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NONLINEAR ROTOR WAKE/STATOR INTERACTION COMPUTATIONS

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Abstract

Rotor wake interactions with stators is an important aspect in turbomachinery noise generation. This paper deals with the time lagged periodic boundary condition (chorochronic periodicity) used in time domain Navier-Stokes equations solvers. The time lag periodic b.c. is used to solve the nonlinear three dimensional N-S equations using realizable k-epsilon turbulence model and time dependent wake defined at the inlet, with only a limited number of blade passages discretized. The time lag periodic b.c. has been validated through a number of 3D test cases. It has been shown that the use of a periodic/temporal damping term is an efficient way to stabilize the time lagged boundary condition without adding any extra spatial dissipation to the computation. The acoustic response from a stator vane with wakes defined at the inlet is presented.

Nomenclature

T	Period
C	Fourier coefficient
Q	Vector with flow variables
ε	Damping factor
m	Tangential mode number
N	Number of blades

Subscripts

k,l	Fourier coefficient index

Superscripts

^	Fourier representation
→	Vector

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Introduction

Aerodynamic interaction between rotor wakes and stators is one of the most important sources in turbomachinery noise generation [1]. The rotor wake, caused by the rotor boundary layer, is seen by the stator as a local change in velocity, rotating with the same rotational speed and pitch as the rotor. The stator is therefore experiencing an unsteady velocity field, and an unsteady pressure distribution on the blades is created. This in turn creates pressure waves, which will take the form of spinning modes in the turbomachinery duct, according to the well known theory of Tyler and Sofrin [2]. The resulting noise is a tone that consist of the blade passing frequency, BPF, and harmonics.

When pressure modes of an arbitrary axial stage are to be predicted, some problems may arise. One is that the use of ordinary CFD can be computationally very expensive if no periodicity can be used to simplify calculations, i.e. number of rotors and number of stators are such that a large number of stators has to be discretized to fit the periodicity of the rotor wakes. One solution to this problem is to use a frequency domain linearized Navier-Stokes equations solvers. A nonlinear mean flow solution is computed and on top of this the wakes are treated as linear perturbations and calculated in frequency domain. This has to be done for each harmonic to the BPF, i.e. each Fourier coefficient in the Fourier series representation of the flow perturbations, that are of interest. The linear response for each harmonic can then be calculated by using pseudo time marching, to reach convergence of the Fourier coefficients. If the period of the rotor wakes does not fit in a single stator vane discretization, it is easily solved by introducing a phase lag at the periodic boundaries. This method has successfully been used at e.g. Volvo Aero [3], but it has some built in limitations. Due to the linearization, the nonlinear effects in the wakes are neglected. These nonlinear effects depend on the amplitude of the flow perturbations, and it is unclear when they become significant. The solution is obtained through many steps, i.e. first a mean flow has to be calculated, then the wake response for each BPF harmonic has to be calculated separately. This can be seen as a drawback too.

Another way of calculating rotor-wake interaction on a single or a few stator vane discretizations is to use ordinary unsteady CFD with a time lagged periodic boundary condition, i.e. chorochronic periodicity [4,5]. A Fourier representation of the information near the periodic boundaries can be made by sampling the flow variables there. The Fourier-series can then be used to insert a time shift into the periodic boundary, which then makes it possible to calculate the rotor wake response of an arbitrary axial stage. In this case the unsteady effects in the perturbations are treated and all the frequencies of the wake response together with the mean flow can be obtained in one calculation. The drawback is that the time it takes to reach periodic convergence can be very long. Anyhow, it can save computational cost compared to a full 360^{o} simulation if that is the only alternative.

Chorochronic periodicity

The interaction between two arbitrary sets of blade rows that are rotating with different angular velocities can be computed by using chorochronic boundary conditions [4,5]. Only the stators of a fan stage is considered is this paper. The wakes from the rotor are modeled at the inflow of the computational domain, and a chorochronic periodic b.c., i.e. time lagged periodic b.c., is used at the pitch wise boundaries as shown in Fig. 1. The fundamental frequency in a fan stage is the blade passing frequency (BPF), i.e. the frequency of the rotor wakes that are entering the domain, and all deterministic flow perturbations that are caused by the rotor wakes can then be described by the BPF and harmonics to it. A rotor wake of arbitrary pitch can, with a chorochronic b.c., fit into an arbitrary number of stator vane discretizations.

The chorochronic b.c. works in three steps; First as a filter, that samples values at each side, to update a Fourier series representation of the flow variables near the boundaries. Then it works as a time shift, by evaluating the Fourier series at a time that corresponds to the phase shift between the periodic sides, and uses the time shifted and rotated flow variables at the other side. Next the Fourier representation is evaluated again, at the current time, and used to damp out unperiodic flow phenomena in the cell where the sampling occurred. A diagram of how this works is shown in Fig. 2. The only thing that changes if two or more stator vanes are discretized is the timelag size/phase shift, and the rotational angle.

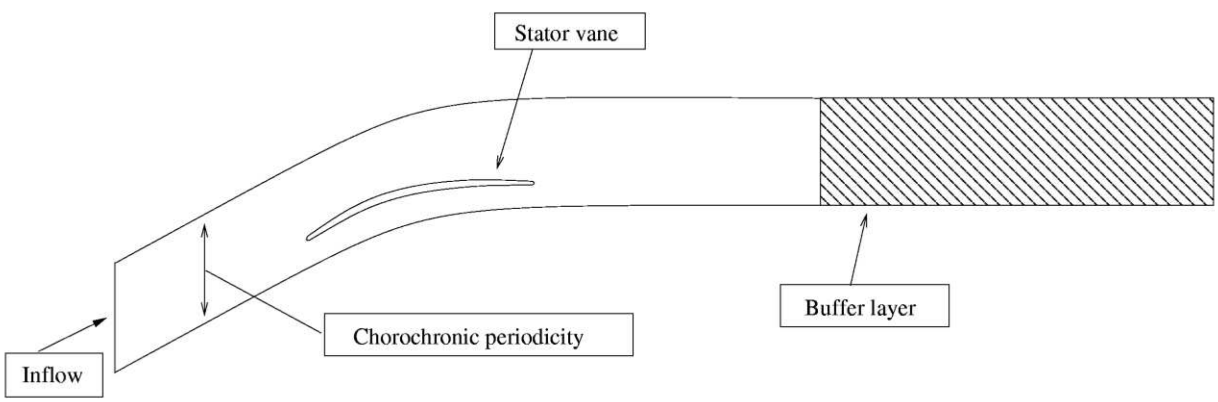

Fig. 1 *Schematic drawing of computational domain*

Step 1: Updating the Fourier coefficients

The Fourier coefficients are updated with a moving average technique [4], and the computation should converge to a periodic solution, where the period is the inverse of the BPF. The Fourier coefficients that should represent the flow variables near the periodic boundaries can be calculated by integration over one period,

$$\vec{C}_k = \frac{1}{T}\int_{t-T}^{t} Q(t')e^{-i\frac{2\pi k}{T}\cdot t'}dt' \quad (1)$$

The time derivatives of these Fourier coefficients can be found by looking at the integration boundaries,

$$\frac{d\vec{C}_k}{dt} = \frac{1}{T}\left[Q(t)e^{-i\frac{2\pi k}{T}\cdot t} - Q(t-T)e^{-i\frac{2\pi k}{T}\cdot(t-T)}\right] \quad (2)$$

where,

$$Q(t-T) \cong \hat{Q}(t) = \sum_k \vec{C}_k e^{i\frac{2\pi k}{T}\cdot t} \tag{3}$$

The approximate time derivative of the Fourier coefficients can then be found with the new sample and the existing set of Fourier coefficients as,

$$\frac{d\vec{C}_k}{dt} = \frac{1}{T}\left[Q(t) - \sum_l \vec{C}_l e^{i\frac{2\pi l}{T}\cdot t}\right] e^{-i\frac{2\pi k}{T}\cdot t} \tag{4}$$

or,

$$\frac{d\vec{C}_k}{dt} = \frac{1}{T}Q(t)e^{-i\frac{2\pi k}{T}\cdot t} - \frac{1}{T}\sum_l \vec{C}_l e^{i\frac{2\pi(l-k)}{T}\cdot t} \tag{5}$$

The Fourier coefficients are updated inside the Runge-Kutta cycle that is used to update the solution in time. The calculation has converged to a periodic solution when the Fourier coefficients are stationary, and the flow through the boundary will not be affected if a sufficient amount of Fourier coefficients is used at each cell close to the boundary.

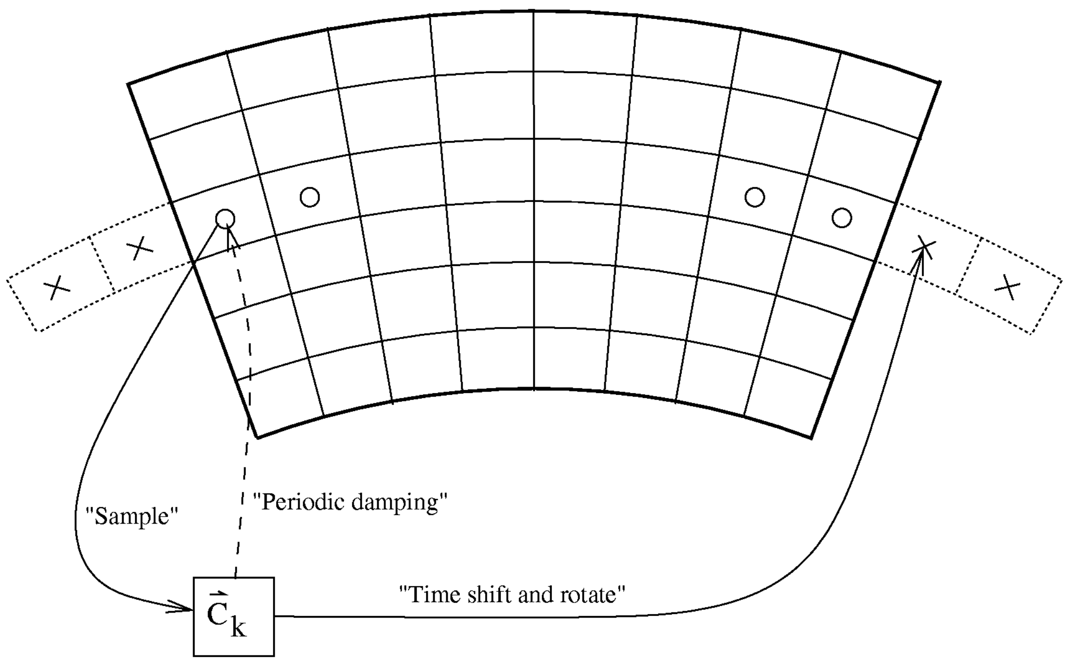

Fig. 2 *Simplified diagram of how the periodic b.c. with time shift works. Rings represent physical cells inside the domain and crosses are ghost cells used to calculate fluxes through the boundary. The Fourier representation of a physical cell near a boundary is updated continuously by sampling. It is used both to damp out unperiodic flow phenomena (errors) in the physical cell and in ghost cells at the other side of the boundary.*

Step 2: Time shift

A Fourier representation of the flow variables in two cell layers at each side of the periodic b.c. is needed, because two cells on each side of a cell face are needed for the convective flux calculation scheme used in this paper. The Fourier series are evaluated at a time that corresponds to the phase shift between the periodic sides, and then the time shifted variables are used in ghost cells at the boundaries at each side. The ghost cells corresponds to physical cells near the other side of the boundary, at a different time. A diagram of how data is transferred from one side to the other is shown in Fig. 2. It works in the same way, the other way around, i.e. on the other side of the boundary.

Step 3: Periodic damping

If only step one and two are used in a periodic b.c. with time shift, it will be unstable. A 1D-Euler test case was made to find out what caused these instabilities. The 1D problem represents a tube with periodic time lagged boundaries, and the time lag was inserted as described in Step 1 and Step 2. The domain and the Fourier coefficients was initialized with an acoustic wave that had a wave length that was 50% longer than the domain. The time lag and the period was specified so that the acoustic wave should be completely resolved. The nonlinear effects will start to deform the wave when the simulation is started. The Fourier coefficients will adapt to the deformed wave, but some reflections will occur at the boundaries, since the deformed wave is not completely represented at once. These reflected waves will travel in the opposite direction compared to the wave that was initialized, and therefore it will not match the specified period and time lag. Nevertheless, these reflections will grow and contaminate the solution and the Fourier coefficients until it diverges. The first solution to this problem was to make the time lagged periodic b.c. absorbing [6]. It was shown to be stable, but it adds some extra spatial dissipation close to the b.c. This extra dissipation was shown to destroy the wake too much, when the absorbing time lag periodic b.c. was tested in a 3D computation. A better solution to stabilize the b.c. is to use temporal damping, by evaluating the Fourier coefficients and use the result in a periodic damping term;

$$\frac{dQ}{dt} = \ldots - \varepsilon\left(Q - \hat{Q}\right) \tag{6}$$

where

$$\hat{Q} = \sum_k \vec{C}_k e^{i\frac{2\pi k}{T}\cdot t} \tag{7}$$

The damping term will damp out unperiodic errors/reflections and stabilize the b.c. When the solution has reached periodic convergence, the damping term will be zero, and no extra spatial dissipation is added. The optimal size of the damping factor is mesh dependent, because we only use periodic damping in two cell layers on each side of the b.c., as shown in Fig. 2. For that reason the size of the cells close to the b.c. will have an influence on the amount of damping. The solution will reach periodic convergence slower if the damping term is set to a higher than optimal value, but on the other hand, if ε is too small there will be instabilities and the computation will either be contaminated with unperiodic flow phenomenon or diverge. Anyhow, for our cases it seemed to work well with a damping factor somewhere between:

$$\frac{10}{T} \le \varepsilon \le \frac{100}{T} \tag{8}$$

Absorbing boundaries

The inflow is defined with the rotor wakes on top of the mean inflow properties, as radial profiles of wake harmonics and mean flow. The inflow is a zero order absorbing boundary condition that absorbs most of the upstream traveling waves. The outlet is defined with a static pressure profile. Upstream of the outlet there is a buffer layer, as shown in Fig. 1, that works as a low pass filter, and it will cancel out all transients before it reaches the outlet, to ensure that no reflection of waves occur.

Numerical schemes

The solver, based on the G3D family of codes [7], solves the unsteady Reynolds-averaged Navier-Stokes equations with realizable k-ε turbulence model, by using a finite volume solver with a standard third-order upwind scheme for the convective fluxes, and a second order centered difference scheme for the diffusive fluxes. The solution is updated with a three-stage Runge-Kutta technique.

Results

A fan stage with a rotor stator vane count ratio of about 1:1.56 is considered. Two stators are here discretized to reduce the time lag size in the chorochronic periodic b.c. This will speed up convergence time [6]. The wakes from the rotors have been calculated in a separate steady state RANS calculation for the rotor blade row. The wakes are modeled at the inflow of the stator domain by an unsteady inflow b.c., and can be seen by plotting entropy contours, as shown in Fig. 3.

Fig. 3 *Entropy contours at a mid-radius-surface and blades, over the two stator passages used in the wake response computation. Wakes are entering from the left through the unsteady inflow b.c. and seen as light strikes that are convected downstream (to the right) with the flow.*

The block structured mesh that has been generated consist of ~$6.3 \cdot 10^6$ nodes, and it has a resolution that should be able to capture acoustic response up to the 3rd BPF harmonic. A sparse mesh, ~$1.0 \cdot 10^6$ nodes, is also created and used to generate a good initial solution for the fine mesh.

Three harmonics is used in the Fourier series representation of the flow through the time lagged periodic b.c.

The wake response from the unsteady (nonlinear) RANS equations solver using chorochronic periodicity is compared to a solution obtained with a frequency domain linearized Navier-Stokes equations solver [6], that has been validated against independent data [3].

The acoustic response is evaluated through a decomposition into the Tyler-Sofrin modes [2] that are of interest:

$$m = n \cdot N_{rotor} + p \cdot N_{stator} \quad (9)$$

where,

$$n = 1,2,\cdots, \quad p = \cdots -1,0,1,\cdots \quad (10)$$

In this case the propagating modes are m=-10 for the BPF, m=-20, m=8 for 2nd BPF harmonic, and m=-30, m=-2, m=26, for the 3rd BPF harmonic.

The calculation on the sparse mesh is simulated for about 80 periods to ensure periodic convergence. Then the solution and the Fourier coefficients is interpolated onto the fine mesh and simulated for another 20 periods, which seemed to be enough for periodic convergence on the acoustically suited mesh.

The maximum amplitude of the pressure modes is about the same or lower in the results from the nonlinear solver compared to the linear frequency domain solver; Fig. 4-9. Also the shape of the pressure modes is about the same for the 1st BPF, Fig. 4, but differs for the higher frequencies, Fig. 5-9. Some discrepancies are expected since nonlinear effects are ignored in the linear solver, and at this stage it can only be said that the results shows promising agreement. It is also unclear if it is enough with a three harmonics representation of the flow through the time lagged periodic b.c., but this will be investigated in a near future.

Results from the linear frequency domain solver is shown to the left and results from the nonlinear solver is shown to the right. This corresponds to all the following plots; Fig. 4-9.

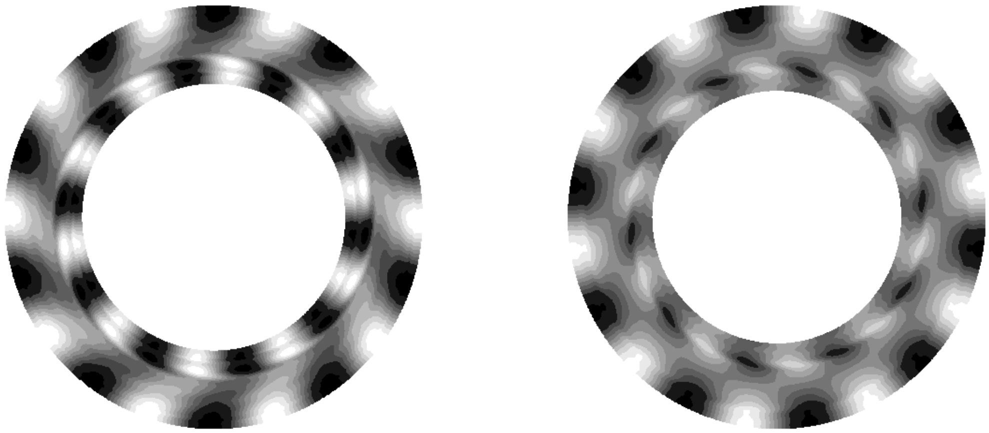

Fig. 4 *Pressure fluctuations downstream of the stator vanes at* ***BPF*** *and tangential mode number* ***m=-10****. The maximum amplitude of the nonlinear results is here about 10 % lower than in the linear results.*

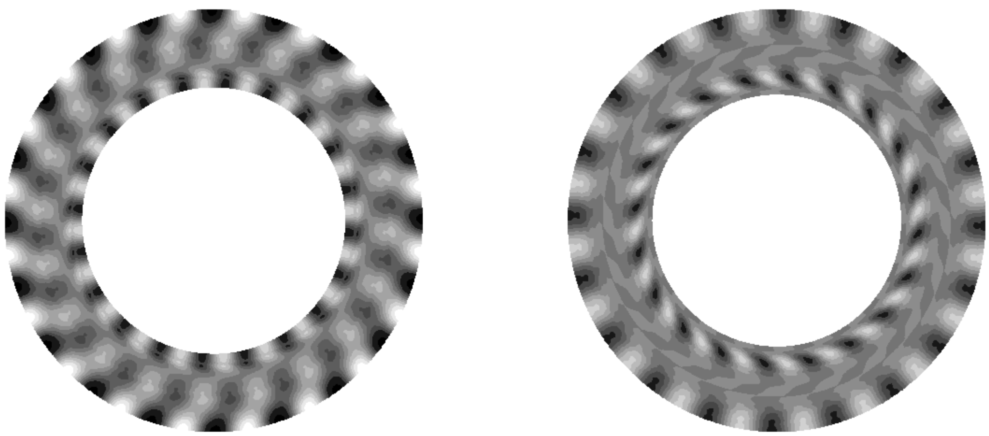

Fig. 5 *Pressure fluctuations downstream of the stator vanes at* ***2nd BPF*** *and tangential mode number* ***m=-20****. The maximum amplitude of the nonlinear results is here about 40 % lower than in the linear results.*

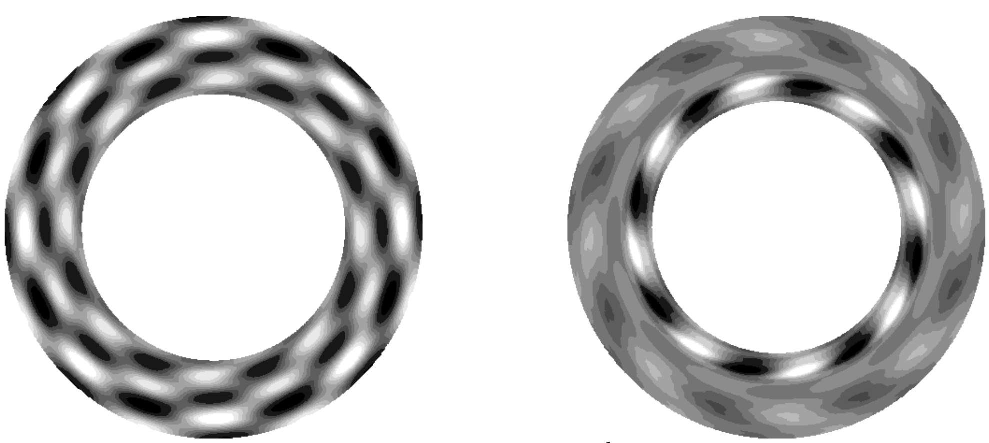

Fig. 6 *Pressure fluctuations downstream of the stator vanes at* ***2nd BPF*** *and tangential mode number* ***m=8****. The maximum amplitude of the nonlinear results is here about the same the in the linear results.*

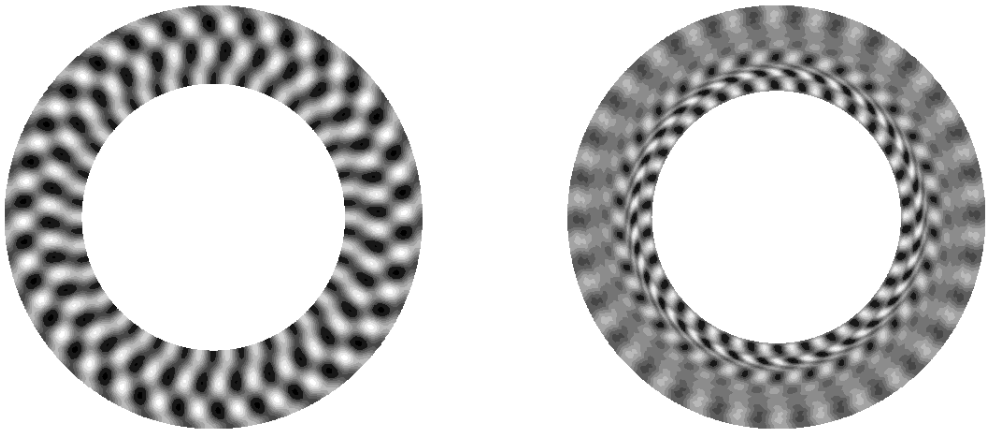

Fig. 7 *Pressure fluctuations downstream of the stator vanes at* ***3rd BPF*** *and tangential mode number* ***m=-30****. The maximum amplitude of the nonlinear results is here about 5 % lower than in the linear results.*

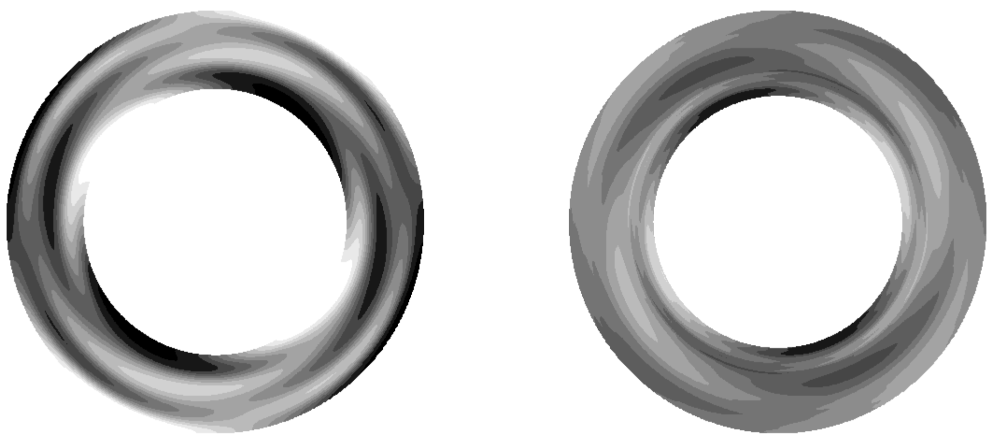

Fig. 8 *Pressure fluctuations downstream of the stator vanes at* ***3rd BPF*** *and tangential mode number* ***m=-2****. The maximum amplitude of the nonlinear results is here about 40 % lower than the linear results.*

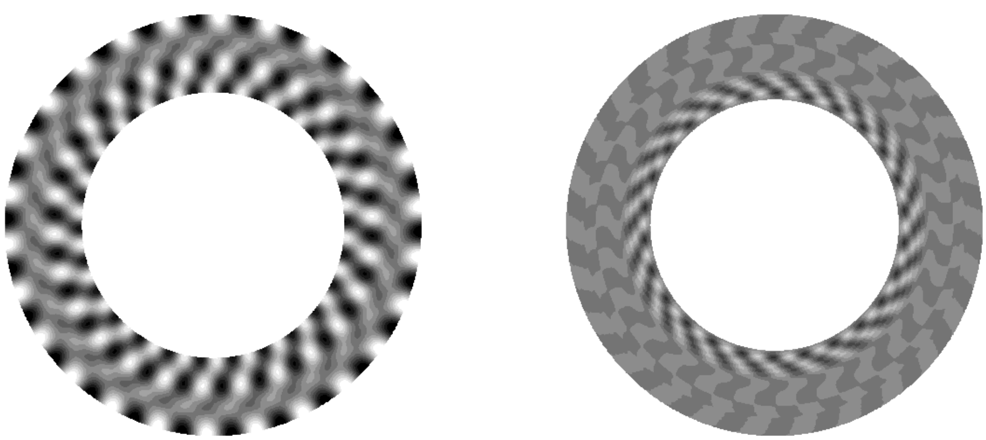

Fig. 9 *Pressure fluctuations downstream of the stator vanes at* ***3rd BPF*** *and tangential mode number* ***m=26****. The maximum amplitude of the nonlinear results is here about 50 % lower than in the linear results.*

Conclusions

The method of calculating acoustic response from rotor wake interactions with stators by using chorochronic periodicity and unsteady CFD has been validated against a frequency domain linearized Navier-Stokes equations solver method.

The use of a periodic damping term is an efficient way to stabilize the time lagged boundary condition without adding any extra spatial dissipation to the computation.

Acknowledgement

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