Cosmological Constraints on Unparticles

JOHN MCDONALD

Cosmology and Astroparticle Physics Group, University of Lancaster, Lancaster LA1 4YB, UK

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Abstract

We study the cosmological constraints on unparticle interactions and the temperature of the Universe for an unparticle sector with dimensional transmutation scale $\Lambda_U \geq 1$ TeV. By considering thermal background quark decay to unparticles via a scalar operator of dimension $d_U$, we show that the condition that the Universe is not dominated by unparticles at nucleosynthesis imposes a lower bound on the scale of the interaction of the unparticle sector, with $M_U \geq 20 - 2600$ TeV for $1.1 \leq d_U \leq 2.0$ and $2 \leq d_{BZ} \leq 4$. The existence of an unparticle sector also imposes an upper bound on the temperature of the Universe during radiation-domination, which can be as low as a TeV for $M_U$ close to its lower bound.

1j.mcdonald@lancaster.ac.uk
1 Introduction

Recently there has been much interest in the possible existence of a conformally-invariant hidden sector consisting of 'unparticles' see [4-11]. In this picture, the conformally-invariant unparticle sector is generated non-perturbatively from a Banks-Zaks (BZ) sector consisting of vector-like non-Abelian gauge fields and massless fermions, which are assumed to interact very weakly with the SM sector via operators suppressed by a mass scale $M_U$. The BZ sector undergoes dimensional transmutation to the unparticle phase, corresponding to a strongly self-coupled conformal field theory (CFT), below an infra-red fixed point energy $\Lambda_U$. This unparticle phase has no particle description but interacts with the SM via unparticle operators of mass dimension $d_U$, which create states made of the conformal fields. The remarkable characteristic of unparticles is that the phase space in decay processes to unparticle stuff is the same as the phase space for decay to $d_U$ massless particles, where $d_U$ can be non-integral. This could provide a distinctive signal for unparticle production in collider experiments in the case where the unparticle description is valid at future collider energies, corresponding to $\Lambda_U \gtrsim 1$ TeV.

The couplings of the unparticle operators and the value of $d_U$ will depend on the underlying BZ operators and the infra-red dynamics of dimensional transmutation. Since these are model-dependent, most studies of unparticle physics have introduced several possible unparticle operators with unknown coefficients and investigated their consequences. Under reasonable assumptions regarding the unknowns, the mass scale $M_U$ can then be constrained by the requirement that the unparticle sector does not conflict with present phenomenological, astrophysical, long-range force and cosmological observations.

Here we consider the effect of an unparticle sector on the cosmology of the SM and the resulting constraints on unparticle interactions and the temperature of the Universe. To do this we calculate the decay and annihilation rates of thermal background SM quarks to unparticles. For clarity, we focus on a specific interaction between SM

\footnote{Similar ideas were proposed earlier in [3].}
quarks and a scalar unparticle operator, leaving a more general operator analysis for future work.

The article is organised as follows. In Section 2 we review unparticles and their interpretation in terms of deconstructing scalars. We also relate these scalars to a physical interpretation of the unparticle states. In Section 3 we calculate the decay and pair annihilation rates of thermal SM quarks to unparticles and derive bounds from requiring that the unparticle energy density does not disturb nucleosynthesis. In Section 4 we present our conclusions.

2 The Model

The unparticle model \[1, 2\] is based on an interaction between Banks-Zaks (BZ) fields and SM fields mediated by exchange of heavy particles of mass \(M_U\)

\[
\frac{1}{M_U^d} O_{SM} O_{BZ} .
\]

\(O_{BZ}\) is an operator with mass dimension \(d_{BZ}\) made out of BZ fields. At an energy scales less than \(\Lambda_U\) the BZ sector undergoes dimensional transmutation and a scale-invariant unparticle sector, corresponding to a strongly self-coupled CFT, is formed. The BZ operators match onto unparticle operators and Eq. (1) becomes

\[
\frac{C_U \Lambda_U^{d_{BZ} - d_U}}{M_U^d} O_{SM} O_U ,
\]

where \(d_U\) is the scaling dimension of the unparticle operators \(O_U\). The value of \(C_U\) and \(d_U\) will be determined by the infra-red dynamics of the Banks-Zaks fields and the form of \(O_{BZ}\). These will be considered as free parameters in the following. To study how the unparticle sector influences the cosmology of the SM, we focus on a specific interaction between SM quarks and a scalar unparticle operator \(O_U\) \[1\],

\[
\frac{i\lambda}{\Lambda_U^{d_{BZ}}} T\gamma_\mu (1 - \gamma_5) \, q' \, \partial^\mu O_U + h.c. .
\]

Here \(\lambda \equiv C_U (\Lambda_U / M_U)^{d_{BZ}}\) is a dimensionless coupling.
A useful conceptual framework for understanding the unparticle sector was presented in [9]. It was shown that by breaking scale-invariance in a controlled way, the continuous energy spectrum of the unparticle sector can be replaced by a discrete tower of deconstructing scalar particles \( \phi_n \) of mass \( M_n \), with mass spacing controlled by a mass parameter \( \Delta \) such that \( M_n^2 = n\Delta^2 \). In this case the unparticle operator \( O_U \) is replaced by a sum over canonically normalized deconstructing scalar fields \( \phi_n \),

\[
\sum_{n=1}^{\infty} \frac{i\lambda F_n}{A_{dU}^2} q \gamma^\mu (1 - \gamma_5) q' \partial^\mu \phi_n + \text{h.c.} ,
\]

where

\[
F_n^2 = \frac{A_{dU}}{2\pi} \Delta^2 (M_n^2)^{dU} - 2
\]

and

\[
A_{dU} = \frac{16\pi^{5/2}}{(2\pi)^{2dU}} \frac{\Gamma (d_U + 1/2)}{\Gamma (d_U - 1) \Gamma (2d_U)} .
\]

\( A_{dU} \) is a conventionally chosen phase space factor for unparticles [1]; only the combination \( C_U A_{dU}^2 \) appears in physical processes. In the limit \( \Delta \to 0 \), the sum over decays \( q' \to q + \phi_n \) for all kinematically allowed values of \( n \) produces an expression for \( d\Gamma/dE_q \) in agreement with the direct unparticle calculation of \( q' \to q + U \) based on \( O_U [9] \). The individual \( q + \phi_n \) final states of energy \( E_q = (m_q'^2 - M_n^2)/2m_q' \) merge into a continuum decay rate as a function of \( E_q \) as \( \Delta \to 0 \), reproducing the unparticle result.

How should the unparticles be interpreted physically? In [10] it was observed that unparticle states can be considered to be particle-like states with a continuous mass. (See also [11].) In the limit \( \Delta \to 0 \) the deconstructing scalar will also have a continuous mass, consistent with this picture. What are these continuous mass scalars? The strongly self-coupled CFT will have no conventional particle interpretation (a characteristic feature of unparticles) since the conformal field quanta are confined by their self-interaction. The unparticle states we observe, which are created by the unparticle operator, will then correspond to composite objects made of the strongly self-interacting CFT fields. These objects can have all kinematically possible masses and radii as a result of the scale-invariance of the underlying CFT, consistent with the continuous mass states proposed in [10]. The continuous mass implies that the
unparticle sector has a large number of effective degrees of freedom, corresponding to
the different confined states which can be formed from the CFT fields.

Motivated by the above interpretation, we will assume in the following that the
unparticles are physically equivalent to continuous mass particles. Ultimately this
picture should be confirmed by the dynamics of the underlying CFT, which will also
determine the parameters of the generic unparticle model $d_U$, $C_U$ and $A_{d_U}$.

3 Cosmological constraints from quark decay and
annihilation to unparticles

In the following we will consider the couplings Eq. (3) and Eq. (4) in the context of
cosmology. In particular, we will calculate the decay and annihilation rates of thermal
background quarks to unparticles to obtain the conditions under which these processes
can produce a significant unparticle energy density and the resulting constraints on
cosmology.

A basic property of the unparticle sector is that the unparticles are stable and
no SM particles are produced by decay of the unparticle energy density [9]. This
can be understood by considering the deconstructing scalars in the limit $\Delta \to 0$.
In this limit the couplings of the deconstructing scalars, which are proportional to
$F_n \propto \Delta^{d_U-1}$, tend to zero if $d_U > 1$. The reason the decay and scattering rates
of SM particles to unparticles is finite in the $\Delta \to 0$ limit is that the decrease in
the rate to a scalar particle final state $\phi_n$ is compensated for by an increase in the
number of kinematically allowed scalar final states $\approx E^2/\Delta^2$, where $E$ is the energy
the decaying or annihilating quark. (We will demonstrate this explicitly for the case of
pair annihilation of SM quarks to deconstructing scalars.) However, if we consider the
decay and scattering of deconstructing scalars to SM particles, then as $\Delta \to 0$ there is
no compensating increase in the number of SM particles. So the rates of all processes
which could result in the decay of the unparticle energy density to SM particles tend
to zero in the unparticle limit $\Delta \to 0$ [9]. The relative stability of the unparticle
states is consistent with the picture of unparticle states having a continuous mass. SM
states can scatter into any kinematically allowed unparticle mass state, analogous to production of different hadron-like states in a confined theory, except here the hadrons have continuous masses. However, in the reverse process, the unparticle states can only scatter into a finite number of SM states. Therefore the rate of unparticle scattering or decay to SM particles will generally be much smaller than the reverse process of SM states scattering or decaying into unparticle final states.

Since the unparticle sector is scale-invariant, the unparticle density evolves with scale factor as $\rho_U \propto a^{-4}$ [8]. If the unparticle density decouples from the SM density at $T_{DEC} \gg T_{BBN} \approx 0.1$ MeV, where $T_{BBN}$ is the temperature at Big-Bang Nucleosynthesis (BBN), then a dilution of the unparticle density relative to the radiation density is possible, due to subsequent photon heating by annihilation of massive SM particles. However, the maximum dilution relative to photons is only by a factor $(g(T_{BBN})/g(T_{dec}))^{1/3}$, since the photon energy density is proportional to $g(T)T^4$ while the unparticle density redshifts as $a^{-4}$ with $g(T)a^3T^3 = constant$. The maximum dilution is obtained if decoupling occurs at a higher temperature than the electroweak phase transition, in which case $g(T_{DEC}) = 106.75$. With $g(T_{BBN}) = 2$ for photons, and adding the neutrino density at BBN which increases the total radiation density by 1.7 relative to the photon component, this gives dilution by at most a factor 0.16. This is not enough to dilute the unparticle density sufficiently unless $\rho_U < 0.4\rho_{SM}$ at $T_{DEC}$, since successful nucleosynthesis requires that any additional energy density be less than 6% of the total at $T_{BBN}$ [15]. Therefore in order that the successful light element abundance predictions of BBN are unaffected, SM quark decay or pair annihilation to unparticles should not produce a large unparticle density.

### 3.1 Quark decay to unparticles

The interaction Eq. (4) implies a differential decay rate to unparticles $q' \rightarrow q + U$ given by [11][9]

$$\frac{d\Gamma}{dE_q} = \frac{|\lambda|^2 m_q^2 A_{du} E_q^2 (m_q^2 - 2m_q E_q)^{du-2}}{2\pi^2 \Lambda_{2du}^2}.$$  

(5)
This is the decay rate in the $q'$ quark rest frame. Integrating this for all energies up to $m_{q'}/2$ gives

$$\Gamma = \frac{|\lambda|^2 A_{d_U} m_q^{2d_U+1}}{8\pi^2 d_U(d_U^2 - 1)\Lambda_{2d_U}^2}.$$  \hspace{1cm} (6)

In the limit $d_U \rightarrow 1$ the decay rate becomes infinite, therefore $d_U > 1$ must be imposed \[^{[1]}\], in agreement with general unitarity arguments for a scalar unparticle operator \[^{[10]}\].

For thermal relativistic quarks of mean energy $E \approx 3T$, the decay rate $\Gamma_d$ is obtained from the rest frame decay rate by dividing by the Lorentz factor $\gamma = 3T/m_{q'}$. Therefore

$$\Gamma_d \approx \frac{|\lambda|^2 A_{d_U} m_q^{2d_U+2}}{8\pi^2 d_U(d_U^2 - 1)\Lambda_{2d_U}^2} \frac{m_{q'}^{2d_U+1}}{3T}.$$ \hspace{1cm} (7)

At $T < T_{EW}$, where $T_{EW}$ is the temperature of the electroweak phase transition, we will assume that the Higgs expectation value is given by the $T = 0$ value\[^{[2]}\] $<H> = v$. At $T > T_{EW}$, we expect the effective mass of the quarks to be given by the temperature, $m_{q'} \approx T$. Therefore

$$\Gamma_d \approx \frac{C_2^2 A_{d_U}}{24\pi^2} \frac{\Lambda_{2d_{BZ} - d_U}^2}{d_U(d_U^2 - 1)M_{d_{BZ}}^2} \frac{m_{q'}^{2d_{BZ} + 1}}{T} ; \hspace{0.5cm} T < T_{EW}$$ \hspace{1cm} (8)

and

$$\Gamma_d \approx \frac{C_2^2 A_{d_U}}{24\pi^2} \frac{\Lambda_{2d_{BZ} - d_U}^2}{d_U(d_U^2 - 1)M_{d_{BZ}}^2} \frac{m_{q'}^{2d_{BZ} + 1}}{T} ; \hspace{0.5cm} T > T_{EW}.$$ \hspace{1cm} (9)

In order not to produce a large unparticle energy density we must then impose that $\Gamma_d < H$, where during SM radiation-domiation $H = k_T T^2/M$ with $k_T = (\pi^2 g(T)/90)^{1/2}$ and $M = M_{Pl}/\sqrt{8\pi}$.

For $T < T_{EW}$, the strongest constraint comes from the case of $t$ quark decay. In this case the condition $\Gamma_d < H$ is strongest at the lowest value of $T$ for which there are relativistic thermal $t$-quarks, $T \approx m_t/3$, which implies that

$$M_{d_{BZ}}^2 \gtrsim \frac{9}{8\pi^2} \frac{k_t m_{t}^{2d_{U}-1}M\Lambda_{2d_{BZ} - d_U}^2}{k_T d_U(d_U^2 - 1)} ,$$ \hspace{1cm} (10)

\[^{2}\text{The Higgs expectation value is temperature dependent, given by }< H > = v(1 - T^2/T_{EW}^2)^{1/2}, \text{ where } T_{EW} \approx 1.2m_h \[^{[17]}\]. \text{ For simplicity in our estimates we will use the approximation that }< H > = 0 \text{ at } T < T_{EW} \text{ and }< H > = v \text{ at } T > T_{EW}.\]
where $k_1 = C_U^2 A_{d_U}$. If this condition is satisfied, then for $m_t/3 \lesssim T < T_{EW}$ there is no significant decay of the SM radiation in unparticles. Once $T \lesssim m_t/3$, the next possible decay is to b quarks, but the condition for this not to occur, which replaces $m_t$ by $m_b$ in Eq. (10), is automatically satisfied if Eq. (10) is satisfied\(^3\). On the other hand, if Eq. (10) is not satisfied, then as $T$ decreases from $T_{EW}$, at some $T > m_t/3$ the decay rate becomes faster than $H$. The SM radiation then loses energy to unparticles, decreasing $T$ and increasing $\Gamma_d$ relative to $H$ (which remains constant throughout since the total energy density is constant) until $T \lesssim m_t/3$. As a result, if $T$ is larger than $m_t/3$, a large fraction of the SM radiation energy will be transferred to the unparticle sector, resulting in an unparticle dominated Universe.

For $T > T_{EW}$, the condition $\Gamma_d < H$ requires that

$$T^{2d_U-1} < \frac{24\pi^2 k_T d_U (d_U^2 - 1) M_{d_BZ}^{2d_BZ}}{k_1 \Lambda_{d_BZ-d_U}^{2d_BZ-1} M}. \quad (11)$$

If this condition is not satisfied, then SM radiation will rapidly decay to unparticles until $T$ has decreased sufficiently that the condition $\Gamma_d < H$ is satisfied. As a result, a large fraction of the SM radiation energy will have transferred to the unparticle sector when the decay rate becomes negligible. Therefore we reach the important conclusion that there is an upper bound on the temperature of the radiation-dominated universe in the presence of an unparticle sector. This is likely to play an important role in the development of a complete unparticle cosmology.

These conclusions are based on the idea that any process which results in energy transfer will always be much more rapid from the SM to the unparticle sector than the reverse process. In the deconstructing scalar picture the unparticle states are stable with respect to decay and annihilation via the interaction Eq. (3). However, it is conceivable that unparticle self-interactions, which will depend on the details of the underlying CFT, could results in processes transferring energy to the SM. Nevertheless, the reverse process from SM to unparticles will always be much more rapid due to the large number of unparticle final states, parameterised by their continuous mass. It is

\(^3\)Decay to other light quarks is also possible, but these decays will be subdominant due to the quark mass factor in Eq. (10).
important to emphasize that our conclusions do not depend on thermal equilibrium of
the unparticle sector, nor do they require a large number of degrees of freedom in the
underlying CFT as in the case of cooling into large $N$ CFTs [18]. The rapid transfer
of energy is due to the continuous mass nature of the unparticle states formed by the
strongly-coupled CFT, which follows from the scale-invariance of the CFT.

### 3.2 Quark pair annihilation to unparticles

We next consider the annihilation of thermal quark pairs to unparticles, by calculating
the scattering cross-section to deconstructing scalars in the $\Delta \rightarrow 0$ limit. The CM
annihilation cross-section $q + q \rightarrow \phi_m + \phi_n$ following from Eq. (4) is given by

$$
\sigma_{mn} \approx \frac{g_m^2 g_n^2 E^2}{M_s^4},
$$

(12)

where $E$ is the energy of the annihilating quarks,

$$
g_n^2 = \frac{|\lambda|^2}{n^{(2-d_U)}}
$$

and

$$
M_s^2 = \frac{2\pi}{A_{dU}} \left( \frac{\Lambda_U}{\Delta} \right)^{2d_U} \Delta^2.
$$

(In this we have written the coupling for each $n$ in Eq. (4) as $g_n/M_s$.) The total
scattering rate to deconstructing scalars is obtained by summing over the kinematically
allowed final states. With quark energy $E$, the final state scalars have $n_{\text{max}} = E^2/\Delta^2$,

therefore

$$
\sigma_{\text{TOT}} = \sum_{n=1}^{n_{\text{max}}} \sum_{m=1}^{m_{\text{max}}} \sigma_{mn}.
$$

(13)

For $n_{\text{max}} \gg 1$ we can replace the sums with integrals,

$$
\sum_{n=1}^{n_{\text{max}}} g_n^2 \approx \int_{1}^{E^2/\Delta^2} g_n^2 dn \equiv \int_{1}^{E^2/\Delta^2} \frac{|\lambda|^2}{n^{(2-d_U)}} dn.
$$

(14)

Therefore

$$
\sum_{n=1}^{n_{\text{max}}} g_n^2 \approx \frac{|\lambda|^2}{(d_U - 1)} \left( \frac{E}{\Delta} \right)^{2(d_U - 1)} - 1.
$$

(15)
Thus summing over \( m \) and \( n \) is Eq. (13) gives
\[
\sigma_{TOT} \approx \frac{E^2}{M_s^4 (d_U - 1)^2} \left[ \left( \frac{E}{\Delta} \right)^{2(d_U - 1)} - 1 \right]^2.
\] (16)

Replacing \( M_s^2 \) by its definition then gives
\[
\sigma_{TOT} \approx \frac{|\lambda|^4 E^2}{(d_U - 1)^2} \left[ \left( \frac{E}{\Delta} \right)^{2(d_U - 1)} - 1 \right]^2 \frac{A_{d_U}^2 \Delta^{4(d_U - 1)}}{(2\pi)^2 \Lambda_U^{4d_U}}.
\] (17)

In the case \( d_U > 1 \), in the unparticle limit \( \Delta \to 0 \) the cross-section tends to a finite limit
\[
\sigma_{TOT} \approx \frac{|\lambda|^4}{(d_U - 1)^2} \frac{A_{d_U}^2 E^{4d_U - 2}}{(2\pi)^2 \Lambda_U^{4d_U}}.
\] (18)

In the case \( d_U \leq 1 \), \( \sigma_{TOT} \to \infty \) as \( \Delta \to 0 \). Therefore, as in the case of the quark decay rate, the quark annihilation rate is singular as \( d_U \to 1 \).

For \( d_U > 1 \) the annihilation rate is then \( \Gamma_{ann} = n_q \sigma_{TOT} \), where \( n_q = 12T^3/\pi^2 \) is the number density of thermal quarks \( q \). Therefore, with \( E \approx 3T \),
\[
\Gamma_{ann} \approx \frac{1}{9\pi^4} \frac{C_U^4 A_{d_U}^2}{(d_U - 1)^2} \frac{(3T)^{4d_U + 1}}{\Lambda_U^{4d_U}}.
\] (19)

In most cases this rate is small compared with the quark decay rate. However, for \( d_U \) sufficiently close to 1, the annihilation rate \( (\propto (d_U - 1)^{-2}) \) can become larger than the decay rate Eq. (7) \( (\propto (d_U - 1)^{-1}) \).

### 3.3 Cosmological constraints on unparticle parameters

We next consider the constraints on \( M_U \) and \( \Lambda_U \) as a function of \( d_U \) following from thermal quark decay to unparticles. The bounds derived above assume that the energy of the quarks is less than \( \Lambda_U \), otherwise the decay and annihilation rates should be calculated for BZ particle final states. Since the most interesting case is where unparticles may be observed in future colliders, we will consider \( \Lambda_U \gtrsim 1 \) TeV in the following. The decay rate depends upon the product \( k_1 \equiv C_U^2 A_{d_U} \), which is determined by the infra-red dynamics of the Banks-Zaks fields [1]. Therefore any conclusions about the cosmology of unparticles is dependent upon assumptions about this product, which
should become better understood as unparticle physics develops. In the following we will assume that $k_1$ is not very large or small compared with 1.

The constraints will depend on the assumed values of the model parameters $d_{BZ}$, $d_U$ and $\Lambda_U$. We first focus on the case where the BZ operator is a quark bilinear, so that $d_{BZ} = 3$, and calculate constraints for the cases $d_U = 1.1$, 1.5 and 2 when $\Lambda_U \gtrsim 1$ TeV.

The most important constraints come from Eq. (10) when $T < T_{EW}$. This gives lower bounds on $M_U$ as a function of $d_U$

$$d_U = 1.1: \quad M_U \gtrsim 190 k_1^{1/6} \left( \frac{\Lambda_U}{1 \text{ TeV}} \right)^{19/30} \text{ TeV}$$

$$d_U = 1.5: \quad M_U \gtrsim 110 k_1^{1/6} \left( \frac{\Lambda_U}{1 \text{ TeV}} \right)^{1/2} \text{ TeV}$$

$$d_U = 2.0: \quad M_U \gtrsim 70 k_1^{1/6} \left( \frac{\Lambda_U}{1 \text{ TeV}} \right)^{1/3} \text{ TeV}, \quad (20)$$

where we have used $k_T = 3$. Thus if $k_1$ is not very large or small compared with 1 and $\Lambda_U \gtrsim 1$ TeV, the mass scale of the $d_{BZ} = 3$ interaction between the BZ and SM fields must be greater than around 100 TeV for $d_U$ between 1.1 and 2.0. These bounds hold if the radiation-dominated era has $T \gtrsim m_t/3$ at some time, otherwise weaker bounds based on lighter quark decays will apply.

Upper bounds on the temperature of the radiation-dominated era when $T > T_{EW}$ can be obtained from Eq. (11). However, these bounds strictly hold only if the energy of the decaying quark $E \approx 3T$ is less than $\Lambda_U$, since for larger temperatures and energies the decay will produce BZ particles rather than unparticles. For $T_{EW} < T \lesssim \Lambda_U/3$ we obtain:

$$d_U = 1.1: \quad T \lesssim 10 \text{ TeV} \left( \frac{1}{k_1} \right)^{5/6} \left( \frac{M_U}{250 \text{ TeV}} \right)^5 \left( \frac{1 \text{ TeV}}{\Lambda_U} \right)^{19/6}$$

$$d_U = 1.5: \quad T \lesssim 2.6 \text{ TeV} \left( \frac{1}{k_1} \right)^{1/2} \left( \frac{M_U}{150 \text{ TeV}} \right)^3 \left( \frac{1 \text{ TeV}}{\Lambda_U} \right)^{3/2}$$

$$d_U = 2.0: \quad T \lesssim 1.3 \text{ TeV} \left( \frac{1}{k_1} \right)^{1/3} \left( \frac{M_U}{100 \text{ TeV}} \right)^2 \left( \frac{1 \text{ TeV}}{\Lambda_U} \right)^{2/3}. \quad (21)$$

The upper bounds on $T$ are sensitive to $M_U$ and the value of $3T$ can easily be above $\Lambda_U$. In this case the decay should be calculated to BZ sector fields using the interaction
Eq. (11). For $d_{BZ} = 3$ and $O_{BZ}$ corresponding to a bilinear of BZ fermions $\Psi \Psi$, the interaction with the SM quarks becomes

$$\frac{1}{M_U^6} O_{SM} O_{BZ} \rightarrow \partial_\mu (\gamma_\mu (1-\gamma_5) q') \overline{\Psi} \Psi.$$ (22)

The decay rate for $q' \rightarrow q \overline{\Psi} \Psi$ is then

$$\Gamma_d \approx \frac{1}{(8\pi)^3} \frac{T^7}{M_U^6}.$$ (23)

The condition $\Gamma_d < H$ then implies that

$$T \lesssim 1.8 \left( \frac{M_U}{100 \text{ TeV}} \right)^{6/5} \text{ TeV}.$$ (24)

If this bound is not satisfied, energy will flow to the BZ sector until thermal equilibrium is established with the BZ sector, resulting in a large BZ sector energy density. For example, for the case of the original Banks-Zaks theory with an $SU(3)$ gauge group and $N_F > 16.5$ Dirac fermions transforming in the fundamental representation of $SU(3)$ [14], the number of thermal degrees of freedom is at least $g(T) = 8 \times 2 + 7/8 \times 4 \times 3 \times 17 = 195.5$, which is larger than the number of degrees of freedom in the SM, $g(T) = 106.75$. Therefore in thermal equilibrium, $\rho_{BZ} \approx 2 \rho_{SM}$. Once $T$ drops below the upper limit in Eq. (24), the SM and BZ sectors decouple. As the BZ fields lose energy via expansion they will evolve into unparticles, leaving an energy density in the unparticle sector which is too large to dilute sufficiently by photon heating in the SM before nucleosynthesis. Therefore, a stable unparticle sector in general implies an upper limit on the temperature of the radiation-dominated Universe, which is of the order of 1 TeV for $M_U$ close to its lower limit.

The above results are for the case $d_{BZ} = 3$. To show the sensitivity to $d_{BZ}$ we calculate the lower bound on $M_U$ from Eq. (10) for the cases $d_{BZ} = 2$ and 4:

$d_{BZ} = 2$

$\mathbf{d_U = 1.1}: \quad M_U \gtrsim 2600 \ k_1^{1/4} \left( \frac{\Lambda_U}{1 \ \text{TeV}} \right)^{9/20} \text{ TeV}$

$\mathbf{d_U = 1.5}: \quad M_U \gtrsim 1100 \ k_1^{1/4} \left( \frac{\Lambda_U}{1 \ \text{TeV}} \right)^{1/4} \text{ TeV}$

$\mathbf{d_U = 2.0}: \quad M_U \gtrsim 520 \ k_1^{1/4} \text{ TeV}.$ (25)
$d_{BZ} = 4$

$d_{U} = 1.1 : \quad M_{U} \gtrsim 43\, k_{1}^{1/8}\left(\frac{\Lambda_{U}}{1\, \text{TeV}}\right)^{29/40} \, \text{TeV}$

$d_{U} = 1.5 : \quad M_{U} \gtrsim 33\, k_{1}^{1/8}\left(\frac{\Lambda_{U}}{1\, \text{TeV}}\right)^{5/8} \, \text{TeV}$

$d_{U} = 2.0 : \quad M_{U} \gtrsim 23\, k_{1}^{1/8}\left(\frac{\Lambda_{U}}{1\, \text{TeV}}\right)^{1/2} \, \text{TeV}.$  \hspace{1cm} (26)

Larger $d_{BZ}$ for a given $d_{U}$ reduces the lower bound on $M_{U}$ for $\Lambda_{U} \approx 1$ TeV, but increases the sensitivity to $\Lambda_{U}$. From Eq. (20), Eq. (25) and Eq. (26) we see that the lower bounds on $M_{U}$ are in the range 20 TeV to 2600 TeV when $1.1 \leq d_{U} \leq 2.0$ and $2 \leq d_{BZ} \leq 4$, assuming that $\Lambda_{U} \gtrsim 1$ TeV and $k_{1}$ is not too different from 1.

### 3.4 Discussion

The first study of unparticle cosmology was presented in [8]. This was based on a dimensional estimate of the rate of production $\Gamma_{\psi}$ of unparticles from SM radiation via a vector unparticle operator interaction of the form $\overline{\psi} \gamma_{\mu} \psi O_{\mu}^{I}$. It was assumed that once $\Gamma_{\psi} > H$, the unparticles and SM particles are in thermal equilibrium at $T$. Once the unparticles decouple, the unparticle sector is assumed to have a temperature $T_{U}$ which is less than the photon temperature $T$ due to photon heating by subsequent annihilations. In order to ensure that $T_{U}$ is sufficiently suppressed that the additional unparticle energy does not affect BBN, it was required that decoupling of the unparticles occurs at $T \gtrsim 1$ GeV, before the QCD phase transition. Requiring this imposes an upper bound on $\Lambda_{U}$. However, as we have shown, the actual suppression of the unparticle density relative to the radiation density is by at most a factor of 0.16, which is ineffective in suppressing a decoupled unparticle density with $\rho_{U} > 0.4 \rho_{SM}$.

The main difference in our discussion of unparticle cosmology is in the interpretation of the condition $\Gamma_{d} > H$. Rather than establishing thermal equilibrium between the SM and unparticles, we interpret this as a rapid flux of energy from SM radiation to the unparticle sector, resulting in an unparticle dominated Universe. Therefore $\Gamma_{d} < H$ must hold for all $T$, imposing an upper bound on $T$ during radiation-domination. Our analysis also differs in the interaction we considered, based on a scalar unparticle op-
erator. The decay and annihilation rates in this case differ from a simple dimensional estimate due to the quark mass factors which must also be included.

Strong constraints on unparticles can be imposed by supernovae \cite{5} and by unparticle-mediated long-range forces \cite{6}. However, these constraints can be evaded by slightly by breaking the conformal invariance, for example by giving the deconstructing scalars a mass $> 30$ MeV to suppress their creation in supernovae.

Since we have shown that $M_U \gtrsim 100$ TeV is typically necessary for compatibility with SM cosmology, it might appear that detection of unparticles in collider experiments is unlikely. However, in the case where unparticle operators couple to the Higgs doublet, it is possible to detect unparticles with such large $M_U$ e.g. by observing the partial Higgs boson decay width to gluon and photon pairs \cite{12}. Such unparticle-Higgs interactions can also break conformal invariance of the unparticles and can modify electroweak symmetry breaking \cite{13}.

4 Conclusions

We have shown that the requirement that unparticles do not dominate the energy density at nucleosynthesis implies that there is a lower bound on the scale $M_U$ of the interaction between the Banks-Zaks fields and the Standard Model and an upper bound on the temperature of the radiation-dominated Universe.

For an interaction between SM quarks and scalar unparticle operators with $d_{BZ} = 3$ and $\Lambda_U \gtrsim 1$ TeV, the lower bound on $M_U$ is typically of the order of 100 TeV for unparticle dimensions $d_U$ in the range 1.1 - 2.0. Varying $d_{BZ}$ widens the range of lower bounds, with $M_U \gtrsim 20 - 2600$ TeV for $2 \leq d_{BZ} \leq 4$. Decays will dominate over annihilations except if $d_U$ is very close to 1. Including other unparticle operators can only strengthen these bounds.

An important consequence of an unparticle sector coupled to the Standard Model is the existence an upper bound on the temperature of the Standard Model radiation-dominated era. In the example with $d_{BZ} = 3$, the upper bound is of the order of 1 TeV for $M_U$ close to its lower bound.
These conclusions are based on the assumption that unparticles are physically equivalent to continuous mass particles. If true, this implies that the rate of scattering of SM particles to unparticle states is generally much larger than that of the reverse process scattering unparticle states to SM particles. As a result, energy will flow from the SM thermal bath to the unparticle sector.

The existing literature on unparticles is based purely on scale-invariance, with the physics of the strongly-coupled CFT being absorbed into the free parameters of the generic unparticle model. In order to establish the conditions under which unparticles are equivalent to continuous mass states, as well as to relate the unparticle parameters to the underlying CFT, a detailed analysis of strongly-coupled CFTs and their relation to unparticle states is needed.

An important issue related to this is the necessary equivalence of the thermodynamics of the CFT, which is determined by its central charge, and the thermodynamics of unparticles in the continuous mass picture. This should shed light on the validity of the continuous mass picture and on the relationship between the parameters of the underlying CFT and the unparticle parameters based on scale-invariance.

There are many other issues to be addressed before a complete understanding of unparticle cosmology can be achieved. One is the effect of breaking the conformal invariance of the unparticle sector on the stability of the unparticle energy density. Another is whether a complete unparticle cosmology, including inflation, reheating and baryogenesis, can be constructed with a low maximum temperature during radiation-domination. Establishing the lowest radiation-domination temperature for which a complete unparticle cosmology is feasible will place strong constraints on unparticle operators, which will in turn have direct consequences for the possibility of observing the unparticle sector experimentally. We will return to these issues in future work.

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References


