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Impedance matrix of a folded dipole pair under eleven configuration

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Abstract: A new decade wideband antenna, the Eleven antenna, has been developed during the last years. The basic configuration of Eleven antenna is two parallel dipoles separated by half wavelength above the ground plane, which is referred to as the eleven configuration. In order to cascade the dipoles in log-periodic array for obtaining wideband performance, folded dipoles are often used. The input impedance of shorted folded dipole in free space has been analysed by using two-mode method: a transmission line mode and an antenna mode. In the present study, the impedance matrix of a pair of folded dipoles as a two-port network under the eleven configuration will be analysed by including the mutual coupling among elements of folded dipoles into the two-mode method. As verification, an Eleven antenna with one pair of folded dipoles has been analysed by the present method, modelled in a commercial software – CST MS and manufactured. The results by using the present method agree well with the measured and simulated one.

1 Introduction

Chalmers University of Technology during the last years has been developing a new decade wideband antenna – the Eleven antenna [1–6]. Successful models have been made for use in different radio telescopes: 150–1500 MHz model for Green Bank [2], 200–800 MHz for GMRT [3] and 500–3000 MHz for RATAN [4]. There is also an interest in using the Eleven antenna as a feed in reflectors for square kilometre array to cover 1–13 GHz, and for VLBI 2010 project in which case it is desirable to go up to 18 GHz. The basic geometry of the Eleven antenna is two parallel dipoles separated by half wavelength and located above the ground plane, which is referred to as the eleven configuration, see Fig. 1. The Eleven antenna has very good features: the nearly constant beamwidth with 10 dBi directivity and the almost fixed phase centre location over the whole bandwidth, low profile and simple geometry. Folded dipoles are often used in the Eleven antenna in order to obtain wideband performance by cascading one after the another in a log-periodic array. Therefore it is important to analyse the characteristics of folded dipole under the eleven configuration in order to design the Eleven antenna efficiently. The input impedance of a shorted folded dipole in free space was analysed in 1980s and 1990s by decomposing the current on folded dipole into two distinct modes: a transmission line mode and an antenna mode [7, 8]. In this paper, we will apply the two-mode method with including the mutual couplings to analyse the characteristics of folded dipole under the eleven configuration and obtain impedance matrix (or ABCD matrix) for analysing cascaded folded dipoles one after another in a log-periodic array. As verification, an Eleven antenna with one pair of folded dipoles has been analysed by the present method, modelled in a commercial software CST Microwave Studio (time-domain finite integration method based) and manufactured. The calculated, simulated and measured data will be presented in the paper.

The purpose of this work is to investigate the two-mode method with including the mutual couplings for the folded dipoles under the eleven configuration. The present paper presents the first step of the work: only one pair of folded dipoles. Our goal is extending the method to full cascaded folded dipoles of Eleven antenna with including all mutual couplings, which will lead to a nearly analytical method for

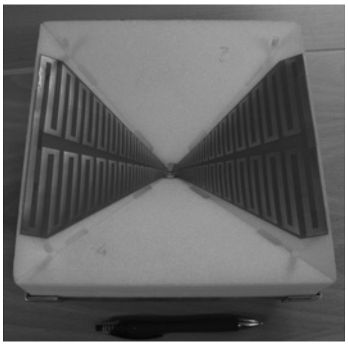

Figure 1 *Configuration of Eleven feed of single polarisation*

analysing Eleven antenna with much less numerical computation cost than a full wave simulation.

2 Impedance matrix of a folded dipole in free space

It is assumed that the folded dipole analysed in the paper is symmetrical: the diameter or the width of the two arms of the dipole is the same and the shape of the folded dipole is rectangular. Then, the currents on the folded dipole can be decomposed into two mode currents: the transmission line mode and the antenna mode [7, 8], see Fig. 2. Each folded dipole arm consists of a shorted transmission line (stub) of characteristic impedance Z_0 and length $L/2$. Therefore the impedance of the transmission line mode seen at input of one such stub is

$$Z_T = jZ_0 \tan\left(k\frac{L}{2}\right) \tag{1}$$

where k is the wavenumber. The transmission line mode current I_T is

$$I_T = \frac{\alpha V}{Z_T} \tag{2}$$

where α is a coefficient to be determined. For the antenna

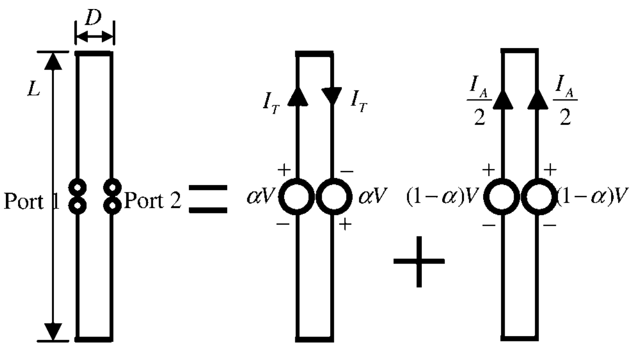

Figure 2 *Decomposition of folded dipole into transmission line mode and antenna mode*

mode current I_A, we have

$$I_A = \frac{(1-\alpha)V}{Z_D} \tag{3}$$

where Z_D is the input impedance of a linear dipole of length L.

2.1 Impedance matrix

We can use the following impedance matrix to describe the two-port network of a symmetrical folded dipole as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{4}$$

where

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \tag{5}$$

when $I_2 = 0$, we have

$$I_2 = \frac{I_A}{2} - I_T = \frac{(1-\alpha)V}{2Z_D} - \frac{\alpha V}{Z_T} = 0 \tag{6}$$

Then, we obtain

$$\alpha = \frac{Z_T}{(Z_T + 2Z_D)} \tag{7}$$

From the above, we have

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{\alpha V + (1-\alpha)V}{I_T + I_A/2}\right|_{I_2=0} = Z_D + \frac{Z_T}{2} \tag{8}$$

By the similar procedure, we can obtain

$$Z_{12} = \frac{Z_D - Z_T}{2} \tag{9}$$

And the impedance matrix of a folded dipole is

$$\boldsymbol{Z} = \begin{bmatrix} \frac{Z_D + Z_T}{2} & \frac{Z_D - Z_T}{2} \\ \frac{Z_D - Z_T}{2} & \frac{Z_D + Z_T}{2} \end{bmatrix} \tag{10}$$

2.2 Admittance matrix

When the folded dipole is half wavelength dipole, the input impedance of transmission line mode becomes infinite, that is, $Z_T = \infty$. Then, we should use admittance matrix to describe the folded dipole as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{11}$$

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where

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}, \quad Y_{12} = \frac{I_2}{V_1}\bigg|_{V_2=0} \tag{12}$$

when $V_2 = 0$, we have $\alpha = 0.5$ and thus

$$Y_{11} = \frac{1}{4Z_{\rm D}} + \frac{1}{2Z_{\rm T}}, \quad Y_{12} = \frac{1}{4Z_{\rm D}} - \frac{1}{2Z_{\rm T}} \tag{13}$$

Therefore the admittance matrix of a folded dipole is

$$\boldsymbol{Y} = \begin{bmatrix} \frac{1}{4Z_{\rm D}} + \frac{1}{2Z_{\rm T}} & \frac{1}{4Z_{\rm D}} - \frac{1}{2Z_{\rm T}} \\ \frac{1}{4Z_{\rm D}} - \frac{1}{2Z_{\rm T}} & \frac{1}{4Z_{\rm D}} + \frac{1}{2Z_{\rm T}} \end{bmatrix} \tag{14}$$

When port 2 is shorted, which is the case discussed in [7], from (14), we can obtain

$$Z_{\rm in} = \frac{1}{Y_{11}} = \frac{4Z_{\rm T}Z_{\rm D}}{Z_{\rm T} + 2Z_{\rm D}} \tag{15}$$

which is the same as in [7]. If $L = \lambda/2$, the result becomes the well-known $Z_{\rm in} = 4Z_{\rm D}$.

After having impedance and admittance matrix, it is easy to obtain the $\boldsymbol{S}$-matrix and ABCD matrix.

3 Impedances of a folded dipole pair under eleven configuration

When a pair of folded dipoles is separated by a distance of S and located above the ground plane with a height of H as shown in Fig. 3, we can use imaging to remove the ground plane and obtain an equivalent problem with a four-folded-dipole array, see Fig. 4. There is no radiation from the transmission line mode due to the opposite current direction on the two shorted wires. Therefore there are no mutual couplings among the transmission line modes of the folded dipoles. The mutual couplings exist only among the antenna modes. Thus, the equivalent problem becomes that the transmission mode current under the eleven configuration is the same as that in free space and the antenna mode current is changed due to the mutual couplings among the elements in the four-folded-dipole array, see Fig. 4. The mutual couplings can be presented by mutual impedances Z_S, Z_{2H} and Z_{S2H} of two parallel dipoles separated by a distance of S, $2H$ and $\sqrt{S^2 + (2H)^2}$, see Fig. 4. Now we can calculate the impedance in the antenna mode in the same way as calculating active impedance of one element in an antenna array defined as [9]

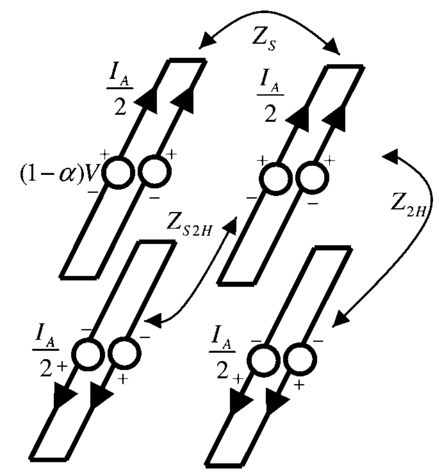

Figure 4 *Equivalent problem of antenna mode for a folded dipole under the Eleven configuration*

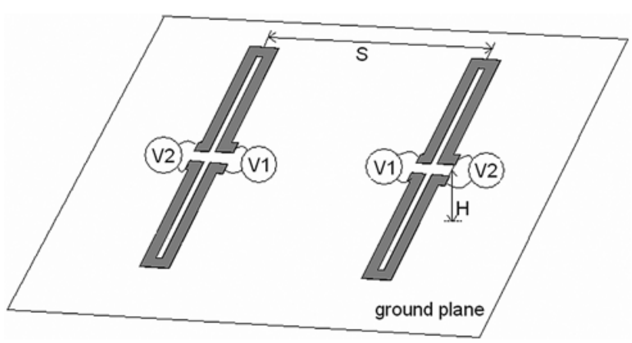

Figure 3 *Folded dipole under the Eleven configuration*

$$Z_m = \sum_{n=1}^{N} Z_{mn}\left(\frac{I_n}{I_m}\right) \tag{16}$$

where Z_m is the active impedance of element m, I_n is the current on element n and Z_{mn} is the mutual impedance between elements m and n. For our case and due to the excitation (both folded dipoles are excited with the same amplitude and phase), we have the currents on all four folded dipoles with the same amplitude but 180° phase difference on the imaging dipoles, see Fig. 4. Then we can obtain the antenna mode impedance of a folded dipole pair under the eleven configuration as

$$Z_{\rm D,Eleven} = Z_{\rm D} + Z_S - Z_{2H} - Z_{S2H} \tag{17}$$

4 Calculation of Z_0 and $Z_{\rm D,Eleven}$

In order to obtain the impedance characteristics of a folded dipole by using the above analysis, we need to calculate the characteristic impedance of the two-conductor transmission line Z_0 and antenna mode impedance $Z_{\rm D,Eleven}$. For the characteristic impedance of the two-conductor line with circular cross section, see Fig. 5, we have [10]

$$Z_0 = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon}}\cosh^{-1}\frac{D}{d} \tag{18}$$

For lines with rectangular cross section, see Fig. 5, we have

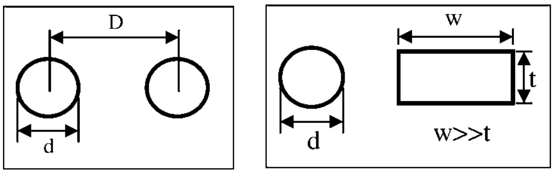

Figure 5 *Two-conductor transmission line with circular cross section (left) and the equivalence between circular and rectangular cross sections (right)*

equivalence between a rectangular and a circular cross section as [11]

$$\frac{d}{w} \simeq \frac{1}{2}\left[1 + \frac{t}{\pi w}\left(1 + \ln\left(4\pi\frac{w}{t}\right)\right)\right] \qquad (19)$$

For antenna mode impedance under the eleven configuration $Z_{\text{D,Eleven}}$, we need to calculate the self-impedance of the dipole in free space Z_{D} and the mutual impedances between dipoles for different separations Z_S, Z_{2H} and Z_{S2H}.We calculate the self- and mutual impedances of the dipoles of convenience by using the classical approach based on known distributed dipole current, refer detailed in [9]. We list the formula here but leave out the details that can be found in [9]. For the self-impedance, we have

$$Z_{\text{D}} = -\int_{-L/2}^{L/2} \boldsymbol{E}_j(\boldsymbol{r}_a) j_l(l)\, \mathrm{d}l \qquad (20)$$

where j_l is the normalised current distribution along the centre line of the dipole and $\boldsymbol{E}_j(\boldsymbol{r}_a)$ is the E field at the surface of the dipole due to j_l. For mutual impedance, we have

$$Z_M = -\int_{-L_1/2}^{L_1/2} \boldsymbol{E}_{12}(\boldsymbol{r}_a) j_1(l_1)\, \mathrm{d}l_1 \qquad (21)$$

where $j_1(l_1)$ is the normalised current distribution on dipole 1 along l_1 and $\boldsymbol{E}_{12}(\boldsymbol{r}_a)$ is the E field on the surface of dipole 1 due to the normalised current distribution $j_2(l_2)$ on dipole 2 along l_2.

5 Calculated, simulated and measured results

In order to verify the present method, a basic Eleven antenna with only one pair of folded dipoles has been manufactured and also modelled in a commercial software – CST Microwave Studio, shown in Fig. 6. The detailed dimensions are listed in Table 1. The metal strips are made of brass with uniform thickness 0.3 mm.The substrate is made of divinicell foam, which at L band is low loss and low relative permittivity (close to 1). It should be noted that we assumed the infinite large ground plane in the above analysis by using imaging. The size of the ground plane in the manufactured basic Eleven antenna and its CST model is $2\lambda \times 2\lambda$ at 1.5 GHz, where λ is the wavelength of the frequency. The effect of the finite

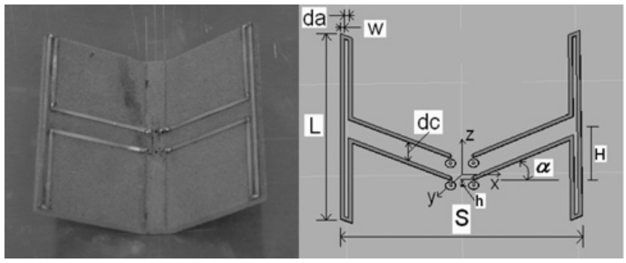

Figure 6 *Manufactured Eleven antenna and its model in CST*

Table 1 Dimensions of the manufactured Eleven antenna

length of dipole *L*, mm	100
width of dipole *w*, mm	2
thickness of all metal lines *t*, mm	0.3
spacing between two parallel dipoles *S*, mm	103.6
arm spacing in folded dipole *da*, mm	2
transmission line spacing *dc*, mm	10
tilted angle α	33°
height *h*, mm	5.2
height *H*, mm	33.6
size of the ground plane, mm	400 × 400

ground plane on the performance of Eleven antenna has been studied by Karandikar [12] and the conclusion is that if the size of the ground plane is larger than $1.2\lambda \times 1.2\lambda$, the effect of the finite dimension of the ground plane is negligible. For a whole Eleven antenna like the one shown in Fig. 1, the size of the ground plane is about $1.8\lambda_{\max} \times 1.8\lambda_{\max}$, where $\lambda_{\max}$ is the wavelength of the lowest operating frequency for Eleven antenna. Therefore the above analysis method is applicable.

By using (18) and (19), the characteristic impedance of the transmission line of the folded dipole in the transmission line mode shown in Fig. 6 can be obtained and the result is

$$Z_0 = \frac{1}{\pi} 377 \cosh^{-1}\frac{4}{1.26} = 218\,\text{Ohm} \qquad (22)$$

The self- and mutual impedances of the folded dipole can be calculated by (20) and (21), and the values are presented in Figs. 7 and 8. Then, using (1), (17) and (15), we can calculate the input impedance Z_{in} of the folded dipole under the Eleven configuration. It should be noted that Z_{D} in (15) should be replaced by $Z_{\text{D,Eleven}}$ calculated by using (17) if the mutual couplings are included. For our manufactured prototype, Z_{in} is equal to $1/Y_{11}$ because the two-port network of the one pair of folded dipoles is shorted at port 2. The calculated Y_{11} is shown in Fig. 10.

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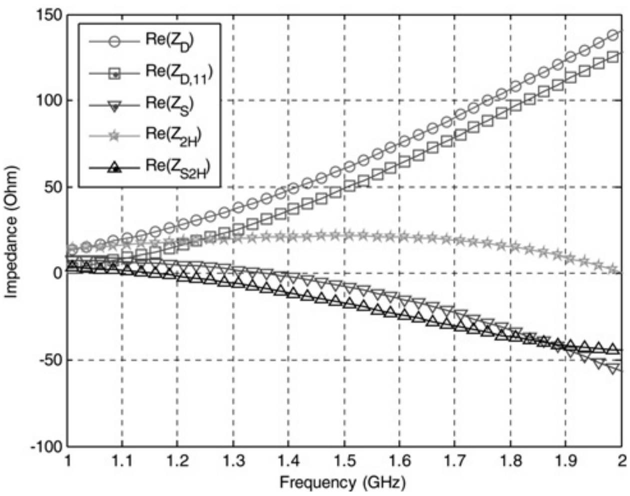

Figure 7 *Real part of self- and mutual impedance of the Eleven antenna defined in Table 1*

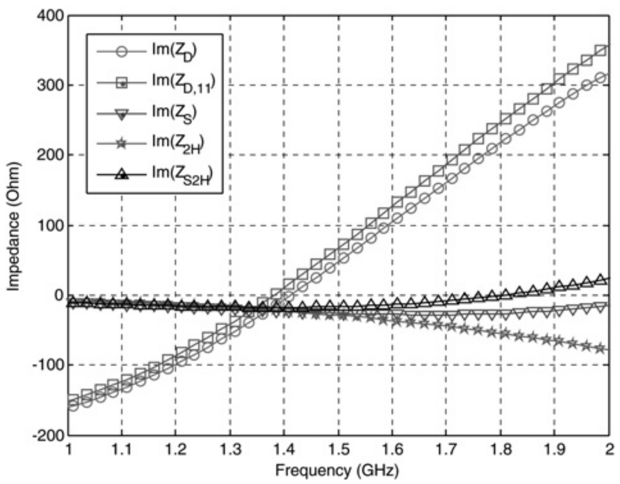

Figure 8 *Imaginary part of self- and mutual impedance of the Eleven antenna defined in Table 1*

Fig. 9 shows the model of the same folded dipole pair in CST. The folded dipoles are half wave dipoles at 1.5 GHz so the ***Y***-matrix is calculated in order to avoid the infinite

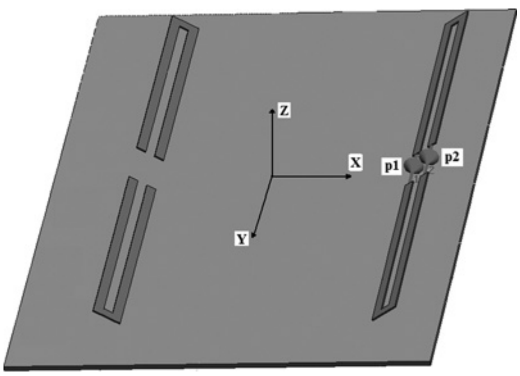

Figure 9 *Folded dipole pair under Eleven configuration as a two-port network modelled in CST*

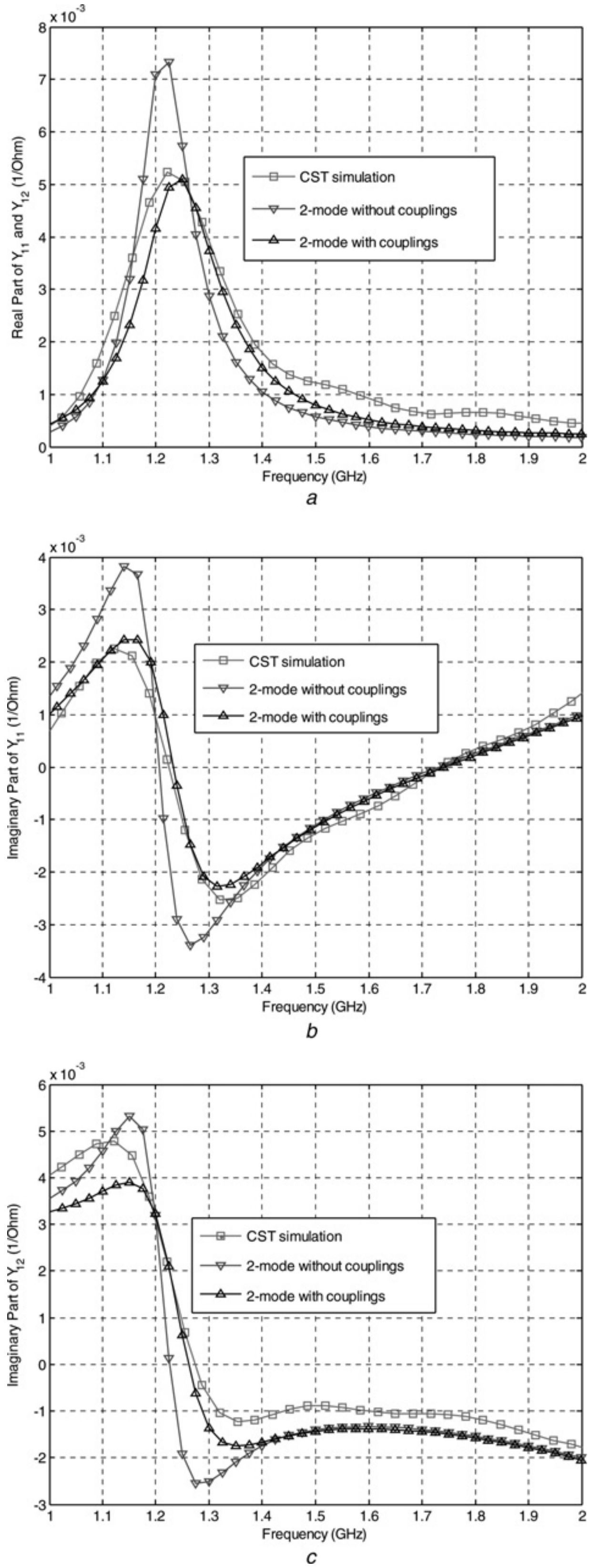

Figure 10 ***Y****-matrix of the folded dipole pair under the Eleven configuration shown in Fig. 9 simulated by CST and calculated by the present method*

a Real part of Y_{11} and Y_{12}
b Imaginary part of Y_{11}
c Imaginary part of Y_{12}

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values of $\boldsymbol{Z}$-matrix over the frequency band of 1–2 GHz. Fig. 10 shows a comparison of the values of $\boldsymbol{Y}$-matrix simulated by CST and calculated by using the present method. A good agreement can be observed from the figures. The calculated values by the present method with including the mutual couplings agree with the simulations better than those without including the mutual couplings.

In the manufactured model, the network of the transmission lines connecting the centre coaxial cables and the folded dipoles is complicated and not easy to be modelled analytically. Therefore we use CST to model it and obtain the S-parameters of the network, see Fig. 11, when ports 5 and 6 are not loaded with Z_{in}. Using the S-parameters, we can calculate the $\boldsymbol{S}$-matrix of the network when the ports 5 and 6 are loaded with Z_{in} calculated by using the present method. The calculated, simulated and measured reflection coefficient is shown in Fig. 12. The reflection coefficient of the Eleven antenna is obtained by using the following formula [5]

$$s_{11,\text{tot}} = s_{11} - s_{12} + s_{13} - s_{14} \tag{23}$$

because the excitation for the Eleven antenna is the same amplitude for all port and with the phase (0°, 180°, 0°, 180°) from port 1 to port 4, respectively. One can observe that the data from the present method with including the mutual couplings agree better with the measured one than the method without including the mutual couplings and all data from calculated by using the present method, simulated from CST and the measured agree well each other in general, which verifies the present method.

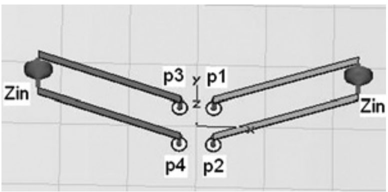

Figure 11 *Model of the center circuit in CST*

p1 stands for port 1 and so on

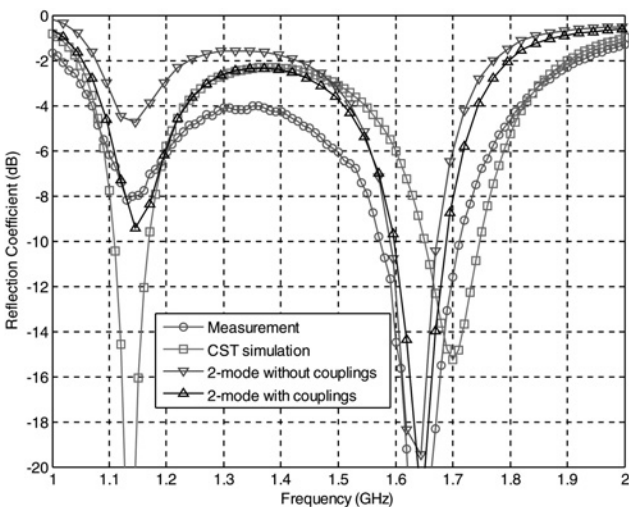

Figure 12 *Calculated, simulated and measured reflection coefficients of the Eleven antenna in Fig. 6*

6 Conclusions

The method of calculating impedance or admittance matrix of a folded dipole under the eleven configuration by using two-mode method including mutual couplings has been presented and the verification shows a good agreement among the calculation, the simulation and the measurement. This method will be very helpful for analysing and designing Eleven antenna efficiently in a nearly analytical way.

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