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Instability in Exchange Rates of the World Leading Currencies: Implications of a Spatial Competition Model among Central Banks (Currencies, Competition, and Clans)

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Abstract

We use a spatial competition based model in a two-stage game setup to assess whether equilibrium in exchange rates among the leading currencies is attainable. We show that a stable equilibrium can be reached in the case of two leading currencies, but not in the case of three. In our model, central banks of leading currencies attract, through the workings of their objective and policy, small currencies that tie with leading currencies via exchange rate regimes. This can be thought of as a competition to link smaller currencies to a leading currency that is motivated by the fact that such a tie greatly reduces volatility within such an informal "currency area". Our theoretical findings are supported by empirical evidence. Since firms, traders, and countries currently recognize three leading currencies and their economic behavior reflects this, we may expect disagreement on overvaluation or undervaluation of certain currencies to continue.

Keywords: exchange rates, exchange rate regimes, central bank policy, monetary union, spatial competition

JEL Classification: C72, E42, E58, N20, O23

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1. Introduction

On February 6-7, 2004, the heads of central banks and the finance ministers of the G-7 countries gathered in Boca Raton, Florida, to discuss the development of the exchange rate between the US dollar and the European common currency, the euro. G-7 finance ministers and central bankers agreed after a two-day meeting to issue a statement that "excess volatility" and "disorderly movements" in exchange rates were undesirable. Translated into plain English, the statement implied that the dollar had fallen against the euro far enough.

Recent developments in the foreign exchange market have formed the basis for complaints that the euro has borne a disproportionate share of the dollar's decline.[1] When looking at the situation from the perspective of a broad basket of currencies, during 2002-2003 the euro rose against the US dollar by roughly twice as much as the Japanese yen, the British pound or the Canadian dollar; but among the main currencies, it is the Australian dollar that actually recorded the biggest gain against the US dollar.. The currencies of emerging Asian economies other than China, meanwhile, moved only a little, while the Mexican peso even fell against the dollar.[2] Overall, the dollar fell by a modest 15% against a broad basket of currencies over the 2002-2003 period. Complaints on overvaluation or undervaluation between the US dollar and the euro have their predecessors in the context of the Deutsch mark and other currencies under the former European Monetary System (EMS) as well as in past disputes on “fair” parity between dollar and yen. In short, the system of exchange rates seems to be out of equilibrium, a cure is hard to find, and as Iida (1999) argues, international cooperation in monetary affairs may likely be very counterproductive.[3] Figure 1 illustrates the deviations among key currencies in the post-war horizon.

In this paper we use an historical account of the post-war existing factual exchange rate regimes, exchange rate development, and evolution of crucial monetary variables to assess the likelihood of attainability of equilibrium among the leading currencies. We build on a model of spatial competition among the central banks of leading currencies in a two-stage game setup and show that stable equilibrium among the existing leading currencies cannot be achieved under the

[1] Since 2001 to early 2004 the dollar has fallen by 33% against the euro and by 15% against the Japanese yen.
[2] Mexico is America's third-largest trading partner.
[3] Based on the theory of model uncertainty, originally proposed by Frankel (1988), the expected benefits and costs of cooperation depend on the model used to forecast the outcome of such cooperation; the intuition behind it is that “if policymakers do not know what they are doing, it is unlikely that cooperation will improve the situation” (Iida 1999, p. 31).

existing world monetary arrangement.[4]

The roots of developments in the exchange rates can be traced to the origin of post-war development that itself can serve as a proxy for stylized facts underlying the motivation for our model. After World War II the international foreign exchange arrangement returned to a relaxed version of the Gold Standard known as the Bretton Woods System.[5] All participating currencies were pegged to the US dollar with a very narrow margin to move up and down. The dollar was then tied to the gold in fixed proportion. However, as Reinhardt and Rogoff (2004) argue, when market-determined rates were used instead of official rates, *de facto* floating was not uncommon even during the Bretton Woods period of pegged regime, and for many countries it was difficult to detect any change in exchange rate behavior between the period of peg and the period of general floating that followed. Since the increasing amount of dollar denominated obligations held by foreign holders exceeded US gold reserves, the Bretton Woods System collapsed in 1973.

After the collapse , the system of exchange rates clearly longed for stability and aimed to repair itself since, during the post-Bretton Woods period, pegs and crawling pegs were the most frequent exchange rate regimes used (33% and 26% respectively). Further, from 1990 to 2001 the crawling peg was the most common type of regime in Asia and the Western Hemisphere, excluding Canada and the US (ibid). Such an empirical finding is supported by an earlier argument of Williamson (1998) that, under the conditions of high capital mobility, the more prudent choice should in most cases be a system of limited flexibility, in the form of a crawling band (a wide band that is adjusted in small steps so as to keep it in line with the fundamentals, but is defended in the traditional ways) or possibly a monitoring band (a wide band with similar properties, which is defended only when the rate goes outside the band). In a similar spirit, Calvo and Reinhart (2002) focus on whether countries that claim their currencies are floating are indeed doing so. They find that countries that say they allow their exchange rate to float mostly do not and conclude, with a coined term, that there seems to be an epidemic case of "fear of floating."

The recent debate regarding stability of international monetary arrangements is rich and discusses the issue from a variety of angles. Ghosh, Gulde, and Wolf (2002) review trends in

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[4] In no respect do we claim that central banks of leading currencies determine exchange rates. On the contrary, we leave this task fully on the shoulders of the market (with the exception of time-limited effect of foreign exchange interventions by central banks).

[5] For a review of the system as well as its lasting impressions on the world monetary landscape see for example Leeson (2003), Battilossi and Cassis (2002), Andrews, Henning, and Pauly (2002).

exchange-rate regime shifts over the post-Bretton Woods period, present a typology of regime changes, and address issues related to voluntary and forced exits from exchange rate regimes. Benassy-Querre, Fontagne, and Lahreche-Revil (2001) empirically show that exchange-rate volatility is detrimental to foreign direct investment (FDI) and that its impact compares with that of misalignments. One policy implication of their work is that the building of currency blocs could be a way of increasing FDI to emerging countries as a whole.[6]

The debate is not concentrated only on Europe or the United States, an impression that may emerge due to the stress on euro versus dollar advances. Bird and Rajan (2002), for example, discuss key aspects of the new Asian financial architecture and focus on the reform of domestic financial systems in Asia, exchange rate regimes, and regional liquidity arrangements. Madden, Savage, and McDonald (2000) discuss stabilizing Asia-Pacific exchange rates by establishing a system of pegs, bands or target zones around the Japanese yen. This requires the compromise of domestic policy autonomy and symmetric reaction to economic shocks to ensure the lowest cost. The authors suggest that the economic preconditions for a yen bloc are not in place yet. Frieden and Stein (2001) provide a systematic understanding of exchange rate issues by analyzing the political economy of currency policy in Latin America and the Caribbean.

The political economy of exchange rates conveys a plain message that any shift in currencies produces winners and losers. Since the US dollar, Japanese yen, Deutsch mark, and recently euro have become the leading currencies in post-war development, they also tend to be most vulnerable to volatility.[7] Exchange rates across the three leading currencies became particularly volatile in the post-1971 period and our earlier account implies the same with respect to the euro.

Following the collapse of the Bretton Woods System, attempts to reduce volatility of exchange rates in economically interconnected Europe led to creating the "Snake"[8] in 1973 and the European Monetary System (EMS) in 1979. The former Deutsch mark was represented by the largest weight in a currency basket that was used to limit volatility of participating European currencies. Further economic integration evolved into the Economic and Monetary Union (EMU), the establishment of the European Central Bank, and the euro being adopted in 1999.

[6] This empirically backed conclusion supports our motivation for the model outlined later in Section 2.

[7] Devereux, Engel, and Tille (2003) show that introduction of the euro could have important positive and normative effects for both Europe and the rest of the world. They also conjecture that the acceptance of the euro will lead European prices to become more insulated from exchange rate volatility.

[8] This group of countries, the so-called "Snake", consisted of Germany, the Netherlands, Belgium, and Denmark; it also included France on several occasions. In 1973, these countries fixed their exchange rates with each other while jointly floating against other countries.

Further enlargement of the EMU is expected, since countries that accessed the EU in 2004 were given no option but to join the EMU at a later date. For a classical in-depth analysis of the EMS, as well its relevance for the rest of the world, see Giavazzi and Giovanninni (1989). For a description of European integration around a common currency and for an explanation as to why many of the EU states have agreed to sacrifice their monetary independence see Overturf (2000).

Aside from a macroeconomic account, instability in exchange rates is the main problem that businesses ultimately face, and with very limited space to maneuver. Various hedging possibilities may prevent losses to some extent but they serve as short-term solutions with a relatively narrow scope. Moreover, hedging instruments, while serving against loss, do not reduce volatility in exchange rates. Furthermore, foreign exchange market-makers may benefit from volatility and help to increase it as well. Firms can adjust in the medium and long term to different levels of exchange rate parities, but they are unable to efficiently react to volatility in the short run. This is due to the fact that some delivery contracts and, more importantly, wage contracts and investment projects are assessed, evaluated, and adopted for at least the medium term. The firms are unable to adapt when volatility in exchange rates erodes links among prices and when formerly established information, based on prices with respect to exchange rates, becomes highly distorted.

As we noted earlier, we cannot expect that stability in the system of currencies is attainable. We aim to show this with the aid of a formal model, which is based on the simple and widely recognized premise that a central bank's objective is characterized by price stability. Interest rate, as a main instrument, is used for conduct of bank policy. (The model and motivation for its use are described in detail in Section 2.) In our model, central banks of leading currencies attract, through the workings of their objective and policy, small currencies that tie with leading currencies via exchange rate regimes. This can be thought of as a competition to link smaller currencies to a leading currency that is motivated by the fact that a tie between small currencies and a leading currency greatly reduces volatility within such an informal "currency area". Reduced volatility in turn promotes international trade and increased stability, further reducing costs of business activity. Further, countries of the small currencies benefit by enhanced price stability if they tie their currencies to a leading currency that experiences lower inflation. Such import of low inflation is theoretically grounded as well as empirically documented (see Giavazzi and Giovannini 1989, among others).

The theoretical model we present is an extension of the model of spatial competition by Eaton and Lipsey (1975), who investigate the principle of minimum differentiation in the model of spatial competition by Hotelling (1929). They show that this principle, which had previously been assumed to be of wide applicability and which states that firms choose to differentiate as little as possible holds only for the case of two firms. In contrast, with three firms there exists no pure strategy equilibrium and with more than three firms the existence of a pure strategy equilibrium (that, if it exists, does *not* imply minimal differentiation) depends on the distribution of customers.

Like firms (in the Eaton and Lipsey model) that compete for customers who are distributed along a line, we can consider a realistic analogy with respect to central banks of the leading currencies. Through their objective of price stability and with interest rate as a positioning instrument in a policy space, they attract small currencies that tie with leading currencies via exchange rate regimes. This process may be understood as if central banks of the leading currencies were competing for shares in currency holdings of small countries (i.e. those whose policy has a negligible impact), whose preferences for the policy of the central bank of a leading currency that they are linked to are distributed along a line. We show that under some changes in the assumptions, the results of the standard spatial competition model that are of interest to us continue to hold. Namely, stability in the case of two leading currencies, but instability in the case of three leading currencies.

The paper is organized as follows. In Section two we present and analyze the formal theoretical model. Section three describes the data and brings empirical extensions to illustrate our point about the quest for equilibrium. A brief conclusion follows.

2. Model and Equilibrium Analysis

There are n leading currencies, each attached to one large country (or to a group of countries that form a monetary union). In addition there is a continuum of small countries, each with their own currency. A small country is defined by its monetary policy having only a negligible influence on world markets. The policy space of the central banks of the leading currencies is one-dimensional. This one-dimensional policy is indeed the result of a set of policy choices, but for simplicity we collapse it into one single variable, an interest rate, which is the dominating policy instrument as well as the most significant loading factor of our formal generalization. Within this policy space, there is a range which fulfills the basic goals of the central bank. Normalize this feasible policy space to [0, 1].[9]

As outlined in the introduction, the objective of a leading central bank in our model is price stability. The interest rate is its main instrument and is naturally used for positioning purposes within the policy space. In reality, usually price stability and, hence, some type of inflation management belong to the explicit goals of a central bank. Implicitly, central banks may be concerned about economic growth or trade deficit, since these are related to the bank's foreign exchange reserves. Hence, these goals also serve to increase price stability, albeit indirectly.[10]

Using standard theory, the origin of a monetary base can be inferred from a country's choice of an exchange rate regime. If a country favors a floating exchange regime then the monetary authority has full control over its monetary policy, by definition has no exchange rate policy, and the origin of the monetary base is entirely domestic. On the other hand, if a country prefers to peg its domestic currency to a foreign one, then the central bank de facto resigns from an independent monetary policy, conducts an explicit exchange rate policy, and the origin of a monetary base is purely foreign.[11] Any exchange rate regime in between the two extremes means a different extent of independence in both monetary and exchange rate policies as well as a mixed origin of the monetary base. Hence, by knowing the (true) adopted exchange regime we may identify the amount of domestic money (of a small currency) linked to a particular leading currency via the exchange rate regime and express this amount in terms of such leading currency.

We define the dependencies of small currencies on leading currencies in the context of arguments given by Reinhardt and Rogoff (2004). Based on a factual exchange regime we are able to trace the preference of small currencies' central banks with respect to leading ones and in this way to classify shares in foreign currency holdings. Formally, let C be the amount of domestic currency expressed in terms of foreign leading currencies to which a domestic currency is linked via particular exchange rate regime, and c_i be the part of C expressed in leading currency i that corresponds to the weight of i in the currency basket.[12] Clearly $\sum_{i=1}^{n} c_i = C$. This

[9] Replacing the interval by an open interval does not change the results.
[10] In any event, we do not want to be too definite about these goals since specific goals vary across central banks and it is our aim to keep the model general.
[11] In this context we can say that a leading currency "absorbs" the currency of a small country.
[12] Technically, C should also include foreign exchange reserves of central banks of small currencies held in leading currencies. However, since those foreign exchange reserves consist of currencies already issued by the leading central banks, we cannot consider them. Aside from this, the structure of foreign exchange reserves held usually

convenient notation covers all possible cases outlined above: 1) when n=0, then C=0 and small currency is floating; 2) when n=1, then small currency is pegged to a leading currency, and 3) when n>1, then small currency is under a currency basket peg regime.[13]

To summarize, leading central banks attract, through their policy choice, small currencies that tie with leading currencies via exchange rate regimes. Small countries have a preference over the location of the currency they are linked to. The most preferred locations of small countries are distributed with respect to a density f on [0, 1]. To assess the impact of such a link, the domestic money stock of a small currency may be expressed in terms of the foreign leading currency (currencies) to which the small currency is linked via specific exchange rate regime.

Our basic assumption is that price stability for a large country i (with a leading currency) depends on two factors: the policy (interest rate) of the central bank, x_i, and the share s_i of domestic currency, expressed in leading foreign currency i, that is held by small countries whose domestic currency is linked via specific exchange rate regime to leading currencies. More precisely, the objective function of a central bank is $G_i(x_i, s_i)$, where G_i is a proxy for price stability and is increasing in s_i but decreasing in the absolute difference between its actual policy x_i and its preferred policy p_i.[14] Therefore, when choosing its policy, a leading central bank has to consider not only the direct effect on price stability, but also the indirect effect via the change in the share of small currencies linked to it.[15]

We analyze this interaction between central banks of leading currencies and those of small countries as a two-stage game. In this game, the central banks of leading currencies first decide simultaneously on their policy, i.e. of their location in the policy space, and then the small countries choose their foreign currency holdings. More precisely, the two stages are as follows:

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reflects the weights of the leading currencies within the exchange rate regime.

[13] If a country favors, say, a currency basket peg, then weights of currencies in a basket are used to determine the importance of leading currencies with respect to small currency holdings. Since currencies in a basket usually represent currencies most frequently used in conduct of international monetary operations of a particular country, such an approach is justified.

[14] The objective function G_i is increasing in s_i since the connection of small currencies to a leading currency decreases volatility in the informal currency area, and hence fosters international trade and further stability. Such effects naturally reduce the costs of business activities.

[15] Devereux, Shi, and Xu (2004) deliver a model of monetary policy under a US dollar standard and describe how to conduct a monetary policy once a leading currency rounds up small currencies. Their work potentially can motivate our own, as well as lend support to our model which, besides other things, describes how to get to the point when small currencies are linked to leading currencies via foreign exchange standards.

1. The n central banks choose simultaneously their locations $x_1,...,x_n$. Assume $x_1 \leq ... \leq x_n$.
2. After observing $x_1,...,x_n$ the small countries choose their basket of leading currencies. They choose a mix between the closest leading currencies. More precisely, let l be the preferred location of a small country; as already mentioned, let C be the amount of domestic currency expressed in terms of foreign leading currencies to which a domestic currency is linked via a particular exchange rate regime, and c_i be the part of C expressed in leading currency i that corresponds to the weight of i in the currency basket ($\sum_{i=1}^{n} c_i = C$). If $l \leq x_1$ then the country will choose a currency basket consisting only of currency 1, $c_1 = C$; in such a case a currency basket reduces to a simple peg. If $l \geq x_n$ then the country will choose $c_n = C$. If $x_i \leq l \leq x_{i+1}$ then the country will choose a mix of currencies i and $i+1$, $c_i = C\frac{x_{i+1}-l}{x_{i+1}-x_i}$, $c_{i+1} = C\frac{l-x_i}{x_{i+1}-x_i}$. Note that $(c_i x_i + c_{i+1}x_{i+1})/C = l$ and that $c_i = C$ if $l = x_i$. If $x_{i-1} = x_i$ then $c_{i-1} = c_i = \frac{C}{2}\frac{x_{i+1}-l}{x_{i+1}-x_i}$ and similarly if $x_{i+1} = x_{i+2}$ then $c_{i+1} = c_{i+2} = \frac{C}{2}\frac{l-x_i}{x_{i+1}-x_i}$ (and correspondingly if more than 2 x are identical).[16]

Assume for simplicity that C is identical for all small countries and normalize $C = 1$. This can be achieved by replacing the density of small countries f by the density of in-foreign-currency-expressed holdings f^* with $f^*(l) = \frac{f(l)C(l)}{\int_0^1 f(l)C(l)dl}$ for all $l \in [0,1]$, where $C(l)$ denotes the average currency holding of the countries whose preferred location is l. Denote by s_i the share of currency i of the total in-foreign-currency-expressed holdings by small countries.

Each leading central bank has a preferred policy p_i. As noted above, the aim of a leading central bank is to maximize $G_i(x_i, s_i)$, where G_i is assumed to be linear increasing in s_i but the costs of deviating from the preferred policy p_i (henceforth location costs) is convex in the absolute difference. More precisely let $G_i(x_i, s_i) = s_i - L(x_i - p_i)$ with $L(-y) = L(y)$, $L'(0) = 0$ and $L''(y) > 0$.[17] Assume furthermore for simplicity that the preferred policies of small countries are distributed according to a uniform distribution on [0,1] and that location costs are quadratic,

[16] Modern monetary history documents that usually the number of currencies in a basket ranges from 2 to 5. The basket of currencies within the former EMS is an exception due to institutional setup.
[17] Note that the cost function is the same for all leading countries.

$L(y) = a(y)^2$ with $a > 0$.[18] Since the leading economies are more alike than the whole spectrum of countries, the preferred locations of leading central banks are assumed to be relatively similar compared to the distribution of preferences of small banks. Furthermore, it appears that small deviations from the preferred locations have a relatively small impact on stability compared to s_i, hence a is assumed to be small enough such that concerns for location costs do not dominate concerns for the share of currency holdings s_i.

This model resembles the spatial competition model by Eaton and Lipsey (1975), but differs in three respects. First and most importantly, we introduce preferences of the leading central banks over their location in the policy space. Second, smaller countries do not exclusively choose the leading currency closest to their own preferred location, but a mix of respective closest currencies on both sides such that the weighted average location of these currencies corresponds to the preferred location *l*. Finally, central banks can choose identical locations in which case the linked countries choose baskets with equal shares in these currencies.[19] We show below that two central results of the basic spatial competition model, namely existence of a pure strategy equilibrium in the case of two central banks, but non-existence of a pure strategy equilibrium in the case of three central banks, continue to hold if location costs are not too high and if the preferred policies of leading central banks are relatively homogeneous compared to the distribution of preferred policies of smaller countries.

Proposition 1: Let there be two leading currencies and let their preferred policies be $p_1 < p_2$. Then

(a) There is an equilibrium $x_1 = x_2 = \frac{1}{2}$ if $p_1 \geq \frac{1}{2} - \frac{1}{4a}$ and $p_2 \leq \frac{1}{2} + \frac{1}{4a}$. The shares in equilibrium are $s_1 = s_2 = \frac{1}{2}$.

(b) If $p_2 - p_1 > \frac{1}{2a}$ and $|1 - p_1 - p_2| \leq a(p_2 - p_1)^2 - \frac{1}{4a}$, then (x_1, x_2) with $x_1 = p_1 + \frac{1}{4a}$ and $x_2 = p_2 - \frac{1}{4a}$ forms an equilibrium. The shares in equilibrium are $s_1 = x_1 + \frac{(x_2 - x_1)}{2} = \frac{(x_2 + x_1)}{2}$ and $s_2 = 1 - x_2 + \frac{(x_2 - x_1)}{2} = 1 - \frac{(x_2 + x_1)}{2}$.

(c) Otherwise there is no equilibrium in pure strategies.

[18] We will elaborate below on generalizations.
[19] Such behavior can be observed during periods of post-war development and is a prominent feature of emerging economies during the last two decades of the 20th century.

Proof. (a) Assume $x_1 = x_2$. In that case all small countries choose $c_1 = c_2 = \frac{1}{2}$. Hence $s_1 = s_2 = \frac{1}{2}$. By deviating to $x_1 - \varepsilon$ or $x_1 + \varepsilon$ with $\varepsilon > 0$ very small, central bank 1 can capture $s_1 = x_1 - \varepsilon$ or $1 - x_1 - \varepsilon$ at a minimal increase in location costs. Hence unless $x_1 = \frac{1}{2}$, central bank 1 has an incentive to deviate (as has central bank 2). Thus the only possible equilibrium with $x_1 = x_2$ is $x_1 = x_2 = \frac{1}{2}$.

This is an equilibrium if the location costs are not too high for any of the banks. Consider first the case $p_1 < \frac{1}{2} < p_2$. Observe that if $x_1 < x_2$, $s_1 = x_1 + \int_{x_1}^{x_2} \frac{x_2 - z}{x_2 - x_1} dz = x_1 + \frac{x_2 - x_1}{2} = \frac{x_1 + x_2}{2}$ and hence $\frac{\partial s_1}{\partial x_1} = \frac{1}{2}$. Thus also the derivative of s_i from the left at $x_1 = x_2 = \frac{1}{2}$ equals $\frac{1}{2}$. Since $G_i'(x_i) = s_i'(x_i) - L'(x_i - p_i)$, we get for the derivative from the left $G_1'\left(\frac{1}{2}\right) > 0$ if $L'\left(\frac{1}{2} - p_1\right) = 2a\left(\frac{1}{2} - p_1\right) \le \frac{1}{2}$ or $p_1 \ge \frac{1}{2} - \frac{1}{4a}$. In that case 1 has no incentive to marginally deviate from $x_1 = \frac{1}{2}$ to $x_1 < \frac{1}{2}$ if $x_2 = \frac{1}{2}$ since its loss in s_1 would not be compensated by a sufficient reduction of location costs. Since $L'' > 0$ a deviation to any $x_1 < \frac{1}{2}$ would not pay. Clearly, a deviation to $x_1 > \frac{1}{2}$ does not pay, because it would yield a smaller s_1 at higher location costs. Likewise, we derive for $p_2 > \frac{1}{2}$ that the necessary and sufficient condition for 2 not to deviate to $x_2 > \frac{1}{2}$ if $x_1 = \frac{1}{2}$ is $L'\left(\frac{1}{2} - p_2\right) = 2a\left(\frac{1}{2} - p_2\right) \ge -\frac{1}{2}$ or $p_2 \le \frac{1}{2} + \frac{1}{4a}$ (since $\frac{\partial s_2}{\partial x_2} = -\frac{1}{2}$ for $x_2 > x_1$).

Similarly, if $p_2 > p_1 > \frac{1}{2}$ then the condition for bank 1 changes to $L'\left(\frac{1}{2} - p_1\right) \ge -\frac{1}{2}$ (which always holds if $L'\left(\frac{1}{2} - p_2\right) \ge -\frac{1}{2}$ since $L'' > 0$) and for $\frac{1}{2} > p_2 > p_1$ the condition for bank 2 changes to $L'\left(\frac{1}{2} - p_2\right) \le \frac{1}{2}$ (which always holds if $L'\left(\frac{1}{2} - p_1\right) \le \frac{1}{2}$ since $L'' > 0$).

For example, if $a = 1$ then there is an equilibrium at $\frac{1}{2}$ if $p_1 \ge \frac{1}{4}$ and $p_2 \le \frac{3}{4}$. The range for p_1 and p_2 such that $x_1 = x_2 = \frac{1}{2}$ is an equilibrium decreases in *a*.

(b) If $p_1 < x_1 < x_2 < p_2$ and $L'(x_1 - p_1) = \frac{1}{2}$ $\left(\Leftrightarrow 2a(x_1 - p_1) = \frac{1}{2} \Leftrightarrow x_1 = p_1 + \frac{1}{4a}\right)$ and $L'(x_2 - p_2) = -\frac{1}{2}$ $\left(\Leftrightarrow 2a(x_2 - p_2) = -\frac{1}{2} \Leftrightarrow x_2 = p_2 - \frac{1}{4a}\right)$ then since, as was shown above, $\frac{\partial s_1}{\partial x_1} = \frac{1}{2}$ and $\frac{\partial s_2}{\partial x_2} = -\frac{1}{2}$, $G_1'(x_1) = G_2'(x_2) = 0$ and hence neither bank 1 nor bank 2 has an incentive to marginally deviate (note that $L'' > 0$ implies that if there is no incentive for a marginal deviation, there is also no incentive for a larger deviation that preserves $x_1 < x_2$). In this case, $x_1 < x_2$ is obviously equivalent to $p_2 - p_1 > \frac{1}{2a}$.

Bank 1 would want to deviate from x_1 to $x_2 + \varepsilon$ only if $(1 - x_2) - \left(x_1 + \frac{x_2 - x_1}{2}\right) > L(x_2 - p_1) - L(x_1 - p_1)$, that is the additional gain in currency holdings by switching to (a position slightly to the right of) x_2 will overcompensate the increase in location costs.[20] Bank 1 would certainly not want to deviate to any larger *x*, because this would imply a smaller share at higher location costs. Note that there can only be an incentive for a deviation to $x_2 + \varepsilon$ if the preferred locations of the two leading central banks are relatively close together but off the median of *f*. Put differently, such an equilibrium exists, if the preferred locations of both banks are located rather symmetrically around $\frac{1}{2}$, are relatively far apart, or location costs are high. Note that $L'(0) = 0$ implies $p_1 < x_1 < x_2 < p_2$ since each bank would be willing to incur some location costs in order to increase its share s_i.

By somewhat tedious, but straightforward computation we can show that $(1 - x_2) - \left(x_1 + \frac{x_2 - x_1}{2}\right) \le L(x_2 - p_1) - L(x_1 - p_1) \Leftrightarrow 1 - p_2 - p_1 \le a(p_2 - p_1)^2 - \frac{1}{4a}$ (note that the right-hand side is > 0 since $p_2 - p_1 > \frac{1}{2a}$).

Similarly, bank 2 has no incentive to deviate to $x_1 - \varepsilon$ if $x_1 - \left(1 - x_2 + \frac{x_2 - x_1}{2}\right) \le L(x_1 - p_2) - L(x_2 - p_2)$ which is equivalent to $p_2 + p_1 - 1 \le a(p_2 - p_1)^2 - \frac{1}{4a}$. Hence no bank has an incentive to deviate if $|1 - p_1 - p_2| \le a(p_2 - p_1)^2 - \frac{1}{4a}$.

Consider again $a = 1$. Then if $p_2 - p_1 > \frac{1}{2}$ there is an equilibrium $x_2 = p_2 - \frac{1}{4}$ and $x_1 = p_1 + \frac{1}{4}$ as long as $|1 - p_1 - p_2| \le (p_2 - p_1)^2 - \frac{1}{4}$. Thus in order for such an equilibrium to exist, the preferences of leading central banks have to be very different, but the range where such an equilibrium exists increases in *a*.

(c) As was shown in (a) no equilibrium exists with $x_1 = x_2 \ne \frac{1}{2}$. (b) states necessary and sufficient conditions for an equilibrium with $p_1 < x_1 < x_2 < p_2$. It is obvious that bank 1 would profit from deviating from an x_1 with $x_1 < p_1 < x_2$, $x_1 < x_2 < p_1$, $x_2 < p_1 < x_1$ or $p_1 < x_2 < x_1$

[20] In the following, we will ignore ε in the share and also in the costs because it can be arbitrarily small.

because 1 could simultaneously increase s_1 and lower location costs. Similarly $x_2 < p_2 < x_1$, $p_2 < x_1 < x_2$, $x_1 < p_2 < x_2$ and $x_2 < x_1 < p_2$ are impossible. This covers all possible constellations of locations. If there is a "smallest policy unit ε", then there could in principle be constellations $x_1 = x_2 - \varepsilon < p_1$. Bank 1 would then not wish to deviate to x_2 (or anything larger) if $s_1 > s_2$. But in that case bank 2 would want to deviate to x_1 (as long as ε is small enough such that the increase in location costs is negligible). This leads to the requirement $\frac{1}{2} - \varepsilon \leq x_1 \leq \frac{1}{2}$ and we are essentially back in case (a). The remaining cases with the smallest possible policy unit would be solved in a similar way. QED

If neither the condition for case (a) nor (b) holds, the intuition for the non-existence of the equilibrium is as follows: there is an *x* with $p_1 < x < \frac{1}{2}$ such that if $x_2 > x$, the marginal location cost for bank 1 at *x* would be $\frac{1}{2}$ such that *x* would be the preferred location of bank 1, but the marginal location costs for bank 2 would be greater than $(-\frac{1}{2})$ so that bank 2 would be willing to push even further to the left to increase s_2, so both would choose the same *x*, which cannot be an equilibrium unless $x = \frac{1}{2}$. Hence non-existence of equilibrium requires sufficient asymmetry of the leading central banks with respect to their preferred locations, and an intermediate level of location costs. If their preferred locations are not too asymmetric with respect to the median of the distribution costs of small countries, then if location costs are low, there is an equilibrium at $\frac{1}{2}$ and if location costs are high, there is an equilibrium with $x_1 \neq x_2$.

The above logic also applies to more general distributions of the preferred locations of small countries and to more general convex location costs. In particular, if *L* is sufficiently small and the preferred policies of the two leading countries are relatively close to the median of *f*, then there is an equilibrium where both choose policies equal to the median. If the preferred locations are rather different and location costs are high, then there is an equilibrium where they choose different locations (which are, however, closer together than their preferred locations). In the first case, where concerns for location costs are dominated by concerns for the share of small countries, the minimal differentiation result remains true, whereas if location costs dominate concerns for shares in small countries, there is an equilibrium with unequal locations.

Note that since conditions (a) and (b) are mutually exclusive, the equilibrium (if it exists) is unique. Hence in a repeated game the equilibrium play will be repeated and the situation is stable in the sense that the policies of the two leading currencies are stable over time and that small countries do not change their exchange rate regimes. In the case of an equilibrium of type (a) the policies will change if, due to external shocks, the preferred policies of the small countries shift. They will, however, change in a parallel fashion provided that the shift is not too radical, because the equilibrium policies will stay at the median as long as the condition in (a) remains fulfilled. The policies will, however, not change if the preferred policies of the leading countries shift as long as condition (a) remains true. But they will shift if the equilibrium is of type (b). Furthermore, if the preferred locations of the leading countries move closer together over time, we can move from an equilibrium of type (b) to one of type (a) (if preferred locations are relatively symmetric to the median of *f*) or to non-existence of a pure strategy equilibrium (if they are highly asymmetric).

Proposition 2: Let $n = 3$ and $p_1 < p_2 < p_3$.

(a) If

(1) $p_2 - p_1 \geq \frac{1}{4a}$,

(2) $p_3 - p_2 \geq \frac{1}{4a}$,

(3) $p_1 + \frac{p_2}{2} - \frac{p_3}{2} + \frac{7}{16a} \leq a(p_2 - p_1)^2$,

(4) $1 + \frac{p_1}{2} - \frac{p_2}{2} - p_3 + \frac{7}{16a} \leq a(p_3 - p_2)^2$,

(5) $-\frac{p_1}{2} - p_2 + \frac{p_3}{2} - \frac{3}{16a} \leq a(p_2 - p_1)^2$,

(6) $-1 - \frac{p_1}{2} + p_2 + \frac{p_3}{2} - \frac{3}{16a} \leq a(p_3 - p_2)^2$,

then $\left(p_1 + \frac{1}{4a}, p_2, p_3 - \frac{1}{4a}\right)$ is an equilibrium.

The equilibrium shares are $s_1^* = x_1 + \frac{p_2 - x_1}{2} = \frac{x_1 + p_2}{2} = \frac{p_1 + p_2 + \frac{1}{4a}}{2}$,

$s_2^* = \frac{x_3 - x_1}{2} = \frac{p_3 - p_1 - \frac{1}{2a}}{2}$, and $s_3^* = 1 - x_3 + \frac{x_3 - p_2}{2} = 1 - \frac{x_3 + p_2}{2} = 1 - \frac{p_3 + p_2 - \frac{1}{4a}}{2}$.

Conditions (5) and (6) are actually not very restrictive and hold quite generally (e.g. if $a \leq 1$, then (5) holds whenever (1) and (3) hold and (6) holds whenever (2) and (4) hold).

(b) Let $x_3 = p_3 - \frac{1}{4a}$ and $x_1 = \frac{x_3}{3}$. If $p_2 \leq x_1 \leq p_1 + \frac{1}{4a}$ and

(7) $1 \leq \frac{8}{9} p_3 + \frac{8}{9} a p_3^{\ 2} - \frac{5}{18a} + \frac{p_2}{3} - \frac{4}{3} a p_2 p_3$,

then (x_1, x_1, x_3) is an equilibrium. The equilibrium shares are $s_1^{\ *} = s_2^{\ *} = x_1 = \frac{p_3}{3} - \frac{1}{12a} < \frac{1}{3}$, $s_3^{\ *} = 1 - x_3 + \frac{x_3 - x_1}{2} = 1 - \frac{2}{3} x_3 = 1 - \frac{2}{3} p_3 + \frac{1}{6a} > \frac{1}{3}$.

Note that independent of *a*, a necessary requirement is $p_3 - p_2 = x_3 - p_2 + \frac{1}{4a} \geq x_3 - x_1 + \frac{1}{4a} = 2x_1 + \frac{1}{4a} \geq 2p_2 + \frac{1}{4a}$, whereas $p_2 - p_1 \leq \frac{1}{4a}$, hence the preferences of bank 2 are much closer to those of bank 1 than to those of bank 3. Thus in equilibrium bank 1 and 2 choosing the same policy resembles the formation of a monetary (policy) union of two banks with similar preferences.

(c) Let $x_1 = p_1 + \frac{1}{4a}$ and $x_3 = \frac{x_1 + 2}{3}$. If $p_3 - \frac{1}{4a} \leq x_3 \leq p_2$ and

(8) $2 \geq 8p_1 + 4ap_1 - 8ap_1^{\ 2} + 4a + \frac{5}{2a} + 3p_2 - 12ap_2 + 12ap_2 p_1$,

then (x_1, x_3, x_3) is an equilibrium. The equilibrium shares are $s_1^{\ *} = x_1 + \frac{x_3 - x_1}{2} = \frac{2}{3} x_1 + \frac{1}{3} = \frac{2}{3} p_1 + \frac{1}{3} + \frac{1}{6a}$, $s_2^{\ *} = s_3^{\ *} = 1 - x_3 = \frac{1}{3} - \frac{p_1}{3} - \frac{1}{12a}$.

This is just a mirror image of case (b).

(d) Otherwise there is no pure strategy equilibrium.

Before proving Proposition 2, consider again the case $a = 1$ for illustration. Conditions (1) and (2) imply that an equilibrium of type (a) only exists if $p_3 - p_2 \geq \frac{1}{4}$ and $p_2 - p_1 \geq \frac{1}{4}$, that is if preferred locations of leading central banks are highly heterogeneous. Conditions (3) and (4) are even more restrictive, implying for example for $p_1 = 0$ and $p_2 = \frac{1}{2}$ that $p_3 \geq \frac{\sqrt{15}}{4}$ or for $p_2 = \frac{1}{2}$ and $p_3 = 1$ that $p_1 \leq 1 - \frac{\sqrt{15}}{4}$. The range where an equilibrium exists increases in *a*. An equilibrium of type (b) only exists if the preferences of banks 1 and 2 are relatively similar and those of bank 3 are quite different. In particular even for $p_3 = 1$ and hence $x_3 = \frac{3}{4}$, $p_1 < p_2 \leq \frac{1}{4}$ is necessary. Hence an equilibrium in pure strategies exists only if preferences are highly heterogeneous or if location costs are very high. If, as we argued above, leading central banks are relatively homogeneous in their preferences compared to small countries and if the weight they attach to small countries being linked to their currency is large compared to the costs of marginal deviations from the preferred policy, there is no equilibrium in pure strategies with $n = 3$, but with $n = 2$ there is an equilibrium where both leading banks choose a policy at the median of *f*, i.e. the basic results of the standard spatial competition model still hold.

Proof of Proposition 2.

(a)

Step 1: bank 2 does not want to deviate:

Note that for all x_2 with $x_1 < x_2 < x_3$, $s_2 = \frac{1}{2}(x_3 - x_1)$ and hence bank 2 has no incentive to deviate to any such x_2 since this will not affect s_2 but will cause positive location costs. If bank 2 deviates to $x_2 = x_1 - \varepsilon$, then its share is x_1 (we will again ignore ε in the share and also in the costs because it can be arbitrarily small). So deviating does not pay if the gain in s_2 is smaller than the incurred location costs, i.e. if $x_1 - s_2^* \le L(x_1 - p_2)$ which is (as again tedious but straightforward computation shows) equivalent to (3). If bank 2 deviates to x_1, then $s_2 = \frac{s_2^*}{2} + \frac{x_1}{2}$, so if deviating to $x_1 - \varepsilon$ does not pay, deviating to x_1 definitely does not pay. If bank 2 deviates to $x_2 = x_3 + \varepsilon$ then $s_2 = 1 - x_3$, so deviating does not pay if $1 - x_3 - s_2^* \le L(x_3 - p_2)$ which is equivalent to (4). If bank 2 deviates to x_3, then $s_2 = \frac{s_2^*}{2} + \frac{1-x_3}{2}$, so if deviating to $x_3 + \varepsilon$ does not pay, then deviating to x_3 definitely does not pay.

Step 2: bank 1 does not want to deviate:

Since $L'(x_1 - p_1) = L'\left(\frac{1}{4a}\right) = \frac{1}{2}$ the marginal location costs of bank 1 at x_1 are equal to the marginal gain in s_1, hence bank 1 has no incentive to marginally deviate and condition (1) is equivalent to $x_1 \le p_2$. Since $L'' > 0$ bank 1 has no incentive to deviate to any $x < p_2$.

Bank 1 does not want to deviate to any *x* with $p_2 < x < x_3$: Note that $s_1 = \frac{x_3 - p_2}{2}$ for all such *x*. Hence 1 would, if anything choose $p_2 + \varepsilon$. Bank 1 will not deviate to $p_2 + \varepsilon$ if $\frac{x_3 - p_2}{2} - \frac{p_2 + x_1}{2} \le L(p_2 - p_1) - L(x_1 - p_1)$, which is equivalent to (5).

Bank 1 does not want to deviate to $x_3 + \varepsilon$ because bank 2 does not want to deviate to $x_3 + \varepsilon$, as can be seen by the following argument. For ease of notation let $A = x_1$, $B = p_2 - x_1$, $C = x_3 - p_2$ and $D = 1 - x_3$. Assume that bank 1 wants to deviate to $x_3 + \varepsilon$, i.e. $D - A - \frac{B}{2} > a(x_3 - p_1)^2 - a(x_1 - p_1)^2$, but bank 2 does not, i.e. $D - \frac{C+B}{2} \le a(x_3 - p_2)^2$. Observe that

$$a(x_3 - p_1)^2 - a(x_1 - p_1)^2 = a(x_3 - p_2 + p_2 - p_1)^2 - a(x_1 - p_1)^2 =$$
$$= a(x_3 - p_2)^2 + a(p_2 - p_1)^2 - a(x_1 - p_1)^2 + 2a(x_3 - p_2)(p_2 - p_1) \geq a(x_3 - p_2)^2 + \tfrac{C}{2}.$$

Hence the above assumptions imply $D - A - \frac{B}{2} > a(x_3 - p_2)^2 + \frac{C}{2} \geq D - \frac{B}{2}$ which can obviously not be true.

By deviating to p_2, bank 1 would obtain the average of the shares that it obtains at $p_2 - \varepsilon$ and $p_2 + \varepsilon$, so if it does not want to deviate to either of these, it does not want to deviate to p_2 either, and by a parallel argument it does not want to deviate to x_3.

Deviating to any other location is dominated because it yields the same or a lower s_1 at a higher location cost than one of the options discussed above.

Step 3: bank 3 does not want to deviate:

The situation of bank 3 is symmetric to that of bank 1 and hence the conditions are derived in a similar way. Condition (2) ensures that $x_3 \geq p_2$ and since $L'\left(\frac{1}{4a}\right) = \frac{1}{2}$ bank 3 would not like to marginally deviate and because of $L'' > 0$ would not want to deviate to any $x > p_2$.

Bank 3 would not want to deviate to $p_2 - \varepsilon$ if $\frac{p_2 - x_1}{2} - \frac{x_3 - p_2}{2} - (1 - x_3) \leq L(p_2 - p_3) - L(x_3 - p_3)$, which is equivalent to (6). Again, because s_3 would be identical for all locations x with $x_1 < x < p_2$, bank 3 would not want to deviate to any such x either.

Bank 3 would not want to deviate to $x_1 - \varepsilon$ if bank 2 does not want to. As above, assume the opposite. This implies (with the above notation) that $A - D - \frac{C}{2} > a(p_3 - x_1)^2 - a(p_3 - x_3)^2$ but $A - \frac{B+C}{2} \leq a(p_2 - x_1)^2$. Note that

$$a(p_3 - x_1)^2 - a(p_3 - x_3)^2 = a(p_3 - p_2 + p_2 - x_1)^2 - a(p_3 - x_3)^2 =$$
$$= a(p_2 - x_1)^2 + a(p_3 - p_2)^2 - a(p_3 - x_3)^2 + 2a(p_3 - p_2)(p_2 - x_1) \geq a(p_2 - x_1)^2 + \tfrac{B}{2}$$

Together this implies $A - D - \frac{C}{2} > a(p_2 - x_1)^2 + \frac{B}{2} \geq A - \frac{C}{2}$, obviously a contradiction.

As for bank 1, deviating to any other location does not pay because it implies a lower or identical share at higher costs than one of the possible deviations discussed above (and deviations to p_2 or x_1 do not pay by an argument parallel to that for bank 1).

The above analysis shows that conditions (1) to (6) are sufficient for (x_1, p_2, x_3) being an equilibrium, but also necessary for an equilibrium with $x_1 < p_2 < x_3$ and $x_1 < x_2 < x_3$.

(b)

Step 1: banks 1 and 2 do not want to deviate: Note that since $3x_1 = x_3$ we have $x_1 = \frac{x_3 - x_1}{2}$. If either bank 1 or bank 2 deviated to $x_1 - \varepsilon$ it would obtain x_1, if it deviated to $x_1 + \varepsilon$ it would obtain $\frac{x_3 - x_1}{2} = x_1$, staying at x_1 yields half of both, hence also x_1. So there is no incentive for a marginal deviation. Note that all x with $x_1 < x < x_3$ yield $s = \frac{x_3 - x_1}{2}$ but since $p_1 < p_2 \le x_1$ the location costs are higher and hence a deviation to any such x does not pay. A deviation to $x < x_1$ implies a reduction of s by $\frac{x_1 - x}{2}$. Since $x_1 - p_2 < x_1 - p_1 \le \frac{1}{4a}$ we have $L'(x_1 - p_2) < L'(x_1 - p_1) \le \frac{1}{2}$. Thus the decrease in location costs is smaller than the loss in s and a deviation to $x < x_1$ does not pay. Finally, a deviation to $x_3 + \varepsilon$ does not pay for bank 2 if $1 - x_3 - x_1 \le L(x_3 - p_2) - L(x_1 - p_2)$, which is equivalent to (7). Since $L'' > 0$ and $p_1 < p_2$, $L(x_3 - p_1) - L(x_1 - p_1) > L(x_3 - p_2) - L(x_1 - p_2)$, and thus deviating to $x_3 + \varepsilon$ does not pay for bank 1 if it does not pay for bank 2.

Step 2: bank 3 does not want to deviate: since $x_3 = p_3 - \frac{1}{4a}$, $L'(x_3 - p_3) = -\frac{1}{2}$ and hence bank 3 does not want to deviate to any $x > x_1$. Since $s_3^* > \frac{1}{3} > x_1$, a deviation to $x_1 - \varepsilon$ or x_1 implies a lower share at a higher location cost and hence bank 3 has no incentive to deviate.

(c) This is just the symmetric situation to (b). The proof is essentially identical.

(d) There are no further equilibria.

Step 1: as established above, the equilibrium in (a) is the only equilibrium with $x_1 < p_2 < x_3$ and $x_1 < x_2 < x_3$. There can be no equilibrium with $x_1 < x_2 < x_3$ but $p_2 \le x_1$, because in that case bank 2 could, by deviating to x with $x_1 < x < x_2$, obtain the same s_2 at lower location costs.[21] By

[21] If there is a "smallest policy unit ε" then there could be an equilibrium where $x_1 < x_2 < x_3$ but $p_2 \le x_1$, namely if $x_2 = x_1 + \varepsilon$ and s_2 is larger than s_1, but in that case bank 1 would have an incentive to deviate to x_2, unless

a parallel argument, there is also no equilibrium with $x_1 < x_2 < x_3$ but $x_3 \le p_2$. Hence the equilibrium in (a) is the only equilibrium with $x_1 < x_2 < x_3$.

Step 2: $x_1 = x_2 < x_3$ implies $x_1 = \frac{x_3 - x_1}{2}$ otherwise bank 1 or 2 could, by a marginal deviation, increase its share at essentially 0 increase in location costs. This then implies $p_1 < p_2 \le x_1$ because any x with $x_1 < x < x_3$ yields the same share, so if $p_2 > x_1$, bank 2 could obtain the same share at lower location costs. Hence the equilibrium in (b) is the only equilibrium with $x_1 = x_2 < x_3$.

Step 3: by the same argument as in step 2, the only equilibrium with $x_1 < x_2 = x_3$ is the equilibrium in (c).

Step 4: $x_1 = x_2 = x_3$ cannot be an equilibrium: in this case $s_i = \frac{1}{3}$ and by a marginal deviation bank i could obtain $\max(x_1, 1 - x_1) \ge \frac{1}{2}$.

Step 5: in equilibrium $x_2 < x_1 \le x_3$ is impossible, because in that case $p_1 < x_1$ or $x_2 < p_2$ and hence one bank could lower its location costs while increasing or retaining its share (note that as was argued in the proof of part (b), if in equilibrium $x_1 = x_3$ then $\frac{x_3 - x_2}{2} = 1 - x_3$, so by deviating to x with $x_2 < x < x_3$, bank 1 would obtain the same s_1, as it is also the case for $x_1 < x_3$). On the other hand, $x_2 = x_1 < x_3$ corresponds to the equilibrium in (b), so all cases $x_2 \le x_1 \le x_3$ are covered (in case of equality of all x, step 4 applies).

Step 6: the argument why any constellation, $x_1 \le x_3 < x_2$, $x_2 \le x_3 \le x_1$, $x_3 \le x_2 \le x_1$, $x_3 \le x_1 \le x_2$ cannot occur in equilibrium is the same as in step 5: at least one bank can reduce its location costs without reducing its share if at least one inequality is strict; otherwise the argument of step 4 applies.

This covers all possible constellations of x_1, x_2, and x_3 and shows that no equilibrium except for those in (a), (b), and (c) exist. QED

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the difference in shares is very small, so this essentially corresponds to the equilibrium in (b).

The qualitative results of proposition 2 should also hold for more general convex cost functions and more general distributions of preferences of small countries.[22] In particular, if central banks' preferences are relatively homogeneous compared to the preferences of small countries and location costs are not excessively high, there is no equilibrium in pure strategies. The logic is the same as in the standard spatial competition model: banks 1 and 3 would like to choose locations close to x_2. In that case s_2 would be small, but bank 2 could increase s_2 at only a small increase in location costs by deviating to $x_1 - \varepsilon$ or to $x_3 + \varepsilon$.

In a repeated game the non-existence of a pure-strategy equilibrium means that the actual choices of central banks in period t are not in equilibrium. Therefore, at least one central bank would like to change its location. Hence the configuration of locations of leading central banks will change from period t to period $t+1$, even without external shocks and even not in a parallel fashion. Put differently, there is only a mixed strategy equilibrium, and these mixed strategies will (in general) yield different realizations and hence different locations of leading currencies in each period. As a consequence, the currency baskets of (at least some of the) smaller countries will also change from period to period.[23]

The difference between the cases $n = 2$ and $n = 3$ can be summarized as follows. If leading central banks' preferences are highly heterogeneous or location costs are very high, then both for $n = 2$ and $n = 3$ there exists an equilibrium where central banks choose different locations. But if, as we assume, leading central banks' preferences are relatively similar compared to the distribution of small countries' preferences and location costs are not very high, then the result of the model without location costs survives, namely that for $n = 2$ there is an equilibrium where both central banks choose a policy at the median of the distribution of small countries' preferences, and if $n = 3$, then there is no equilibrium in pure strategies. Corresponding results for $n > 3$ can be derived by a similar extension of Eaton and Lipsey (1975). This, however, is beyond the scope of this paper.

[22] In the latter case, $x_2 = p_2$ would in general not hold any more in equilibrium, because the density *f* is not constant and hence s_2 is not the same for all x_2 with $x_1 < x_2 < x_3$.

[23] Such behavior can be observed in case of emerging countries in our sample.

3. Data and Statistical Inference

3.1 Data and Quantitative Evidence

We collected data on exchange rates of domestic currencies with respect to the US dollar, to the Deutsch mark/ECU/euro, and the Japanese yen. Furthermore, we assembled data on monetary aggregate (M2), short-term and long-term interest rate, and nature of exchange rate regime for 30 OECD countries plus Russia. Short-term interest rates are defined as three-month money market rates, or rates on similar financial instruments. The span of our yearly data is from 1953 to 2002, with the exception of emerging countries where meaningful data are available only from the mid-1980s. All data were assembled from OECD Economic Outlook statistics, International Financial Statistics of the IMF and, for particular missing data, from the central banks and ministries of finance of the respective countries. In order to observe dependencies of small currencies on leading currencies we used the description of *de facto* (true) exchange regimes provided in Reinhardt and Rogoff (2004). We consider the OECD countries, due to their economic capacity and derived amount of monetary aggregate used, as a proxy for the world.[24]

The overall situation from the 1950s to 2002 is captured in Figures 2-5. They illustrate how the share of monetary aggregate linked to leading currencies as well as the share of countries linked through their exchange rate regimes to leading currencies evolved over time. We see a massive outflow from the US dollar after the collapse of the Bretton Woods System, and an equally pronounced gain in Europe. While the share of currencies linked to the US dollar stabilized in the late 1980s, European currency has been solidifying its share steadily. The share of currencies not linked to any leading currency never exceeds 30% and meanders with time. The share of monetary aggregate of these countries tends to be negligible. The Japanese yen represents a significant share of money that is linked to it, hovering around 30% of the total.

Figure 6 presents the total amount of monetary aggregate of all considered currencies divided among three groups in terms of the exchange rate regime link. The link of a currency is either to the US dollar, to the Deutsch mark/ECU/euro, or to the Japanese yen. In the context of our model we see a clearly dominant position of the US dollar from the 1950s to 1971. This equals a situation in which the number of leading currencies is just one; $n = 1$. Period 1971-1977 represents a transition after the Bretton Woods System ceased to work. We see a departure from state $n = 1$ towards $n > 1$. During this period there exist no obvious candidates that would firmly

[24]Because of this, we do not incorporate into our sample a number of small countries.

establish a situation of two leading currencies in which $n = 2$. Figure 7 illustrates the development after 1978: the amount of monetary aggregate of currencies that are linked neither to the US dollar nor to the Deutschmark/ECU/euro indicates that there are not two dominating currencies. Most of the non-linked aggregate originates in Japan, but not all. Hence, the post-1978 period represents a situation in which, in the framework of our model, unquestionably $n > 2$ and no equilibrium exists.

Short-term interest rates (Figure 8) allow us to detect changes in the positioning of central banks of the leading currencies in one-dimensional space. Short-term interest rates vary extensively and they do not move in a parallel fashion. The differences among the short-term rates are relatively small and the differential between Japanese and US/European rates becomes slightly pronounced only in the 1990s. Such behavior is consistent with our model specification, which assumes that central banks use interest rate for positioning purposes in their one-dimensional space. Evidence shows that interest rates interact over time; disparities due to central banks altering the interest rate have been quickly adjusted for.

3.2 Empirical Econometrics and Statistical Inference

In order to verify the prediction of the model we test the following hypothesis using the data described. We assume in our model that the central bank of a leading currency uses interest rate as a policy instrument for its positioning purposes within one-dimensional space. Our model predicts that a change in this instrument ultimately leads to a change in the choice of small countries with respect to their ties to leading currencies. Hence, based on our model we should witness a link from a central bank (of a leading currency) interest rate (i_t) to the extent of money tied to a leading currency via exchange rate regime (m_t). We proceed to verify the model's prediction by using the concept of Granger-causality.

Since Granger (1969) introduced his definition of 'causality', the test of Granger-type causality has been applied frequently in empirical work, including studies on links among a wide array of macroeconomic variables.[25] This methodology for testing linkages has become standard and well known. In general, we say that "$\{x_t\}$ causes $\{y_t\}$" if the present value of y_t can be predicted significantly better when past values of x_t are included in our specification. Usually the

[25] For applications using monetary variables see Thornton and Batten (1985), Sauer and Scheide (1995), Hess and Porter (1993), Masih and Masih (1998), and Lee (1997) among others.

notion of ‘causality’ in economic systems is limited to linear relations between observed time series. The Granger causality is then tested via an autoregressive representation in the form:

$$(9)\quad \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \nu_t \end{pmatrix},$$

where L denotes the lag operator.

Because disturbances are serially uncorrelated, the test for the direction of causality between $\{x_t\}$ and $\{y_t\}$ can be turned into standard tests of whether $b(L)$=0 and $c(L)$=0, respectively.[26] The testing can proceed only if some restrictions on the autoregressive form (x_t) are specified before the actual estimation. Particularly, the length of autoregression should be identified prior to the estimation of (x_t). We applied Hsiao's (1981) two-step approach to determine the length of the lag structure. The “optimal lengths” were estimated applying standard information criteria (See Akaike, 1969; Hannan and Quinn, 1979; Schwarz, 1978), all of them suggesting that only one lag of both variables be used.

Hence, our model for testing the potential causal relationship between interest rate and money is specified as follows:

$$(10)\quad \begin{aligned} m_t &= \alpha m_{t-1} + \beta i_{t-1} + \varepsilon_t \\ i_t &= \gamma m_{t-1} + \delta i_{t-1} + \nu_t \end{aligned},$$

where i_t is a short-term interest rate and m_t is a money aggregate (as defined in Section 2). Within the framework of the Granger causality the first hypothesis is formulated that a change in interest rate does not Granger-cause a change in the amount of money linked to a particular leading currency ($H_0 : \beta = 0$). Analogously, the second hypothesis captures no causal link from money to short-term interest rate ($H_0 : \delta = 0$). Results are presented in Table 1.

[26] The test of the hypothesis ‘$\{x_t\}$ causes $\{y_t\}$’ is equivalent to the test of the restriction $b(L)$=0. Similarly, the opposite direction of causality can be tested via the restriction $c(L)$=0.

Table 1
Interest rate of a central bank of a leading currency and total amount of money linked to a particular leading currency: Granger-causality results

Variable linked to currency of:	Granger Causality Link	Estimated equation
Europe	Link from interest rate to money	$m_t = \underset{(0.044)}{0.927^{**}} m_{t-1} + \underset{(0.002)}{0.004^{*}} i_{t-1}$
	No link from money to interest rate	$i_t = \underset{(0.047)}{0.992^{**}} i_{t-1} - \underset{(1.356)}{0.260} m_{t-1}$
Japan	Link from interest rate to money	$m_t = \underset{(0.026)}{0.956^{**}} m_{t-1} + \underset{(0.001)}{0.002^{*}} i_{t-1}$
	No link from money to interest rate	$i_t = \underset{(0.040)}{0.998^{**}} i_{t-1} - \underset{(1.047)}{0.833} m_{t-1}$
USA	Link from interest rate to money	$m_t = \underset{(0.038)}{0.867^{**}} m_{t-1} + \underset{(0.002)}{0.005^{*}} i_{t-1}$
	No link from money to interest rate	$i_t = \underset{(0.050)}{0.957^{**}} i_{t-1} + \underset{(0.867)}{0.533} m_{t-1}$

** and * denote significance at 1% and 5% levels, respectively.

Results from Table 1 show that for all leading currencies we have found a statistically significant link from interest rate to money tied to leading currencies via exchange rate regime. In addition, we have not found a reverse causal relationship. These two sets of results combined are fully consistent with and support predictions of our theoretical model. In all models we verified the co-integration of the time series studied by performing the ADF test on residuals. In all cases we have rejected the null hypothesis of no co-integration at 1% significance level. Further, by using the Durbin-Watson h-alternative test, residuals were detected to be free of autocorrelation.

4. Final remarks

The theoretical model that we present in this paper is based on the idea of spatial competition and rests on a set of realistic assumptions related to the behavior of central banks, the working of exchange rate regimes, and international monetary arrangements in general. We show that although stable equilibrium of exchange rates can arise in the case of two leading currencies, instability is a prominent feature in the case of three leading currencies.

We support the implications of our model with both quantitative evidence and formal statistical inference. Our empirical results back up the predictions of our theoretical model. We find a statistically significant link from the interest rate to money tied to leading currencies via exchange rate regime; we do not find a reverse causal relationship, though.

Our results have stirring implications with respect to recent developments. As the euro has gained value against the dollar, central banks in Japan, China, and other Asian countries have bought dollars to hold down the value of their own currencies. The total reserves of the four largest Asian economies - China, Japan, South Korea, and Taiwan - have more than doubled over the 2001-2003 period and reached 1.5 trillion US dollars, most of it held in American government securities. China itself keeps its currency tightly pegged to the US dollar, which greatly upsets non-dollar allied Europe that appeals to China to let its currency float and to Japan to discontinue its interventions on the yen-dollar market. Its rationale behind such a claim is to enhance the stability among exchange rates of the leading currencies. Our conclusions would indicate just the opposite. In fact if China keeps its link to the dollar and Japan pegs the yen in some way, then our model predicts that the overall situation will lean towards the two-currency equilibrium.

Since firms, traders, and countries currently recognize three leading currencies and their economic behavior reflects this, we may expect disagreement on overvaluation or undervaluation of certain currencies to continue. Under current monetary arrangements, attaining stability among the exchange rates of leading currencies is about as likely as squaring a circle.

References

Akaike, H. 1969. Fitting Autoregressive Models for Predictions. *Annals of Mathematical Statistics*, 243-47.

Andrews, David M., C. Randall Henning, and Louis W. Pauly. 2002. *Governing the world's money*. Ithaca: Cornell University Press.

Battilossi, Stefano and Youssef Cassis. 2002. *European banks and the American challenge: Competition and cooperation in international banking under Bretton Woods*. New York: Oxford University Press.

Benassy-Quere, Agnes, Lionel Fontagne, and Amina Lahreche-Revil. 2001. Exchange Rate Strategies in the Competition for Attracting Foreign Direct Investment. *Journal of the Japanese and International Economies*, 15(2): 178-98.

Bird, Graham and Ramkishen S. Rajan. 2002. The evolving Asian financial architecture. *Essays in International Economics*, no. 226. Princeton: Princeton University, Department of Economics, International Economics Section.

Calvo, Guillermo A. and Carmen M. Reinhart. 2002. Fear of Floating. *Quarterly Journal of Economics*, 117(2): 379-408.

Devereux, Michael, B., Shi Kang, and Xu Juanyi. 2004. Global Monetary Policy under a Dollar Standard. *CEPR Discussion Paper* No. 4317.

Devereux, Michael, Charles Engel, and Cédric Tille. 2003. Exchange Rate Pass-Through and the Welfare Effects of the Euro. *International Economic Review*, 44(1), 223-242.

Eaton, B. Curtis and Richard G. Lipsey. 1975. The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition. *Review of Economic Studies,* 42(1), pp. 27-49.

Frankel, Jeffrey, A. 1988. *Obstacles to International macroeconomic Policy Coordination.* Princeton Studies in International Finance 64 (Princeton: International Finance Section, Department of Economics, Princeton University.

Frieden, Jeffry and Ernesto Stein (eds.) 2001. *The currency game: Exchange rate politics in Latin America*. Washington, D.C.: Inter-American Development Bank; distributed by Johns Hopkins University Press, Baltimore.

Ghosh, Atish R, Anne-Marie Gulde, and Holger C. Wolf. 2002. *Exchange rate regimes: Choices and consequences*. Cambridge and London: MIT Press.

Giavazzi, Francesco and Alberto Giovannini. 1989. *Limited exchange rate flexibility: The European monetary system*. Cambridge, Mass. and London: MIT Press, 1989

Granger, Clive. J., 1969. Investigating Causal Relationships by Econometrics Models and Cross Spectral Methods. *Econometrica*, 37: 425-35.

Hannan, E. J. and B. G. Quinn. 1979. The Determination of the Order of an Autoregression. *Journal of Royal Statistical Society*, 41: 190-195.

Hess, Gregory D. and Richard D. Porter. 1993. Comparing Interest-Rate Spreads and Money Growth as Predictors of Output Growth: Granger Causality in the Sense Granger Intended. *Journal of Economics and Business*, 45(3-4): 247-68.

Hotelling, Harold. 1929. Stability in Competition, *Economic Journal,* 39, pp. 41-57.

Hsiao, C. 1981. Autoregressive Modeling and Money-Income Causality. *Journal of Monetary Economics,* 7: 85-106.

Iida, Keisuke. 1999. *International Monetary Cooperation among the United States, Japan, and Germany*. Norwell: Kluwer Academic Publishers.

Lee, Jim. 1997. Money, Income and Dynamic Lag Patterns. *Southern Economic Journal*, 64(1), 97-103.

Leeson, Robert. 2003. *Ideology and international economy: The decline and fall of Bretton Woods*. Basingstoke: Palgrave Macmillan.

Madden, Gary, Scott Savage, and Andrew McDonald. 2000. Assessing the Economic Preconditions for a Yen Bloc. *Australian Economic Papers*, 39(1): 25-32.

Masih, Abul M. M. and Rumi Masih, 1998. Does Money Cause Prices, or the Other Way Around? Multi-country Econometric Evidence Including Error-Correction Modelling from South-East Asia. *Journal of Economic Studies*, 25(2-3): 138-60.

Overturf, Stephen-Frank. 2000. *Money and European Union*. New York: St. Martin's Press.

Reinhart, Carmen and Kenneth Rogoff. 2002. Modern history of exchange rate arrangements: A reinterpretation. *Quarterly Journal of Economics*, 119(1): 1-48.

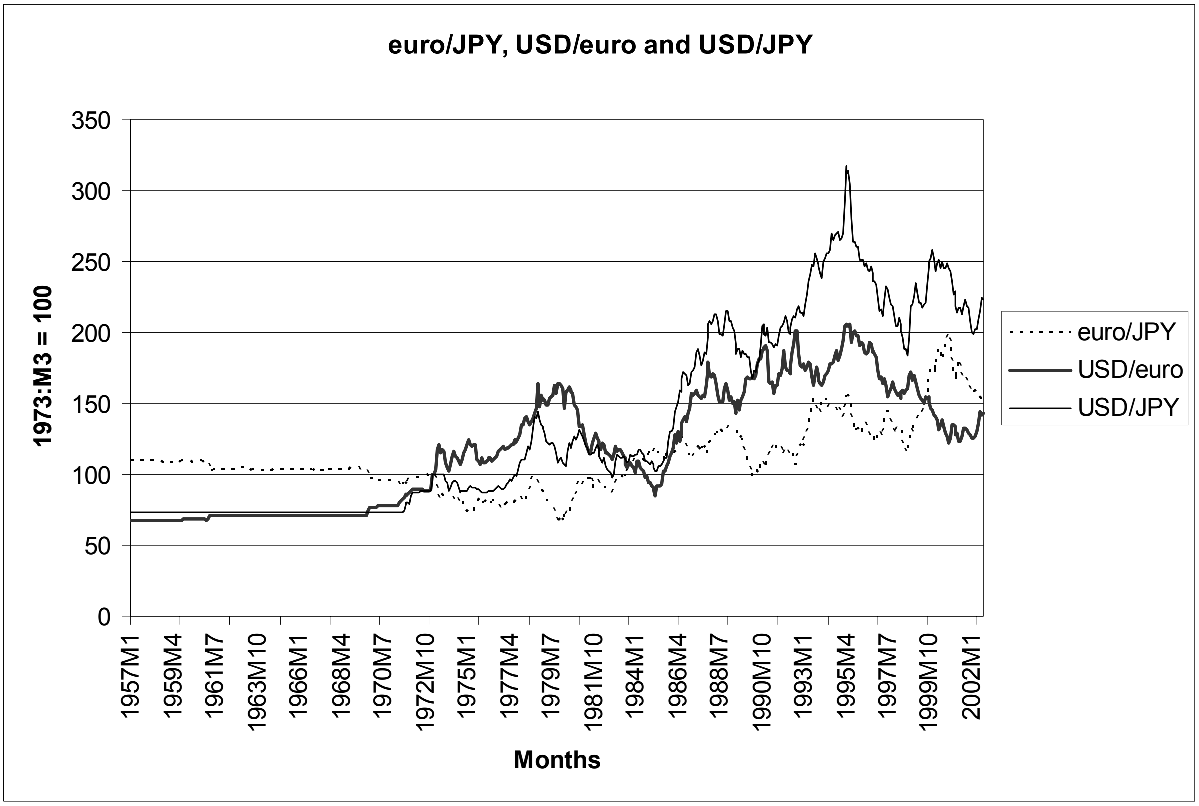
Sauer, Christine and Joachim Scheide. 1995. Money, Interest Rate Spreads, and Economic Activity, *Weltwirtschaftliches Archiv*, 131(4): 708-22

Shwarz, G. 1978. Estimating the Dimension of a Model. *Annals of Statistics*, 6: 461-464.

Thornton, D and D. Batten. 1985. Lag-length Selection and Tests of Granger Causality between Money and Income. *Journal of Money, Credit, and Banking*, 17: 164-78.

Williamson, John. 1998. Crawling Bands or Monitoring Bands: How to Manage Exchange Rates in a World of Capital Mobility. *International Finance*, 1(1): 59-79.

Figure 1. Exchange Rate Deviations. (March 1973 = 100)



Note: euro refers to euro from 1999 onwards and to DEM and ECU prior to this date. Official fixed parities are used to calculate respective exchange rate. Since the figure is in deviations, it doesn't matter whether DEM, ECU, or euro is used as a common denominator.

Figure 2. Inter-temporal Relative Share of Money and Countries Linked to the US Dollar

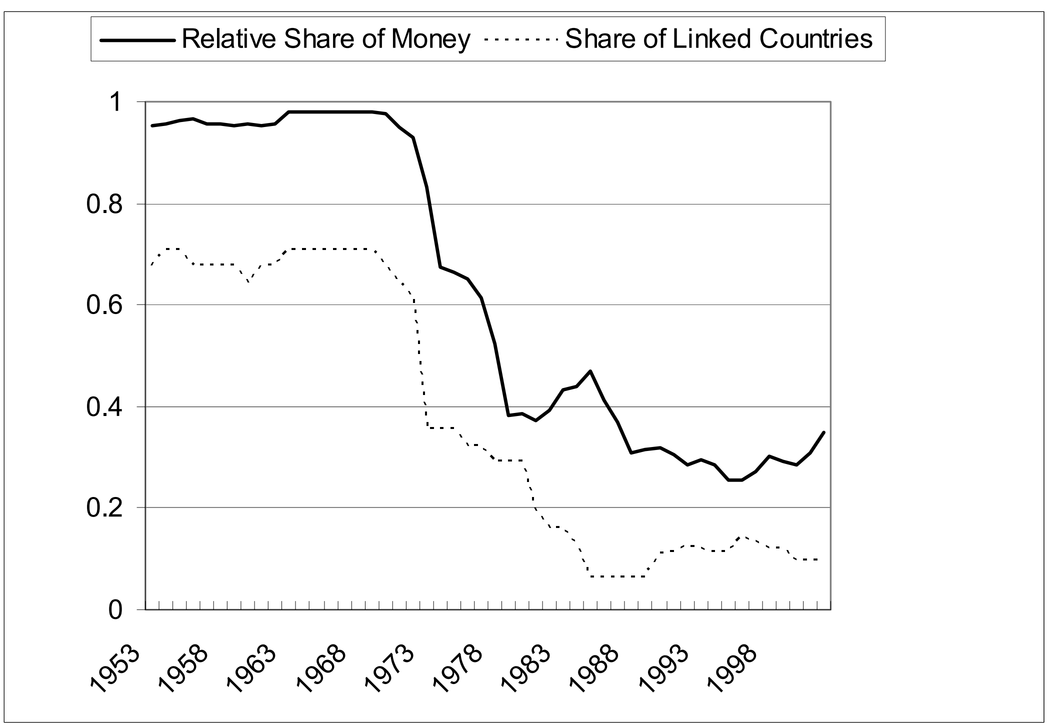


Figure 3. Inter-temporal Relative Share of Money and Countries Linked to the DEM/ECU/euro

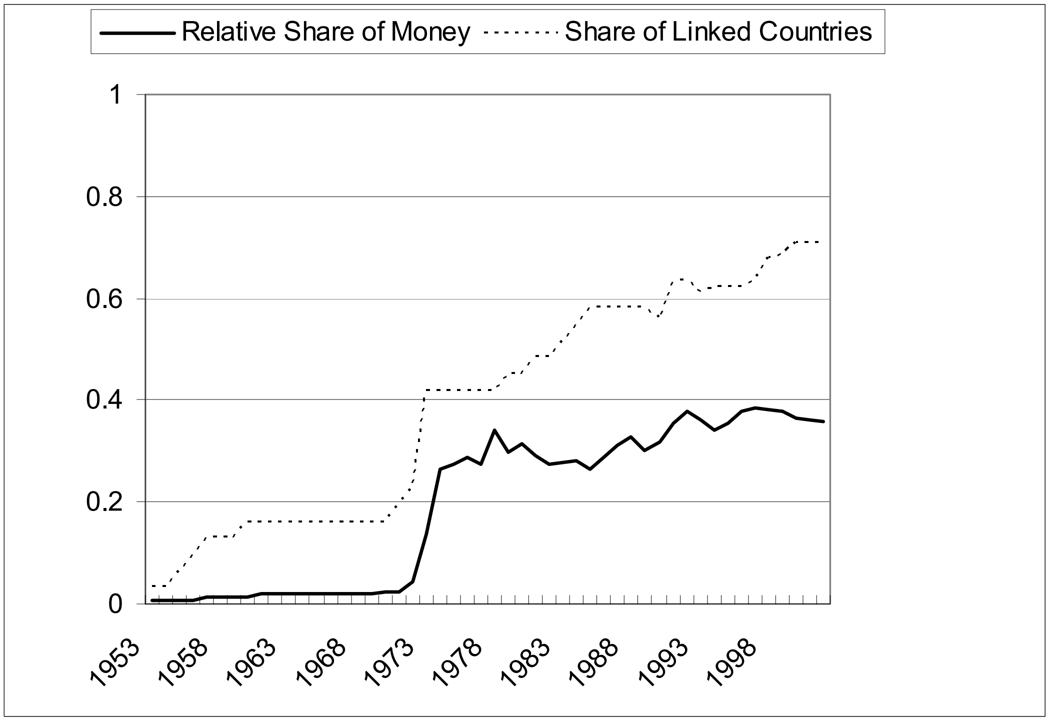


Figure 4. Inter-temporal Relative Share of Money and Countries Linked to the Japanese yen

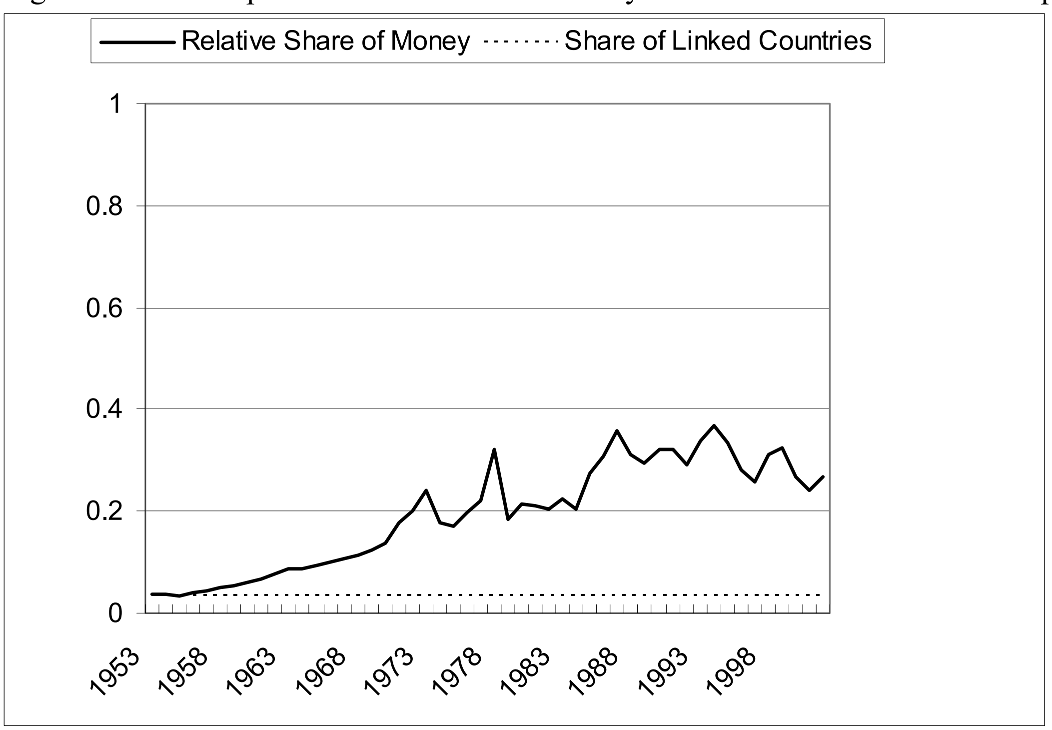


Figure 5. Inter-temporal Relative Share of Money and Countries Not-linked to any of the Leading Currencies

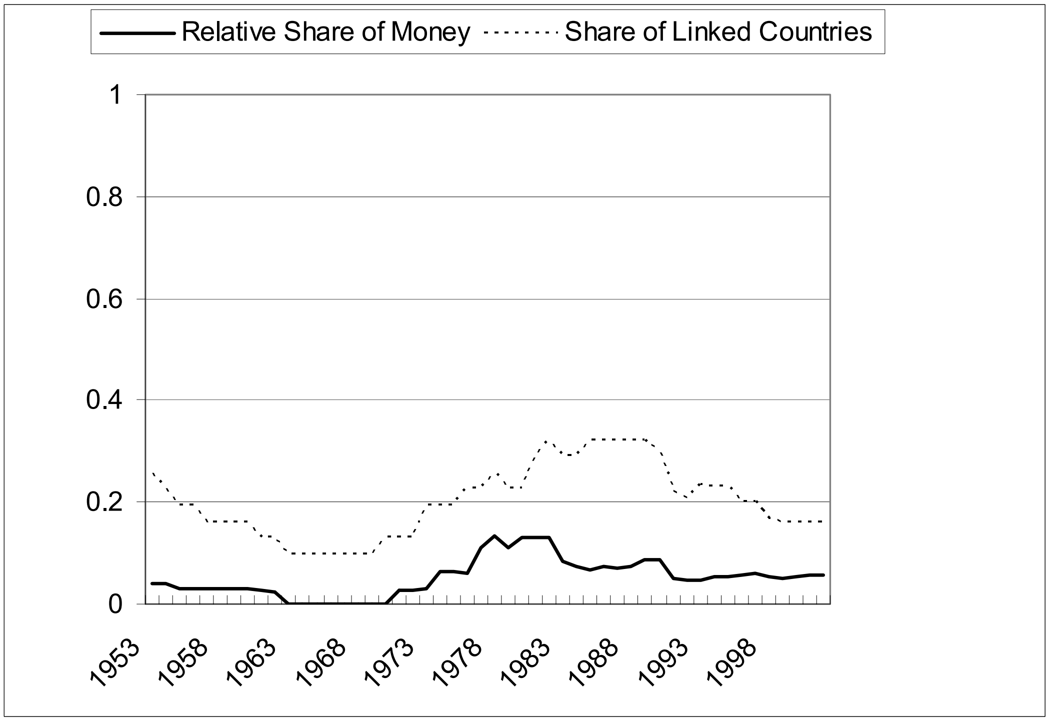


Figure 6. Relative Share of Money Linked to Leading Currencies (Long Period, 1953-2002)

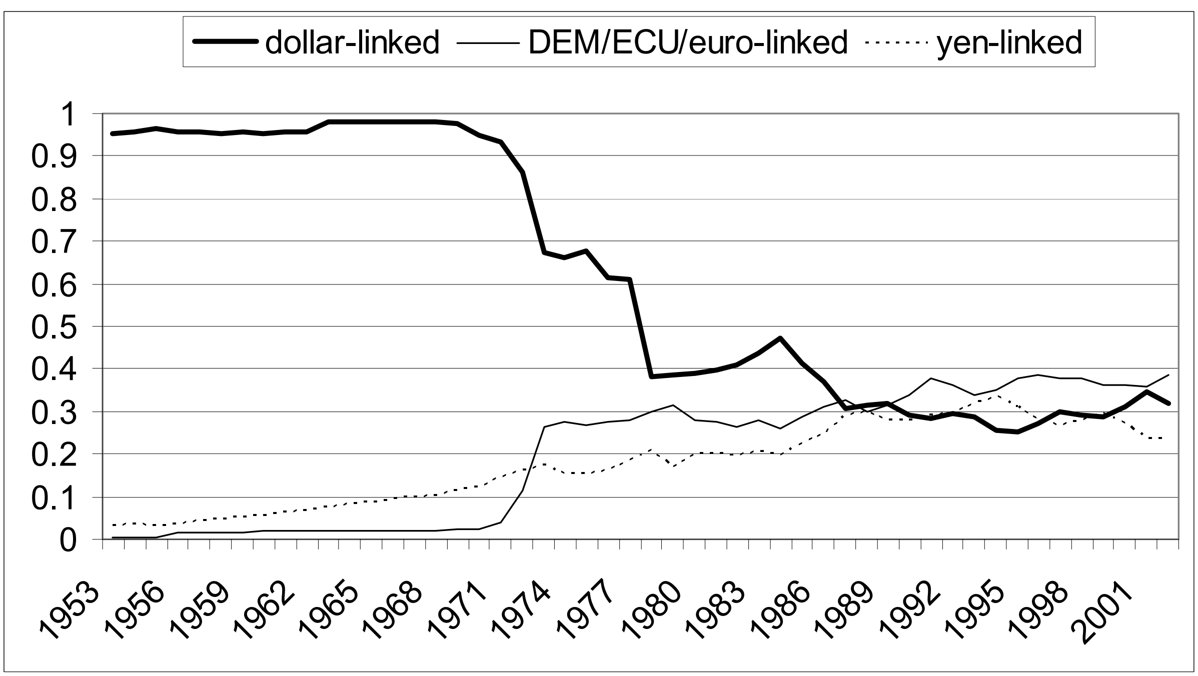


Figure 7. Relative Share of Money Linked to Leading Currencies (Short Period, 1978-2002)

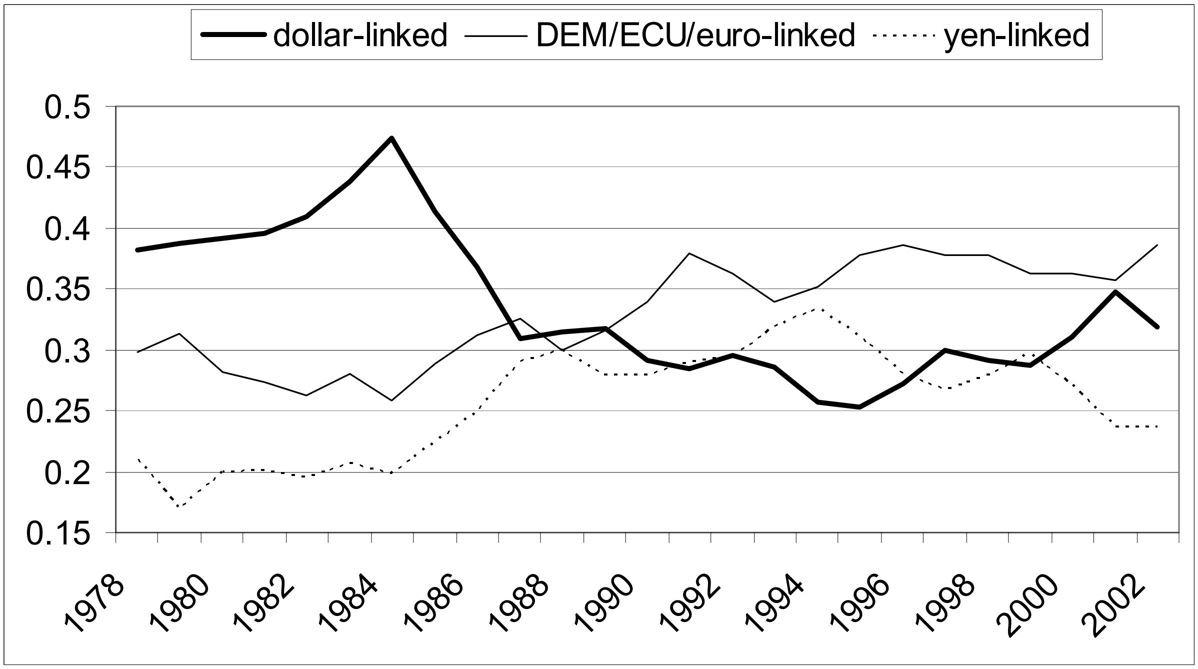
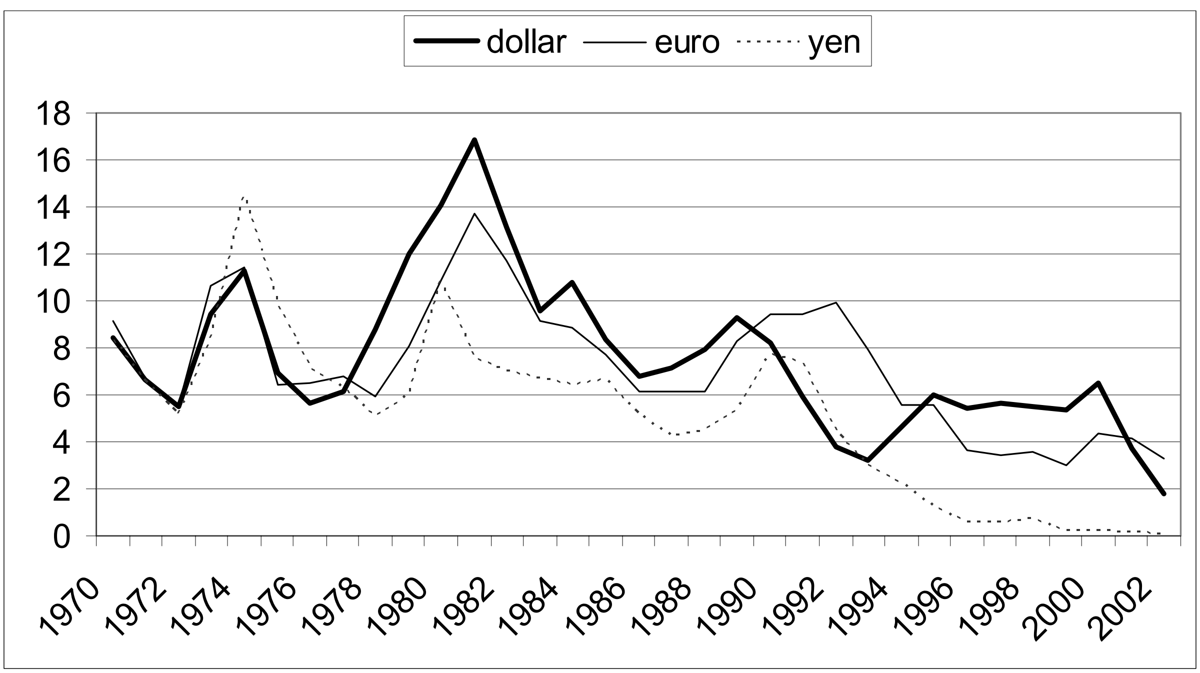
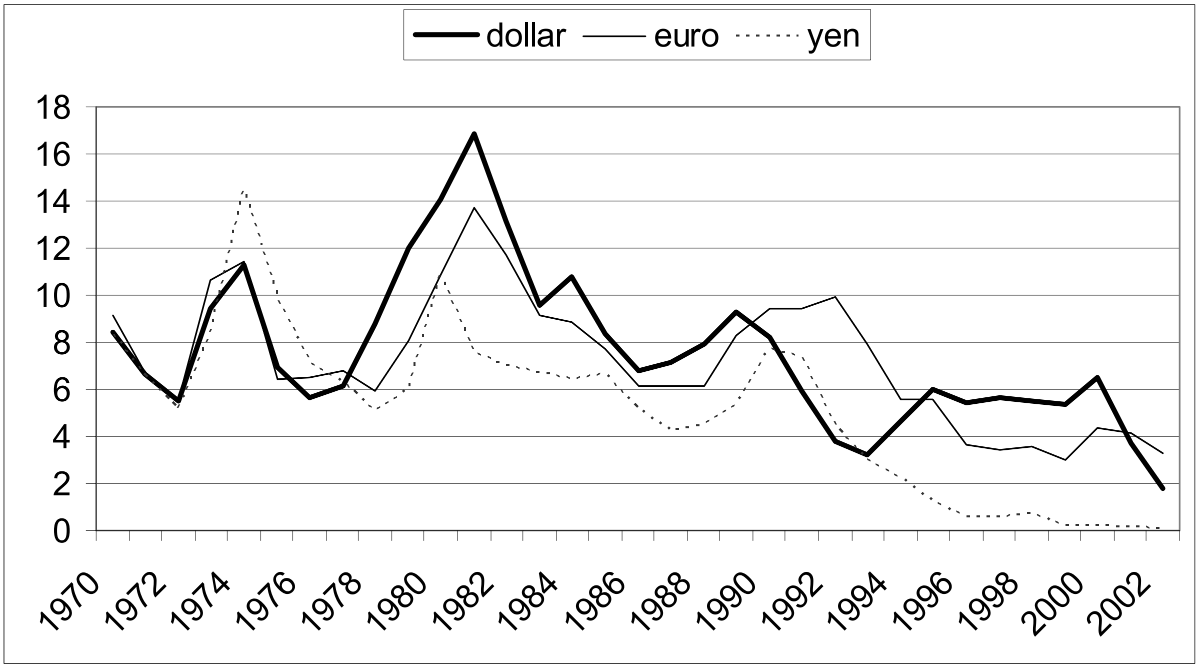


Figure 8. Short-term Interest Rates of the Leading Currencies



Note: euro refers to euro from 1999 onwards and to DEM and ECU prior to this date. Short-term interest rates for the period prior to 1999 are associated with DEM.

Figure 8. Short-term Interest Rates of the Leading Currencies



Note: euro refers to euro from 1999 onwards and to DEM and ECU prior to this date. Short-term interest rates for the period prior to 1999 are associated with DEM.

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