from Figs. 2, and 5 that necessary conditions for convergence of CMA are that $|\Delta k_1| > 5$ and $|\Delta k_2| > 1$.

B. Autocorrelation

From the above, it is rather difficult to generalize the choice of the delays which guarantees CMA convergence. But to gain a deeper insight into how the different values of Δk_1 and Δk_2 affect the correlation between data symbols, we have examined the autocorrelation of the 16-QAM signal using a single infinitely long i.i.d. bit sequence. The autocorrelation is obtained as

$$\begin{aligned}\mathbb{E}[a_k a_{k+\tau}^*] &= \frac{5}{2}\delta(\tau) - \delta(\tau - \Delta k_1) - \delta(\tau + \Delta k_1)\\ &+ j[\frac{5}{4}\delta(\tau - \Delta k_2) - \frac{5}{4}\delta(\tau + \Delta k_2)\\ &- \frac{1}{2}\delta(\tau - \Delta k_1 - \Delta k_2) + \frac{1}{2}\delta(\tau + \Delta k_1 + \Delta k_2)\\ &- \frac{1}{2}\delta(\tau + \Delta k_1 - \Delta k_2) + \frac{1}{2}\delta(\tau - \Delta k_1 + \Delta k_2)].\end{aligned}$$

By examining the autocorrelation function, it is clear that there will always be peaks in the autocorrelation function of the 16-QAM symbols, despite the use of an i.i.d. sequence of bits to drive the modulator. This infers that there will always be a correlation between the data symbols using the 16-QAM modulator shown in Fig. 1. Therefore, for the CMA to converge, we have to avoid strong correlation for low values of $|\tau|$ by using proper delays. In particular, Δk_1 and Δk_2 should be chosen such that the impulse in the autocorrelation closest to lag $\tau = 0$ is as far away as possible. The above could be mathematically formulated by optimizing the values of Δk_1 and Δk_2 such that the objective function

$$F(\Delta k_1, \Delta k_2) = \min(|\Delta k_1|, |\Delta k_2|, |\Delta k_2 \pm \Delta k_1|) \quad (2)$$

is maximized. By solving this optimization problem, we end up with feasible regions shown in Fig. 6.

It could then be concluded that the desired values of Δk_1 and Δk_2 that maximize $F(\Delta k_1, \Delta k_2)$ are such that $\Delta k_1 = \pm 2\Delta k_2$ or $\Delta k_2 = \pm 2\Delta k_1$. Therefore, it could be concluded that it is beneficial to choose both Δk_1 and Δk_2 to be as large as possible, and to set Δk_1 to be roughly double of Δk_2 in order to guarantee the CMA convergence.

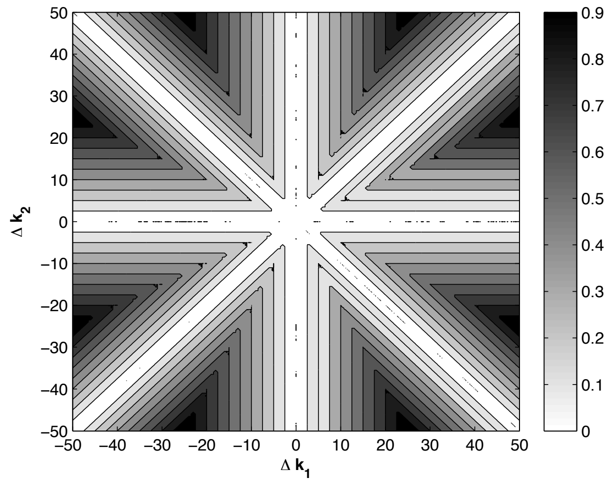

Fig. 6: Contour plot of the function $F(\Delta k_1, \Delta k_2)$.

C. Other constellations

Our results can be easily applied to other constellations. For example, to generate QPSK, we can consider a model as in Fig. 1, without the 6dB attenuator and Δk_1. The autocorrelation is now given by

$$\mathbb{E}[a_k a_{k+\tau}^*] = \quad 2\delta(\tau) + j\left[\delta(\tau + \Delta k_2) - \delta(\tau - \Delta k_2)\right].$$

To avoid strong autocorrelation, it follows that we should maximize Δk_2. This is common practice in the testing of QPSK modulators.

IV. Conclusion

A strong requirement to guarantee CMA convergence is that the transmitted symbols are independent. We have shown that generating 16-QAM data from a single PRBS to drive the I/Q modulator in laboratory experiments potentially leads to CMA misconvergence, due to correlation in adjacent symbols. We have evaluated the performance of CMA for various delays in a specific choice of modulator and found that by correct selection of these delays, proper operation of the CMA can be achieved. However, care needs to be taken as there are specific values for the delays, dependent on the particular PRBS sequence, that cause incorrect constellations to be generated. Further study should be conducted to find good values for delays when generating higher-order constellation (e.g., 64-QAM) from a single PRBS.

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