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An extended substitute-sources method for a turbulent atmosphere: Calculations for upward refraction

Running title: An extended substitute-sources method

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Abstract

The substitute-sources method (SSM) was previously implemented for a single noise barrier in a turbulent atmosphere by applying a substitute surface between the barrier and the receiver [1, 2]. Here, the method is extended, aiming to more general applicability to traffic noise propagation in urban environments. In the method, multiple substitute surfaces are used along the propagation path. The atmospheric turbulence causes a transfer of the initially coherent field into a residual, random field along the propagation path. The mean sound level at the receiver position is found from uncorrelated addition of the substitute surfaces' contributions. The calculation of each contribution is based on a mutual coherence function (MCF) for a turbulent atmosphere. The strength of the substitute sources and the Green functions to the received pressure are calculated for a non-turbulent atmosphere, here by using a fast field program (FFP). A special MCF for the residual field is derived. Examples are calculated for a turbulent atmosphere with upward refraction or without refraction. The results are compared with those from a parabolic equation method (PE) for the refractive cases and with an analytical solution otherwise. The results show good agreement, which indicates that the SSM could be useful for predictions of outdoor sound propagation.

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1 Introduction

A substitute-sources method (SSM) was previously developed to predict the increased noise level behind a single barrier due to a turbulent atmosphere [1, 2]. The approach presented here aims to be applicable to more variations along the propagation path, e.g. multiple barriers and range dependent ground and medium properties. Of main interest is the prediction of noise propagation in urban environments, for instance for city planning purposes. In urban situations the propagation is expected to be influenced by many things: atmospheric turbulence, sound speed profiles that may vary with range, multiply reflecting and diffracting buildings and barriers, and range varying ground properties. Parabolic equation methods (PE) are largely applicable to such situations [3, 4, 5]. Potentially applicable methods are those based on finite elements (FEM) or finite differences, boundary element methods (BEM) [6, 7] and fast field programs [8].

The approach with substitute sources presented here enables calculations for steep geometries, for instance when a high barrier is located close to the source or to the receiver [2]. PE methods are in general limited to not too steep geometries, but a high barrier can be modelled if the diffraction is calculated by other means and inserted into the PE solution. The SSM models the propagation outward from the source, as also the PE does. This means that backscattering is neglected, unless calculated separately and added (as can be done in the PE [5]).

In the SSM, the sound field due to an original source is represented by a distribution of sources on a plane surface. The surface is called a substitute surface and the sources are called substitute sources, which can be seen as Huygens' secondary sources. Here, many substitute surfaces are put between the source and the receiver, with separation distances large compared with the wavelength. (See Figure 1.) The propagation is calculated in steps from one surface to the next for a non-turbulent atmosphere. The effect of turbulence is that it causes a loss in coherence of the sound field. Within each step the unperturbed, coherent field loses power into a residual, random field. The coherent field is further propagated toward the receiver and at each substitute surface the residual, random part is taken out. The contributions from different surfaces are assumed to be uncorrelated, and the total power at the receiver is found by adding the power from the coherent field to the powers from the residual fields. The strengths of the substitute sources are calculated as for a non-turbulent atmosphere and scaled in amplitude to fulfill the power of the residual field. The Green function for the sound pressure at the receiver due to each substitute source is also found for a non-turbulent atmosphere. From one substitute surface, each of the sources' contributions is decomposed into a direct and a ground reflected part. All the contributions are summed up to give the estimated power by taking into account their mutual coherence due to the turbulence. A mutual coherence function for the residual field is derived.

Since the calculation of the source strengths and the Green functions do not involve turbulence, many methods could be used. For instance ray-methods would be efficient for a homogeneous atmosphere or a linear sound speed profile. Here, a fast field program (FFP) is used throughout. The following section describes the theory and thereafter a few examples are calculated. The examples are for a hard and a finite impedance ground surface, with or without an upward refracting atmosphere. All calculations are for two-dimensional situations.

2 Theory

The description of the theory is divided into five subsections. The use of the Rayleigh integral for representing the sound field is described in the first subsection. Thereafter the mutual coherence function (MCF) or transverse coherence function, Γ, and the extinction coefficient, γ, for a turbulent atmosphere are shown. In the third subsection the MCF for the residual field is derived, and thereafter the calculation scheme is described. The fifth subsection is about the Gaussian turbulence model that is used here.

2.1 The Rayleigh integral

The Rayleigh integral can be used to calculate the sound pressure level in a medium provided that the particle velocity in normal direction to a plane surface and the Green function for the medium are known. The propagation through either a snapshot of a turbulent atmosphere or through a non-turbulent atmosphere, with for instance refraction, could be described by a Green function. The plane surface is here one of the substitute surfaces S_i, $i = 1 \ldots N$, and the normal component of the velocity, v_i, of the sound field at the surface S_i is the strength of the substitute sources. In the two-dimensional (2-D) implementation used here, the surface is transformed into a line, but still referred to as a substitute surface. The resulting pressure amplitude at a receiver position, $p(x, y)$, from the velocity on surface S_i can be written

$$p(x,y) = \frac{\mathrm{j}\omega\rho_0}{2\pi} \int_l v_i(y_S) G(x_S, y_S, x, y) \mathrm{d}y_S, \tag{1}$$

where l is the line of integration, ω is the angular frequency of a time-oscillation, $\mathrm{e}^{\mathrm{j}\omega t}$, with time t, and ρ_0 the medium density. In equation (1) G is the Green function which in general depends on the position (x_S, y_S) on the surface and on the receiver position. The velocity, v_i, and the Green functions are here found from FFP calculations.

2.2 Coherence in a turbulent atmosphere

The subject of line-of-sight propagation in a random medium has been studied extensively (e.g. [9, 10, 11, 12, 13]), and the theoretical results most useful here relate to the correlation between acoustic pressure signals that have travelled from monopole sources through different parts of the medium. In Figure (2) a geometry with two sources and two receivers are shown; ρ' and ρ are transverse separations and L is the longitudinal distance or range. For the case where the pressure p_1 is only due to source 1 and pressure p_2 is only due to source 2, the mutual coherence function for p_1 and p_2 can be written as

$$\Gamma_{12} = \frac{\langle p_1 p_2^* \rangle + \langle p_1^* p_2 \rangle}{\langle \hat{p}_1 \hat{p}_2^* \rangle + \langle \hat{p}_1^* \hat{p}_2 \rangle}, \tag{2}$$

where the complex conjugate is denoted by an asterisk (*), p_1 and p_2 are the fluctuating pressure amplitudes in the turbulent atmosphere and $\hat{p}_1$ and $\hat{p}_2$ are the amplitudes without turbulence (e.g. [14]). In the usual definition of the MCF there is only one source (i.e. coinciding source positions in Figure 2). This MCF is here referred to as $\Gamma^0(\rho, L)$, where ρ is the distance between the receivers and L is the range. The reciprocal problem has the same MCF, i.e. when there are two sources and one receiver.

The extinction coefficient, γ, is related to the decay over distance of the mean field in a turbulent atmosphere. If the pressure amplitude due to a point source in free field is $\hat{p}$ without turbulence, then the mean amplitude in turbulence will be

$$p_c = \langle p \rangle = \hat{p}\mathrm{e}^{-\gamma L}, \tag{3}$$

where L is the distance of propagation [13]. The mean pressure amplitude, p_c, is also called the coherent field. The total field is the sum of the mean field and the residual, fluctuating field, $p = \langle p \rangle + p_r$, with $\langle p_r \rangle = 0$.

In the case the two paths are largely separated, the fluctuations in p_1 and p_2 will be independent. The mutual coherence will then not depend on the separation, ρ, and can be written as a function of the extinction coefficient γ [13]:

$$\Gamma_{12} = \mathrm{e}^{-2\gamma L}. \tag{4}$$

If the situation of propagation involves a ground surface, the effect of turbulence is that it makes the interference less strong. The received pressure amplitude is the sum of the direct and the ground reflected contributions. Without turbulence the amplitude is constant, $\hat{p} = \hat{p}_1 + \hat{p}_2$, whereas with turbulence the amplitude will fluctuate, $p = p_1 + p_2$. The long-term average of the square of the pressure amplitude is then computed as [15, 16, 17]

$$\langle |p|^2 \rangle = |\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2| \cos\left[\arg\left(\frac{\hat{p}_2}{\hat{p}_1}\right)\right] \Gamma_{12}(\rho, L). \tag{5}$$

The case with a ground surface is special since the ground reflected ray changes direction. A good approximation of the MCF in equation (5) is taking $\rho = \rho_{\max}$, where $\rho_{\max}$ is the maximum vertical separation between the direct and the ground reflected rays; $\rho_{\max} = 2h_S h_R/(h_S + h_R)$, where h_S and h_R are the source and receiver heights. Hence Γ_{12} in equation (5) is calculated as for no ground but with the adjusted transverse distance: $\Gamma_{12} = \Gamma^0(\rho_{\max}, L)$ [18].

That Γ_{12} is chosen as $\Gamma^0(\rho_{\max}, L)$ can be schematically explained as follows. It can be shown that the MCF can be generally written as

$$\Gamma^0(\rho, L) = \mathrm{e}^{-LU(\rho)}, \tag{6}$$

where U is a function that depends on the turbulence model and on ρ. (See Appendix for a more detailed description.) For a flat geometry with equal source and receiver heights ($h_S = h_R \ll L$), the distance of propagation is approximately L, and the ground reflection is at range $\frac{L}{2}$. At the range $\frac{L}{2}$ the separation between the two rays is $\rho_{\max}$ and the MCF is $\Gamma^0(\rho_{\max}, \frac{L}{2}) = \exp[-\frac{L}{2}U(\rho)]$ there. The decorrelation has a multiplicative property over range, whereby the total MCF, at the range of the receiver, will be $\Gamma^0(\rho_{\max}, \frac{L}{2})^2 = \Gamma^0(\rho_{\max}, L)$, using equation (6). The argument can be generalised and extended to situations where the source and the receiver are not at the same height (see Appendix). The calculation of other special cases are also shown in the Appendix, for instance the coherence between a direct wave from one source and a ground reflected wave from another source. (The results for these cases are not applied in the implementation of the SSM used here, but can be useful in further implementations.)

For M contributions p_j, $j = 1 \ldots M$, the long-term average of the square of the total pressure amplitude can be computed as [16]

$$\langle |p_{tot}|^2 \rangle = \sum_{j=1}^{M} |\hat{p}_j|^2 + 2 \sum_{j=1}^{M-1} \sum_{k=j+1}^{M} |\hat{p}_j \hat{p}_k| \cos\left[\arg\left(\frac{\hat{p}_k}{\hat{p}_j}\right)\right] \Gamma^0_{jk}. \tag{7}$$

The equation corresponding to equation (7) but for a continuous source distribution can be written as

$$\langle |p_{tot}|^2 \rangle = \langle \int \int p(y) p^*(y') dy dy' \rangle = \tag{8}$$

$$\int \int |\hat{p}(y)\hat{p}(y')| \cos\left[\arg\left(\frac{\hat{p}(y')}{\hat{p}(y)}\right)\right] \Gamma^0 dy dy',$$

where y and y' are positions on the substitute surface. If there would be a homogeneous atmosphere, $\Gamma^0 \equiv 1$, equation (8) could be seen as the same as the square of the Rayleigh integral in equation (1).

The quantity $\langle |p_{tot}|^2 \rangle$ is proportional to the power of the signal at the receiver. In the following, the quantity $\frac{1}{2}\langle |p_{tot}|^2 \rangle$ is referred to as the power. (The sound pressure level is then found as $L_p = 10\,\lg \frac{\frac{1}{2}\langle |p_{tot}|^2 \rangle}{2\cdot 10^{-5}}$ dB re $2 \cdot 10^{-5}$ Pa.) Hence, when there is a turbulent atmosphere between a substitute surface and the receiver, the received mean power can be calculated as

$$W_{tot} = \frac{1}{2}\langle |p_{tot}|^2 \rangle = \frac{1}{2}\left(\frac{\omega\rho_0}{2\pi}\right)^2 \int_l \int_l |vGv'G'| \cos\left[\arg\left(\frac{v'G'}{vG}\right)\right] \Gamma^0 \mathrm{d}y\,\mathrm{d}y'. \quad (9)$$

2.3 Coherence of the residual field

In Figure (3) a situation is described where only a part, L', of the range of propagation is through turbulence. This case could be formulated as

$$\langle |p_{tot}|^2 \rangle = |\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2| \cos\left[\arg\left(\frac{\hat{p}_2}{\hat{p}_1}\right)\right] \Gamma', \quad (10)$$

where Γ' is the MCF for a turbulent layer with thickness L'. If $\rho' \ll L' + L$, the propagation distance through the turbulence will be approximately L', and then the decrease in power in the coherent field can be approximated by the factor $\mathrm{e}^{-2\gamma L'}$, using equation (3). At the receiver, the contribution due to the coherent field can then be written as

$$\langle |p_c|^2 \rangle = \mathrm{e}^{-2\gamma L'} \left(|\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2| \cos\left[\arg\left(\frac{\hat{p}_2}{\hat{p}_1}\right)\right] \right). \quad (11)$$

Since the coherent pressure field, p_c, and the residual pressure field, p_r, are uncorrelated, one gets $\langle |p_{tot}|^2 \rangle = \langle |p_c|^2 \rangle + \langle |p_r|^2 \rangle$, and the residual contribution is given by:

$$\begin{aligned} &\langle |p_r|^2 \rangle = \langle |p_{tot}|^2 \rangle - \langle |p_c|^2 \rangle = \quad (12) \\ &\left(1 - \mathrm{e}^{-2\gamma L'}\right) \left(|\hat{p}_1|^2 + |\hat{p}_2|^2 + 2|\hat{p}_1\hat{p}_2| \cos\left[\arg\left(\frac{\hat{p}_2}{\hat{p}_1}\right)\right] \frac{\Gamma' - \mathrm{e}^{-2\gamma L'}}{1 - \mathrm{e}^{-2\gamma L'}} \right). \end{aligned}$$

From the above equation a MCF for the residual field, $\tilde{\Gamma}$, can be generally defined as

$$\tilde{\Gamma} = \frac{\Gamma^0 - \mathrm{e}^{-2\gamma L}}{1 - \mathrm{e}^{-2\gamma L}}. \quad (13)$$

It can be noted that, as Γ^0 approaches the value $\mathrm{e}^{-2\gamma L}$ for large ρ, $\tilde{\Gamma}$ approaches zero. The equations (12) and (13) constitute the main theoretical result of this paper, and can be seen as describing the transfer of the coherent field into a random field and how the contribution from the random field is calculated.

For the case with a turbulent layer (as in equation 12), Γ' can be found from the multiplicative property as

$$\Gamma' = \frac{\Gamma^0(\rho', L' + L)}{\Gamma^0(\rho, L)}, \tag{14}$$

with $\Gamma^0(\rho', L' + L)$ and $\Gamma^0(\rho, L)$ calculated as for the same turbulence throughout the source-to-receiver range.

If there is turbulence also after the range L', the MCF for the residual field should be multiplied by $\Gamma^0(\rho, L)$. For the coherent field the contribution is then found from multiplying the mixed term in equation (11) (the last term in the parentheses) by the same function, $\Gamma^0(\rho, L)$.

2.4 Numerical method

In the SSM the sound field is represented by a distribution of sources on each substitute surface. The above derived mutual coherence function for the residual field is applied to each pair of source contributions when calculating the powers from all but the last surface. The power $\tilde{W}_i$ from the residual field from surface S_i can be written

$$\tilde{W}_i = \frac{1}{2}\langle |p_i|^2 \rangle = \frac{1}{2}\left(\frac{\omega\rho_0}{2\pi}\right)^2 \int_l \int_l |v_i G_i v_i' G_i'| \cos\left[\arg\left(\frac{v_i' G_i'}{v_i G_i}\right)\right] \tilde{\Gamma}\Gamma^0 \mathrm{d}y\, \mathrm{d}y', \tag{15}$$

where $\tilde{\Gamma}$ is the MCF for the residual field and Γ^0 for the medium after the next surface, S_{i+1}. The contribution, W_N^0, from the last surface, i.e. from the coherent source field, is found from setting $\tilde{\Gamma} = 1$ in the above equation. When the integrals in equation (15) are discretised, the solution takes the form of equation (7).

In equation (15) the distribution of power between different surfaces is ignored. Taking into account the correct loss in power to the residual field within each step, the total power at the receiver can be found as

$$W_{tot} = \tag{16}$$

$$\left(1 - \mathrm{e}^{-2\gamma L}\right)\left(\tilde{W}_1 + \mathrm{e}^{-2\gamma L}\tilde{W}_2 + \mathrm{e}^{-4\gamma L}\tilde{W}_3 + \ldots + \mathrm{e}^{-2(N-2)\gamma L}\tilde{W}_{N-1}\right)$$

$$+\mathrm{e}^{-2(N-1)\gamma L}W_N^0 = \left(1-\mathrm{e}^{-2\gamma L}\right)\left(\sum_{i=1}^{N-1}\mathrm{e}^{-2(i-1)\gamma L}\tilde{W}_i\right)+\mathrm{e}^{-2(N-1)\gamma L}W_N^0,$$

where L is the distance between the substitute surfaces and $\mathrm{e}^{-2(N-1)\gamma L}W_N^0$ is the power from the last surface.

In the present formulation of the method the assumption that the contributions from different substitute surfaces are uncorrelated relies on L being large compared with the correlation length of the turbulence. For smaller values of L it might be possible to find corrections to γ and $\tilde{\Gamma}$ if necessary. The study of this can belong to future work.

2.5 Gaussian turbulence model

To describe the turbulence, it is assumed to be homogeneous and isotropic, that is, the fluctuations are assumed to follow the same statistics for all points and the statistics are independent of rotation. In the Gaussian turbulence model the temperature fluctuations of the medium are described by a Gaussian spectrum, and here the velocity fluctuations are omitted. All the above assumptions simplify the turbulence modelling and results in a poor description of realistic situations. The turbulence modelling can however be improved in future implementations of the SSM, but the simplifications used here facilitate a first evaluation of the method. For the Gaussian turbulence model the mutual coherence function can be written as

$$\Gamma^0(\rho, L) = \exp\left[-2\gamma L\left(1-\frac{\phi(\rho/l)}{\rho/l}\right)\right], \tag{17}$$

where l is usually referred to as the correlation length or the scale of the fluctuations, $\phi(\rho/l) = \int_0^{\rho/l}\exp(-t^2)dt$ and

$$\gamma = \frac{\sqrt{\pi}k^2 l}{2}\mu_0^2, \tag{18}$$

where μ_0^2 is the variance of the index of refraction fluctuations [19, 20, 13]. (It could be noted that $\mu_0^2 = \frac{1}{4}\sigma_T^2$, where σ_T^2 is the variance of the relative temperature fluctuations.) Following the same references, the MCF for plane wave propagation can be written

$$\Gamma^{\mathrm{pl}}(\rho, L) = \exp\left[-2\gamma L \left(1 - \mathrm{e}^{-\rho^2/l^2}\right)\right]. \tag{19}$$

(In equations (17) and (19) it can be seen that for large separations, $\rho \gg l$, the MCF becomes independent of ρ and reduces to equation (4).) The MCF (17) is deduced for a three-dimensional turbulence. It can however be shown that the MCF for two dimensions is the same for the Gaussian turbulence model [14, 21].

3 Implementation and calculated examples

A few examples are calculated to study the behaviour of the SSM, involving upward refraction and no refraction for a hard or a soft, grass-like ground surface. The calculations without turbulence are made using a fast field program (FFP) implemented according to Salomons [17]. In the FFP the sound field is transformed into a wave number domain and this is used to efficiently calculate the velocity from the pressure derivative which is transformed into a multiplication.

The parts of the non-turbulent sound field that are wanted are the normal velocities at the surfaces, i.e. the strength of the substitute sources, and the Green functions for the received pressure due to these velocities. In a model using the mutual coherence function all contributions are seen as rays, for which the separation, ρ, and the range, L, can be defined. The received pressure due to a source on a surface is therefore decomposed into a direct and a ground reflected contribution. The decomposition is done by making FFP calculations both with and without a ground surface. Subtracting the two results gives the ground reflected contribution and the calculation without the ground gives the direct contribution. The values of the input parameters ρ and L for the coherence $\tilde{\Gamma}$ are here approximated as for straight rays. Taking into account the curvature of a pair of rays due to the refraction would alter the values of ρ and L. Here, however, it is assumed that the corresponding error in $\tilde{\Gamma}$ is negligible, but this could be improved in future implementations.

For upward refraction a shadow region is formed and, in general, the sound field cannot be decomposed into defined rays. In the SSM, however, the approach is different since the field due to the original source is substituted by a surface of sources. The sources at large enough height will be above the limiting ray of the shadow region and will thereby have direct rays to the receiver. Hence, it is assumed that the dominating contribution comes from the substitute sources that are above the limiting ray to the receiver, whereby the error from applying the MCF $\tilde{\Gamma}$ also to the other rays is negligible.

For the results shown here the FFP is used to calculate directly the velocities at all surfaces. Another possible approach is to calculate a matrix of Green functions relating all velocities on one surface to all the velocities on the following surface, and apply the matrix repeatedly. This approach would be motivated for instance if there is a an impedance jump of the ground surface, or other range varying properties. Concerning computational demands, the SSM and the PE are fairly similar, and in the SSM the computation time is dominated by the FFP calculations.

For the cases with upward refraction, a logarithmic effective sound speed profile is used up to 30 m height. Above that height the sound speed is taken as constant to improve the numerical stability of the FFP. The logarithmic sound speed profile is $c(y) = c_0 - b \cdot \ln(y/y_{\text{rough}} + 1)$, with $b = 0.43$ m/s and the roughness height $y_{\text{rough}} = 0.05$ m [17]. For the turbulence the correlation length $l = 1$ m and the variance of the index of refraction $\mu_0^2 = 2 \cdot 10^{-6}$ or $5 \cdot 10^{-5}$ are used. The larger value for μ_0^2 models a strong turbulence and is chosen for the examples without refraction to give a strong turbulence effect at relatively short propagation range.

In the calculations with the SSM presented here, only the plane wave MCF is used. The motivation is that between one substitute surface and the next, most pairs of rays toward the receiver can be approximated as parallel. The MCF for the residual field is then calculated as $\tilde{\Gamma} = [\Gamma^{\text{pl}} - \exp(-2\gamma L)]/[1 - \exp(-2\gamma L)]$. Moreover, in the examples calculated here, the MCF $\tilde{\Gamma}$ dominates the decorrelation and $\tilde{\Gamma}\Gamma^0$ is approximated as $\tilde{\Gamma}$, i.e. $\Gamma^0 = 1$ is used.

The discretisation in height in the SSM, i.e. the distance between the discrete substitute sources is $\lambda/5$ for the non-refractive cases and $\lambda/10$ for the cases with upward refraction, where λ is the sound wavelength. These values were found from numerical tests without including the turbulence effects. The height used for the substitute surfaces is about half the maximum propagation range, and the top third is windowed to give a smooth decay with height of the source strengths. The windowing is used to reduce spurious oscillations in the solution [1]. The separation in range between the substitute surfaces, L, is 10 m in all examples, and the results are calculated every 5 meters.

The calculations for the soft ground uses a normalised ground impedance of 3.71 − j3.68 at the frequency 500 Hz and 5.56 − j6.10 at 1000 Hz. These values are for a grass-like surface and come from the Delany and Bazely model using a flow resistivity of 200 kNs/m^4 (e.g. [16]). (The sign of the imaginary part of the impedance is consistent with the time-dependence used here and with a normal vector pointing into the ground.)

The calculated results are shown in Figures (4–9). Figures (4–7) are for a non-refractive atmosphere, for hard and soft ground, and for the frequencies $f = 500$ Hz and $f = 1000$ Hz. Figures (8–9) are for upward refraction, for hard and soft ground, for $f = 500$ Hz. (Other data are given in the Figure captions.) The results are plotted as sound pressure level relative to free field as a function of propagation range from the source.

For the calculations without refraction (Figures 4–7) the two thicker curves show the results for a turbulent atmosphere. The solid line is for the SSM and the dashed line is the analytical solution using the MCF for the direct and ground reflected rays from the source to the receiver, as described in Subsection 2.2. The two thinner curves are for no turbulence; the dashed line for the FFP directly and the solid line for the SSM without turbulence, i.e. where all power is calculated from the last substitute surface without decorrelation. (These two curves are almost indistinguishable in the Figures.)

In the examples with upward refraction (Figures 8 and 9), a comparison is made with a parabolic equation method (PE). In the PE calculations 50 realisations of the turbulence are used to estimate the power at the receiver. (The PE implementation used here is mainly based on Ref. [3] and is described in detail in Ref. [22].) The dash-dotted curves show the PE results, with and without turbulence.

For an atmosphere without refraction, the effect of turbulence is mainly that it reduces the interference, as can be seen in Figures (4–7). Most of the SSM results show good agreement with the analytical in these examples. However, for the soft ground at $f = 500$ Hz (Figure 5), there is a significant discrepancy. To provide an additional comparison, the PE method is applied also to this case, and the results are shown in the same figure. The PE results are similar to those from the SSM, which indicates that the analytical solution may give a significant error in this case. A possible explanation is that the turbulence scattering results in a larger loss into the ground than what is given by the analytical ray model. For upward refraction the main effect of the turbulence is that is limits the acoustic shadow, as can be seen in Figures (8) and (9). The difference between the SSM and the PE results is around 2 dB at most, which for situations with strong shadowing can be considered as good agreement.

4 Conclusions

The good agreement shown in the comparison with the other methods indicate that the extended substitute-sources method presented here could be a useful tool for predictions of outdoor sound propagation. The approach also enables application to steep geometries, as shown in a previous implementation for a single barrier [2].

In future work, range dependent properties of the atmosphere and the ground could be taken into account. For example, with small changes in the method, a ground impedance that is step-wise constant over range could be modelled.

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Appendix

The mutual coherence function (MCF) $\Gamma^0(\rho, L)$, or transverse coherence function, describes the correlation between the pressure amplitudes at two receiver positions at the range L from a monopole source. The range L is also called the longitudinal distance. The transverse separation between the receivers is ρ. In this paper the MCF is defined as in equation (2). The main assumption made when finding the analytical solution for the MCF is weak scattering due to the turbulence. (For a thorough description the reader is referred to Refs. [19, 20, 13].) With the notation used here, the MCF for a homogeneous and isotropic turbulence can be written as

$$\Gamma^0(\rho, L) = \exp\left[B_\chi(\rho, L) - B_\chi(0, L) + B_\phi(\rho, L) - B_\phi(0, L)\right], \tag{20}$$

where B_χ and B_ϕ are correlation functions of the log-amplitude fluctuations, χ, and the phase fluctuations, ϕ (e.g. [19]). The relation to the fluctuating pressure amplitude, p, is $p = \hat{p}\exp(\chi + \mathrm{j}\phi)$, where $\hat{p}$ is the amplitude without turbulence. For χ_1 and χ_2 at two receivers with separation ρ at range L from a single source, the correlation function for the log-amplitude fluctuations can be written as $B_\chi(\rho, L) = \langle\chi_1\chi_2\rangle$. The correlation function for the phase fluctuations is defined in the same way: $B_\phi(\rho, L) = \langle\phi_1\phi_2\rangle$. As a physical explanation of what equation (20) describes, the sum of variances $B_\chi(0, L) + B_\phi(0, L) = \langle\chi^2\rangle + \langle\phi^2\rangle$ can be seen as causing a loss of the coherent wave into the residual field. The correlation $B_\chi(\rho, L) + B_\phi(\rho, L)$ results in a compensating factor because some of the fluctuations are the same for both rays.

For a spherical wave the correlation functions can be found as

$$B_{\chi,\phi}(\rho, L) = \tag{21}$$
$$\frac{\pi^2 k^2}{2}\int_0^L\int_0^\infty K\left\{1 \mp \cos\left[\frac{K^2 x}{k}\left(1 - \frac{x}{L}\right)\right]\right\} J_0\left(K\rho\frac{x}{L}\right)\Phi_{\mathrm{eff}}(0, K)\mathrm{d}K\mathrm{d}x,$$

where B_χ is for the minus-sign of $\mp$ and B_ϕ for the plus-sign and where J_0 is the Bessel function of zero order [19, 20, 13]. As defined in Refs. [19, 13] $\Phi_{\mathrm{eff}}(K_x, K_y, K_z)$ is a three-dimensional spectral density for a moving random medium, and in equation (21) $K_x = 0$ and $K = \sqrt{K_y^2 + K_z^2}$. (For a detailed description the reader is referred to Chapter 7 in Ref. [13].) For the special case where Φ_{eff} models Gaussian index of refraction fluctuations in a non-moving atmosphere, the equations (17–19) are arrived at, which can be seen as a model for the temperature fluctuations in turbulence.

After integrating equation (21) over K, the resulting MCF can be written as

$$\Gamma^0(\rho, L) = \exp\left[-\int_0^L u(\rho\frac{x}{L})\mathrm{d}x\right], \tag{22}$$

where the function u depends on only one variable. Making the substitution $t = x/L$ gives

$$\Gamma^0(\rho, L) = \exp\left[-L\int_0^1 u(\rho t)\mathrm{d}t\right], \tag{23}$$

which can be written as $\exp[-LU(\rho)]$. Using equation(22), the properties of the MCF can be studied and the special problems of interest here can be solved.

If the propagation through the turbulence is divided into two ranges, the multiplicative property of the MCF can be shown. (See Figure 10a.) The total MCF can be written

$$\Gamma_{\text{tot}} = \Gamma^0(\rho_2, L_1 + L_2) = \exp\left[-\int_0^{L_1+L_2} u(\rho_2\frac{x}{L_1 + L_2})\mathrm{d}x\right] = \tag{24}$$

$$\exp\left[-\int_0^{L_1} u(\rho_2\frac{x}{L_1 + L_2})\mathrm{d}x\right] \exp\left[-\int_{L_1}^{L_1+L_2} u(\rho_2\frac{x}{L_1 + L_2})\mathrm{d}x\right].$$

Since $\rho_2/(L_1 + L_2) = \rho_1/L_1$, the first factor in the last step in equation (24) is equal to $\Gamma^0(\rho_1, L_1)$. If there is turbulence only after range L_1, the MCF is given by only the last factor in equation (24), and the result can be written: $\Gamma_{\text{tot}} = \Gamma^0(\rho_2, L_1 + L_2)/\Gamma^0(\rho_1, L_1)$.

The situation of a direct and a ground reflected ray is shown in Figure (10b). For a flat geometry ρ is much smaller than L and the distance of propagation is approximately equal to L for both rays. Using the multiplicative property over range (as in equation 24), the total MCF can be written $\Gamma_{\text{tot}} = \Gamma^0(\rho, L_1)\Gamma^0(\rho, L_2)$, and using equation (23) gives $\Gamma^0(\rho, L_1)\Gamma^0(\rho, L_2) = \Gamma^0(\rho, L_1 + L_2)$.

For the case when a direct and a ground reflected ray do not meet (Figure 10c), the total MCF can be found as

$$\Gamma_{\rm tot} = \frac{\Gamma^0(\rho, L_1 + L_1')}{\Gamma^0(\rho_1, L_1')} \frac{\Gamma^0(\rho, L_2 + L_2')}{\Gamma^0(\rho_2, L_2')}. \tag{25}$$

The situation with crossing rays (Figure 10d) can be solved as $\Gamma_{\rm tot} = \Gamma^0(\rho_1, L_1)\Gamma^0(\rho_2, L_2)$. If there are both crossing rays and a ground reflection, the solution can be found after dividing the propagation into three ranges.

If the pair of sources (or receivers) are not at the same range, it can be seen as an additional independent decorrelation (Figure 10e). The solution for that case can be written: $\Gamma_{\rm tot} = \Gamma^0(\rho, L_1)\exp(-\gamma L_2)$. This result could be used in the SSM to model the rays that deviate largely from horizontal direction, as shown in Figure (10f).

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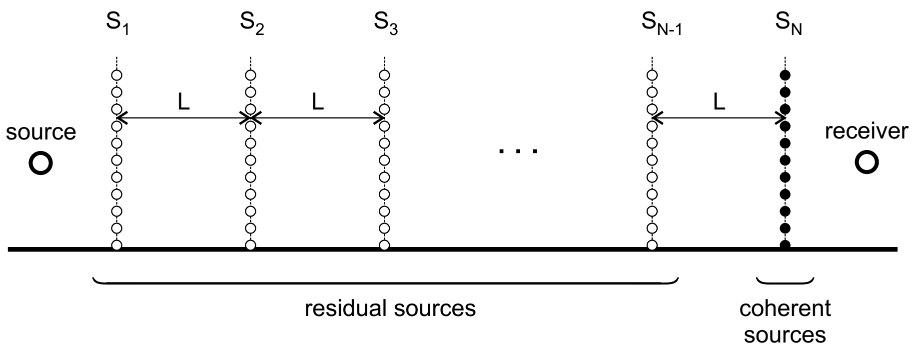

Figure 1: Substitute surfaces S_i, with separation distance L.

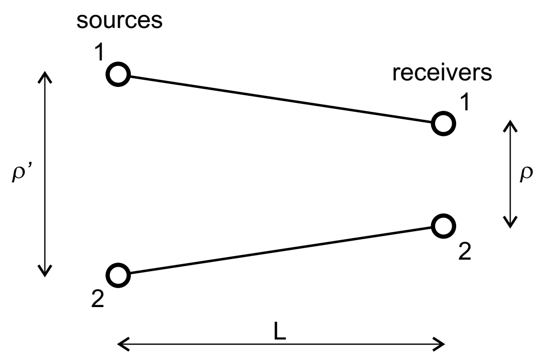

Figure 2: A pair of sound rays with transverse separation ρ' at the start and ρ at range L.

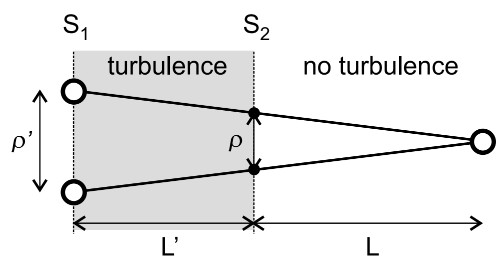

Figure 3: Propagation through a turbulent layer.

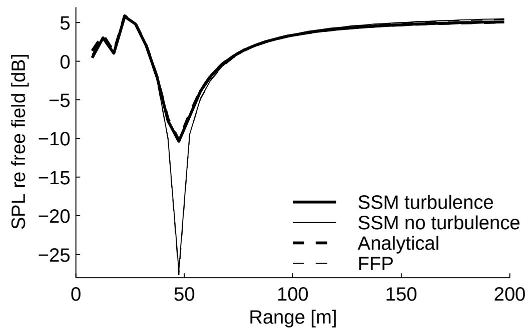

Figure 4: Result for hard ground, $f = 500$ Hz, $\mu_0^2 = 5 \cdot 10^{-5}$, $h_S = 2$ m, $h_R = 4$ m.

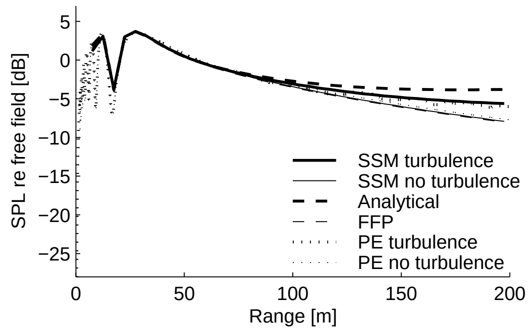

Figure 5: Result for soft ground, $f = 500$ Hz, $\mu_0^2 = 5 \cdot 10^{-5}$, $h_S = 2$ m, $h_R = 4$ m.

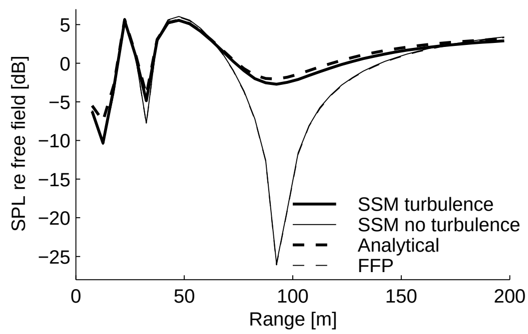

Figure 6: Result for hard ground, $f = 1000$ Hz, $\mu_0^2 = 5 \cdot 10^{-5}$, $h_S = 2$ m, $h_R = 4$ m.

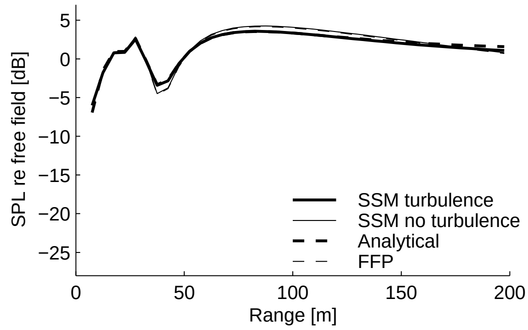

Figure 7: Result for soft ground, $f = 1000$ Hz, $\mu_0^2 = 5 \cdot 10^{-5}$, $h_S = 2$ m, $h_R = 4$ m.

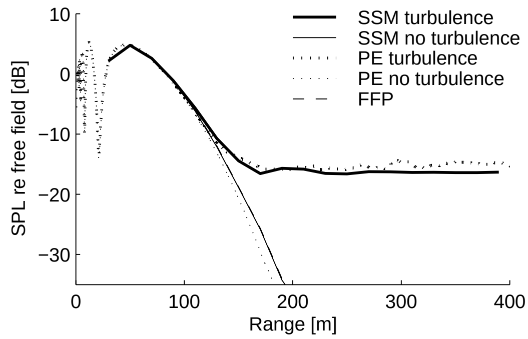

Figure 8: Result for hard ground and upward refraction, $f = 500$ Hz, $\mu_0^2 = 2 \cdot 10^{-6}$, $h_S = 2$ m, $h_R = 2$ m.

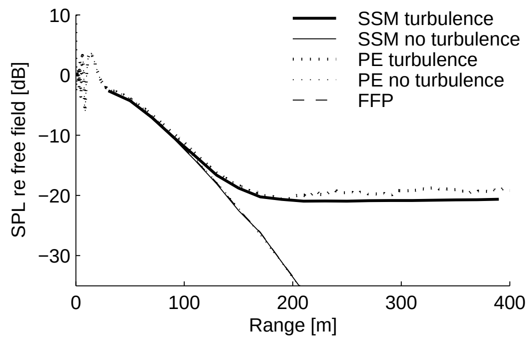

Figure 9: Result for soft ground and upward refraction, $f = 500$ Hz, $\mu_0^2 = 2 \cdot 10^{-6}$, $h_S = 2$ m, $h_R = 2$ m.

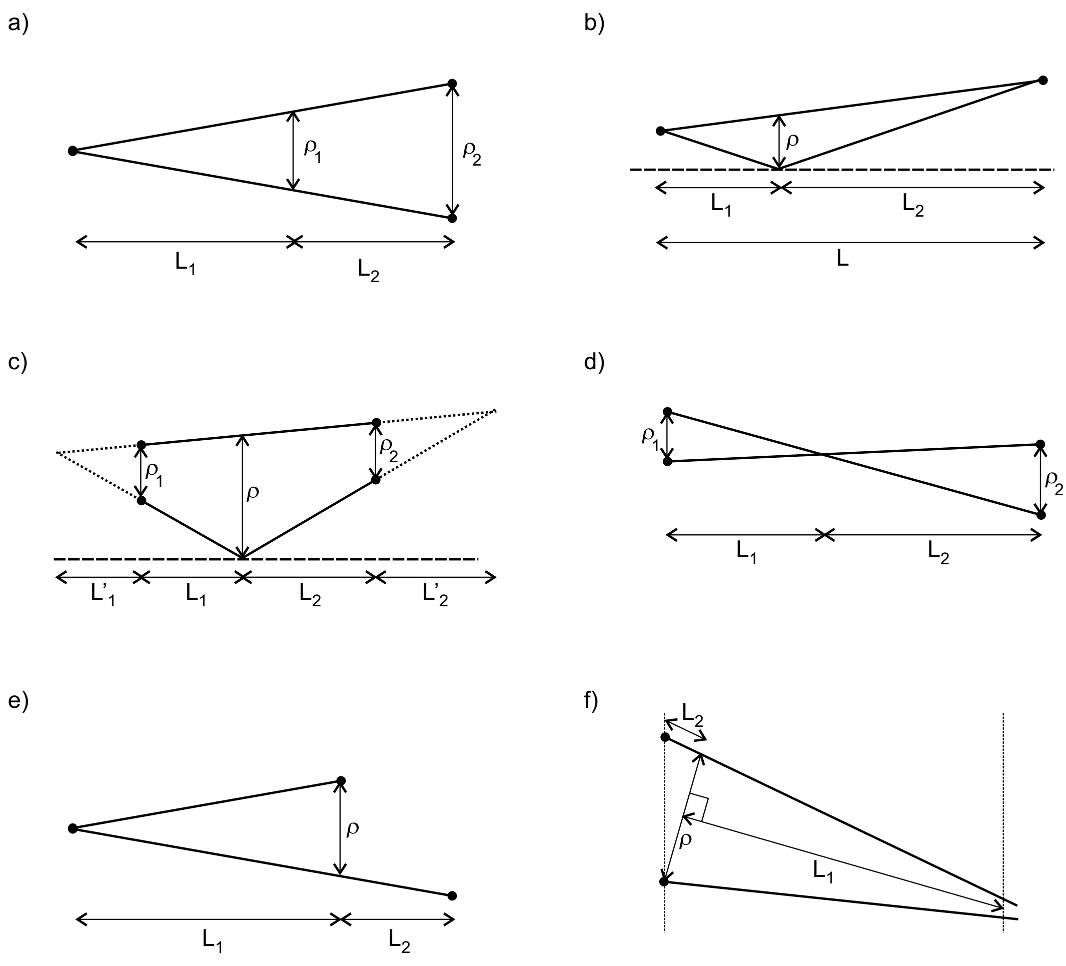

Figure 10: Different cases of ray pairs.