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AN EQUILIBRIUM MODEL OF SORTING IN AN URBAN HOUSING MARKET: THE CAUSES AND CONSEQUENCES OF RESIDENTIAL SEGREGATION

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An Equilibrium Model of Sorting in an Urban Housing Market: The Causes and Consequences of Residential Segregation

Patrick Bayer, Robert McMillan, and Kim Rueben

Abstract

This paper presents a new equilibrium framework for analyzing economic and policy questions related to the sorting of households within a large metropolitan area. We estimate the model using restricted-access Census data that precisely characterize residential and employment locations for households the San Francisco Bay Area, yielding accurate measures of preferences for a wide variety of housing and neighborhood attributes across different types of household. We use these estimates to explore the causes and consequences of racial segregation in general equilibrium. Our results indicate that, given the preference structure of households in the Bay Area, the elimination of racial differences in income and wealth would *significantly increase* the residential segregation of each major racial group, as the equalization of income leads, for example, to the formation of new wealthy, segregated Black and Hispanic neighborhoods. We also provide evidence that sorting on the basis of race itself (whether driven by preferences or discrimination) leads to large reductions in the consumption of housing, public safety, and school quality by Black and Hispanic households.

JEL Classification: H0, J7, R0, R2

Keywords: Segregation, Sorting, Housing Markets, Locational Equilibrium, Residential Choice, Discrete Choice

1 INTRODUCTION

A number of important features of the landscape of an urban housing market are determined by the way that households sort among its neighborhoods. This sorting affects residential stratification on the basis of race, income, and other family attributes, the congestion of the transportation network, and the distribution of school quality, crime, property tax bases, and housing prices throughout the urban area. It also has important welfare implications. A full understanding of these implications requires knowledge of the preferences of the heterogeneous households in the metropolitan region and a model that describes how these preferences aggregate to form an equilibrium. The primary goal of this paper is to provide these necessary components.

To that end, we develop an equilibrium model of sorting in an urban housing market and provide a general strategy for identifying preferences in the presence of social interactions in the location decision. Building on McFadden's (1978) discrete choice framework, our model allows households to have preferences for a wide variety of housing and neighborhood attributes, including many that depend explicitly on the way that households sort across neighborhoods in equilibrium (e.g. the quality of local schools, the neighborhood crime rate, and the sociodemographic composition of the neighborhood). Each household's preferences are allowed to vary with its own characteristics, including its wealth, income, education, race, employment location (taken as given), and family composition. The model also provides a well-defined characterization of how these preferences aggregate to determine the equilibrium in an urban housing market; under a set of reasonable assumptions, we demonstrate that a sorting equilibrium always exists in this framework.

The model is estimated using newly available, restricted Census micro-data that provide precise geographic information on the residential locations of a quarter of a million households in the San Francisco Bay Area in 1990. Because the sorting equilibrium is not generically unique, we develop an estimation strategy that permits estimation in the presence of multiple equilibria, exploiting the fact that in any equilibrium, each household chooses its location optimally conditional on the decisions made by the other households in the metropolitan region. This strategy does not require us to compute the equilibrium as part of the estimation procedure, thereby allowing the estimation of hundreds of heterogeneity parameters in a computationally feasible manner.

Following Berry, Levinsohn, and Pakes (1995), we allow explicitly for unobserved differences in the quality of houses and neighborhoods. In so doing, we bring an important endogeneity problem to the

forefront of the analysis – namely that the value (or rent) of a house and any other neighborhood attributes determined by the sorting of households are likely to be highly correlated with unobserved house and neighborhood attributes. And we provide a general strategy for identifying the model in the face of this endogeneity problem, developing an appropriate set of instruments for endogenous choice characteristics.

We show that instruments rise naturally out of the logic of the choice model itself. Because each household's location decision is affected by the full set of available alternatives, the housing prices and sociodemographic composition of any particular neighborhood will be partly dependent on the wider availability of choices in the urban housing market. In particular, characteristics of the housing stock and land use in surrounding neighborhoods can be expected to influence, through the market equilibrium, prices and sociodemographics of a given neighborhood. At the same time, as long as the surrounding neighborhoods are sufficiently distant, it is unlikely that their fixed characteristics are correlated with the unobserved features of a given neighborhood that affect household utility, allowing them to serve as valid instruments.

The estimated model yields precise measures of the full set of preference parameters which, along with the characterization of how these preferences aggregate to determine the market equilibrium, can be used to explore a wide variety of economic questions concerning sorting in the urban housing market. The model is particularly useful for carrying out urban policy analysis, providing a way to measure the general equilibrium effects of a policy in terms of its impact on the sociodemographic composition, house values (and rents), school quality, and crime rates of each neighborhood of the metropolitan region, its impact on the intensity of usage of the transportation network, and clear measures of the policy's distributional consequences in terms of income, race, and other household attributes.

Relation to Previous Models of Sorting in an Urban Housing Market

Our framework draws on two main lines of research in the empirical urban economics literature. Following the seminal work of McFadden (1973, 1978), many researchers have used a discrete choice framework to study residential location decisions, as this framework provides a natural way to estimate heterogeneous preferences for housing and neighborhood attributes.[1] Relative to this literature, the key contribution of our approach is that we explicitly control for the fact that housing prices and neighborhood sociodemographic characteristics are determined as part of the sorting equilibrium, both

[1] Important applications of this framework can be found in Anas (1982), Anas and Chu (1984), Quigley (1985), and Gabriel and Rosenthal (1989), Nechyba and Strauss (1998), and Duncombe, Robbins, and Wolf (1999).

when estimating the model and conducting counterfactual simulations.[2] In formally characterizing the sorting equilibrium, we build on a vast theoretical literature in urban and public economics[3] and most directly on the empirical work of Epple and Sieg (1999), which estimates an equilibrium model of community sorting. The key contribution of our framework relative to Epple and Sieg's analysis lies in the flexible form that we adopt for utility, in essence expanding their vertical model of locational differentiation to a more flexible horizontal model of differentiation.[4] By combining what we see as the best features of these two lines of the literature, our goal is to provide a general and flexible framework useful for analyzing a wide range of economic and policy questions in urban economics and local public finance.

The Causes and Consequences of Segregation

The main economic analysis of the paper uses the estimated equilibrium model of sorting to explore the causes and consequences of racial segregation in the housing market. As the seminal work of Thomas Schelling (1969, 1971, 1978) makes clear, a number of distinct microeconomic forces may contribute to an aggregate phenomenon such as segregation. Most obviously, racial segregation could be driven by individual residential choices related to race, either because of direct preferences for the race of one's neighbors or through discrimination in the housing market. The correlation of race with other household characteristics that influence residential sorting - income, wealth, language, immigration experience, and education – could also give rise to a sizeable amount of segregation if these other characteristics are important in shaping residential location decisions. As Schelling noted, "color is

[2] Developed concurrently with our paper is a closely related study by Bajari and Kahn (2001), which, following Bayer (1999), incorporates error terms that capture the unobserved quality of each location. In their analysis, the authors do not formally model the sorting equilibrium and do not address the correlation between the sociodemographic composition of a community and its unobserved quality, a correlation that is implied by the model.

[3] Important contributions to this literature date back to the work of Tiebout (1956) and include the work of Epple, Filimon, and Romer (1984, 1993), Benabou (1993, 1996), Fernandez and Rogerson (1996), and Nechyba (1997, 1999) among others.

[4] In practice, the vertical model constrains households with different characteristics and income to make the same trade-offs between community characteristics so that workers employed in the suburbs, for example, are restricted to have the same preferences for central residential locations relative to other community characteristics as workers employed in the central city. The problem of considering preferences for neighborhood sociodemographics is complicated within the Epple and Sieg framework by the fact that preferences for these characteristics may differ quite non-monotonically across households of different races and ethnicities. Simply including them as part of a public goods index would place undue constraints on these preferences. Epple and Sieg (1999) do not include such measures in their analysis, which may seriously bias estimates of preferences for local public goods, as these sociodemographic characteristics are likely to be highly correlated with the observed local public goods.

correlated with income, and income with residence; so even if residential choices were color-blind and unconstrained by organized discrimination, whites and blacks would not be randomly distributed across residences" (page 144, Schelling (1971)). Other basic mechanisms such as shared social networks or across-race differences in preferences for housing or neighborhood attributes may also contribute to observed segregation patterns. Our equilibrium model of sorting allows us to account for a variety of these potential causes of segregation explicitly.

We begin our analysis of racial segregation by using the equilibrium model to better understand the forces underlying the observed level of racial segregation. In addition to distinguishing the causes of segregation, we also provide evidence on a potentially important consequence of segregation that arises because the single residential location decision simultaneously determines consumption of housing, commuting, and a wide variety of local goods (including neighborhood racial composition). In the presence of this bundled consumption decision, strong preferences in any dimension (e.g., for neighborhood racial composition) distort consumption in other dimensions, especially when the available set of housing options is limited in some important way. In the presence of segregating preferences, it may be difficult for a household to simultaneously satisfy its preferences for neighborhood racial composition and other local goods when the number of households of the same race is relatively small and particularly when the household has significantly different preferences from the majority of households of the same race. The second goal of our analysis of racial segregation is to shed light on this issue, examining the extent to which racial interactions in the location decision accentuate differences in the consumption of housing, school quality, and public safety between white households and those of other races.

Relation to Previous Segregation Literature

Our analysis departs from most of the prior segregation literature in both its focus and methodology. Much of the prior literature has been concerned with documenting segregation patterns, particularly between black and white households, and how these have changed over time.[5] Recent studies that explore the extent to which segregation might be driven by the correlation of race with other household characteristics include Borjas (1998) and Bayer, McMillan, and Rueben (2002). Both papers examine how the propensity of households to live in segregated neighborhoods varies with other

[5] See Massey and Denton (1987, 1989, 1993), Miller and Quigley (1990), and Harsman and Quigley (1995), for instance.

household attributes, including income, education, language, and immigration experience, providing an indication of the extent to which these other household characteristics affect segregation. In forming exact predictions as to how the observed segregation patterns would change if the correlation of race and other household attributes were altered, these studies necessarily condition on features of the urban housing market that are not likely to be primitives. The predictions of the equilibrium model, in contrast, are built on more reasonable primitives of the urban housing market – the underlying distribution of choices in the urban area and preferences across different types of household.

While we do not attempt to make such a distinction in this paper, a number of studies have focused on distinguishing whether segregation arises because of centralized discriminatory practices or the decentralized residential location decisions made by the households of a metropolitan area, each with preferences defined over the race of their neighbors. These studies have typically used data characterizing differences in the prices paid for comparable houses by households of different races to distinguish whether segregation is decentralized sorting on the basis of preferences or discrimination. These studies have focused exclusively on race-based explanations for segregation.[6] The consequences of segregation have also been explored in another body of research that assesses, for example, how across-MSA differences in the degree of segregation affect important outcomes such as educational attainment and wages.[7] None of these papers, however, examines the effect of segregation on racial gaps in the consumption of housing and local public goods.

Data and a Preview of Results

Our analysis is facilitated by access to newly available restricted-access Census data, as mentioned above. Unlike publicly available Census data, which match each household with a PUMA (a Census area of at least 100,000 residents), these provide a household's residential and employment locations at the level of a Census block (a Census area with approximately 100 residents), allowing us to characterize each household's actual neighborhood much more accurately than has been possible in past studies. The Census data also provide us with detailed information on the households in the sample,

[6] Notable papers in this line of research include King and Mieszkowski (1973), Schnare (1976), Yinger (1978), Schafer (1979), Follain and Malpezzie (1981), Chambers (1992), Kiel and Zabel (1996), and Cutler, Glaeser, and Vigdor (1999). Perhaps the most definitive study is by Cutler, Glaeser, and Vigdor (1999), which examines segregation patterns over the full course of the 20th century, concluding that centralized racism was much more important in driving segregation in the earlier part of the century.

[7] See Borjas (1995) and Cutler and Glaeser (1997) for important contributions.

including each household member's race, education, income, age, immigration status, employment status, and job location. Using these new Census data as a centerpiece, we have assembled an extensive data set characterizing the housing market in the San Francisco Bay Area. This combines housing and neighborhood sociodemographic data drawn from the Census with neighborhood-level data on schools, air quality, climate, crime, topography, geology, land use, and urban density.

The estimated model provides the most complete picture of the preferences of the households in a major metropolitan region to appear in the literature to date. We obtain precise estimates of the mean valuations across all households of a variety of house and neighborhood attributes, including attributes determined by the way households sort across neighborhoods. The latter include the racial composition of neighborhoods by households of different levels of wealth and education. We also obtain a series of estimates showing how preferences across these choice characteristics vary with household characteristics. In particular, our estimates of racial interactions indicate that there is a strong tendency of households of a given race to be willing to pay much more to live in neighborhood with households of the same race.[8]

The main economic analysis of the paper uses this estimated preference structure along with the equilibrium model to calculate the new sorting equilibrium that arises as the result of a change in the model's primitives. To explore the causes and consequences of racial segregation, we conduct counterfactuals that eliminate racial differences in income, education, and employment locations as well as experiments that eliminate the preferences that give rise to social interactions in the residential location decision – for instance, preferences for living with households of the same and other races. Our results indicate that the elimination of racial differences in income and wealth (or education) would lead to a *significant increase* in the segregation of each major racial group in the Bay Area given the preferences of the current residents. This result and others associated with eliminating racial difference in education and the geographic distribution of employments leads to one of the fundamental conclusions of our analysis: given the relatively small fractions of Asian, Black, and Hispanic households in the Bay Area (each around 10%), the elimination of racial differences in income/wealth (or, education or employment geography) spreads households in these racial groups much more evenly across the income distribution, allowing more racial sorting to occur at all points in the distribution – e.g., leading to the formation of

[8] It is important to stress that we cannot distinguish whether the estimated racial interactions in the residential location decision are due to the preferences of each race for living with neighbors of the same race or to discrimination in the housing market. We discuss this issue at greater length in Section 5.1 below.

wealthy, segregated Black and Hispanic neighborhoods. The partial equilibrium predictions of the model, which do not account for the fact that neighborhood sociodemographic compositions and prices adjust as part of moving to a new equilibrium, lead to the opposite conclusion, emphasizing the value of the general equilibrium approach developed in the paper.

Our analysis also provides evidence that sorting on the basis of race itself (whether driven by preferences directly or discrimination) leads to large reductions in the consumption of public safety and school quality by all Black and Hispanic households, and large reductions in the housing consumption of upper-income Black and Hispanic households.[9] When the portion of the preference structure that generates racial interactions in the location decision is eliminated, upper-income Black and Hispanic households in particular are much more likely to choose owner-occupied housing, larger houses, and neighborhoods with much higher levels of school quality and public safety, neighborhoods that also have a much higher fraction of other high-income and white neighbors. These results therefore point to a fundamental consequence of racial sorting in the housing market – namely, a distortion in the consumption of housing and local public goods by members (especially wealthy members) of racial groups with a small numbers of individuals in parts of the income distribution.

The remainder of this paper is organized as follows: In Section 2, we set out the modeling framework and describe the equilibrium properties of the model. The extensive new data set that we have assembled for the analysis is described in Section 3, and estimation of the model is discussed in Section 4. Here, we also relate our model to other methods of estimating willingness-to-pay measures for house and neighborhood attributes. Section 5 discusses issues of identification and interpretation that arise in our sorting model. The next two sections of the paper present our empirical analysis: the parameter estimates of the model are given in Section 6, and Section 7 characterizes the pattern of racial segregation in the Bay Area, before setting out results from our general equilibrium simulations. Section 8 concludes.

[9] As noted previously, we remain agnostic throughout this paper as to whether these interactions arise as the result of the preferences of each race for living with neighbors of the same race or discrimination in the housing market. While this distinction has important welfare implications, the point made here concerning the impact of racial interactions on the consumption of local public goods by a population with relatively small numbers remains regardless of which explanation prevails.

2 AN EQUILIBRIUM MODEL OF SORTING IN THE URBAN HOUSING MARKET

We begin our analysis by setting out an equilibrium model of the housing market, first describing the central component of this model - a discrete choice framework that governs each household's residential location decision - before developing the equilibrium properties of the model.

2.1 The Residential Location Decision

The residential location decisions of all households in the San Francisco Bay Area are modeled as a discrete choice of a single residence. The utility function specification is based on the random utility model developed in McFadden (1978) and the specification of Berry, Levinsohn, and Pakes (1995), which includes choice-specific unobservable characteristics.

In the model, each household chooses its residence h to maximize its utility, which depends on the observable and unobservable characteristics of its choice. Let X_h represent the observable characteristics of house h other than price that vary with the household's housing choice and let p_h denote its price. The observable characteristics of a housing choice include characteristics of the house itself (e.g., size, age, and type), its tenure status (rented vs. owned), and the characteristics of its neighborhood (e.g., sociodemographic composition, school, crime, topography, and air quality). Household i's optimization problem is given by:

$$(2.1) \qquad \underset{(h)}{Max} \quad V_h^i = \alpha_X^i X_h - \alpha_D^i D_h^i - \alpha_p^i p_h + \xi_h + \varepsilon_h^i$$

where ξ_h is the unobserved quality of each housing unit, including the unobserved quality of the corresponding neighborhood. The $\alpha^i{}_D D^i{}_h$ term in the utility function captures the disutility of commuting – the negative impact of the distance between household i's workplace and house h. The final term of the utility function, $\varepsilon^i{}_h$, is an idiosyncratic error term that captures unobserved variation in household i's preference for a particular housing choice.

Each household's valuation of choice characteristics is allowed to vary with its own characteristics, Z_i, including education, income, race, employment status, and household composition. We also assume that each working household is initially endowed with a primary employment location, l_i. We treat employment status and employment location as exogenous variables throughout this paper.[10]

[10] We discuss the impact of these assumptions on the parameter estimates in Section 5 below.

Each parameter associated with housing characteristics, distance to work, and price, α^i_j, for $j \in \{X, D, p\}$, is allowed to vary with a household's own characteristics,

$$\alpha^i_j = \alpha_{0j} + \sum_{r=1}^{R} \alpha_{rj} Z^i_r \,, \tag{2.2}$$

so equation (2.2) describes household i's preference for choice characteristic j. The first term captures the taste for the choice characteristic that is common to all households and the other terms capture observable variation in the valuation of these choice characteristics across households with different socioeconomic characteristics. This heterogeneous coefficients specification allows for great variation in preferences across different types of household.[11]

The specification of utility given in equations (2.1)-(2.2) contains two stochastic components that allow the model flexibility in explaining the observed data. The first component is the house-specific unobservable, ξ_h. This term captures the common value of unobserved (to the econometrician) aspects of a particular house and its neighborhood, that is, value shared by all households. Because many housing and neighborhood attributes are likely to be unobserved in any data set, specifications of the utility function that do not include such unobserved characteristics are likely to lead to biased parameter estimates. The houses in neighborhoods with high levels of unobserved quality, for example, will generally command higher prices and attract higher income households, *ceteris paribus*. Thus analyses that do not account for unobserved characteristics will tend to attribute their impact on utility to observed characteristics with which they are correlated.

The second stochastic component of the utility function is the idiosyncratic term ε^i_h, which is assumed to be additively separable from the rest of the utility function. We assume that it is distributed according to the Weibull distribution, giving rise to the multinomial logit model. With this assumption, the probability that household i selects house h, P^i_h, is given by the expression:

$$P^i_h = \frac{\exp(\alpha^i_X X_h - \alpha^i_D D^i_h - \alpha^i_p p_h + \xi_h)}{\sum_k \exp(\alpha^i_X X_k - \alpha^i_D D^i_k - \alpha^i_p p_k + \xi_k)} \tag{2.3}$$

where k indexes all possible house choices.

[11] While it would also be possible to include random coefficients, i.e., a stochastic term in the preference specification of equation (2.2), which would allowed for unobserved heterogeneity in tastes for each house and neighborhood characteristics, we do not include stochastic terms in the analysis presented in this paper.

The multinomial logit assumption implies that the ratio of the probabilities between any two choices is independent of the characteristics of the remaining set of alternatives – the IIA property. This property is usually thought to be undesirable, as conveyed by the well-known 'red bus-blue bus' example.[12] In housing markets, however, the IIA property helps capture a key feature that is difficult to model directly: the fact that the houses on the market at any time may be thin relative to the full housing stock. Given that a household is limited to purchasing houses that are on the market at the time of search, an increase in the stock of a certain type of housing may significantly increase a household's probability of choosing that type of house, and perhaps even in a way that resembles the substantial increase generally implied under the multinomial logit assumption.

Two additional elements of the specification given in equations (2.1)-(2.2) limit the impact of the IIA property on the substitution patterns implied by this model. First, the inclusion of the commuting distance term in the utility function ensures that a household is more likely to substitute among choices located near its place of work, giving rise to reasonable substitution patterns in geographic space. Second, the heterogeneous coefficients specification shown in equation (2.2) ensures that while the IIA property holds at the individual level, it does not hold in the aggregate, allowing the model specified in equations (2.1)-(2.2) to give rise to more plausible aggregate substitution patterns. If highly educated households, for example, have a particularly strong taste for school quality, the introduction of a new house in a high quality school district will tend to attract highly educated households, thereby drawing demand away from other houses in high quality school districts. Similarly, houses that are located near each other in geographic space will also tend to be relatively close substitutes in the aggregate, so that the introduction of a new neighboring house will tend to be attractive to those working nearby - the same set of households who presumably found the initial houses attractive in the first place.

2.2 Equilibrium: Definition and Properties

While the random utility specification developed above is flexible from an empirical point of view, it also has a convenient theoretical interpretation. Without the idiosyncratic error component, e^i_h, this specification would suggest that two households with identical characteristics and employment locations would make identical location decisions. Since this is unlikely to be true in the data, a useful

[12] In this example, the introduction of an additional though redundant choice takes probabilities away evenly from existing choices, leaving the ratio of probabilities among existing choices unchanged, even though one such choice may be a far closer substitute for the 'new' choice than others.

interpretation of e^i_h is that it captures unobserved heterogeneity in preferences across otherwise identical households. Thus for a set of households with a given set of observed characteristics, the model predicts not a single choice but a probability distribution over the set of housing choices. By working with these choice probabilities rather than the discrete decision observed for each household in the sample, it is straightforward to define and explore the properties of a sorting equilibrium for the class of models depicted in equations (2.1)-(2.2). Throughout our analysis, we assume that each household's vector of idiosyncratic preferences e^i is observable to all of the other households in the model and we use a Nash equilibrium concept.[13]

Given the household's problem described in equations (2.1)-(2.2), household i chooses house h if the utility that it gets from this choice exceeds the utility that it gets from all other possible house choices - that is, when:

$$(2.4) \qquad V^i_h > V^i_k \quad \Rightarrow \quad W^i_h + e^i_h > W^i_k + e^i_k \quad \Rightarrow \quad e^i_h - e^i_k > W^i_k - W^i_h \qquad \forall \quad k \neq h$$

where W^i_h includes all of the non-idiosyncratic components of the utility function V^i_h. As the inequalities depicted in (2.4) imply, the probability that a household chooses any particular choice depends in general on the characteristics of the full set of possible house choices. In this way, the probability P^i_h that household i chooses house h can be written as a function of the full vectors of house characteristics (both observed and unobserved) and prices $\{\mathbf{X}, \mathbf{p}, \mathbf{x}\}$:

$$(2.5) \qquad P^i_h = f_h(Z^i, \mathbf{X}, \mathbf{p}, \mathbf{x})$$

as well as the household's own characteristics Z^i.[14]

When the set of draws $\{e^i_h\}$ for each household observed in the data is interpreted as idiosyncratic heterogeneity in preferences for each house, working with choice probabilities is equivalent to assuming

[13] It is important to point out that other interpretations concerning the exact nature of the idiosyncratic preferences are possible within this framework. We could, for example, treat each household's idiosyncratic preferences as private information and drop the assumption that each household observed in the data stands in for a continuum of other households. In developing the theoretical properties of our model and the estimator, however, we work with the single, consistent interpretation of **e** specified here, attempting to point out in footnotes when other assumptions would be equally valid.

[14] For simplicity of exposition, we have included the household's employment location in Z^i and the location of the house in X_h. Note also that the h subscript on the function f simply indicates that we are solving for the probability that household i chooses house h not that the form of the function itself varies with h.

that each household that we observe in our sample represents a continuum of households with the same observable characteristics. The choice probabilities depict the distribution of location decisions that would result for a continuum of households with a given set of observed characteristics as each household responds to its particular idiosyncratic preferences. Let the measure of the continuum of households be m. This assumption concerning the distribution of households requires a similar assumption about the set of housing choices observed in the sample. In order to make the model coherent, therefore, we also assume that each house observed in the sample represents a continuum of identical houses, and that this continuum also has measure m.

Market Clearing Conditions

Aggregating the probabilities in equation (2.5) over all households yields the predicted number of households that choose each house h, $\hat{N}_h$:

$$(2.6) \qquad \hat{N}_h = m \bullet \sum_i P_h^i$$

where again m represents the measure of the continuum of households with the same observable characteristics as household i. In order for the housing market to clear, the number of households choosing each house h must equal the measure of the continuum of houses that each observed house represents:[15]

$$(2.7) \qquad \hat{N}_h = m, \quad \forall h \quad \Rightarrow \quad \sum_i P_h^i = 1, \quad \forall h$$

It is a straightforward extension of the central proof in Berry (1994) to show that under a simple set of assumptions, a unique vector of housing prices clears the market. In particular, we can state the following proposition:

Proposition 2.1: If U^i_h is a decreasing, linear function of p_h for all households and $\boldsymbol{e}$ is drawn from a continuous distribution, a unique vector of housing prices (up to a scaleable constant) solves the system of

[15] Note that the measure m drops out of the market-clearing condition depicted in equation (2.7) and, consequently, simply serves as a rhetorical device for understanding the use of the continuous choice probabilities shown in equation (2.5) in defining equilibrium rather than the actual discrete choices of the individuals observed in the data.

equations depicted in (2.7), conditional on a set of households **Z** and houses **X**, **x**. *Proof:* See Technical Appendix.

Building on Proposition 2.1, the following lemma is also useful for characterizing the properties of a sorting equilibrium in the housing market:

Lemma 2.1: If in addition to the assumptions specified in Proposition 2.1, U^i_h is continuous in a house characteristic x_h for each household i, the unique vector of housing prices that clears the market is continuous in **x**. *Proof:* See Technical Appendix.

In proving Proposition 2.1, we show that it is possible to write the solution to (2.7) as a contraction mapping in **p**.[16] Thus, starting from any vector **p**, an iterative process that increases the prices of houses with excess demand and decreases the prices of houses with excess supply at each iteration leads ultimately to an even spread of households across houses. Writing this market-clearing vector of prices as $\mathbf{p}^*(\mathbf{Z}, \mathbf{X}, \mathbf{x})$, the probability that household i chooses house h can be written:

$$(2.8) \qquad P^i_h = f_h\left(Z^i, \mathbf{X}, \mathbf{p}^*(\mathbf{Z}, \mathbf{X}, \hat{\mathbf{i}}), \hat{\mathbf{i}}\right)$$

where the notation $\mathbf{p}^*(\mathbf{Z}, \mathbf{X}, \mathbf{x})$ indicates that the set of market-clearing prices is generally a function of the full matrices of the household **Z** and house and neighborhood characteristics $\{\mathbf{X}, \mathbf{x}\}$ that are treated as the primitives of the sorting model.

If the entire set of house and neighborhood characteristics that households value were not affected by the sorting of households across residences, a sorting equilibrium would simply be defined as the set of choice probabilities in equation (2.8) along with the vector of market clearing prices, $\mathbf{p}^*$. In this case, since a unique set of prices clears the housing market, the sorting equilibrium would also be unique.

[16] The conditions stated in Proposition 2.1 provide sufficient but not necessary conditions for the existence of a unique vector of market clearing prices. For example, while reasonable, the condition that p_h enters U^i_h in a negative manner for *every* household is much more stringent than is actually necessary to ensure the uniqueness result. Ensuring that it is possible to write the solution to the system of equations depicted in (2.8) as a contraction in **p** is as important in practice as proving this system of equations has a unique solution. It is this feature that makes it possible to solve for the unique vector of prices conditional on a set of house and household characteristics in a computationally feasible way.

Defining a Sorting Equilibrium with Social Interactions

For the analysis undertaken in this paper, however, we allow households to have preferences for the sociodemographic characteristics of their neighbors. Such preferences may arise through multiple channels as households may value the characteristics of their neighbors directly and also value other neighborhood attributes such as public safety and school quality that are influenced by neighborhood sociodemographic characteristics. In general, the sociodemographic composition of neighborhood *n(h)* can be written in terms of the probability that each household observed in the data chooses each house in that neighborhood. Thus the contribution to the sociodemographic composition of neighborhood *n(h)* made by household *j* is given by:

$$(2.9) \qquad Z^{j}_{n(h)} = \sum_{k \in n(h)} Z^{j} \bullet P^{j}_{k}$$

and the sociodemographic composition of neighborhood *n(h)* can be characterized by the vector of these individual components: $\mathbf{Z_{n(h)}}$.

If household *i*'s utility from choosing house *h* depends explicitly on a function of the sociodemographic characteristics of the occupants of other houses in the same neighborhood *n(h)*, $g(\mathbf{Z_{n(h)}})$,[17] we can write the choice probability defined in equation (2.8) as an explicit function of this function of neighborhood sociodemographic characteristics:

$$(2.10) \qquad P^{i}_{h} = f_{h}\left(\mathbf{g(Z_{n(h)}), Z^{i}, X, p^{*}, \hat{\imath}}\right)$$

Having made the non-price social interactions explicit in the sorting model, we are in a position to define an equilibrium. In particular, a *sorting equilibrium* is defined as a set of choice probabilities $\{P^{i\,*}_{h}\}$ and a vector of housing prices $\mathbf{p}^{*}$ such that the following two conditions hold:

i. The housing market clears according to equation (2.7).

ii. The set of choice probabilities $\{P^{i\,*}_{h}\}$ is a fixed point of the mapping defined in (2.10), where $\mathbf{g(Z_{n(h)})}$ is formed by explicit aggregation of $P^{j\,*}_{k}$ $\forall\,(j,k)$ according to equation (2.9).

[17] For expositional simplicity, we assume that $g(Z_{n(h)})$ captures both the direct and indirect channels through which neighborhood sociodemographic characteristics affect utility just described. Note, however, that this function does not capture the impact that neighborhood sociodemographic characteristics have on utility through their effect on house price.

The second condition in this definition ensures that, in equilibrium, each household makes its optimal location decision given the location decisions of all other households.[18]

Existence

While the equilibrium is defined in terms of the set of optimal household choices and market clearing conditions, it is easier to prove that an equilibrium exists by transforming the problem into a fixed-point problem in the vector of neighborhood sociodemographic characteristics $\mathbf{g(Z_{n(h)})}$. By rewriting equation (2.9) as:

$$(2.11) \quad Z^j_{n(h)} = \sum_{k \in n(h)} Z^j \cdot P^j_k = \sum_{k \in n(h)} Z^j \cdot f_k\left(\mathbf{g}(Z_{n(h)}), Z^i, \mathbf{X}, \mathbf{p}^*(\mathbf{g}, \mathbf{Z}, \mathbf{X}, \hat{\imath}), \hat{\imath}\right)$$

it is easy to see that since g is defined over the vector $\mathbf{Z_{n(h)}}$, the elements of which are given in equation (2.11), this mapping along with the definition of the function **g** implicitly defines $\mathbf{g(Z_{n(h)})}$. Any fixed point of this mapping, $\mathbf{g}^*$, is associated with a unique vector of market clearing prices $\mathbf{p}^*$ and a unique set of choice probabilities $\{P^{i*}_h\}$ that together satisfy the conditions for a sorting equilibrium. In this way, finding a vector of prices $\mathbf{p}^*$ and choice probabilities $\{P^{i*}_h\}$ that give rise to a sorting equilibrium can be transformed into a fixed-point problem in $\mathbf{g(Z_{n(h)})}$. We are now able to state the following proposition concerning the existence of an equilibrium:

***Proposition 2.2*:** If the assumptions of Proposition 2.1 hold, (i) U^i_h is continuous in $g(Z_{n(h)})$, (ii) **g** is a continuous function of $Z^j_{n(h)}$ $\forall$ j, and (iii) **g** is bounded both above and below, a sorting equilibrium exists. *Proof:* See Technical Appendix.

In the empirical analysis below, we assume that the utility that a household receives from choosing a house is linear in the average sociodemographic characteristics of its neighbors. This assumption ensures that U^i_h is continuous in $g(Z_{n(h)})$, $g(Z_{n(h)})$ is a continuous function of $Z^j_{n(h)}$ $\forall$ j, and $g(Z_{n(h)})$ is bounded by

[18] Notice that while each household actually makes a discrete location decision, we define the equilibrium in terms of the vector of choice probabilities $\{P^i_h\}$. These choice probabilities represent the distribution of location decisions made in equilibrium by the continuum of households that each household i represents. Note that the alternative assumption that **e** is observed only privately along with a symmetric Bayesian Nash equilibrium concept would allow us to define the equilibrium in terms of discrete location decisions rather than working with the choice probabilities. Existence would continue to hold under this interpretation concerning **e**.

the maximum and minimum values of each household characteristic observed in the data. Thus, if the assumptions of Proposition 2.1 hold, a sorting equilibrium always exists for this class of models.

Uniqueness

While it is straightforward to establish the existence of an equilibrium for the class of models described above, a unique equilibrium need not arise. Consider an extreme example in which two types of households that have strong preferences for living with neighbors of the same type must choose between two otherwise identical neighborhoods. In this case, it is easy to see that the model has multiple equilibria. In particular, two stable equilibria arise with households sorting across neighborhoods by type. When the neighborhoods are identical except for their sociodemographic composition, the matching of each household type with a particular neighborhood is not uniquely determined in equilibrium. Thus, uniqueness is not a generic property of the class of models developed above.

This extreme example, however, gives an unduly pessimistic impression of the likelihood that multiple equilibria arise in this model. Extending the simple example just described, imagine that households of one type have significantly more income than households of the other type, that the quality of one of the neighborhoods is significantly better than that of the other neighborhood in some fixed way, and that households have preferences for neighborhood quality. In this case, while strong preferences to segregate certainly ensure that households again sort across neighborhoods by type, the matching of household type and neighborhood is made much clearer by the marked differences in income and neighborhood quality. In general, a unique equilibrium will arise when the meaningful variation in the exogenous attributes of households, neighborhoods, and houses $\{Z^i, X_h, x_h\}$ is sufficiently rich relative to the role that preferences for neighborhood sociodemographic composition play in the location decision.[19]

Using Choice Probabilities to Define Equilibrium

In defining a sorting equilibrium, we work with continuous choice probabilities rather than the discrete decisions made by the households observed in the sample. As mentioned, we assume that each

[19] See Bayer and Timmins (2002) for a formal analysis of the conditions under which unique equilibria arise in these models. The discussion here echoes results found earlier in the network and social effects literatures; Katz and Shapiro (1994), for example, write that "consumer heterogeneity and product differentiation tend to limit tipping and sustain multiple networks. If the rival systems have distinct features sought by certain consumers, two or more systems may be able to survive by catering to consumers who care more about product attributes than network size." Likewise, in a closely related model of neighborhood sorting, Nechyba (1999) points out that when "communities are sufficiently different in their inherent desirability, the partition of households into communities is unique."

household observed in the data represents a continuum of household with identical observable characteristics but distinct idiosyncratic locational preferences. Under this assumption, the sorting equilibrium that arises is not affected by the particular idiosyncratic preferences $\{e^i_h\}$ of any single household. The attractiveness of this assumption is obvious as it is the continuity of the choice probabilities that we exploit in proving that a unique vector of prices clears the market and that a sorting equilibrium always exists. If, on the other hand, we interpreted our sample as the literal extent of the housing market, the set of prices that would clear the market (conditional on any finite set of individuals) would no longer be unique.[20] In essence, if an individual had a particularly high draw of e for some house, any price high enough to keep everyone else from preferring this house to others in the market and low enough to keep this house as the optimal choice for this individual could work. Despite this range of prices, the existence of an equilibrium would continue as this framework fits within the class of models analyzed by Nechyba (1997, 1999).[21]

As we discuss in Section 4, the same assumptions that allow us to develop the theoretical properties of the model in terms of choice probabilities also play an important role in our estimation strategy. In particular, because uniqueness is not a generic property of the class of models developed above, it is not possible to estimate the model using Maximum Likelihood. We develop instead a GMM estimation procedure that requires that households do not react to the idiosyncratic preferences of any other households in particular. Thus, by ensuring that households can effectively integrate out over $\mathbf{e}$, the assumption that we maintain concerning $\mathbf{e}$ plays an important role in generating a coherent estimation strategy.

Finally, the use of choice probabilities does not affect the attractive properties of the underlying discrete choice framework related to self-selection. Consider the set of choice probabilities P^i_h for a

[20] This is true as long as $\mathbf{e}$ continued to be interpreted as individual heterogeneity and each household's idiosyncratic preferences were common knowledge. An alternative assumption that would generate similar equilibrium properties for our model would be to assume that each household's idiosyncratic preferences were not common knowledge. This would again ensure that households could not react to the particular idiosyncratic preferences of other households in the market.

[21] It is important to note, however, that given any data set, a researcher would not be able to back out a unique vector $\mathbf{e}^i$ for each household i from an observed set of market clearing prices and location decisions. Each household's equilibrium location decision only reveals that its idiosyncratic preferences for its chosen house exceeded its idiosyncratic preferences for each other house by a certain threshold value. In this way, the *particular* vector $\mathbf{e}$ for any finite set of households is unidentified, making counterfactual simulations based on calculations of a new equilibrium under alternative assumptions for a *particular* set of households impossible. Knowing the range in which each household's vector $\mathbf{e}^i$ must lie, one could conduct counterfactual simulations by randomly drawing a vector $\mathbf{e}^i$ for each household. This assumption is exactly equivalent to the assumption that we maintain concerning $\mathbf{e}$ throughout our analysis.

particular household observed in the data, which represent the distribution of the discrete decisions made by the continuum of households that the observed household represents. Among this continuum of households, however, those households that choose each particular house h will be those that get a relatively high draw of e^i_h relative to the other houses in the sample. In this way, the set of households predicted to choose each type of house observed in the data are those that place the highest value on it, as governed by both observable household characteristics and idiosyncratic preferences.

2.3 A Restricted Version of the Model – A Standard Hedonic Price Regression

Before turning to issues involved with the identification and estimation of the equilibrium model of sorting, it is helpful to examine a restricted version of the model. In particular, consider a specification of the utility function in which all households share the same value for each house except through the idiosyncratic error term:

$$(2.12) \quad U^i_h = \alpha_{0X} X_h - \alpha_{0p} p_h + \xi_h + e^i_h$$

Relative to the broader specification described above, this specification eliminates all non-idiosyncratic heterogeneity in preferences and endowments (e.g., employment locations). In this case, the market clearing condition implies that prices adjust so that the mean utility of each alternative is identical and, consequently:

$$(2.13) \quad \alpha_{0X} X_h - \alpha_{0p} p_h + \xi_h = K \quad \Rightarrow \quad p_h = {}^{\alpha_{0X}}\!/_{\alpha_{0p}}\, X_h + {}^{1}\!/_{\alpha_{0p}}\, \xi_h$$

Equation (2.13) is a standard hedonic price regression. This equivalence makes clear that a hedonic price regression returns the mean valuation of housing and neighborhood attributes when the underlying assumptions of the sorting model specified above (which include the assumption of a fixed stock of housing) are combined with the additional assumption that households have identical preferences for houses and locations.[22]

In the presence of heterogeneity in household preferences for housing and neighborhood characteristics as well as locations, housing units generally provide unequal levels of mean utility in

[22] This condition holds no matter what assumption is made concerning the distribution of the idiosyncratic error term and, in fact, holds in the absence of such idiosyncratic preferences.

equilibrium. The equilibrium mean utility that a house returns is governed by the relative scarcity of its attributes as well as its location within the urban housing market. Consider, for example, a house with a spectacular view of the Golden Gate Bridge. Such a view is scarce. In this case, we would expect the equilibrium price to reflect the valuation of the view by a very wealthy individual rather than the mean individual, thereby implying a relatively low level of mean utility in equilibrium. If such a view were less rare, however, the price for such a house would be lower and the level of mean utility higher in equilibrium. Consequently, in the presence of heterogeneous preferences, an adjustment must be made to the price regression of equation (2.13) in order to return mean preferences. As we show in Section 5 below, such an adjustment arises naturally in the course of estimating the equilibrium model.

3 DATA

Our analysis is conducted using an extensive new data set built around restricted Census microdata for 1990. These restricted Census data provide detailed individual, household, and housing variables found in the public-use version of the Census, but unlike the public-use data, also include information on the location of individual residences and workplaces at a very disaggregate level. In particular, while the public-use data specify the PUMA (a Census region with approximately 100,000 individuals) in which a household lives, the restricted data specify the Census block (a Census region with approximately 100 individuals). The restricted Census microdata thus allow us to identify the local neighborhood each individual inhabits and to determine the characteristics of that neighborhood far more accurately than has been previously possible with such a large-scale data set.

Our study area consists of six contiguous counties in the San Francisco Bay Area: Alameda, Contra Costa, Marin, San Mateo, San Francisco, and Santa Clara. We focus on this area for three main reasons. First, it is reasonably self-contained. Examination of Bay Area commuting patterns in 1990 reveals that a very small proportion of commutes originating within these six counties ended up at work locations outside the area; and similarly, a relatively small number of commutes to jobs within the six counties originated outside the area. Second, the area contains a racially diverse population. And third, the area is sizeable along a number of dimensions, including over 1,100 Census tracts, and almost 39,500

Census blocks, the smallest unit of aggregation in our data.[23] Our final sample consists of about 650,000 people in just under 244,000 households.

The Census provides a wealth of data on the individuals in the sample – race, age, educational attainment, income from various sources, household size and structure, occupation, and employment location (also provided at the Census block level). Throughout our analysis, we treat the household as the decision-making agent and characterize each household's race as the race of the 'householder' – typically the household's primary earner. We assign households to one of four mutually exclusive categories of race/ethnicity: Hispanic, non-Hispanic Asian, non-Hispanic Black, and non-Hispanic White.[24] To ensure that our sample is representative of the overall Bay Area population, we employ the individual weights given in the Census. Accordingly, 12.3 percent of households are categorized as Asian, 8.8 percent as Black, 11.2 percent as Hispanic, and 67.7 percent of households as White. The full list of the household characteristics used in the analysis, along with means and standard deviations, is given in the upper portion of the first column of the Appendix Table 1.

Characterizing Housing Choices

Households in the model have preferences defined over housing choices, each of which is described by the location of the housing unit,[25] a vector of house characteristics, and a vector of neighborhood characteristics that includes sociodemographic characteristics as well as other information about the neighborhood. The Census data provide a variety of housing characteristics: whether the unit is owned or rented, the corresponding rent or owner-reported value, property tax payment, number of rooms, number of bedrooms, type of structure, and the age of the building.

In constructing neighborhood characteristics, we calculate measures describing the stock of housing in the neighborhood surrounding each house. We also construct neighborhood racial, education

[23] Our sample consists of all households who filled out the long-form of the Census in 1990, approximately 1-in-7 households. In our sample, Census blocks contain an average of 6 households, while Census block groups – the next level of aggregation up - contain an average of 92 households.

[24] The task of characterizing a household's race/ethnicity gives rise to the issue of what to do with mixed race households. One solution would be to assign a household with, for instance, one white and one Hispanic individual a 0.5 measure for both categories while a second option would be to use the characteristics of the household head to define the race/ethnic makeup of the household. We use this second definition and have also omitted the households that do not fit into one of these four primary racial categories (0.7 percent of all households). The results of our analysis are not sensitive to these decisions. Our final sample consists of the 243,350 households that fit into these four racial categories and live in a Census block group that contains at least one other household in our sample.

[25] The latitude and longitude of each house is known at the level of the block, a Census region that contains approximately 100 housing units.

and income distributions based on the households within the same block group, a Census region containing approximately 500 housing units.[26] We merge additional data with each house record related to air quality, climate, crime rates, land use, local schools, topography, and urban density. For each of these measures, a detailed description of the process by which the original data were assigned to each house is provided in a Data Construction Appendix.[27] In generating the climate and air quality data at the Census block level, for example, we make use of locally weighted regression techniques to assign data on climate stations and air quality monitoring stations to a lower level of aggregation (in this case, a Census block), as there are far fewer climate stations than Census blocks. The full list of house and neighborhood variables, along with means and standard deviations is given in the lower portion of the first column of Appendix Table 1.

Employment Access Measures

Two variables related to employment access are also constructed. First for employed households, we calculate a measure of the distance from the household's principal workplace (defined as the workplace of the individual with the highest labor earnings in the household) to each house in the sample. Here, the location of a house and a job is given by the centroid of the Census block in which each is found, a household's work location also being given in the restricted-access Census data at the block level. Then for every house in the sample, we construct a series of employment access measures based on the local density of jobs that employ households in each of five education categories (<HS, HS Degree, Some College, BA Degree, Advanced Degree). Specifically, for each of these education categories E, we calculate the access measure A_h^E, given by:

$$(3.1) \qquad A_h^E = \sum_{j \in E} \frac{1}{(d_{jh} + 1)}$$

where d_{jh} measures the distance from house h to each job j in education category E. This employment access measure for an education category will be large when a house is surrounded by many jobs

[26] In principle, as we know the location of each house very precisely, neighborhoods could be defined to include all houses within a given radius of the house. In practice, the use of such measures yielded very similar results to those based on conventional Census boundaries, (e.g., Census blocks, block groups, or tracts), and consequently, we use Census block groups when constructing neighborhood sociodemographic measures to facilitate comparison with past research.

[27] This Appendix is available online at www.economics.utoronto.ca/mcmillan/dca.htm.

employing individuals in that education category. Given the education level of each householder, we use the corresponding employment access measure associated with each house to help characterize the quality of the choice.

Refining the House Price Variables Provided in Census

For a variety of reasons, the house price variables reported in the Census are ill-suited for our analysis. House values are self-reported and top-coded, and rents may reflect substantial tenure discounts. Moreover, because we have implicitly defined the model and developed its equilibrium properties in terms of a single price variable for both owner-occupied and rental properties, we must relate house values to rents in some way.[28] Consequently, we make four adjustments to the housing price variables reported in the Census aiming to get a single measure for each unit that reflects what its monthly rent would be at current market prices. We describe the reasoning behind each adjustment here, leaving a detailed description of the methodology for the Data Construction Appendix.[29]

Because house values are self-reported, it is difficult to ascertain whether these prices represent the current market value of the property, especially if the owner purchased the house many years earlier. Fortunately, the Census also contains other information that helps us to examine this issue and correct house values accordingly. In particular, the Census asks owners to report a continuous measure of their annual property tax payment. The rules associated with Proposition 13 imply that the vast majority of property tax payments in California should represent exactly 1 percent of the transaction price of the house at the time the current owner bought the property or the value of the house in 1978. Thus, by combining information about property tax payments and the year that the owner bought the house (also provided in the Census in relatively small ranges), we are able to construct a measure of the rate of appreciation implied by each household's self-reported house value. We use this information to modify

[28] This requirement may seem more restrictive than it actually is. Note that we treat ownership status as a fixed feature of a housing unit in the analysis. Thus, whether a household rents or owns is endogenously determined within the model by its house choice. In the model, we allow households to have heterogeneous preferences for home-ownership (a positive interaction between household wealth and ownership, for example, will imply that wealthier households are more likely to own their housing unit, as we find below) and other house characteristics. Moreover, the model could incorporate heterogeneous elasticities of demand for features of a house or neighborhood depending on whether the unit is owned or rented. The use of a single house price variable does not impose any serious restrictions on the model.

[29] In Section 5, we discuss the implications of using current market prices in estimating the model along with the related issues of moving costs, rent control, and potential lock-in effects associated with Proposition 13.

house values for those individuals who report values much closer to the original transaction price rather than current market value.

A second deficiency of the house values reported in the Census is that they are top-coded at $500,000, a top-code that is often binding in California. Again, because the property tax payment variable is continuous and not top-coded, it provides information useful in distinguishing the values of the upper tail of the value distribution.

The third adjustment that we make concerns rents. While rents are presumably not subject to the same degree of misreporting as house values, it is still the case that renters who have occupied a unit for a long period of time generally receive some form of tenure discount. In some cases, this tenure discount may arise from explicit rent control, but implicit tenure discounts generally occur in rental markets even when the property is not subject to formal rent control. In order to get a more accurate measure of the market rent for each rental unit, we utilize a series of locally based hedonic price regressions in order to estimate the discount associated with different durations of tenure in each of over 40 sub-regions within the Bay Area.

Finally, we construct a single price vector for all houses, whether rented or owned. In order to make owner- and renter-occupied housing prices as comparable as possible, we seek to determine the implied current annual rent for the owner-occupied housing units in our sample. Because the implied relationship between house values and current rents depends on expectations about the growth rate of future rents in the market, we estimate a series of hedonic price regressions for each of over 40 sub-regions of the Bay Area housing market. These regressions return an estimate of the ratio of house values to rents for each of these sub-regions and we use these ratios to convert house values to a measure of current monthly rent. Again, the procedure is described in detail in the Data Construction Appendix.

4 ESTIMATION

Having specified the theoretical framework and described the data, we now present the procedure that we use to estimate the model. We begin by introducing some notation that simplifies the exposition. The terms of the utility function specified in equations (2.1)-(2.2) can be divided into a *choice-specific constant*, d_h, an *interaction component*, m^i_h, which includes any parts of the utility function that interact household and choice characteristics, and the *idiosyncratic error term*, e^i_h. Thus the utility function can be rewritten as:

$$(4.1) \qquad V^i_h = d_h + m^i_h + e^i_h.$$

where:

$$(4.2) \qquad d_h = a_{0X} X_h + a_{0Z} \overline{Z}_h - a_{0p} p_h + x_h$$

$$(4.3) \qquad m^i_h = \left(\sum_{r=1}^{R} a_{rX} Z^i_r \right) X_h + \left(\sum_{r=1}^{R} a_{rZ} Z^i_r \right) \overline{Z}_h - \left(a_{0D} + \sum_{r=1}^{R} a_{rD} Z^i_r \right) D^i_h - \left(\sum_{r=1}^{R} a_{rp} Z^i_r \right) p_h$$

In these equations, r indexes household characteristics, and we have explicitly separated the vector of average neighborhood sociodemographic characteristics $\overline{\mathbf{Z}}$ from $\mathbf{X}$. The choice-specific constant d_h captures the portion of the utility provided by house h that is common to all households. In the same way, x_h, the unobservable component of d_h, captures the portion of unobserved preferences for house h that is correlated across households, while e_h^i represents unobserved idiosyncratic preferences over and above this shared component.[30] Denoting the full set of parameters q, we subdivide these into two sets in later discussion: the set of interaction parameters in m^i_h, q_m, and the set of parameters in d_h, q_d. Here, it is worth recalling the assumption from Section 2 that each household i observed in the sample represents a continuum of otherwise identical households with different idiosyncratic locational preferences and that each house h observed in the sample represents a continuum of identical houses of the same measure as this continuum of households.

4.1 Estimation without a Generically Unique Equilibrium

[30] Another way to describe x_h is that it captures the shared portion of the quality of house h (including the quality of its neighborhood) that is observed by the households in the data but not the econometrician.

Because uniqueness is not a generic feature of the sorting equilibrium, it is clearly not possible to estimate the parameters of the model using Maximum Likelihood. That is, for a set of exogenously given household characteristics Z^i and house/neighborhood characteristics X_h, some regions of parameter space give rise to multiple equilibria and therefore do not map uniquely to the set of endogenous variables, which include the matrix of choice probabilities $\{P^i_h\}$, the equilibrium vectors of house prices $\mathbf{p}$, and neighborhood sociodemographic characteristics $\overline{\mathbf{Z}}$. Consequently, we develop a strategy for estimating the parameters using the Generalized Method of Moments (GMM). In this case, the underlying theoretical sorting model need not have a unique equilibrium. Instead, we base the estimation of most of the model's parameters on the assumption that the observed location decisions are individually optimal, given the collective choices made by other households and the vector of market-clearing prices. Our estimation strategy therefore relies on the assumption that an equilibrium in the sorting model exists and is observed, but not the fact that this equilibrium is unique.[31]

In particular, we form moments based on maximizing the probability that each household chooses its observed location conditional not only on $\mathbf{X}$ but also on $\mathbf{p}$ and $\overline{\mathbf{Z}}$, ignoring the fact that these latter variables are determined as part of the sorting equilibrium. This procedure mirrors an assumption that is typically made when researchers estimate discrete choice models with micro-data – namely that each household is small relative to the whole population and therefore takes all variables as fixed when making its own location decision, even those variables that are endogenously determined. More formally, the validity of our approach derives from our assumption that each household observed in the data represents a continuum of households with distinct idiosyncratic locational preferences. This assumption ensures that households can each effectively integrate out the idiosyncratic preferences of all others when making their own location decisions and consequently that no household's particular idiosyncratic preferences affect the equilibrium. In this way, the vector of idiosyncratic preferences $\mathbf{e}$ is uncorrelated with the prices and neighborhood sociodemographic characteristics that arise in any equilibrium.

Fitting the observed individual location decisions permits the estimation of the parameters of the interaction term m^i_h and the vector of choice-specific constants, $\mathbf{d}$. However, the set of observed residential choices provides no information that distinguishes the elements of the choice-specific constant $\mathbf{d}$. Consequently, it is necessary to bring additional econometric information to bear on the problem. Given the estimate of $\mathbf{d}$ obtained from fitting the observed individual location decisions, equation (4.2) is

[31] Note that this estimation procedure does not require the explicit calculation of an equilibrium, which has the attractive effect of significantly reducing the computational burden involved.

simply a regression equation. The most obvious approach to identifying the parameters of this equation involves forming moments based on covariance restrictions between the observed choice characteristics and x_h. It is immediately obvious, however, that forming covariance restrictions between x_h and p_h, $\overline{Z}_h$, or any other choice characteristic that depends on neighborhood sociodemographic composition (such as local school quality) is not consistent with the logic of the choice model, as any increase in the unobserved quality of a house typically raises demand for a house and in turn its equilibrium price. Similarly, an increase in the unobserved quality of a neighborhood will tend to increase the price of houses in that neighborhood and alter the sociodemographic composition of the households living there in equilibrium. In estimating equation (4.2), therefore, it is necessary to find a vector of additional instruments **W** for the housing prices and neighborhood sociodemographic characteristics that are determined endogenously in the sorting model. We discuss the specific instruments used in the analysis in Section 5 below.[32]

4.2 The Estimation Procedure

The estimation procedure just outlined is straightforward to implement. For any combination of interaction parameters and house-specific constants, d_h, the model predicts the probability that each household i chooses house h:

$$(4.4) \qquad P_h^i = \exp(d_h + \hat{m}_h^i) \Big/ \sum_k \exp(d_k + \hat{m}_k^i)$$

Maximizing the probability that each household makes its correct housing choice, conditioning on the full set of observed household characteristics Z^i and choice characteristics $\{X_h, p_h, \overline{Z}_h\}$, gives rise to the following log-likelihood function:

$$(4.5) \qquad \ell = \sum_i \sum_h I_h^i \ln(P_h^i)$$

[32] In some empirical settings, researchers may not be interested in distinguishing the components of the vector of choice-specific constants, **d**. It is necessary to do so, however, if one is interested in calculating *any* individual's willingness-to-pay for *any* choice characteristic or if one wants to carry out any counterfactuals – or make any predictions – since any systematic changes in location preferences affect **p** and $\overline{\mathbf{Z}}$ in equilibrium. Distinguishing the components of **d** immediately forces one to address the correlation of **x** with **p** and $\overline{\mathbf{Z}}$, giving rise to the need for instruments.

where I^i_h is an indicator variable that equals 1 if household i chooses house h in the data and 0 otherwise. The first step of the estimation procedure consists of searching over the interaction parameters and vector of choice-specific constants to maximize ℓ,[33] returning estimates of the interaction parameters $\hat{\pi}_m$ and the vector of choice-specific constants $\hat{\delta}$. The second step of the estimation procedure uses $\hat{\delta}$ along with a set of appropriate instruments to estimate equation (4.2) via instrumental variables.[34]

A Computational Shortcut – Enforcing the Market Clearing Condition

When the size of the choice set grows large, it becomes infeasible to search freely over the full set of parameters that need to be estimated in the first step of the estimation procedure (δ, π_m). Conditional on the data and π_m, however, it is possible to 'back-out' an estimate of δ by enforcing the market clearing conditions specified in equation (2.7). As it turns out, this procedure returns the vector $\boldsymbol{\delta}$ that maximizes the likelihood function in equation (4.5), conditional on the set of interaction parameters, π_m.[35] Operationally, Berry (1994) demonstrates that for any π_m, a unique vector of choice-specific constants $\boldsymbol{\delta}$ (up to a scaleable constant) satisfies the market-clearing conditions and Berry, Levinsohn, and Pakes (1995) provide a contraction mapping that solves for $\boldsymbol{\delta}$. For our application, the contraction mapping is simply:

$$(4.6) \qquad \delta_h^{t+1} = \delta_h^t - \ln(\hat{N}_h^t)$$

where t indexes the iterations of the contraction mapping and $\hat{N}_h^t$ is the predicted number of households that choose each house. Using this contraction mapping, it is possible to solve quickly for an estimate of the full vector $\hat{\delta}$ even when it contains a large number of elements, and consequently the likelihood

[33] Recall that the likelihood function defined in equations (4.5) conditions on the full set of choice characteristics, including those that are endogenously determined, so that this procedure is not a Maximum Likelihood procedure, despite appearances.

[34] While estimating the two steps of the estimation procedure jointly would increase the efficiency of the estimator if the instruments used in the second stage of the analysis were valid, consecutive estimation ensures that the estimates obtained in the first stage are consistent regardless of the consistency of the instruments used in the second stage. In all of the analysis presented in this paper, we use the two-step procedure.

[35] The derivative of the likelihood function given in equation (4.5) with respect to δ_h is:

$$\partial \ell / \partial \delta_h = \sum_{i=h} \partial \ln(P_h^i) / \partial \delta_h + \sum_{i \neq h} \partial \ln(P_h^i) / \partial \delta_h = \sum_{i=h} (1 - P_h^i) + \sum_{i \neq h} (-P_h^i) = 1 - \sum_i (P_h^i)$$

function can be concentrated as: $\ell(\delta, \theta_m) = \ell^c(\hat{\delta}(\theta_m), \theta_m)$. This reduces our free parameter search to θ_m, thereby dramatically reducing the computational burden in the first step of the estimation procedure.

Because the issue can lead to confusion, it is important to point out that the vector of choice-specific constants can be estimated even if the number of housing alternatives in the sample is as large as the number of households. In essence, estimation is possible because an increase in any particular δ_h increases the probability that each household in the sample chooses house h. This increases the probability that the model correctly predicts the house choice for the household that actually chooses house h, but decreases the probability that all of the other households in the sample make the correct choice, as is apparent from (4.4). As the logic of the market clearing condition makes clear, the likelihood function is maximized when the model predicts equal demand for each house observed in the sample. In this way, the estimate of δ_h for each house h is governed by the total demand for the house rather than solely on the demand of the individual that purchases the house.[36]

Using a Random Sample of Alternatives

Despite the shortcut for estimating the full vector of choice-specific constants introduced in the previous subsection, calculating choice probabilities for each household-house pair at each iteration of the optimization routine quickly becomes computationally infeasible when the number of housing alternatives grows large. Consequently, in order to estimate the model, we employ a sampling framework specified in McFadden (1978) in which a randomly chosen subset of the full set of alternatives is used in calculating each household's contribution to the likelihood function in equation (4.5). Using this sampling framework, McFadden shows that one can obtain consistent estimates of the model's parameters without calculating the full matrix of choice probabilities.

The particular procedure that we use is as follows. We first draw a large sample of households S^I and their corresponding houses S_H at random from the full Census data set.[37] This initial draw ensures that S^I and S_H are random samples, each representative of the entire San Francisco Bay Area and that the actual house chosen by each household in S^I is in the full sample of houses S_H. For each household i

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and, consequently, setting the vector $\partial\ell/\partial\delta$ equal to zero produces the market clearing conditions specified in equation (2.7).

[36] This discussion also makes clear why $\boldsymbol{\delta}$ is subject to a free normalization, as an increase in each element of $\boldsymbol{\delta}$ has no effect on any household's demand for any house.

[37] Census sample weights are used in this step of the analysis, ensuring that our initial sample is representative of the households and houses with the San Francisco Bay Area.

observed in this sample, we then construct a subset S^i_H of the full set of houses in the Bay Area that consists of the household's chosen house and a random sample of the remaining alternatives in S_H. In this way, the choice probabilities for each household that are used in the constructing the likelihood function shown in equation (4.5) are given by:

$$(4.7) \qquad P^i_h = \exp(\delta_h + \hat{m}^i_h) \Big/ \sum_{k \in S^i_H} \exp(\delta_k + \hat{m}^i_k)$$

where the sum in the denominator is now taken over only those alternatives in the subset associated with household *i*.

In practice, because we estimate choice-specific constants for each house, the precision of the estimation procedure increases greatly if we ensure that each alternative appears in the choice set of the same number of households. To this end, we employ the following random sampling procedure: Starting with the assignment of each household's chosen house, we assign each household a first additional (not chosen) alternative by randomly re-shuffling the full set of houses across households. We then repeat this random re-shuffling of houses as many times as is necessary to generate the desired size of the sample of additional (not chosen) alternatives. In this way, with an additional random draw for each household, we ensure that each alternative is sampled exactly once.

The sampling framework developed by McFadden (1978) also justifies the initial sampling process that generates the full set of households S^I and houses S_H used in the analysis (i.e., the use of less than the full census of houses) as long as the IIA property holds at the individual level. That is, if households actually choose from the full census of houses including, for example, those that are not sampled in the long form of the Census, no researcher will observe each household's full choice set in the data. In this case, the assumption that ε^i_h is distributed according to the Weibull distribution, which gives rise to the IIA property at the individual level, ensures that the econometrician obtains consistent estimates of the model's parameters despite observing only a random sample of the full set of alternatives that each household faces. While other assumptions could be made to justify the use of a sub-sample of the full census of alternatives (i.e., that the observed sample spans the full choice set), the underlying assumption concerning the distribution of $\boldsymbol{\varepsilon}$ justifies the use of the sample without such assumptions.[38]

[38] It is worth noting that this does not prevent a researcher from including other stochastic elements in the utility function such as random coefficients, which allow for unobserved differences across households in willingness-to-pay for choice characteristics. All that is required to justify the general use of this kind of sampling procedure is that the final idiosyncratic component of location preferences be distributed according the Weibull distribution.

The use of a random sample of the full census of alternatives for each household necessitates a slight adjustment to the calculation of the predicted number of households that choose each house that is used in the contraction mapping (4.6). In particular, because the sampling procedure ensures that each household's actual choice is included in the subset of alternatives when calculating the choice probabilities shown in equation (4.7), the predicted number of households that chooses each house used in equation (4.6) must be corrected for this inherent over-sampling. This requires the following straightforward adjustment:

$$(4.8) \qquad \hat{N}_h = \frac{(C+1)}{N} \sum_{i=h} P_h^i + \frac{(C+1)}{N} \frac{(N-1)}{C} \sum_{h \in S_H^i, i \neq h} P_h^i$$

where N is the total number of alternatives in the full census, C is the number of additional (not chosen) alternatives sampled for each household and, consequently, the number of times house h appears in other household's choice set, and the notation $i=h$ refers to the household that actually chooses house h. In equation (4.8), the first term captures the contribution to $\hat{N}_h$ made by the household who actually chose house h, while the second term sums the contributions of the other households in the sample which could have chosen house h (i.e., the house was in the household's randomly drawn choice set) but did not.

If one takes the full set of houses in the metropolitan area to be the relevant choice set, N is very large, on the order of 1.5 - 2 million for the San Francisco Bay Area. Even if one counts only the number of houses in the full set S_H used in the analysis, N is typically large relative to C, and consequently, equation (4.8) effectively reduces to:[39]

$$(4.9) \qquad \hat{N} = \frac{(C+1)}{C} \sum_{h \in S_H^i, i \neq h} P_h^i$$

In calculating $\hat{N}_h$ for use in backing out the vector of choice-specific constants that ensures that the housing market clears, we essentially do not count the contribution of the household that actually chooses the house. Dropping the choice probability for the household that actually chooses a house from this calculation is intuitive as the sampling framework described above includes this individual-house pair in the analysis in a non-random way.

[39] For a large enough sample of houses, one might assume that this sample effectively spans the full census of houses in the metropolitan area. In this way, the full census of choices would be represented in the full choice set S_H drawn initially from the data.

Summary of the Full Estimation Procedure

The full estimation procedure that we employ can be summarized as follows:

1. Sample and Choice Set Construction

   i. Draw a large sample of households S^I and their corresponding houses S_H at random from the full Census data set. The sample of households is used directly in the analysis, while the sample of houses is used in constructing subsets of alternatives for each household in (1.ii).

   ii. For each household i observed in this sample, construct a subset S^i_H consisting of the household's chosen house and a random sample of the remaining alternatives in S_H. These sets are held fixed throughout the remainder of the estimation.

2. Estimation of Interaction Parameters and Choice-Specific Constants

   i. For a given set of interaction parameters (those in m^i_h), solve for the vector of choice-specific constants $\boldsymbol{d}$ that implies that the housing market clears for each house (i.e., that equation (2.7) holds).

   ii. Using the vector of house-specific constants d_h and m^i_h, calculate the log-likelihood function given in equation (4.5).

   iii. Search over the interaction parameters until the objective function calculated in (2.ii) is maximized. The estimated choice-specific constants are those calculated in (2.i) at the final iteration.

3. Estimation of Choice-Specific Constant Regression

i. Using the estimated choice-specific constants from (2), estimate equation (4.2) using instrumental variables.

4.3 Asymptotic Properties of the Full Estimator

Because our estimation procedure is somewhat non-standard, we discuss briefly the conditions that ensure the consistency and asymptotic normality of our estimates. We begin by clarifying the maintained assumptions concerning the data generating process. The model is estimated on data drawn from a single, large metropolitan area. The complete metropolitan area housing market consists of a total of I individuals who must choose from H distinct types of housing, with H assumed to be less than I. Each individual i is characterized by a set of characteristics Z^i and a set of idiosyncratic preferences $\{e^i{}_h\}$ defined over the full set of distinct housing alternatives, and identically and independently distributed across choices according to the Weibull distribution. Households are assumed to follow the model's decision rule at the true parameter vector. Each distinct housing type h is characterized by the set of characteristics $\{X_h, x_h\}$. The $\{X_h, x_h\}$ vectors are assumed to be exchangeable draws from some larger population of possible house types.

We do not observe the full census of households and houses in the metropolitan area, but instead observe a random sample of households S^I and their corresponding houses S_H of size I^S and H^S respectively. Moreover, the actual house type chosen by each household in S^I appears in the sample of houses S_H. Because the sample of houses is drawn randomly from the full sample of houses, the relative market share of each house observed in S_H is exactly k_h/H^s, where k_h is the number of times that a house of type h is sampled. Thus, within the observed sample of housing alternatives, the relative market share of each house type is known exactly. Finally, the random sampling technique described above is used to draw a subset of housing alternatives for each household i in sample S^I that consists of the household's chosen house and a random sample of the remaining alternatives in S_H of size C. With this characterization of the data generating process, our problem fits within a class of models for which the asymptotic distribution theory has been developed. In the remainder of this section, therefore, we summarize the requirements necessary for the consistency and asymptotic normality of our estimates and provide some intuition for these conditions.

In general, there are three dimensions in which our sample can grow large: as H^S, I^S, or C grow large. For any full sample of housing alternatives of size H^S and any random sampling of these alternatives of size C, the consistency and asymptotic normality of the first-stage estimates (δ, q_m) follows directly as long as I^S grows large. This is the central result of McFadden (1978), justifying the use of a random sample of the full census of alternatives. Intuitively, even if each household is assigned only one randomly drawn alternative in addition to its own choice, the number of times that each house is sampled (the dimension in which the choice-specific constants are identified) grows as a fixed fraction of I^S.

If the true vector $\mathbf{d}$ were used in the second stage of the estimation procedure, the consistency and asymptotic normality of the second-stage estimates q_d would follow as long as $H^S \to \infty$.[40] In practice, ensuring the consistency and asymptotic normality of the second-stage estimates is complicated by the fact the vector $\mathbf{d}$ is estimated rather than known. Berry, Linton, and Pakes (2002) develop the asymptotic distribution theory for the second stage estimates q_d for a broad class of models that contains our model as a special case and, consequently, we employ their results. In particular, the consistency of the second-stage estimates follows as long as $H^S \to \infty$ and I^S grows fast enough relative to H^S such that $H^S \log H^S / I^S$ goes to zero, while asymptotic normality at rate $\sqrt{H^S}$ follows as long as $(H^S)^2 / I^S$ is bounded. Intuitively, these conditions ensure that the noise in the estimate of $\mathbf{d}$ becomes inconsequential asymptotically and thus that the asymptotic distribution of q_d is dominated by the randomness in $\mathbf{x}$ as it would be if $\mathbf{d}$ was known.

Calculating Standard Errors

Estimation of the standard errors is straightforward conceptually. The covariance matrix for the parameters q (q_m and d) obtained in the first stage is given by the standard likelihood formulation and can be estimated using the outer product of the gradient:

$$(4.10) \quad V_q = E\left[\frac{\partial \ell}{\partial q}\frac{\partial \ell}{\partial q}'\right]$$

When the number of choice-specific constants is large, we can again use the concentrated likelihood function $\ell(d, q_m) = \ell^c\left(\hat{d}(q_m), q_m\right)$ to our advantage. Using the market-clearing conditions to back out $\hat{d}$

[40] This condition requires certain regularity conditions. See Berry, Linton, and Pakes (2002) for details.

given q_m, we can calculate the gradient $\partial \ell^c / \partial q_m$. If $\hat{d}(q_m)$ were a deterministic function of q_m, the covariance matrix would be given by:

$$(4.11) \quad V_{q_m} = E\left[\frac{\partial \ell^c}{\partial q_m}\frac{\partial \ell^c}{\partial q_m}'\right]$$

Since $\hat{d}(q_m)$ provides only an estimate of $\mathbf{d}$, however, we must add another term to equation (4.11) to capture the impact of this randomness on $\hat{q}_m$. As long as the sample of households and houses used in the analysis is drawn at random from the full census of alternatives, this source of randomness will be independent of the variance term shown in equation (4.11) and, consequently, the full variance matrix for q_m is given by:

$$(4.12) \quad V_{q_m} = E\left[\frac{\partial \ell^c}{\partial q_m}\frac{\partial \ell^c}{\partial q_m}'\right] + Var_d\left[\frac{\partial \ell^c}{\partial q_m}\right]$$

To estimate the second term of this equation, we use a Monte Carlo procedure following Berry, Levinsohn, and Pakes (1995). Specifically, we randomly re-draw the sample of additional (not chosen) housing alternatives for each household independently a number of times. Using our estimate of q_m and this new sample of housing alternatives, we calculate the gradient of the likelihood function with respect to q_m and use the empirical variance of these calculate gradients as the estimate of Var_d.

An estimate of the covariance matrix for the second stage is given by the expression:

$$(4.13) \quad V_{q_d} = (X' P_W' X)^{-1} X' P_W \Omega P_W' X (X' P_W' X)^{-1}$$

where X represents all included regressors, W represents all instruments including the regressors that are not instrumented, and $P_W = W'(W'W)^{-1}W$ and Ω is the covariance matrix for the error in regression of $\hat{d}$ on X. Note that because the randomness in $\hat{d}$ enters the second-stage regression as noise in the dependent variable, it is accounted for in the covariance matrix of (4.13) when a two-step procedure is used.

5 ISSUES OF INTERPRETATION AND IDENTIFICATION

This section of the paper has two main parts. The first discusses a number of key assumptions that underlie the identification of the model and the interpretation of its parameters. In so doing, we attempt to draw a clear distinction between the assumptions that can be weakened in future work versus those that are more fundamental. We also discuss the likely direction and magnitude of any potential biases. The second part of this section develops an instrumental variables strategy for identifying the choice-specific constant regression of equation (4.2). We begin by developing a specific set of instruments for price and neighborhood sociodemographic characteristics, providing a clear argument for their appropriateness as instruments. We relate this identification strategy to the classic hedonic identification problem and conclude the section by developing a set of 'quasi-' optimal instruments.

5.1 Assumptions and Interpretations

The Housing Market and Moving Costs

A number of features of the housing market suggest that the assumption that each household resides in its optimal location may not hold in all cases. First, search costs and the fact that the complete census of houses is not available at the time a household makes its location decision imply that households may actually choose from a subset of the full set of houses in the Bay Area. Second, the cost of moving (broadly defined) may cause a household to continue living in a housing unit that differs from its currently most preferred housing option. A household that chooses a neighborhood with good schools when it has school-aged children, for example, may continue to live in the same house even after the children have finished school. Gaps between actual and optimal choices may also arise because of changes in set of available options, as new, possibly more-preferred houses may be added to the housing stock over time. In these cases, the root cause of the gap between preferences and location is the cost of changing residence, which includes search costs, transaction costs, and other benefits to having a longer tenure in a house such as relationships built up with one's neighbors. Transactions costs alone can easily sum to nearly a tenth of the value of a house and California's Proposition 13 creates an additional 'lock-in' effect as property taxes are assessed based on the most recent transaction price of the house. Tenure discounts also arise in the rental markets for a variety of reasons, including formal rent control policies.

The presence of moving costs distorts our estimated preference parameters in a number of ways, especially as they relate to changes in the household's position in the lifecycle. To the extent that

households anticipate future needs when making housing decisions as young adults and do not move quickly to re-optimize at later stages in life, our analysis will understate the degree to which current housing preferences change with age. One approach that has been used in the literature to deal with this issue is to estimate the model using only a sample of recent movers.[41] To the extent that new movers are systematically different than households who have lived in housing units for a longer period, however, this approach leads to preference parameter estimates that are not representative of the full set of households in the housing market. An alternative way to incorporate moving costs within the static framework developed here is to estimate each household's location decision while discounting the price of the currently occupied house to reflect tenure discounts, transactions costs, and the 'lock-in' effect of Proposition 13. In this way, for households with large implicit moving costs, the model would be able to explain their current location as resulting partially from this implicit price discount as opposed to preferences for the features of that house/location relative to others. Because a full characterization of the way that these important dynamic aspects of the decision process affect the equilibrium in the urban housing market is fairly involved, we leave a broader treatment of these issues for future work.

Discrimination in the Housing Market

Another reason that actual housing choices may differ from optimal housing choices relates to discrimination in the housing market. If minority households face severe obstacles trying to locate in certain neighborhoods, this may cause some households to choose houses other than their unconstrained first choices. Consider, for example, two houses that are identical in every way except that one is located in a predominantly white neighborhood and costs less than the other, which is located in a predominantly black neighborhood. If racial discrimination in the housing market forces a black household to choose the house in the predominantly black neighborhood despite the household's preference to purchase the cheaper house in the predominantly white neighborhood, our model will interpret this choice as a signal that the black household is willing to pay a premium to live with black versus white neighbors. Thus, discrimination in the housing market leads to location decisions that, from the point of view of the current model, are observationally equivalent to decisions based on active preferences for neighborhood racial composition. Throughout our analysis, we recognize this limitation and interpret the parameters that

[41] See for example Bajari and Kahn (2000), Duncombe, Robbins, and Wolf (1999), and Quigley (1985).

multiply the interactions of household race and neighborhood racial composition as representing the combination of preferences and discrimination.[42]

Whether the sizes of the parameters that multiply the interactions of household race and neighborhood racial composition result from preferences or discrimination, these parameters inform us about the importance of sorting on the basis of race in the housing market. If one thinks of discrimination as an expression of the preferences of the discriminating group concerning the group discriminated against, then our model essentially mis-assigns these preferences to the group discriminated against. Thus, while the model's estimate of the preferences of black households to live with other black households may be overstated, the difference between this preference measure and that of white households remains informative. Because it is the *differences* in estimated preferences that drive the equilibrium predictions of the model, our inability to distinguish discrimination from direct preferences for the race of one's neighbors does not seriously affect the a key aim of our analysis, namely to gauge the relative importance of racial versus non-racial factors in driving segregation.

In this way, it is worth emphasizing that the racial interactions that we estimate combine the effects of (i) discrimination in the housing market (e.g., discrimination against recent immigrants from China on the part of household of other races), (ii) direct preferences for the race of one's neighbors (e.g., preferences on the part of a recent immigrant from China to live with other Chinese immigrants), and (iii) preferences for race-specific portions of unobserved neighborhood quality (e.g., preferences for Chinese groceries which are located in neighborhoods with a high fraction of Chinese residents).[43]

Housing Supply

[42] It is worth stressing that one would generally expect discrimination to primarily affect the estimates of the interaction of household race with neighborhood racial composition rather than the interaction of household race with other choice characteristics.

[43] In related work (Bayer et al. (2002b)), we extend the framework used in this paper to more directly address the issue of distinguishing preferences for self-segregation in the presence of discrimination. In particular, if discrimination were perfect, the estimation of the parameters of the model would reduce to estimating separate models for each race using only the houses inhabited by households of that race. This extension also implicitly allows household of each race to place different values on the unobservables associated with housing and neighborhood quality. In essence, this strategy uses only within-race price variation to identify the preferences of households of a given race, while continuing to control for the likely correlation of the neighborhood racial composition with unobserved housing and neighborhood quality. While the presence of discrimination in the housing market is exceedingly difficult to determine, this empirical strategy allows to us to estimate preferences under a series of alternative assumptions about discrimination – thereby, providing reasonable bounds on the actual underlying preferences.

Another key assumption underlying the analysis presented in this paper is that the supply of housing is fixed. In solving for the new equilibrium that results from in a counterfactual setting, then, our analysis does not account for any effects that come through a change in housing supply. In estimating the model, this assumption has the advantage of capturing the durability of housing and allows us to treat housing characteristics as exogenously rather than endogenously determined. In future work, it is certainly possible to weaken this assumption by estimating a housing supply function along with the utility function, conditioning the supply of new housing on the lagged supply of housing (observable in the Census). While providing a more reasonable assumption regarding the identification of the model, this approach also allows the housing supply to adjust as part of calculating a new counterfactual equilibrium, thereby bringing an important dynamic aspect of the housing market into the analysis.

Employment Locations and Geography

A final important assumption that underlies the analysis is that each household's employment location(s) is not affected by its residential location decision. Conditioning on employment location explicitly brings the geography of the housing market into the analysis, giving rise to reasonable substitution patterns in geographic space and producing housing price gradients that reflect the advantages of proximity to the many employment centers within the metro area. Another major advantage of conditioning on employment location is that it allows for the identification of more sophisticated stochastic structures (including nested logit and random coefficients) and, importantly, the identification of social interactions in the sorting model.[44] To distinguish these more complex stochastic and interaction structures from a simple multinomial logit specification, one essentially needs to observe observably identical households choosing from different choice sets. By conditioning each household's decision on its employment location, we assign each household a distinct geographic bliss point, thereby giving rise to variation in the choice set across otherwise identical households.

To see how this assumption affects the estimated parameters, consider an alternative assumption – that households first choose their residential location and subsequently find work near their residence. In this case, while a household's residential choice actually reveals that choice to be preferred to all other

[44] In general, the presence of social interactions in the sorting process gives rise to substitution patterns that can be distinguished from any standard random utility model. Thus, variation in the choice set, which allows the econometrician to learn about substitution patterns, is essential to the identification of social interactions – if this identification is not to be based on distributional assumptions alone. For a formal treatment of this issue see Bayer and Timmins (2002).

houses in the metropolitan area, the eventual proximity of its workplace provides an alternative explanation for this residential choice. Generally, then, our analysis will tend to understate the strength of preferences for the particular attributes of the house and neighborhood that gave rise to the household's choice in the first place. It is worth emphasizing that the particular assumption concerning employment location used here could be considerably weakened while continuing to incorporate the geography of the Bay Area and generate variation in the choice set across households. A weaker assumption, for example, would involve conditioning on the geographic distribution of employment by industry, occupation, and education, with households choosing residential locations based on access to jobs for which they are qualified rather than a specific job location.

5.2 Identifying Preferences in the Presence of Social Interactions and Equilibrium Prices

As the above discussion makes clear, the assumptions made in this paper related to moving costs, discrimination in the housing market, housing supply, and employment locations can all be weakened in future work. The central identification issue in the paper concerns the endogeneity of housing prices, p_h, and neighborhood sociodemographic characteristics, $\overline{Z}_h$. As we discussed briefly in describing the estimation procedure, the identification of the choice-specific constant regression (equation (4.2)) requires a set of instruments that are correlated with p_h and $\overline{Z}_h$ but not with unobserved housing/neighborhood quality, x_h. In developing appropriate instruments for $\overline{Z}_h$, we provide a general solution to the problem of identifying preferences in the presence of social interactions, while in developing appropriate instruments for p_h, we provide a solution to the classic problem of distinguishing preferences from the equilibrium (hedonic) price function.

The instruments that we construct rise naturally out of the sorting model when households value only the characteristics of their chosen house and the features of the surrounding neighborhood, as long as the geographic extent of this 'neighborhood' is reasonably small relative to the full metropolitan area. In this case, the attributes of houses and neighborhoods which are positioned just beyond the region that households value directly make ideal instruments for housing prices and neighborhood sociodemographic composition. In particular, for each house h, we form instruments that characterize the housing stock and general land usage patterns (percent residential, commercial, industrial, etc.) beyond a threshold distance from house h. At the same time, we are careful to include variables that describe the housing stock and land usage within this threshold distance directly in the utility function. In developing this set of

instruments, we exploit an inherent feature of the sorting process – that the overall demand (as well as relative demand of different types of households) for houses in a particular neighborhood is affected by not only the features of the neighborhood itself, but also by the way these features relate to the broader landscape of houses and neighborhoods in the region. In this way, we assume that the exogenous attributes of nearby but not immediately proximate neighborhoods influence the equilibrium in the housing market, thereby affecting prices and neighborhood sociodemographic composition, but have no direct effect on utility.

To demonstrate the general logic of this first category of instruments, the first column of Table 5.1 reports the results of a first-stage regression that relates the price of house *h* to a series of attributes of the house and neighborhood as well as a set of variables that characterize the housing stock and land usage in concentric rings surrounding house *h*. Here, we make use of the highly disaggregated geographic information in the dataset and for each Census block construct variables that characterize the housing stock and land usage in rings with radii of 0-to-1, 1-to-3, and 3-to-5 miles.[45] The regressions reported in Table 5.1 are all conducted on a sample of 200,000 houses randomly drawn from the full set of houses observed in the Census sample.[46] The results of the regression shown in the first column of Table 5.1 highlight the logic of our instrumental variables strategy. Consider, for example, the impact of industrial land usage on price. While industrial land usage within a mile of a house has a significantly negative impact on prices, this effect drops considerably in the 1-to-3 mile ring, and then turns *positive* in the 3-to-5 mile ring. Interpreted in the context of the equilibrium model, this sign change suggests the presence of market forces at work – that is, the presence of more industrial land 3-5 miles away reduces the supply of and lowers the quality of houses in relatively close geographic proximity to the neighborhood in question, thereby increasing the equilibrium price of houses in that neighborhood.[47]

Comparing the estimated parameters for the land use and neighborhood housing stock variables within a mile to the estimated parameters for these same variables 3-to-5 away, statistically significant sign changes are occur for the all of other land use measures and four of the six variables that characterize the housing stock. In the two cases in which the coefficient on the housing variable does not change sign,

[45] Summary statistics for these variables are presented in Appendix Table 1.
[46] This sub-sample of 200,000 houses was used in the analysis rather than the full sample of approximately 250,000 due to the limitations of computer memory space that constrained subsequent portions of the analysis. The same random sample of households and houses was used throughout the analysis whenever the number of observations reported equals 200,000.
[47] Note that this regression controls directly for a series of employment access variables that control for the fact that increased industrial or commercial land usage 3-5 miles out may simply pick up better employment access.

the estimated coefficient declines by an order of magnitude or more. In estimating the model, then, we select 3 miles as our threshold distance, using variables that characterize the land use and housing stock in the 3-to-5 mile ring as instruments for price and neighborhood sociodemographic characteristics, including variables that characterize the land use and housing stock within 3 miles of the house directly in the utility function. These instrumental variables are jointly significant in first stage regressions, as the F-statistics reported in Table 5.1 *(F = 181.98)* makes clear.

In sum, then, in estimating the model we treat neighborhood sociodemographic characteristics and price as endogenous variables and topography, housing characteristics, air quality, climate, and land use as exogenous variables, using variables that characterize the land usage and housing stock 3-to-5 miles away as instruments. While the assumption that attributes related to topology, location, and climate are fixed certainly seems reasonable at first glance, it is important to point out that these characteristics will still be correlated with the unobservable if the unobservable itself depends on the way that household sort across neighborhoods. Consequently, in treating *any* variables as exogenous, we assume that having included a set of neighborhood sociodemographic characteristics in the utility function, the remaining unobserved portion neighborhood quality is a fixed attribute that does not depend on how households sort in equilibrium.[48] In essence, then, we assume that all relevant sociodemographic characteristics are included in the model and, consequently, while we allow crime and school quality to depend in equilibrium on the included neighborhood sociodemographic characteristics, we assume that school quality and crime are uncorrelated with any remaining unobserved neighborhood quality.[49]

The Classic Hedonic Identification Problem

While typically framed in a slightly different equilibrium framework, the identification of our model (ignoring the additional burdens of identifying the components related to neighborhood sociodemographics) is essentially the same as the identification problem that underlies a hedonic model with unobserved choice characteristics – i.e., that of distinguishing the equilibrium price gradient from the

[48] If, for example, households care about the average income of their neighbors and this is not included in the model, the unobservable is likely to be correlated with every included choice characteristic. In light of this problem, the best we can do is to include as many variables that describe the sociodemographic composition of the community as possible in order to limit the size of this potential bias. Importantly, this issue argues strongly against leaving neighborhood sociodemographic characteristics out of the model as a way of avoiding dealing with their endogeneity.

[49] In practice, this assumption is needed to keep the number of variables that we instrument for to a manageable number.

marginal utility of households.[50] Solutions to the problem of identifying hedonic models with unobserved choice characteristics have only recently begun to appear in the literature and, consequently, we present a short discussion of the issues related to the identification of our model in order to facilitate comparison of the assumptions underlying identification.[51]

Following Rosen (1974), the classic strategy for estimating hedonic models involves estimating an equilibrium (hedonic) price function in a first-stage regression and using the gradient of this price function as the dependent variable in the household's first-order conditions in a second stage. This second stage of this classic estimation strategy returns both the mean and variation in preferences. Our estimation strategy begins instead by estimating the heterogeneity in preferences along with a vector of mean utilities d in the first stage, identifying mean preferences in the second stage - the choice-specific constant regression.[52] In this way, the identification of our model is tied to these two stages of estimation.

Consider first the identification of the second stage regression, which uses the vector of choice-specific constants d estimated in the first stage. Ignoring the endogeneity of neighborhood sociodemographic characteristics for the purposes of this discussion, equation (4.2) contains two endogenous variables: housing price, p_h, and mean utility, d_h. Recall from Section 2, that when households are identical except for their idiosyncratic locational preferences, the market clearing condition implies that the mean utility of each house must be identical in equilibrium. In this case, then, one of the endogenous variables is eliminated and mean preferences can be recovered by moving price to the other side of equation (4.2) and simply regressing price on characteristics. In other words, without household heterogeneity in preferences or geography, and with the assumption that housing supply is fixed, the equilibrium price function simply reflects mean preferences. From the point of view of the market clearing conditions, the logic is simple: any increase in the unobserved quality of a house, x_h, is immediately offset by an increase in p_h, leaving mean utility and individual location decisions unchanged.

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[50] The hedonic literature typically uses a continuous choice framework. Important contributions include Rosen (1974), Brown and Rosen (1982), Epple (1987), Bartik (1987), and Ekeland, Heckman, and Nesheim (2002). While the underlying assumptions implicit in standard discrete choice estimation differ from the standard estimation of a hedonic model (which generally assumes that the observed decision satisfies a first order condition w.r.t. each choice attribute) the fundamental nature of the identification problem, distinguishing the equilibrium price function from preferences, is the identical in both approaches.

[51] A new working paper by Bajari and Benkhard (2002) presents alternative methods for identifying hedonic models of demand with unobserved product characteristics. It should also be noted that the general nature of the portion of our identification strategy related to the hedonic identification problem is derived from a long line of research in IO starting with the work of Bresnahan (1981, 1987) and Berry, Levinsohn, and Pakes (1995).

[52] Throughout this discussion, we assume that the mean has been subtracted from all included household characteristics so that the choice-specific constant regression returns mean preferences.

When heterogeneity in preferences or geography is allowed for, an increase in the unobserved quality of a choice alters the decisions that are made in equilibrium. If, as we find in our analysis, unobserved quality is a normal good, an increase in the unobserved quality of a house/neighborhood generally increases the income of the households that locate there in equilibrium. In this context, then, the presence of d_h in equation (4.2) provides the appropriate adjustment to the hedonic price equation to return mean household preferences:

$$(5.1) \qquad p_h - \tfrac{1}{\alpha_{0p}}\delta_h = \tfrac{\alpha_{0X}}{\alpha_{0p}} X_h + \tfrac{1}{\alpha_{0p}}\xi_h$$

This mean-utility adjustment generally depends in equilibrium on the distribution of households and their tastes, the geographic distribution of employment, and the geographic distribution of houses and neighborhoods and their characteristics. It is in providing a valid instrument for p_h, then, that we identify the second stage of our two-stage estimation procedure.

The identification of heterogeneous preferences and the vector of choice-specific constants in the first stage of the estimation procedure is guaranteed in the analysis presented in this paper by our assumption concerning the distribution of ε (i.i.d. Weibull). In general, however, it is possible to identify much more flexible forms of this distribution, allowing, for example, individuals to have unobserved tastes for particular choice characteristics (random coefficients). From a practical point of view, the identification of such random coefficients derives from the same source that generates variation in our proposed instruments for price – namely the geography of the location decision. In particular, in our framework, the geographic distribution of employment within the Bay Area gives each household a distinct geographic bliss point, leading to tremendous variation in a household's perception of the choice set depending on the location of its workplace(s) within the metropolitan area. It is this variation that makes choices closer in geographic space closer substitutes for one another, thereby giving our instruments, which describe the local housing and labor market conditions for each residential choice, empirical content. This variation also allows us to learn about substitution patterns by observing households with identical observable characteristics choosing from different sets of choices in different parts of the metropolitan area. By estimating substitution patterns, one is able to reject the substitution patterns predicted by the multinomial logit in the data and thereby distinguish more flexible forms for the stochastic structure of the preferences. In this way, the geographic variation in preferences introduced by

conditioning on the distribution of employment within the Bay Area is essential for the identification of the model. One final point is worth emphasizing: while we estimate the model using data from a single large metropolitan area, the problem is not what is usually referred to as identification using data from a single market, which typically implies that the researcher does not observe variation in the choice set.

Optimal Instruments

In forming an instrument for housing prices and neighborhood sociodemographic characteristics, we exploit the fact that the overall demand (and relative demand of different types of households) for houses in any particular neighborhood is determined not only by the features of that neighborhood, but also by how these features relate to the broader landscape of housing/neighborhood choices. The instruments developed above along with the first stage regression for housing prices shown in the first column of Table 5.1 are important precisely because they demonstrate the logic of this identification strategy. In general, however, because we would like to instrument for a large number of choice characteristics, the precision of the estimation is improved significantly with the use of a parsimonious set of instruments that approximate the optimal instruments for price and neighborhood sociodemographic characteristics. This sub-section of the paper characterizes the optimal instruments and develops computable instruments that approximate them.

The optimal instruments for p_h and $\overline{Z}_h$ in the choice-specific constant regression (equation (4.2)) are given by:

$$\begin{aligned} E\left(\partial \boldsymbol{x}_h \big/ \partial \boldsymbol{a}_{0p}\right) &= E\left(p_h \mid \Omega\right) \\ E\left(\partial \boldsymbol{x}_h \big/ \partial \boldsymbol{a}_{0Z}\right) &= E\left(\overline{Z}_h \mid \Omega\right) \end{aligned} \tag{5.2}$$

that is, the expected value of p_h and $\overline{Z}_h$ conditional on the information set Ω, which contains the full distribution of *exogenous* choice (X_h) and individual characteristics (Z^i). Notice that these instruments implicitly incorporate the impact of the full distribution of the set of choices in exogenous characteristic space as well as information on the full distribution of observable household characteristics into a single instrument for each endogenous variable.

Because the equilibrium in the sorting model is not generically unique, however, this expectation is not well-defined. In particular, the calculation of this expectation requires computing the equilibrium for the full distribution of possible parameter values and the vector of unobserved choice characteristics,

x. Since some parameter values give rise to multiple equilibria, the expectation cannot be calculated without some way of determining how an equilibrium is chosen in these cases. For this reason, we use a well-defined instrument that maintains much of the inherent logic of this optimal instrument while being straightforward to compute. This 'quasi-' optimal instrument is based on the predicted vector of market-clearing prices and the distribution of neighborhood sociodemographic characteristics calculated for an initial consistent estimate of the parameter values with (i) the vector of unobserved characteristics $\boldsymbol{x}$ set identically equal to **0** and (ii) the social interaction parameters (those that multiply $\overline{Z}_h$) set equal to zero. The first condition corresponds to using the prediction at the mean instead of the expected value while the second condition corresponds to using the predictions based only on the exogenous choice and individual characteristics, ignoring the role of social interactions.

The calculation of these instruments requires a consistent estimate of the model's parameters. Notice, however, that we can use the specific instruments developed above to provide an initial consistent estimate of these. Operationally, then, the estimation proceeds as follows:

1. While controlling for characteristics of housing stock and land usage within 3 miles as well as employment access for household's education category, use instruments that characterize the housing stock and land usage 3-5 miles away and employment access in multiple education categories to estimate choice-specific constant regression.

2. Using the resulting consistent parameter estimates, setting $x_h=0$ for all h, and eliminating all social interactions, calculate the vector of housing prices that clears the market, $\hat{p}^*(X_h, Z^i)$. The notation here is intended to indicate the predicted vector of market clearing prices conditional solely on the observable, exogenous characteristics of households and houses/neighborhoods.

3. Using $\hat{p}^*$ along with the observable, exogenous characteristics of houses/neighborhoods, calculate the predicted choice probabilities for each household and, in the aggregate, the predicted sociodemographic composition of each neighborhood, $\hat{\overline{Z}}\left(\hat{p}^*(X_h, Z^i), X_h, Z^i\right)$.

4. Using $\hat{p}^*$ and $\hat{\overline{Z}}$ as instruments for p and $\overline{Z}$, estimate the choice-specific constant regression.

Like the optimal instruments, the instruments that we propose provide a measure of the way that the full landscape of possible choices affects the overall and relative demand for each house/neighborhood. In essence, these instruments extract additional information from Ω than is contained in the vectors of choice characteristics **X**, which are already used directly in estimating equation (4.2). Moreover, the single 'quasi-' optimal instrument for each endogenous variable combines this information in a concise manner

that is consistent with the logic of the sorting model. The final two columns of Table 5.1 show first-stage price regressions that include the 'quasi-' optimal instruments, both with and without the standard set of variables shown in the first column of the table. Even conditional on the full set of standard instruments, the optimal instruments have strong predictive power (the t-statistic on the optimal price instrument is greater than 100) and tests for the joint significance of the full set of instruments and the optimal instruments alone strongly reject the null in both specifications.

6 PARAMETER ESTIMATES

The estimation of the model's parameters proceeds in two stages. The first stage returns estimates of the interaction parameters and the set of choice-specific constants. Due to computational constraints, these parameters are estimated using a sample of 10,000 households and their corresponding houses drawn at random from the full Bay Area sample of nearly a quarter of million households. Summary statistics describing the households and houses/neighborhoods in the sample of 10,000 and the sample of 200,000 households used in separate parts of the estimation (see below) are shown in Table Appendix 1.

The interaction parameter estimates are shown in Table 6.1. As the number of parameters in the table indicates, the heterogeneous coefficients model of equations (2.1)-(2.2) allows for great variation in the preferences of households for the characteristics of their housing choice. Each estimate describes a specific taste parameter associated with the household characteristic shown in the row for the housing choice characteristic shown in the column. The first entry in the table, for example, shows the interaction between household income and house price. The fact that this term is positive implies that, controlling for all of the other factors included in the model, an increase in a household's income increases its demand for housing versus other forms of consumption. While this initial table is helpful for gauging the sign of each interaction parameter and the precision with which it is estimated, the exact interpretation of these parameters in terms of a household's marginal willingness-to-pay for specific housing and neighborhood attributes depends on the results of the second stage of the estimation procedure.

Mean Marginal Willingness-to-Pay Measures

The second stage of the estimation procedure uses the estimated choice-specific constants as the dependent variable in the regression equation shown in equation (4.2). Given estimates from the first

stage of the estimation, it is computationally possible to estimate the second stage using a much larger sample of households and houses. Recall that the choice-specific constants that maximize the first stage likelihood function are those that ensure that the housing market clears. For the second stage of the estimation procedure, therefore, we expand the sample to 200,000 households and houses drawn randomly from the full sample, backing out the set of choice-specific constants that clears the housing market and estimating the choice-specific constant regression using this much larger sample. In estimating the regression, we instrument for house prices and neighborhood sociodemographic composition first using the standard set of instruments and then using the 'quasi-' optimal set of instruments developed in Section 5.

The parameter estimates for a number of specifications are reported in Appendix Table 2. In order to interpret these parameters more easily, it is helpful to transform the choice-specific constant regression in order to report a mean marginal willingness-to-pay measure for each housing and neighborhood attribute, α_{0X}/α_{0p}:

$$(6.1) \qquad p_h = \frac{1}{\alpha_{0p}}\delta_h + \frac{\alpha_{0X}}{\alpha_{0p}}X_h + \frac{1}{\alpha_{0p}}\xi_h$$

The results of this transformation are reported in Table 6.2 along with a standard hedonic price regression, i.e., an OLS regression of price on housing and neighborhood attributes. All marginal willingness-to-pay measures reported in Table 6.2 and subsequently in Table 6.3 are in terms of monthly rent (or imputed monthly rent when a house is owner-occupied).[53]

The first column of Table 6.2 reports mean marginal willingness-to-pay measures when OLS is used in estimating the choice-specific constant regression. Column 2 uses the 'quasi-' optimal set of instruments developed in Section 5, endogenizing price, Column 3 uses these same instruments adding neighborhood sociodemographic characteristics as endogenous variables. The specification reported in Column 3 is our preferred specification and is used in all subsequent analysis. Finally, the results for a standard hedonic price regression are shown in Column 4.

In examining the parameters estimates reported in Table 6.2, notice first the implied coefficient on **d** reported in the first row of each specification, which is the inverse of the estimated coefficient on price, $1/\alpha_{0p}$. This coefficient provides the link between the estimated mean marginal willingness-to-pay

[53] See Section 3 for details concerning the price variables.

measure reported in this table and the interaction parameters reported in Table 6.1. As the implied coefficient on delta decreases, the relative magnitude of the interaction parameters reported in Table 6.1 decreases, indicating in the limit (as $1/\alpha_{0p} \rightarrow 0$) that all households have the same preferences. Notice that in this extreme case, the hedonic price regression accurately returns mean marginal willingness-to-pay measures in the absence of heterogeneity in preferences, as the δ_h 'correction' simply drops out of equation (6.1).

When OLS is used to estimate the **d** regression, the estimate of α_{0p} is generally positively biased due to the correlation of price with unobserved housing/neighborhood quality. Consequently, the implied coefficient on δ_h in Column 1 is too large in magnitude, leading to an overstatement of the importance of the heterogeneity in tastes across households. When instruments are used for price (Columns 2-3), the implied coefficient on δ_h falls significantly and the estimated mean marginal willingness to pay measures move closer to the estimates of the hedonic price regression shown in the final column of the table. The proper estimation of α_{0p} thus provides the key to accurately measuring the mean and heterogeneity in household preferences for housing and neighborhood attributes, and thus it is in finding suitable instruments for price that we provide a full solution to the standard hedonic identification problem in the context of our model. Examining the remaining parameter estimates shown in Table 6.2, a clear pattern emerges with our preferred mean MWTP estimates obtained via IV estimation of the choice-specific constant regression falling in almost all cases between the extremes of the hedonic price regression and the OLS estimate of the choice-specific constant

By instrumenting for price, we control properly for the general heterogeneity in preferences as well as the differences in geographic preferences governed by the work location. The other major difference between our preferred estimates of Column 3 and the standard hedonic price regression of Column 4 is the fact that our preferred approach accounts directly for the potential correlation between unobserved housing/neighborhood amenities and the sociodemographic composition of a neighborhood. Because a simple comparison of our preferred specification and the hedonic price regression confounds this change with other changes, the impact of accounting for the correlation between sorting-dependent neighborhood amenities and unobserved characteristics can be seen most clearly by comparing the specification reported in Column 3, which instruments for both price and neighborhood sociodemographic characteristics, with that reported in column 2, which instruments only for price. Instrumenting for neighborhood sociodemographic characteristics increases the estimated coefficients on

Percent Black by about 5% and Percent Hispanic by about 50%, suggesting that these variables are conditionally negatively correlated with unobserved housing and neighborhood quality. It also reduces the estimated coefficients on Percent Asian by about 20% and average income by about 10%, suggesting that these variables are positively correlated with unobserved quality. Interestingly, the coefficient on Percent with College Degree also increases upon instrumenting for neighborhood sociodemographic characteristics, implying a negative correlation of this variable with unobserved housing and/or neighborhood quality conditional on the average income and racial composition of the neighborhood.

Heterogeneity in Marginal Willingness to Pay

Given the estimates from both stages of the estimation procedure, we are now in a position to interpret the full distribution of preferences. Table 6.3 reports a series of calculations that describe how each household attribute affects a household's marginal willingness-to-pay (MWTP) for each housing/neighborhood attribute. The first row of the table describes the mean MWTP for the change listed in the top of each column heading. For example, the mean MTWP for owner-occupied housing is $209, for an additional room is $132**,** for an additional standard deviation of school quality is $27, and for a standard deviation decrease in crime or pollution is $13 (all per month). The remaining rows of the table report the difference in MWTP associated with the difference in household characteristics listed in the row heading relative to the baseline. The second row, for example, reports the change in MWTP associated with a $10,000 increase in household income, while the fourth row summarizes the difference in MWTP for a Black versus White household. All of the measures reported for a change in a particular household characteristic in Table 6.3 are calculated holding all other household characteristics at the mean.

Starting with the second row of the table, the results indicate that with each $10,000 increase in household income, a household's MWTP increases by $40 per month for owner-occupied vs. rental housing, $32 for housing built in the 1980s versus pre-1960, $1.30 for a standard deviation increase in public school quality, $3.60 for a standard deviation decrease in pollution, $5.80 for a standard deviation decrease in the crime rate, and a result particularly interesting to urban economists, a zero income elasticity of demand for commuting distance. When income derives from dividends, interest, and capital gains - a good indication of wealth - our estimates present a slightly different picture of the relationship of financial well-being to demand for housing and neighborhood attributes. Comparing income from capital

sources vs. non-capital sources, an increase in capital income further increases demand for owner-occupied housing but not newer housing. An increase in capital income also increases distaste for commuting (while an increase in non-capital income does not) and especially the demand for housing near good public schools and in safer neighborhoods.

The next three rows report differences associated with household race, revealing strong segregating racial interactions in the residential location decision. These interactions take the expected form – with each race having a sizeable and significant increased demand for houses in neighborhoods with higher fractions of households of the same race. In reading the numbers associated with racial interactions, it is important to keep in mind that these numbers represent the difference between willingnesses to pay of households of the race specified in the row heading and households of other races for the change in racial composition shown in the row heading. Thus, the $111 per month that characterizes the difference between the MWTP of Hispanic versus non-Hispanic households for a 10 percentage point increase in the fraction of Hispanic versus White neighbors reflects the sum of what a Hispanic household is willing to pay for this increase and what non-Hispanic households are willing to pay for a 10 percent increase in the fraction of White versus Hispanic neighbors. As discussed at length in Section 5, these differences in MWTP measures are all that are identified in the data – that is, it is impossible to distinguish whether the estimated preference parameters associated with neighborhood racial compositions arise because of direct preferences or discrimination in the housing market. In either case, the size of the differences in the marginal willingness to pay measures reported in Table 6.3 implies strong, segregating racial interactions in the residential location decision. The results also indicate strong differences in the willingness of households of different races to pay for housing characteristics, with Asian households in particular willing to pay a great deal for owner-occupied and newer (but not larger) houses.

It is also important to point out that the instrumental variables strategy used in the estimation of the model's parameters addresses the correlation of neighborhood racial composition with any part of unobserved quality valued by households of all races. Our estimation strategy does not, however, address the fact that households of different races may place different values on some unobserved neighborhood attributes. A recent immigrant from China, for example, may gain a great deal of utility from living near Chinese groceries, restaurants, and other shops, while a recent immigrant from Mexico would not gain the same amount of utility. Hence, the estimate differences in marginal willingness-to-pay measures reported

here capture both direct preferences for the race of one's neighbors along with preferences for any other race-specific unobservable neighborhood attributes that come along with these neighbors. The estimates do, however, control for the correlation of overall unobserved house and neighborhood quality with neighborhood sociodemographic characteristics.

An increase in the householder's educational attainment raises demand for school quality and especially highly educated neighbors and also increases distaste for commuting. An increase in the number of adults in a household decreases demand for home-ownership and newer housing, while an increase in the number of children is estimated to have almost no net effect on housing demand. Demand for house size increases with both additional adults and children, but the effect is stronger when an adult is added to the household. In general, an increase in household size also leads a household to live in more densely populated neighborhoods with higher fractions of minority households. When the increase in household size is the result of adding a child, however, these effects are diminished and even reversed in the case of population density. In general, the estimated parameters provide a reasonably complete picture of the preference structure that underlies the equilibrium in the Bay Area housing market.

7 GENERAL EQUILIBRIUM SIMULATIONS

7.1 Simulation Basics

We now use the estimated utility function parameters to conduct a series of general equilibrium simulations designed to explore the causes and consequences of residential segregation on the basis of race. Each of these simulations begins by changing a key primitive of the model; in one of the simulations, for example, we give all households a random draw from the empirical income/wealth distribution of the Bay Area sample. In this new counterfactual environment, we calculate a new equilibrium for the model, the conditions for which are the same as those outlined in Section 2. Specifically, an equilibrium consists of a set of location decisions for each household and a set of housing prices such that (i) each household's decision is optimal given the decisions of all other households, and (ii) the set of housing prices clears the market. To calculate a new equilibrium, then, we must find a set of housing prices that clears the market and find a fixed point of the neighborhood sociodemographic mapping defined in equation (2.11). That is, given a set of sociodemographic characteristics for each

neighborhood and the vector of market clearing prices, the household location decisions predicted by the model must aggregate up to these same sociodemographic characteristics for each neighborhood.[54]

The basic structure of the simulations consists of a loop within a loop. The outer loop calculates the sociodemographic composition of each neighborhood, given a set of prices and an initial sociodemographic composition of each neighborhood. The inner loop calculates the unique set of prices that clears the housing market given an initial sociodemographic composition for each neighborhood. Thus for any change in the primitives of the model, we first calculate a new set of prices that clears the market; as discussed in Section 2, Berry (1994) ensures that there is a unique set of market clearing prices. Using these new prices and the initial sociodemographic composition of each neighborhood, we then calculate the probability that each household makes each housing choice, and aggregating these choices to the neighborhood level, calculate the predicted sociodemographic composition of each neighborhood. We then replace the initial neighborhood sociodemographic measures with these new measures and start the loop again – i.e., calculate a new set of market clearing prices with these updated neighborhood sociodemographic measures. We continue this process until the neighborhood sociodemographic measures converge. The set of household location decisions corresponding to these new measures along with the vector of housing prices that clears the market then represents the new equilibrium.

Summary of the Calculation of the New Equilibrium[55]

1. Incorporate change to the primitives of the model corresponding to simulation, start with initial measures of sociodemographic characteristics for each neighborhood.

2. Calculate unique vector of housing prices that clears the housing market (i.e., ensures that a total of one household chooses each house).

3. Using new vector of housing prices, calculate housing choice probabilities for each household.

[54] Note that, as in the development of the equilibrium definition and properties in Section 2, all of the calculations in this section of the paper are based on the aggregation of each household's predicted probabilities over the set of available houses rather than a single location decision. In this way, each household used in the simulations stands in for a continuum of households with different idiosyncratic preferences. See Section 2 for a more extensive discussion of this issue.

[55] While this procedure always converges to an equilibrium, the model does not guarantee that this equilibrium is generically unique. In all of the calculations presented in this paper, we report results that start from the initial equilibrium and follow the procedure summarized here. Experimenting with other starting values reasonably close to the initial equilibrium led to the same new equilibrium each time.

4. Aggregating these probabilities to the neighborhood level, calculate new sociodemographic characteristics for each neighborhood. Replace existing measures of neighborhood sociodemographics in utility function with these new measures.

5. Repeat steps (2)-(4) until the sociodemographic characteristics calculated in (4) converge.

It is important to point out that because the model itself does not perfectly predict the housing choices that individuals make, the neighborhood sociodemographic measures initially predicted by model, $\overline{Z}_n^{PREDICT}$, will not match the actual sociodemographic characteristics of each neighborhood, $\overline{Z}_n^{ACTUAL}$. Consequently, before calculating the new equilibrium for any simulation we first solve for the initial prediction error associated with each neighborhood *n*:

$$(7.1) \qquad w_n = \overline{Z}_n^{ACTUAL} - \overline{Z}_n^{PREDICT}$$

In solving for the new equilibrium, we add this initial prediction error w_n to the sociodemographic measures calculated in each iteration before substituting these measures back into the utility function.

Adjusting Crime Rates and Average Test Scores

Because some neighborhood amenities, such as crime rates and school quality, depend in part on the sociodemographic composition of the neighborhood, it is natural to expect these neighborhood characteristics to adjust as part of the movement to a new sorting equilibrium. Getting precise measures of the impact of neighborhood sociodemographic characteristics on crime rates and test scores is, of course, an exceedingly difficult exercise, as selection problems abound. For example, an OLS regression of crime rates on neighborhood sociodemographic characteristics almost certainly overstates the role of these characteristics in producing crime as it ignores the fact that households sort non-randomly across neighborhoods. As a result, we take an approach that seeks to provide bounds for the characteristics of the new equilibrium that results for each of our simulations. For one bound, we calculate a new equilibrium without allowing crime rates and average test scores in each neighborhood to adjust with the changing neighborhood sociodemographic compositions. For the other bound, we calculate a new equilibrium, adjusting crime rates and average test scores in each neighborhood according the adjustments implied by an OLS regression of the crime rate and average test score on neighborhood sociodemographic composition. These simple production functions are shown in Appendix Table 3, with all of the variables

constructed to have mean zero and standard deviation one. The first bound will understate the impact of sociodemographic shifts on the implied crime rate and average test score in each neighborhood, while the second bound will tend to overstate the impact of these sociodemographic shifts. As the results below indicate, these bounds provide a reasonable range for the predictions from our simulations.

7.2 Observed Segregation Patterns

To provide a benchmark for our simulation results, we begin by characterizing the patterns of racial segregation in the Bay Area. Figure 7.1 provides a map of the central portion of the Bay Area – a region that focuses on San Francisco and parts of Marin County and the East Bay. The map is divided into Census block groups and the shading indicates block groups that have a majority of Asian, Black, or Hispanic households and block groups that have more than 80% White households.[56] For comparison, the racial composition of the full Bay Area is 12.3 percent Asian, 8.8 percent Black, 11.2 percent Hispanic, and 67.7 White. As the map indicates, while Black households make up only 9 percent of the Bay Area population, a large number of block groups have a racial composition that contains at least 50 percent black households. Predominantly White populations are clustered in Marin County to the north and the more suburban areas of other counties while majority Asian block groups are concentrated in San Francisco and Oakland. Note that while there are more Hispanic households than Black households in the Bay Area, there are far fewer predominantly Hispanic Census block groups.

We describe the general pattern of segregation in the Bay Area by examining exposure rates. Specifically, we define dummy variables, r^i_j, that take the value one if household i is of race j, and zero otherwise. For a particular neighborhood definition, we calculate the fractions of households in each of the four racial categories that reside in the same neighborhood as a given household; let the upper-case notation R^i_k signify the fraction of households of race k in household i's neighborhood. By averaging these neighborhood measures over all households of a given race, we construct measures of the average neighborhood racial composition for households of that race. Put another way, we construct measures of the average exposure, $E(r_j,R_k)$, of households of a race j to households of race k:

$$E(r_j, R_j) = \frac{\sum_i r_j^i R_k^i}{\sum_i r_j^i} \tag{7.2}$$

[56] Figure 7.1 is derived from information based on the public use Census data set.

A variety of segregation measures are available,[57] and while no single measure is perfect, we choose to work with the exposure rate measures because they are easy to interpret and can be decomposed in a variety of meaningful ways. It is straightforward, for example, to calculate exposure rates for various subsets of households within each broad category (*e.g.* households of the same race but with different income levels), rates that must as a matter of necessity aggregate back up to the average exposure rate for the whole group. Exposure rates are also convenient for exploring segregation patterns across multiple categories of race and ethnicity.

A variety of different definitions of 'neighborhood' and thus a variety of different ways of defining R^i_k are available to us. As noted in Section 3, a neighborhood can be defined as Census geographic area or alternatively all households within a given radius surrounding a particular house.[58] Throughout our analysis we report exposure rate measures calculated for Census block groups. Table 7.1 reports measured exposure rates in three panels: Panel A – full Bay Area sample, Panel B – sample of 10,000 used in the first-stage of the estimation procedure and the simulations, and Panel C – the predictions of the model at the estimated parameters.

Reading across the first row of the Panel A, these measures imply that Asian households in the Bay Area live in Census block groups that have on average 23 percent Asian, 8 percent Black, 12 percent Hispanic, and 57 percent White households. Comparing these numbers to the population of the Bay Area as a whole - 12 percent Asian, 9 percent Black, 11 percent Hispanic, and 68 percent White - it is immediately obvious that Asian households typically live in Census block groups with approximately twice the fraction of Asian households than would be found if they were uniformly distributed across the

[57] Among the alternatives (see Massey and Denton (1989) for a description), entropy measures summarize the degree to which the racial distributions of neighborhoods within a region differ from the region's overall racial distribution, entropy being maximized for the region when the racial distributions at lower levels of aggregation are the same as that for the region overall (see, for instance, Harsman and Quigley (1995)). Dissimilarity indices, ranging between zero and one, provide information about the residential concentration of one race relative to others, specifically the share of one population that would need to move in order for the races in a region to be evenly distributed (see Cutler *et al.* (1999) for a definition). Borjas (1998) makes use of individual data, constructing a measure of segregation that takes the value one if the proportion of the individual's own ethnic group in the neighborhood is more than twice the proportion that would be expected under random assignment of individuals, an approach that loses information about the precise extent of local segregation.

[58] We considered both methods of defining neighborhoods, as the first corresponds to the approach most commonly used in the literature and the second might provide a better approximation to a household's neighborhood in certain cases. The usual method of looking for segregation patterns across well-defined geographic units like Census tracts might give misleading results if, for example, households are sorted within the tract so that they match up with the households in neighboring tracts. However, analyses examining neighborhoods defined as observations falling within 0.25, 0.5 and 1-mile radii of a Census block produced results similar to those for Census block groups (.25 miles) and tracts (.5 and 1-mile radii).

Bay Area. In this case, the additional fraction of Asian households in Census block groups in which Asian households reside is almost exactly offset by a reduction in the fraction of White households in these neighborhoods.[59]

Examining the remaining exposure measures at the Census block group level, a clear pattern emerges, with households of each race residing with households from the same race in proportions significantly higher than their proportions for the full Bay Area. Not surprisingly given the patterns in Figure 7.1, the most striking example of the 'over-exposure' of households to other households of the same race occurs for Black households, who on average live in neighborhoods that have almost 5 times the fraction of Black households as the Bay Area and over 8 times the fraction of Black households as the neighborhoods in which White households typically live. White households on average live in blocks with a lower proportion of each other race than would be found if all racial groups were evenly spread across block groups. The 'under-representation' of Black households (5 percent vs. 9 percent) in neighborhoods in which White households reside, however, is more sizeable than that of Asian (10 percent vs. 12 percent) and Hispanic (9 percent vs. 11 percent) households.

Before examining the results of our simulations, it is important to examine how well the model predicts segregation patterns. Panels B of Table 7.1 reveals that the sample used in the estimation appears to be slightly more segregated than the full sample, while Panel C indicates the model tends to predict location patterns that are slightly less segregated, especially for black households. Overall, the model gives rise to patterns that fit the segregation pattern of the Bay Area well, despite the fact that the estimation procedure does not fit neighborhood racial compositions directly.

7.3 Exploring the Causes and Consequences of Segregation

In presenting the results of our simulations, we report both partial and general equilibrium predictions in order to provide insight into the mechanisms that drive the general equilibrium predictions of the model. The *partial equilibrium predictions* are based on allowing each household to adjust its location decision (i.e., its probability distribution over the set of houses) while holding the price of each house and the sociodemographic composition of each neighborhood fixed at the values observed in the

[59] It is worth noting that other segregation measures such as dissimilarity indices would miss the fact that the increased exposure of typical Asian, Black, and Hispanic households to other households of the same race is almost completely offset by a decreased exposure to White households.

data. The *general equilibrium predictions* allow neighborhood sociodemographic characteristics and prices to adjust according to the five-step procedure described in Section 7.1

Eliminating Racial Differences in Income and Wealth

First, we consider the impact of eliminating racial differences in both non-capital income and capital income, which will assume throughout this discussion to be a good proxy for household wealth.[60] Operationally, this first simulation is conducted by replacing each household's actual income and wealth with a random draw from the empirical income/wealth distribution for the Bay Area. Table 7.2 summarizes the distributions of capital and non-capital income as well as other household characteristics by race, indicating that Black and Hispanic households in the Bay Area have much lower levels of income and wealth than Asian and especially White households.

Table 7.3 summarizes the impact of eliminating racial differences in income and wealth on segregation patterns by reporting three sets of exposure rate measures. Panel A repeats the measures based on the predictions of the model reported in Table 7.1. Panels B and C report the partial and general equilibrium predictions respectively. The partial equilibrium predictions of the model imply a reduction in segregation of 15-20% for Black, Hispanic, and White households (as measured by the over-exposure to households of the same race) and of 2% for Asian households. These predictions mirror those generally found in the previous literature, which indicate that differences in income explain only a modest amount of the observed pattern of racial segregation.[61] In essence, the partial equilibrium predictions reflect the fact that eliminating racial differences in income and wealth leads to more similar demands for housing and neighborhood attributes across race. The partial equilibrium predictions do not move even further in the direction of reducing segregation primarily because racial interactions in the housing market dampen the propensity of middle- and upper-income Black and Hispanic household to move into houses in what had been middle- and upper-income neighborhoods with high fractions of White households.

In calculating the general equilibrium predictions reported in Panel C of Table 7.3, the sociodemographic characteristics and housing prices in each neighborhood are allowed to adjust with the change in the distribution of income and wealth across race. In direct contrast to the partial equilibrium

[60] Note that even though we do not control directly for property wealth in our analysis, the estimated coefficients associated with income form capital sources will do a good job of capturing a wealth effect as long as property and non-property wealth are sufficiently correlated.

[61] See, for example, Bayer, McMillan, and Rueben (2002) and, for a more complete summary of results, Massey and Denton (1993).

predictions, the general equilibrium predictions of the model imply a significant increase in the segregation of each race, increasing the over-exposure of households of each race to households of the same race by 40%-110%.

In order to understand why this overall pattern of increased segregation emerges, Table 7.4 reports the exposure of households to others of the same race broken out by income category based on (i) the predictions of the model at the estimated parameters – the pre-simulation equilibrium, and (ii) the general equilibrium predictions following the equalization of income and wealth across race. Results are reported for households at the bottom of the income distribution ($0-$8500) and households at the top of the income distribution ($65,000+).[62] The results indicate increases in segregation across the board but especially for upper-income Asian and Black households. The average exposure of upper-income Black households to other Black households increases from 23% to 36%, while the average exposure of upper-income Asian households to other Asian households increases from 20% to 35%. These increases are roughly double the increased exposure of lower-income Black and Asian households to households of the same race.

The fundamental effect of eliminating racial differences in income and wealth is to more evenly spread households of each race across the income distribution, thereby allowing more sorting on the basis of race to occur at each income level. This is especially apparent at the top end of the income distribution, where the elimination of racial differences in income and wealth significantly increases the number of Hispanic and especially Black households with high levels of income and wealth. This leads to the formation of upper-income, segregated Black and Hispanic neighborhoods, neighborhoods that are not prevalent in the current Bay Area equilibrium due primarily to the small number of households in these race-income categories. Given the increased segregation of upper-income Black and Hispanic households, it is not surprising that upper-income Asian and White households also experience an increase in own-race exposure.

For lower-income households, eliminating racial differences in income and wealth also has the effect of substantially increasing segregation. Black households in the Bay Area are over-represented in the lower portions of the income distribution. For example, while 20 percent of the Black population falls into the bottom quintile of the Black income distribution (by definition), only 6 percent of the White population, 9 percent of the Asian population, and 10 percent of the Hispanic population have incomes

[62] Results are reported using the Black income distribution in order to ensure that enough Black households are included in the upper quintile, a requirement for the disclosure of the Census data.

this low. Thus, eliminating racial differences in income and wealth has the effect of significantly increasing the fraction of Asian, Hispanic, and White households among the very poor. This more even racial distribution of the poor has an effect similar to that of upper-income households, leading to more pronounced segregation of lower-income members of each race. Notice, for example, that the biggest change in the exposure to other races for poor Black households is the decreased exposure to Hispanic and Asian households post-simulation.

Returning to the partial equilibrium results, the reason for the difference in predictions is now clear. Holding the racial composition of each neighborhood constant in the partial equilibrium case, the elimination of racial differences in income gives households of each race much more similar preferences (excepting the racial interactions in the location decision), leading, for example, to a modest movement of upper-income Black and Hispanic households into what had been predominantly White, upper-income neighborhoods. The general equilibrium simulations carry this further, adjusting the racial composition of these neighborhoods and forming new predicted location decisions. When predictions are then based on these new racial compositions, even more upper-income Black and Hispanic households move into these higher quality neighborhoods. As the racial mix of the upper-income neighborhoods evens out, the segregating, racial interactions implicit in the parameter estimates for households of all income levels lead to the segregation of upper-income as well as lower-income households.

The general equilibrium results of additional simulations that equalized education across race are similar to the income results presented here. We report the predicted exposure rates for these simulations in Appendix Table 4. Taken together, the results of these simulations imply that the complete elimination of racial differences in socioeconomic characteristics is not likely to lead to dramatic reduction in segregation, unless the elimination of racial differences in income, wealth, and education also changes preferences, which are assumed to be a primitive in our model. In the current environment, the estimated parameters (and the model allows for flexibility on this) give rise to strong segregating, racial interactions at all levels of education and income and, consequently, equalizing income (education) across race has the effect of creating a critical mass of individuals of each race in each part of the income (education) distribution, leading to additional racial sorting in all parts of the distribution.

Table 7.5 further clarifies the general equilibrium impact of eliminating racial differences in income and wealth, reporting a number of consumption measures for the pre-simulation predictions of the model, and the general equilibrium predictions of the model following the equalization of income and

wealth across race. The rows of the table report the average monthly house price,[63] home-ownership rate, average commuting distance, and the average consumption of house size, school quality, and crime for each racial group. The equalization of income and wealth across race leads, as one would expect, to an overall convergence in the consumption of all housing and neighborhood attributes as the gaps in the average house price paid fall across the board, although never to zero. In particular, the Black-White gap in housing prices closes from $360 to $120 and the Hispanic-White gap declines form $240 to $110. Interestingly, commuting distances decline for all households suggesting that with a more equalized distribution of income across race, households of each race have an easier time finding a desirable neighborhood near their place-of-work.[64] The gap in home-ownership rates between Black and White households falls from 24 percentage points to 10, while the gap between Hispanic and White households falls from 19 percentage points to 8. The gap in house size consumption between White and Black households falls by more than 75 percent, while the White-Hispanic gap falls about 45 percent.

The last four rows of Table 7.5 report results for the consumption of school quality and crime based on the two different methods described in Section 7.1. The numbers reported in the rows marked 'unadjusted' are based on a calculation of a new equilibrium that does not adjust the crime rates and average test scores in each neighborhood with the changing neighborhood sociodemographic compositions. The numbers reported in the rows marked 'adjusted' are based on a calculation of a new equilibrium that adjusts crime rates and average test scores in each neighborhood according the adjustments implied by an OLS regression of the crime rate and average test score on neighborhood sociodemographic composition.[65] The results suggest a decrease in the Black-White school quality consumption gap of 0%-33% and a decrease in the Hispanic-White school quality consumption gap of 30%-55%. Similarly, the results suggest a decrease in the Black-White crime gap of 0-45% and a decrease in the Hispanic-White crime gap of 10%-60%. In sum, as expected, the elimination of racial differences in income and wealth dramatically decreases the observed gaps in the consumption of housing and other local goods, the degree to which this is true for local public goods depending on the extent to

[63] Note prices are normalized to have same mean pre- and post-experiment. This normalization does not affect location decisions due to the linear form of the indirect utility function.

[64] It is important to note that the work location of each household is held fixed in this experiment. We consider the randomization of job locations in a separate simulation below.

[65] While we could report the full set of results for both assumptions concerning how crime and school quality adjust, we report the results for only the 'unadjusted' experiment to conserve space in the paper. The primary impact that this distinction makes on the results comes in the predictions concerning the consumption of school quality and crime reported here.

which crime and school quality are purely functions of neighborhood sociodemographic characteristics, including race.

Eliminating Racial Interactions in the Location Decision

We next consider the general equilibrium predictions of counterfactual simulations that eliminate all racial interactions in the location decisions – that is, setting all of the utility parameters that govern preferences for neighborhood racial characteristics to zero. Table 7.6 reports three sets of exposure rate measures. Panel A repeats those based on the pre-experiment predictions of the model. Panel B reports those that arise with the elimination of racial interactions and Panel C reports the results of a simulation that eliminates both racial interactions *and* racial differences in income and wealth. Two things are immediately apparent from these exposure rate measures. First, the elimination of racial interactions has an enormous effect in reducing segregation, reducing the own-race 'overexposure' of Asian households to other Asian households by 86 percent, of Black households by 85 percent, of Hispanic household by 78 percent, and White households by 83 percent. Keeping in mind that the racial interactions that we estimate combine the effects of (i) discrimination in the housing market (e.g., discrimination against recent immigrants from China on the part of household of other races), (ii) direct preferences for the race of one's neighbors (e.g., preferences on the part of a recent immigrant from China to live with other Chinese immigrants), and (iii) preferences for race-specific portions of unobserved neighborhood quality (e.g., preferences for Chinese groceries which are located in neighborhoods with a high fraction of Chinese residents), the magnitudes of these reductions leave little explanatory role for the other factors included in the model. The second notable result in Table 7.6 is that eliminating racial differences in income and wealth reduces the segregation of three of the four races once racial interactions have been eliminated, highlighting the importance of racial interactions in driving the additional racial sorting at each income level following the elimination of racial differences in income and wealth in the results reported in Table 7.3-7.5.

The elimination of racial interactions in the location decision also has important consequences for the consumption of households of each race. Table 7.7 follows the same format as Table 7.5 reporting predictions concerning the average monthly house price, home-ownership rate, average commuting distance, and the average consumption of house size, school quality, and crime for each racial group. As in the random income-wealth simulation, the elimination of racial interactions leads to an overall

reduction in commuting distances. In this case, without needing to satisfy preferences in a racial dimension, households are able to more easily find suitable locations in other dimensions. Eliminating racial interactions has little overall effect on the consumption of housing, as measured by tenure and house size. The most striking results for this simulation, however, pertain to the consumption of local public goods. In this case, the Black-White gap in school quality consumption is reduced by 40%-60% and the Hispanic-White gap by 37%-50%. Likewise, the Black-White gap in exposure to crime is reduced by 50%-70% and the Hispanic-White gap is reduced by 40%-60%. Again, the ranges for these estimates reflect the results of two simulations that differ in the manner school quality and crime are adjusted with the changing neighborhood sociodemographic composition. The striking feature of these results is that these substantial reductions in racial differences in the consumption come about simply by eliminating racial interactions in the housing market - that is, without changing household income, wealth, education, etc. In fact, the impact of eliminating racial interactions on local public good consumptions is more substantial than the effect of eliminating racial differences in income and wealth!

To provide more perspective on these results, Table 7.8 breaks out the results of Table 7.7 by income, again reporting results for high and low income households. As the upper panel of the table makes clear, in the actual Bay Area equilibrium, racial interactions in the location decision (whether they result from preferences or discrimination) lead the relatively small fraction of upper-income Black and Hispanic households to consume significantly lower levels of housing and local goods in order to live in more segregated neighborhoods. In fact, in the current Bay Area, the racial gaps in housing consumption are more dramatic for upper versus lower-income households. In this context, the elimination of racial interactions causes upper-income Black and Hispanic households to move more readily into what had been upper-income, predominantly White neighborhoods, greatly increasing the consumption of housing and especially school quality and crime by these households.

The impact of eliminating racial interactions for lower-income households is more nuanced, leading to an *increase* in housing consumption gaps and a substantial decrease in racial differences in the consumption of local public goods. In general, the elimination of racial interactions leads to increased income stratification, therefore isolating the poorest households in the Bay Area to an even greater extent, albeit in a racially integrated way. This has the effect of reducing housing consumption (as measured by house size) for households of *each* race, but especially Black households, who have the lowest level of income even among households in the lowest quintile. At the same time, the increased integration of

households dramatically reduces the substantial gaps in the consumption of local public goods that are present in the current Bay Area equilibrium.

Geography

One final table helps to illustrate the importance of racial interactions in the housing market. Table 7.9 reports the results of two simulations: the first eliminates racial differences in job locations among working households, while the second eliminates racial differences in job locations and increases the disutility of commuting ten-fold. To eliminate racial differences in job locations, we simply assign each working household a new job location drawn at random from the distribution of employment in the Bay Area. Importantly, this experiment only changes the geography of the location decision, not a household's income. As the top panel of Table 7.9 illustrates, eliminating racial differences in job locations actually leads to an increase in the segregation of each race in equilibrium! This result is especially striking because one might have thought the geographic distribution of employment is also racially concentrated with the Bay Area. As it turns out, however, randomly distributing the job locations of households of each race throughout the Bay Area leads to a substantial increase in segregation. This result occurs because the current racial distribution of employment in the Bay Area leaves many sub-regions of the Bay Area with a limited number of Asian, Black, or Hispanic households. In these sub-regions, households of these racial groups have little choice but to live in neighborhoods with relatively few households of the same race. With a more even spatial distribution of jobs, however, racially segregated neighborhoods form throughout the metropolitan area, significantly increasing the overall level of racial sorting.[66]

8 CONCLUSION

This paper makes two related contributions to the literature. First, it develops a new framework for analyzing the sorting of households across the neighborhoods of a large metropolitan area, taking equilibrium considerations explicitly into account. The framework's discrete choice model enables

[66] The lower panel of Table 7.9 reports results for an analogous experiment that also increases the disutility of commuting ten-fold. In this case, the segregation of households of each race is reduced as households put more weight on locations nearer their place-of-work. This additional simulation illustrates an important point, namely that the results of the job randomization simulation are highly dependent on the strength of the estimated racial interactions relative to the disutility of commuting. In the limit, as the disutility of commuting goes to – , the randomization of job locations across race would lead to perfect integration.

household preferences to be specified in a very flexible way, preferences that vary with a wide range of house, neighborhood, and household characteristics. Among the choice characteristics, we allow household utility to depend on choice-specific unobservables – unobservable, that is, to the econometrician – forcing us to confront an important endogeneity problem: house prices and neighborhood compositions, over which households have preferences, will be correlated with unobserved housing and neighborhood quality. To identify preferences in the face of this, we provide a general instrumental variables strategy and demonstrate that instruments arise naturally out of the logic of the housing market equilibrium. Using an extremely rich dataset based on restricted access Census micro-data, the resulting parameter estimates provide the most comprehensive view in the literature to date of household preferences over housing and neighborhood characteristics, and of the heterogeneity in these preferences. In general, this framework can be readily applied to answering a variety of economic and policy questions in urban economics, local public finance, and the economics of education where a consideration of the housing market equilibrium and the non-random sorting of households across locations play an important role.

The second main contribution of the paper is to use these estimated parameters along with the equilibrium model of sorting to explore a series of issues related to segregation in the housing market. Despite controlling for the likely correlation of neighborhood racial composition and unobserved housing and neighborhood quality, we estimate strong segregating racial interactions in the location decision, and while it is not possible to distinguish whether these preferences are a manifestation of discrimination or strong tastes for self-segregation, it is very clear that racial factors play a significant role in the residential location decision of many households. Using the estimated preferences, we then conduct a series of general equilibrium counterfactual simulations designed to shed new light on the causes and consequences of residential segregation.

A number of the findings from our general equilibrium analysis are striking. First, removing racial differences in income and wealth or education leads to a marked *increase* in the segregation of each race. Here, it is noteworthy that partial equilibrium calculations using the model estimates produce the opposite conclusion, failing to account for the compounding effects of adjustments in neighborhood socio-demographics. The key insight that the general equilibrium approach affords is that given the combination of strong, segregating racial interactions and the relatively small fractions of Asian, Black, and Hispanic households in the Bay Area, the elimination of racial differences in income/wealth (or,

education) spreads households in these racial groups much more evenly across the income distribution, causing more racial sorting to occur at all points in the distribution – e.g., leading to the formation of wealthy, segregated Black and Hispanic neighborhoods. Moreover, contrary to research that points to the geographic distribution of employment as an important segregating force (e.g., Bajari and Kahn (2001)), we find that randomizing job locations throughout the Bay Area would in fact lead to increases in segregation. As the finding indicates, the current employment distribution combined with the cost of commuting tends to lead different types of household to live together; spreading jobs out would permit greater segregation, in line with self-segregating preferences. It is worth emphasizing that this result again is due in no small part to the relatively small fraction of Asian, Black, and Hispanic households in the Bay Area.

Eliminating the portion of the preference structure relating to preferences for neighborhood race almost entirely eliminates segregation on the basis of race, leaving little explanatory role for factors related to the other features included in our analysis. The elimination of racial interactions also provides clear evidence that certain household types under-consume housing and school quality in order to live in communities with the racial composition that they prefer. For example, we find that upper income Black and Hispanic households would consume significantly more school quality in the absence of any preferences for neighborhood race. This finding has important implications for policy. It indicates that decoupling housing choice from public school assignment could lead to improvements in education quality for a significant subset of minority households, with all the associated benefits that would bring.

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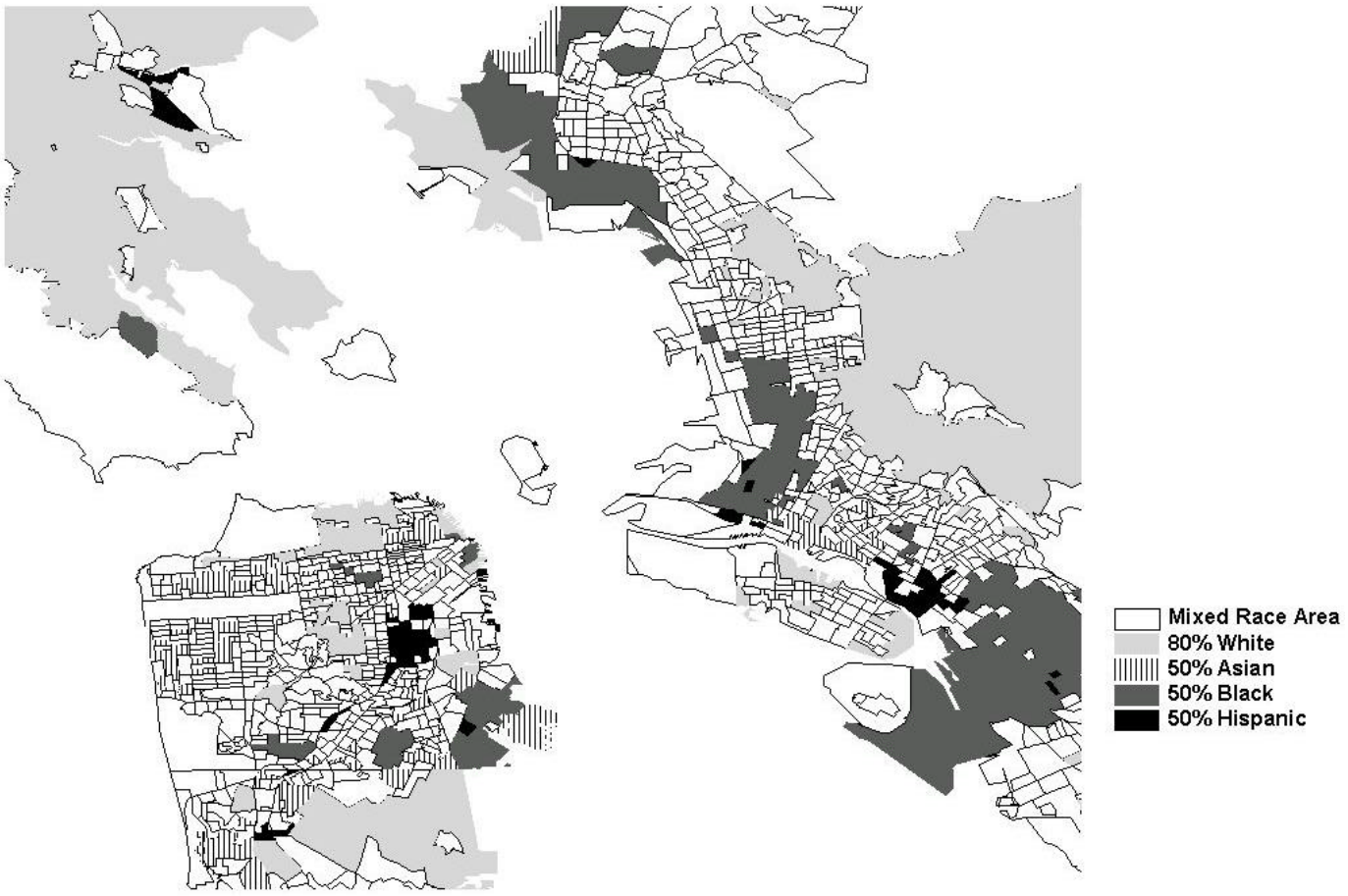
Figure 7.1: Segregation Patterns in the Bay Area



Technical Appendix

Proof of Proposition 2.1:[67] Following the assumptions of Proposition 2.1, consider a utility specification that is a linear, decreasing function of p_h:

$$(A.1) \qquad V_h^i = W_h^i(Z^i, X_h, \boldsymbol{x}_h) - \boldsymbol{a}_p^i p_h + \boldsymbol{e}_h^i = W_h^i - \boldsymbol{a}_p^i p_h + \boldsymbol{e}_h^i$$

If $\mathbf{e}$ is drawn from a continuous distribution, the probability $P^i{}_h$ that household i chooses house h as:

$$(A.2) \qquad P_h^i = f_h(Z^i, \mathbf{X}, \mathbf{p}, \mathbf{x})$$

is continuous and differentiable in $\mathbf{p}$ with derivatives that obey the following strict inequalities: $\partial P_h^i / \partial p_h < 0$ and $\partial P_h^i / \partial p_k > 0,\ k \neq h$, if $-\boldsymbol{a}_p^i$ is negative for each household i. Aggregating these probabilities over all households yields the predicted number of households that choose each house h, $\hat{N}_h$:

$$(A.3) \qquad \hat{N}_h = m \bullet \sum_i P_h^i$$

Given the properties of $P_h{}^i$ just described, $\hat{N}_h$ is also continuous and differentiable in $\mathbf{p}$ with derivatives that obey the following strict inequalities: $\partial \hat{N}_h / \partial p_h < 0$ and $\partial \hat{N}_h / \partial p_k > 0,\ k \neq h$. In order for the housing market to clear, the number of households choosing each house h must equal the measure of the continuum of houses that each observed house represents:

$$(A.4) \qquad \hat{N}_h = m, \quad \forall h$$

Also note that for any finite values of $\{p_k, k \neq h\}$, $\hat{N}_h$ approaches arbitrarily close to zero as p_h goes to $+\infty$, while $\hat{N}_h$ approach arbitrarily close to mN as p_h approaches $-\infty$.

Holding the price of one house fixed (without loss of generality set $p_0 = 0$), we will show that a unique vector of prices clears the market, i.e., that a unique vector $\mathbf{p} = \hat{N}^{-1}(m)$ exists. We begin by defining the element-by-element inverse $r_h(\mathbf{p}, \hat{N}_h)$. This function is defined as the price of house h such that the predicted value $\hat{N}_h$ exactly equals m That is, $\mathbf{r}$ is implicitly defined as:

$$(A.5) \qquad \hat{N}_h(p_1, p_2, ..., r_h(\mathbf{p}, \hat{N}_h), ..., p_H) = m \quad \forall h$$

Given the properties of the function $\hat{N}_h$ defined in (A.3), this element-by-element inverse exists and is continuous and differentiable in $\mathbf{p}$. Note that r_h is strictly increasing in p_k and does not depend on p_h. Also define the vector values $\mathbf{r} = (r_1, \ldots, r_N)$.

The element-by-element inverse allows us to transform the problem of solving for the vector inverse into a fixed-point problem, for a vector $\mathbf{p}$ satisfies equation (A.4) if and only if $\mathbf{p} = \mathbf{r}(\mathbf{p}, \hat{\mathbf{N}})$. The method of proof is to use a slight variant of Brouwer's fixed-point theorem to prove existence of a fixed point of the element-by-element inverse. It is then necessary to show that there cannot be two such fixed points.

To establish existence, first hold $p_0 = 0$ and note that $r_h(\mathbf{p}, \hat{N}_h)$ has an upper bound. This upper bound is $r_h(\mathbf{p}', \hat{N}_h)$ with $\mathbf{p}'$ set equal to any vector in $\mathbf{R}^{N+1}$ such that $p_k = +\infty$ for $k \neq (h, 0)$. Define $\bar{p}$ as the largest values

[67] This proof follows directly the structure of the proof that appears in the technical appendix of Berry (1994). We simply modify it here for our problem.

across houses of these upper bounds. There is no lower bound for p_h, but the following lemma allows one to establish existence in the absence of a lower bound.

Lemma. There is a value $\underline{p}$, with the property that if one element of **p**, say p_h, is lower than $\underline{p}$, then there is a house k such that $r_k(\mathbf{p}, \hat{\mathbf{N}}) > p_k$.

Proof of Lemma. To construct $\underline{p}$, again set $p_k = +\infty$, $\forall\, k \neq (h,0)$. Then define $\underline{p}_h$ as the value of p_h that sets $\hat{N}_0 = m$. Define $\underline{p}$ as any value lower than the minimum of the $\underline{p}_h$. Now, if for the vector **p** there is an element h such that $p_h < \underline{p}$, then $\hat{N}_0(\mathbf{p}) > m$, which implies $\sum_{h=1}^{N} \hat{N}_h(\mathbf{p}) < (N-1) \bullet m$, so there is at least one element k with $\hat{N}_k(\mathbf{p}) < m$. For this k, $r_k(\mathbf{p}, \hat{\mathbf{N}}) > p_k$. ***Q.E.D.***

Now define a new function that is a truncated version of r_h: $\tilde{r}_h(\mathbf{p}, \hat{\mathbf{N}}) = \max\{r_h(\mathbf{p}, \hat{\mathbf{N}}), \underline{p}\}$. Clearly $\tilde{\mathbf{r}}(\mathbf{p}, \hat{\mathbf{N}})$ is a continuous function which maps $[\underline{p}, \bar{p}]^N$ into itself, so by Brouwer's fixed–point theorem, $\tilde{\mathbf{r}}(\mathbf{p}, \hat{\mathbf{N}})$ has a fixed point, **p***. By the definition of $\underline{p}$ and $\bar{p}$, **p*** cannot have a value at the lower bound, so **p*** is in the interior of $[\underline{p}, \bar{p}]^N$. This implies that **p*** is also a fixed point of the unrestricted function $\mathbf{r}(\mathbf{p}, \hat{\mathbf{N}})$, which establishes existence.

A well-known sufficient condition for uniqueness is $\sum_k |\partial r_h / \partial p_k| < 1$, which establishes that **r** is a contraction mapping. By the implicit function theorem, $\partial r_h / \partial p_k = -[\partial \hat{N}_h / \partial p_k] / [\partial \hat{N}_h / \partial p_h]$. From this $\sum_k |\partial r_h / \partial p_k| < 1$ if and only if the following dominant diagonal condition holds:

$$\text{(A.6)} \qquad \sum_{k \neq (h,0)} \left|\partial \hat{N}_h / \partial p_k\right| < \left|\partial \hat{N}_h / \partial p_h\right|$$

To establish this condition, note that increasing all prices (including p_0) by the same amount will not change the demand for any house. Then (A.6) follows from:

$$\sum_{k \neq h} \left|\partial \hat{N}_h / \partial p_k\right| - \left|\partial \hat{N}_h / \partial p_h\right| = 0 \;\Rightarrow \sum_{k \neq (0,h)} \left|\partial \hat{N}_h / \partial p_k\right| = -\partial \hat{N}_h / \partial p_0 + \left|\partial \hat{N}_h / \partial p_h\right| < \left|\partial \hat{N}_h / \partial p_h\right|.$$

Q.E.D.

Proof of Lemma 2.1. Lemma 2.1 follows directly if we can show that the mapping that defines the fixed-point problem above is continuous in **x**, as the *unique* fixed-point $\mathbf{p}^*$ of a mapping continuous in both **x** and **p** is also continuous in **x**. The assumption that utility is continuous in x_h along with assumption about the continuous distribution of **e** implies that P_h^i is continuous in **x** for all i, which in turn implies that $\hat{N}_h$ is continuous in **x**, which in turn implies that the element-by-element inverse defined in (A.5) is continuous in **x**. ***Q.E.D.***

Proof of Proposition 2.2. Conditional on any vector **g** and the primitives of the model {**Z, X, x**}, Proposition 2.1 implies that a unique set of housing prices clears the market and assumption (i) ensures that this vector of market-clearing prices is continuous in **g**. Assumptions (ii) and (iii) in turn imply that that equation (2.11), along with the definition of the function **g**, implicitly defines **g** and represents a continuous mapping of a closed interval into itself. The existence of fixed point of this mapping, $\mathbf{g}^*$, follows directly from Brouwer's fixed-point theorem. Any fixed point, $\mathbf{g}^*$, is associated with a unique vector of market clearing prices $\mathbf{p}^*$ and a unique set of choice probabilities $\{P_h^{i\,*}\}$ that together satisfy the conditions for a sorting equilibrium. Consequently, the existence of a fixed point, $\mathbf{g}^*$, implies the existence of a sorting equilibrium. ***Q.E.D.***

Table 5.1: First-Stage Price Regressions

		First-Stage Price Regressions					
Dependent Variable		Monthly house price		Monthly house price		Monthly house price	
Observations		200,000		200,000		200,000	
Includes full set of standard variables		Shown below		No		Yes	
		Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error
'Quasi-' Optimal Instruments							
Price (/1000)				385.99	3.69	378.62	3.68
Percent Black				-43.7	2.03	-25.85	2.33
Percent Hispanic				-14.13	1.83	6.95	1.99
Percent Asian				6.3	2.12	30.50	2.36
Percent College Degree or More				55.96	2.62	41.36	2.83
Average Neighborhood Income				274.72	2.75	269.14	2.79
Standard Variables							
Owner-Occupied		201.09	4.51				
Number of Rooms		198.87	1.16				
Builts 1980s		271.45	8.77				
Built 1960-79		15.30	5.28				
Pollution Index		-0.82	0.59				
Population Density		-120.95	6.00				
Elevation		0.64	0.01				
Distance to Bay		-4.65	0.45				
Distance to Ocean		-14.12	0.31				
Avg Min Jan Temp		-15.18	1.37				
Employment Accessibility Index		156.30	10.50				
Industrial land use	within 1 mile	-351.43	25.20				
percentage	1-3 miles	-43.69	28.47				
	3-5 miles	221.33	26.29				
Commercial land use	within 1 mile	-354.97	18.30				
percentage	1-3 miles	431.65	43.40				
	3-5 miles	273.37	41.72				
Open space	within 1 mile	83.16	13.67				
percentage	1-3 miles	-60.80	14.32				
	3-5 miles	-28.18	14.37				
Other urban land use	within 1 mile	25.46	31.79				
percentage	1-3 miles	513.04	68.58				
	3-5 miles	-278.64	55.63				
Owner-occupied, large sing-fam homes	within 1 mile	120.70	2.55				
number with 7 or more rooms (/100)	1-3 miles	6.08	0.74				
	3-5 miles	7.33	0.59				
Owner-occupied, small sing-fam homes	within 1 mile	-37.77	2.40				
number with 6 or fewer rooms (/100)	1-3 miles	-6.89	0.82				
	3-5 miles	-6.64	0.66				
Owner-occupied, non-sing-fam homes	within 1 mile	-77.07	3.58				
number (/100)	1-3 miles	0.42	1.43				
	3-5 miles	28.98	1.28				
Renter-occupied, sing-fam homes	within 1 mile	-101.30	6.13				
number (/100)	1-3 miles	6.04	2.26				
	3-5 miles	10.05	1.64				
Renter-occupied, in small apart building	within 1 mile	15.43	1.58				
number (/100)	1-3 miles	-10.42	0.70				
	3-5 miles	-8.25	0.46				
Renter-occupied, in large apart building	within 1 mile	5.08	1.15				
number (/100)	1-3 miles	1.47	0.67				
	3-5 miles	-0.90	0.46				
F-Tests for joint significance		F-stat	P-value	F-stat	P-value	F-stat	P-value
Standard variables -- Variables in 3-5 mile ring		181.98	0.000				
'Quasi-' optimal instruments				6843.67	0.000	5096.50	0.000
Variables in 3-5 mile ring and 'quasi-' optimal instruments						1295.73	0.000

Note: In the subsequent analysis, standard instruments refer to the set of land use and neighborhood housing stock variables in the 3-5 mile ring. In all subsequent analysis, the land use and neighborhood housing stock variables within 1 mile and in the 1-3 mile ring are included in the utility function. The 'quasi-' optimal instruments are based on the predicted vector of market-clearing house prices and neighborhood sociodemographic characteristics using only the exogenous components of the sorting model -- full details are provided in Section 5 of the text.

Table 6.1: Interaction Parameter Estimates

	House Characteristics					Neighborhood Sociodemographic Composition					Other Neighborhood Attributes								Access to Work		
	Monthly House Price (/1000)	Owner-Occupied	Number Rooms	Built in 1980s	Built in 1960-1979	% Black	% Hispanic	% Asian	% College Degree or More	Average Income (/10000)	Average Math Score (/100)	Pollution Index	Elevation (/100)	Population Density	Distance to Bay	Distance to Ocean	Crime Index	Avg Min Temp	Distance To Work	Dist-Work Squared (/100)	Employment Access Index (/1000)
Hhld Income (/10,000)	**0.120** (0.005)	**0.328** (0.02)	**0.050** (0.001)	**0.274** (0.047)	**0.093** (0.035)	**-0.122** (0.035)	-0.315 (0.256)	0.064 (0.047)	**0.351** (0.025)	**0.011** (0.002)	**0.023** (0.012)	**-0.007** (0.001)	0.002 (0.004)	**-0.069** (0.009)	**0.005** (0.001)	**0.004** (0.001)	**-0.004** (0.001)	**0.025** (0.003)	**0.002** (0.)	**-0.011** (0.001)	**0.033** (0.014)
Captital Income (/10,000)	**0.054** (0.017)	**0.309** (0.032)	0.009 (0.012)	-0.089 (0.054)	0.031 (0.356)	**-1.249** (0.064)	-0.064 (0.122)	**-1.002** (0.087)	**0.303** (0.058)	**-0.015** (0.003)	**0.151** (0.003)	**0.020** (0.002)	-0.001 (0.022)	**0.010** (0.02)	**-0.016** (0.002)	**-0.028** (0.001)	**-0.007** (0.001)	**-0.044** (0.005)	**-0.027** (0.001)	**0.006** (0.001)	**0.021** (0.006)
Black	0.129 (0.182)	**-1.135** (0.192)	0.045 (0.106)	0.068 (0.58)	**0.685** (0.239)	**16.376** (0.522)	7.073 (5.422)	**7.435** (0.932)	**2.753** (0.593)	0.013 (0.037)	-1.350 (0.202)	0.025 (0.026)	0.047 (0.066)	**-0.859** (0.16)	**-0.071** (0.015)	**0.047** (0.012)	**0.030** (0.009)	0.020 (0.043)	**-0.048** (0.006)	**0.055** (0.006)	0.015 (0.272)
Hispanic	0.198 (0.125)	-0.010 (0.244)	**-0.434** (0.068)	0.011 (0.733)	-0.240 (0.184)	**4.242** (0.685)	**9.571** (0.888)	**2.658** (0.463)	**1.924** (0.335)	0.039 (0.028)	0.007 (0.276)	0.022 (0.027)	-0.088 (0.208)	0.239 (0.142)	0.008 (0.002)	**0.033** (0.008)	0.011 (0.032)	0.004 (0.068)	**0.063** (0.005)	**-0.173** (0.014)	**-0.134** (0.04)
Asian	**0.593** (0.129)	**1.897** (0.287)	**-0.807** (0.213)	1.198 (1.173)	**0.621** (0.176)	**3.852** (0.426)	1.611 (0.62)	**17.974** (0.641)	**0.956** (0.475)	-0.020 (0.026)	-0.366 (0.246)	0.015 (0.025)	-0.060 (0.051)	-0.118 (0.211)	0.022 (0.016)	-0.004 (0.009)	-0.001 (0.012)	-0.043 (0.056)	**0.002** (0.006)	0.013 (0.008)	-0.022 (0.037)
Some College	**0.487** (0.205)	-0.130 (0.314)	0.081 (0.105)			-0.004 (0.406)	**-1.516** (0.549)	0.171 (0.633)	**1.934** (0.401)		**0.963** (0.114)			0.093 (0.142)			-0.005 (0.025)		-0.004 (0.005)	0.008 (0.009)	-0.014 (0.003)
College Degree	**0.491** (0.139)	**0.287** (0.113)	0.068 (0.069)			**2.112** (0.413)	**-0.771** (0.367)	-0.951 (0.587)	**8.219** (0.429)		0.558 (0.159)			**0.461** (0.097)			**0.015** (0.006)		**0.012** (0.002)	0.001 (0.009)	**0.093** (0.02)
Working	**0.286** (0.083)	0.723 (0.554)	0.017 (0.056)			**-1.549** (0.481)	**-0.870** (0.402)	**-1.309** (0.469)	**-1.281** (0.292)										**-0.429** (0.001)	**0.413** (0.004)	**-0.222** (0.022)
Age	**0.010** (0.002)	**0.096** (0.007)	**0.005** (0.002)			-0.003 (0.009)	**-0.100** (0.016)	0.018 (0.051)	-0.023 (0.035)		**0.013** (0.005)								**-0.003** (0.001)	**0.001** (0.)	0.000 (0.001)
# Persons in Hhld	**0.214** (0.035)	-0.289 (0.189)	**0.324** (0.013)	**-0.763** (0.103)	**-0.262** (0.034)	**1.231** (0.152)	**2.229** (0.129)	**1.035** (0.203)	**-1.163** (0.178)		0.223 (0.039)			0.060 (0.036)					**-0.009** (0.002)	**0.017** (0.005)	**-0.032** (0.009)
# of Kids in Hhld	**-0.224** (0.035)	**0.157** (0.047)	**-0.030** (0.049)	**0.862** (1.054)	**0.445** (0.05)	**-0.871** (0.226)	**-1.600** (0.192)	-0.070 (0.94)	-0.158 (0.586)		-0.011 (0.062)			**-0.311** (0.055)			0.002 (0.002)		**0.018** (0.002)	**-0.040** (0.005)	**-0.057** (0.015)
Income x Black						**-0.330** (0.15)															
Income x Hispanic							**0.701** (0.138)														
Income x Asian								-0.154 (0.096)													
College Degree x Black						**-4.037** (1.313)															
College Degree x Hispanic							-0.786 (1.463)														
College Degree x Asian								**-3.206** (0.828)													

Note: The parameters shown describe the elements of the utility function that interact household characteristics, shown in row headings, with choice characteristics, shown in column headings. Standard errors in parentheses. Bold-face indicates statistical significance at 95 percent level.

Table 6.2: Implied Mean Marginal Willingness-to-Pay (MWTP) Measures

	Implied Mean MWTP Measures			Hedonic Price Regression
Regression Method	OLS	IV	IV	OLS
Dependent Variable in Underlying Regression	Choice-specific constant	Choice-specific constant	Choice-specific constant	House Price
Endogenous Variables in Underlying Regression	None	Price	Price, N'hood sociodemogs.	None
Instruments		'Quasi-' Optimal	'Quasi-' Optimal	
Observations	200,000	200,000	200,000	200,000
Implied coefficient on choice-specific constant	**-1.90** (0.02)	**-0.15** (0.01)	**-0.15** (0.01)	0.00 (---)
Percent Black	**-6305.62** (54.47)	**-605.58** (11.52)	**-590.52** (13.38)	**-144.02** (11.01)
Percent Hispanic	**-4654.13** (84.07)	**-59.13** (18.01)	-30.95 (24.43)	**364.00** (16.41)
Percent Asian	**-2266.60** (6.44)	**-155.90** (13.34)	**-186.22** (15.86)	**76.70** (12.59)
Percent College Degree +	**-711.41** (55.93)	**574.01** (13.09)	**660.81** (17.59)	**668.48** (10.84)
Average Income (/10,000)	**-20.96** (2.68)	**3.96** (0.55)	**3.58** (0.55)	**13.54** (3.22)
Crime Rate	**-31.70** (1.03)	**-1.18** (0.23)	**-1.10** (0.23)	0.27 (0.22)
Average Math Score	**-8.85** (0.03)	**0.85** (0.06)	**0.73** (0.06)	**1.76** (0.05)
Owner-Occupied	**866.76** (16.76)	**208.82** (3.67)	**208.95** (4.07)	**149.97** (3.25)
Number of Rooms	**46.83** (4.57)	**132.59** (1.57)	**132.21** (2.07)	**140.77** (0.83)
Built in 1980s	**439.58** (32.90)	**174.71** (6.88)	**171.11** (4.61)	**103.71** (4.27)
Built in 1960s, 1970s	**247.16** (19.49)	**25.84** (4.00)	**25.63** (3.14)	**11.70** (3.09)
Pollution Index	**-15.16** (1.98)	**-2.89** (0.42)	**-3.02** (0.41)	**-1.35** (0.41)
Population Density	**-231.62** (22.86)	**-56.87** (4.69)	**-49.09** (4.68)	**-43.21** (4.46)
Elevation (in 100 foot units)	**-111.64** (4.78)	**14.43** (1.06)	**13.12** (1.07)	**20.45** (0.91)
Distance to Bay	**27.96** (1.38)	0.39 (0.31)	0.42 (0.31)	-0.04 (0.27)
Distance to Ocean	**24.11** (1.10)	**-3.62** (0.24)	**-3.30** (0.24)	**-6.58** (0.22)
Avg. Min. Jan. Temperature	**48.58** (4.60)	-0.88 (0.99)	-0.40 (0.95)	**-6.50** (0.90)
Employment Access	**-54.83** (3.61)	**-4.46** (0.29)	**-4.44** (0.29)	**3.13** (0.35)
Distance to Work	**-598.33** (4.08)	**-48.67** (0.33)	**-48.43** (0.33)	
Distance to Work - Squared	**576.50** (6.05)	**46.90** (0.49)	**46.66** (0.49)	
Includes variables describing land use and housing stock within 1 mile, and 1-3 mile ring	Yes	Yes	Yes	Yes

Note: The first three columns of the table report the mean marginal willingness-to-pay (MWTP) measures implied by three different estimates of the choice-specific constant regression (equation 4.2). The regressions underlying these mean MWTP measures are shown in columns 1, 4, and 5 in Appendix Table 2. The first column reports the estimates that result when the choice-specific constant regression is estimated via OLS; the middle two columns report IV estimates that use the 'quasi-' optimal instruments, instrumenting for price, and price and neighborhood sociodemographic variables respectively. The final column returns the estimate of a standard hedonic price regression. Standard errors in parentheses. Bold-face indicates statistical significance at 95 percent level.

Table 6.3: Heterogeneity in Marginal Willingness-to-Pay Measures

Change Reported:	Owner Occupied vs. Rental	Number Rooms +1 room	Built 1980s vs. pre-60s	Built 60s-70s vs. pre-60s	% Black vs. White +10%	% Hisp vs. White +10%	% Asian vs. White +10%	% College Degree +10%	Average Income +$10,000	Average Math Score +1 st.dev.	Pollution Index +1 st.dev.	Elevation +1 st.dev.	Population Density +1 st.dev.	Crime Index +1 st.dev.	Distance To Work +1 mile (6-7 miles)
Mean Willingness-to-Pay	**209**	**132**	**114**	**21**	**-59**	**-3**	**-19**	**66**	**36**	**27**	**-13**	**23**	**-26**	**-13**	**-42**
Household Income (Non-Capital sources) (+10,000)	**40**	**8**	**32**	**11**	**-2**	-4	1	**5**	**14**	**1**	**-4**	1	**-4**	**-6**	**-1**
Household Income (Capital sources) (+10,000)	**76**	**9**	**23**	**14**	**-17**	-4	**-11**	**4**	**-4**	**7**	**6**	1	**-4**	**-15**	**-4**
Black vs. White	**-127**	7	9	**79**	**187**	**81**	**85**	**32**	18	**-57**	**12**	9	**-51**	**38**	**-6**
Hispanic vs. White	4	**-47**	4	-27	**48**	**111**	**30**	**23**	44	**1**	**10**	-17	14	14	**6**
Asian vs. White	**245**	**-88**	**153**	**77**	**42**	**19**	**216**	**16**	-22	**-14**	**7**	-11	-9	-3	-3
College Degree vs. HS degree or less	**47**	16			**22**	**-9**	-13	**99**		**26**	**27**		**27**	**19**	**-1**
Working vs. Not working	91	6			**-20**	**-10**	**-16**	**-12**							
Age (+10 years)	**111**	**7**			-1	**-11**	2	-2		**6**					**-4**
Number of Adults (+1 adult)	-28	**41**	**-86**	**-30**	**13**	**26**	**12**	**-12**		**10**			3		**-2**
Number of Children (+1 child)	-16	**34**	7	**19**	**5**	**8**	**11**	**-15**		**9**			**-14**	3	**3**

Note: The first row of table reports the mean marginal willingness-to-pay (MWTP) for the change reported in the column heading.
The remaining rows report the difference in willingness to pay associated with the change listed in the row heading holding all other factors equal.
All measures are in terms of monthly house price. Bold-face indicates statistical significance at 95 percent level.

Table 7.3: Eliminating Racial Differences in Income and Wealth - Segregation Patterns

A: Baseline: **Pre-Simulation Equilibrium (Predictions of the Model)**

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**22.3%**	8.5%	9.5%	59.2%
HH - Black	11.5%	**31.9%**	13.6%	42.5%
HH - Hispanic Origin	9.9%	10.4%	**21.8%**	57.2%
HH - White	9.8%	5.2%	9.1%	**75.3%**
Overall	11.0%	8.8%	11.7%	68.5%

B: Simulation: **Randomizing Income and Wealth Partial Equilibrium**

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage Reduction in Own-Race 'Over-Exposure'**
HH - Asian	**22.2%**	8.8%	9.7%	59.0%	**0.9%**
HH - Black	12.0%	**27.4%**	12.2%	47.9%	**19.5%**
HH - Hispanic Origin	10.2%	9.5%	**19.8%**	59.8%	**19.8%**
HH - White	9.8%	6.0%	9.4%	**74.2%**	**16.2%**
Overall	11.0%	8.8%	11.7%	68.5%	

C: Simulation: **Randomizing Income and Wealth General Equilibrium**

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage Increase in Own-Race 'Over-Exposure'**
HH - Asian	**33.7%**	7.5%	8.2%	50.3%	**101.2%**
HH - Black	10.1%	**41.7%**	10.7%	37.1%	**42.4%**
HH - Hispanic Origin	8.9%	7.8%	**27.5%**	55.0%	**56.4%**
HH - White	7.9%	4.3%	8.2%	**79.0%**	**54.4%**
Overall	11.0%	8.8%	11.7%	68.5%	

Note: Each entry in the table shows the average fraction of households of the race shown in the column heading that reside in the same neighborhood as households of the race shown in the row heading. Panel A repeats the exposure rates predicted by the model -- the pre-simulation equilibrium. Panels B and C report the partial and general equilibrium results of a simulation that replaces each household's actual income and wealth with a income-wealth combination drawn at random from the empirical distribution, thereby eliminating racial differences in income and wealth.

Table 7.2: The Distribution of Selected Household Characteristics for Households of Each Race

Variable	Asian	Black	Hispanic	White	Overall
Household head (HH) is high school dropout	0.19	0.23	0.39	0.10	0.16
HH graduated from high school	0.14	0.22	0.22	0.18	0.18
HH has some college	0.18	0.28	0.19	0.23	0.23
HH has bachelor's degree	0.33	0.20	0.16	0.32	0.29
HH has advanced degree	0.16	0.06	0.05	0.17	0.14
Household income less than $12K	0.13	0.26	0.14	0.10	0.12
Household income $12-20K	0.09	0.14	0.12	0.08	0.09
Household income $20-35K	0.18	0.23	0.24	0.19	0.20
Household income $35-50K	0.18	0.16	0.21	0.18	0.18
Household income $50-75K	0.23	0.14	0.19	0.22	0.21
Household income $75-100K	0.12	0.05	0.07	0.11	0.10
Household receives public assistance income	0.13	0.21	0.11	0.05	0.08
Household has capital income	0.48	0.17	0.25	0.56	0.48
HH over 65	0.13	0.17	0.11	0.21	0.18
HH divorced	0.07	0.20	0.14	0.16	0.15
Number of adults in the household	2.48	1.85	2.40	1.86	2.00
Number of pre-kindergarten children in household	0.31	0.27	0.40	0.17	0.22
Number of children grades K-8 in household	0.46	0.41	0.54	0.22	0.30
umber of children grades 9-12 in household	0.14	0.11	0.14	0.06	0.08
Spanish spoken in household	0.01	0.04	0.68	0.03	0.10
Asian language spoken in household	0.76	0.01	0.02	0.01	0.11
HH not a US citizen	0.15	0.01	0.12	0.01	0.04
HH a naturalized citizen	0.04	0.00	0.03	0.00	0.01
HH entered the US in 1980s	0.33	0.02	0.15	0.02	0.07
HH entered the US in 1970s	0.26	0.01	0.14	0.02	0.06
Number of Observations	30271	18501	26675	167897	243344

Table 7.1: Racial Segregation - Full Sample, Sample Used in Analysis, Model's Predictions

Average Racial Composition of Census Block Group

A: Full Sample

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**22.5%**	8.3%	11.7%	57.4%
HH - Black	11.6%	**40.1%**	11.4%	36.9%
HH - Hispanic Origin	12.9%	9.1%	**21.8%**	56.2%
HH - White	10.4%	4.8%	9.3%	**75.5%**
Overall	12.3%	8.8%	11.2%	67.7%

B: Sample of 10,000

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**23.6%**	8.0%	10.4%	58.0%
HH - Black	11.1%	**41.9%**	11.4%	35.6%
HH - Hispanic Origin	11.4%	9.0%	**24.0%**	55.6%
HH - White	10.1%	4.5%	8.8%	**76.6%**
Overall	11.0%	8.8%	11.7%	68.5%

C: Predictions of Model

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**22.3%**	8.5%	9.5%	59.2%
HH - Black	11.5%	**31.9%**	13.6%	42.5%
HH - Hispanic Origin	9.9%	10.4%	**21.8%**	57.2%
HH - White	9.8%	5.2%	9.1%	**75.3%**
Overall	11.0%	8.8%	11.7%	68.5%

Note: Each entry in the table shows the average fraction of households of the race shown in the column heading that reside in the same neighborhood as households of the race shown in the row heading. Panel A reports the exposure rate measures using the full sample of 243,344 households; Panel B reports measures using the sample of 10,000 used in the first-stage of the estimation procedure; Panel C reports the same measures based on the predictions of the model.

Table 7.4: Eliminating Racial Differences in Income and Wealth - Segregation Patterns by Income

Simulation: Randomizing Income and Wealth
General Equilibrium

	Exposure to Other Households of Same Race							
	Asian		Black		Hispanic		White	
	lowest quintile	highest quintile	lowest quintile	highest quintile	lowest quintile	highest quintile	lowest quintile	highest quintile
Pre-Simulation Equilibrium	26.0%	20.2%	38.3%	22.6%	20.4%	21.4%	72.0%	77.2%
Post-Simulation Equilibrium	34.7%	35.0%	45.0%	35.6%	27.0%	29.1%	77.0%	80.4%
Difference	8.7%	14.8%	6.7%	13.0%	6.6%	7.7%	5.0%	3.2%

Note: The figures shown report the exposure of households to other households of the same race in equilibrium before and after the random income simulation. Results are reported for households at the bottom ($0-$8500) and top ($65,000) of the income distribution respectively.

Table 7.5: Eliminating Racial Differences in Income and Wealth - Consumption Patterns

	Simulation: Randomizing Income and Wealth General Equilibrium					
	Asian	Black	Hispanic	White	% Reduction in Black-White gap	% Reduction in Black-Hisp gap
			Monthly House Price			
Pre-Simulation Equilibrium	1081.3	752.8	871.9	1115.9		
Post-Simulation Equilibrium - Unadjusted	1093.8	950.6	960.7	1073.6	66%	54%
			Commuting Distance (miles)			
Pre-Simulation Equilibrium	10.70	9.86	11.42	10.80		
Post-Simulation Equilibrium - Unadjusted	10.06	9.84	10.91	10.24	57%	-8%
			Home Ownership Rate			
Pre-Simulation Equilibrium	0.624	0.368	0.418	0.605		
Post-Simulation Equilibrium - Unadjusted	0.612	0.480	0.500	0.580	58%	57%
			House Size (rooms)			
Pre-Simulation Equilibrium	4.67	4.46	4.45	5.21		
Post-Simulation Equilibrium - Unadjusted	4.64	4.94	4.69	5.12	76%	43%
			School Quality			
Pre-Simulation Equilibrium	203.3	181.3	196.7	212.7		
Post-Simulation Equilibrium - Unadjusted	202.5	189.8	203.6	210.7	33%	56%
Post-Simulation Equilibrium - Adjusted	202.0	181.4	201.0	212.2	2%	30%
			Crime Rate			
Pre-Simulation Equilibrium	10.02	16.88	10.73	6.85		
Post-Simulation Equilibrium - Unadjusted	10.28	13.11	9.01	7.55	45%	62%
Post-Simulation Equilibrium - Adjusted	11.44	16.90	9.82	6.76	-1%	21%

Note: This table reports the consumption of housing and local pubic goods by households of each race. Figures are reported for the pre-simulation equilibrium and the equilibrium that raises in a simulation that replaces each household's actual income and wealth with a income-wealth combination drawn at random from the empirical distribution, thereby eliminating racial differences in income and wealth.

Table 7.6: Eliminating Racial Interactions - Segregation Patterns

A: Baseline: **Predictions of the Model**

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**22.3%**	8.5%	9.5%	59.2%
HH - Black	11.5%	**31.9%**	13.6%	42.5%
HH - Hispanic Origin	9.9%	10.4%	**21.8%**	57.2%
HH - White	9.8%	5.2%	9.1%	**75.3%**
Overall	11.0%	8.8%	11.7%	68.5%

B: Simulation: **Eliminating Racial Interactions**
General Equilibrium

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage Reduction in Own-Race 'Over-Exposure'**
HH - Asian	**12.5%**	8.8%	10.7%	67.5%	**86.4%**
HH - Black	11.6%	**12.3%**	11.7%	63.7%	**84.8%**
HH - Hispanic Origin	11.4%	9.7%	**14.0%**	64.2%	**77.7%**
HH - White	11.4%	8.2%	10.2%	**69.6%**	**83.2%**
Overall	11.0%	8.8%	11.7%	68.5%	

C: Simulation: **Eliminating Racial Interactions *and* Randomizing Income**
General Equilibrium

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage Additional Reduction in Own-Race 'Over-Exposure' with Randomized Income**
HH - Asian	**12.1%**	8.8%	11.0%	67.5%	**3.8%**
HH - Black	11.6%	**10.8%**	10.7%	66.4%	**6.8%**
HH - Hispanic Origin	12.4%	9.1%	**15.2%**	62.6%	**-12.0%**
HH - White	11.6%	8.6%	9.7%	**69.5%**	**1.8%**
Overall	11.0%	8.8%	11.7%	68.5%	

Note: Each entry in the table shows the average fraction of households of the race shown in the column heading that reside in the same neighborhood as households of the race shown in the row heading. Panel A repeats the exposure rates predicted by the model -- the pre-simulation equilibrium. Panel B reports the general equilibrium results of a simulation that eliminates the protion of household preferences associated with neighborhood racial composition -- that is, eliminating racial interactions in the location decision. Panel C reports the general equilibrium results of a simulation that eliminates racial interactions *and* randomizes income/wealth across households.

Table 7.7: Eliminating Racial Interactions - Consumption Patterns

Simulation: Eliminating Racial Interactions in Location Decision
General Equilibrium

	Asian	Black	Hispanic	White	% Reduction in Black-White gap	% Reduction in Black-Hisp gap
			Monthly House Price			
Pre-Simulation Equilibrium	1081.3	752.8	871.9	1115.9		
Post-Simulation Equilibrium - Unadjusted	1102.5	890.1	895.9	1090.5	45%	20%
			Commuting Distance (miles)			
Pre-Simulation Equilibrium	10.70	9.86	11.42	10.80		
Post-Simulation Equilibrium - Unadjusted	10.38	9.31	10.22	10.44	-20%	135%
			Home Ownership Rate			
Pre-Simulation Equilibrium	0.624	0.368	0.418	0.605		
Post-Simulation Equilibrium - Unadjusted	0.636	0.364	0.431	0.609	-3%	5%
			House Size (rooms)			
Pre-Simulation Equilibrium	4.67	4.46	4.45	5.21		
Post-Simulation Equilibrium - Unadjusted	4.75	4.63	4.55	5.19	25%	16%
			School Quality			
Pre-Simulation Equilibrium	203.3	181.3	196.7	212.7		
Post-Simulation Equilibrium - Unadjusted	205.8	196.6	201.5	209.6	59%	49%
Post-Simulation Equilibrium - Adjusted	205.8	192.9	201.4	211.4	41%	38%
			Crime Rate			
Pre-Simulation Equilibrium	10.02	16.88	10.73	6.85		
Post-Simulation Equilibrium - Unadjusted	8.75	10.96	9.62	7.98	70%	58%
Post-Simulation Equilibrium - Adjusted	9.29	12.61	9.73	7.42	48%	40%

Note: This table reports the consumption of housing and local pubic goods by households of each race. Results are reported for the pre-simulation equilibrium and the equilibrium that arises in a simulation that eliminates the portion of household preferences associated with neighborhood racial composition.

Table 7.8: Eliminating Racial Interactions - Consumption Patterns by Income

Simulation: Eliminating Racial Interactions in Location Decision
General Equilibrium

	Asian		Black		Hispanic		White	
	lowest quintile	highest quintile	lowest quintile	highest quintile	lowest quintile	highest quintile	lowest quintile	highest quintile
				Home Ownership Rates				
Pre-Equilibrium	0.391	0.785	0.253	0.548	0.209	0.628	0.397	0.758
Post-Simulation - Unadjusted	0.407	0.803	0.185	0.614	0.197	0.665	0.400	0.763
				House Size (rooms)				
Pre-Equilibrium	3.471	5.544	3.779	5.432	3.422	5.408	3.993	6.043
Post-Simulation - Unadjusted	3.424	5.653	3.656	5.792	3.400	5.609	3.970	6.036
				School Quality (average math score)				
Pre-Equilibrium	191.5	212.1	173.5	195.8	189.9	204.1	202.0	220.8
Post-Simulation - Unadjusted	193.3	215.3	188.2	211.1	192.6	208.7	197.7	217.9
Post-Simulation - Adjusted	191.1	214.5	184.1	210.0	193.6	209.0	198.5	220.6
				Crime Rate				
Pre-Equilibrium	15.62	6.77	20.70	12.29	14.01	8.01	10.81	5.00
Post-Simulation - Unadjusted	12.68	5.84	14.52	7.19	13.60	6.72	12.68	5.74
Post-Simulation - Adjusted	13.79	6.26	16.33	7.85	13.63	6.64	12.51	4.88

Note: The figures shown report the consumption of housing and local public goods by households of each race in equilibrium before and after a simulation that eliminates racial interactions in the location decision. Results are reported for households at the bottom (0-$8500) and top ($65,000) of the income distribution respectively.

Table 7.9: Eliminating Racial Differences in Job Locations -- Segregation and Commuting Patterns

A: Baseline: **Predictions of the Model**

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**22.3%**	8.5%	9.5%	59.2%
HH - Black	11.5%	**31.9%**	13.6%	42.5%
HH - Hispanic Origin	9.9%	10.4%	**21.8%**	57.2%
HH - White	9.8%	5.2%	9.1%	**75.3%**
Overall	11.0%	8.8%	11.7%	68.5%

B: Simulation: **Randomizing Job Locations**
General Equilibrium

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage *Increase* in Own-Race 'Over-Exposure'**
HH - Asian	**33.3%**	7.2%	8.2%	50.9%	**97.3%**
HH - Black	10.0%	**42.1%**	12.1%	35.3%	**44.1%**
HH - Hispanic Origin	8.4%	9.1%	**29.7%**	52.1%	**78.1%**
HH - White	8.1%	4.1%	7.7%	**79.6%**	**62.5%**
Overall	11.0%	8.8%	11.7%	68.5%	

	Average Commutes			
	Asian	Black	Hispanic	White
Pre-Simulation Equilibrium	11.42	9.86	10.70	10.80
Post-Simulation Equilibrium	10.37	9.99	10.76	10.16

C: Simulation: **Randomizing Job Locations and Increasing Disutility of Commuting Tenfold**
General Equilibrium

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage *Reduction* in Own-Race 'Over-Exposure'**
HH - Asian	**19.5%**	8.2%	10.6%	61.3%	**24.5%**
HH - Black	10.1%	**25.0%**	12.2%	51.7%	**29.7%**
HH - Hispanic Origin	10.6%	10.0%	**18.0%**	60.5%	**37.7%**
HH - White	10.0%	6.3%	9.4%	**73.7%**	**24.3%**
Overall	11.0%	8.8%	11.7%	68.5%	

	Average Commutes			
	Asian	Black	Hispanic	White
Pre-Simulation Equilibrium	11.42	9.86	10.70	10.80
Post-Simulation Equilibrium	5.74	5.61	5.27	5.49

Note: Each entry in the table shows the average fraction of households of the race shown in the column heading that reside in the same neighborhood as households of the race shown in the row heading. Panel A repeats the exposure rates predicted by the model -- the pre-simulation equilibrium. Panel B reports the general equilibrium results of a simulation that randomizes the job locations of working hhlds. Panel C also randomizes job locations but also increases the disutilitof commuting by tenfold.

Appendix Table 1: Summary Statistics

	Random Sample: 10,000*		Random Sample: 200,000*	
Variable Description	Mean	S.D.	Mean	S.D.
household income	53,026	52,065		
capital income - dividends, interest, capital gains	5,031	27,652		
1 if householder - Black	0.088	0.284		
1 if householder - Hispanic	0.110	0.313		
1 if householder - Asian	0.117	0.322		
1 if education - HS Degree or less	0.341	0.474		
1 if education - some college	0.222	0.416		
1 if education - BA Degree or more	0.437	0.496		
1 if someone in household working	0.688	0.464		
householder age (years)	47.7	16.9		
number of persons in household	2.61	1.57		
number of children under 18 in household	0.61	1.04		
monthly house price	1,052	710	1,047	732
1 if unit owned	0.565	0.496	0.559	0.497
number of rooms	5.00	2.01	4.98	2.00
1 if built in 1980s	0.142	0.349	0.145	0.235
1 if built in 1960s or 1970s	0.394	0.489	0.389	0.396
% census block group black	0.086	0.169	0.088	0.170
% census block group Hispanic	0.112	0.115	0.112	0.115
% census block group Asian	0.121	0.117	0.122	0.119
% census block group college degree or more	0.435	0.195	0.434	0.195
average block group income	51751	29037	54900	28663
average math score	207	37	207	37
pollution index	24.1	4.2	24.1	4.2
elevation	206	170	206	174
population density	0.454	0.514	0.465	0.535
distance to Bay	6.13	6.00	6.12	6.16
distance to ocean	17.09	9.85	16.91	9.95
crime rate	8.53	11.19	8.84	11.52
avg min Jan temperature	47.2	1.9	47.6	1.9
distance to work	6.11	8.32	6.12	8.28
employment accessibility index	0.185	0.226	0.186	0.207
Number of owner occupied, single family homes, rooms < 7, within 1 mile			307	205
Number of owner occupied, single family homes, rooms > 6, within 1 mile			173	105
Number of owner occupied, non-single family homes, within 1 mile			105	169
Number of renter occupied, single family homes, within 1 mile			116	72
Number of renter occupied, unit in small apartment building, within 1 mile			268	423
Number of renter occupied, unit in large apartment building, within 1 mile			331	711
Percent land usage -- commercial, within 1 mile			0.147	0.126
Percent land usage -- industrial, within 1 mile			0.039	0.087
Percent land usage -- open space, within 1 mile			0.110	0.194
Percent land usage -- other urban land use, within 1 mile			0.052	0.070
Percent land usage -- residential, within 1 mile			0.652	0.445

Note: Summary statistics are reported for the two samples, randomly drawn from the full sample of 243,344 households used in the analysis. Due to computational constraints, the first-stage of the estimation procedure (which returns the interaction parameters and choice-specific constants) uses the sample of 10,000 and the second-stage of the estimation procedure (the choice-specific constant regressions) uses the sample of 200,000. Housing stock and land use variables are not reported for the sample of 10,000 as these variables are not used in the first-stage of the estimation, while household characteristics are not reported for the sample of 200,000 as these characteristics are not used in the second-stage of the estimation.

Appendix Table 2: Choice-Specific Constant Regressions

	Estimates of House-Specific Constant Regressions				
Regression Method	OLS	IV	IV	IV	IV
Endogenous Variables	none	House price	House price N'hood racial char. N'hood educ level	House price	House price N'hood racial char. N'hood educ level
Instruments		Standard	Standard	Quasi-' Optimal	Quasi-' Optimal
Observations	200,000	200,000	200,000	200,000	200,000
Monthly House Price	**-0.525**	**-1.933**	**-2.423**	**-6.456**	**-6.488**
	(0.006)	(0.138)	(0.169)	(0.058)	(0.059)
Percent Black	**-0.769**	**-0.775**	**-0.919**	**-0.907**	**-0.889**
	(0.006)	(0.008)	(0.036)	(0.017)	(0.019)
Percent Hispanic	**-0.384**	**-0.310**	0.081	**-0.060**	-0.032
	(0.007)	(0.011)	(0.045)	(0.017)	(0.025)
Percent Asian	**-0.193**	**-0.196**	**-0.218**	**-0.164**	**-0.196**
	(0.001)	(0.006)	(0.025)	(0.013)	(0.017)
Percent College Degree +	**-0.099**	**0.183**	-0.072	**0.987**	**1.142**
	(0.007)	(0.026)	(0.057)	(0.022)	(0.029)
Average Income	**-0.043**	-0.004	-0.001	**0.100**	**0.091**
	(0.005)	(0.007)	(0.008)	(0.013)	(0.013)
Crime Rate	**-0.262**	**-0.207**	**-0.202**	**-0.120**	**-0.112**
	(0.008)	(0.011)	(0.021)	(0.022)	(0.023)
Average Math Score	**-0.234**	**-0.132**	0.036	**0.276**	**0.238**
	(0.001)	(0.015)	(0.020)	(0.018)	(0.018)
Owner-Occupied	**0.309**	**0.452**	**0.507**	**0.915**	**0.920**
	(0.006)	(0.016)	(0.019)	(0.015)	(0.016)
Number of Rooms	**0.067**	**0.602**	**0.828**	**2.340**	**2.345**
	(0.006)	(0.053)	(0.066)	(0.027)	(0.026)
Built in 1980s	**0.074**	**0.129**	**0.204**	**0.362**	**0.356**
	(0.005)	(0.009)	(0.013)	(0.014)	(0.014)
Built in 1960s, 1970s	**0.070**	**0.059**	**0.910**	**0.090**	**0.090**
	(0.005)	(0.006)	(0.008)	(0.013)	(0.014)
Pollution Index	**-0.046**	**-0.064**	**-0.034**	**-0.107**	**-0.113**
	(0.006)	(0.007)	(0.008)	(0.015)	(0.015)
Population Density	**-0.089**	**-0.139**	**-0.227**	**-0.268**	**-0.233**
	(0.008)	(0.010)	(0.018)	(0.021)	(0.022)
Elevation	**-0.139**	**-0.060**	**0.054**	**0.221**	**0.202**
	(0.006)	(0.011)	(0.015)	(0.016)	(0.016)
Distance to Bay	**0.123**	**0.099**	**-0.029**	0.021	0.023
	(0.006)	(0.008)	(0.013)	(0.016)	(0.017)
Distance to Ocean	**0.172**	**0.081**	**-0.070**	**-0.318**	**-0.291**
	(0.008)	(0.015)	(0.020)	(0.020)	(0.021)
Avg. Min. Jan. Temperature	**0.067**	**0.029**	0.016	-0.015	-0.007
	(0.006)	(0.007)	(0.009)	(0.016)	(0.016)

Note: This table reports parameter estimates for five specifications of the choice-specific constant regression (equation 4.2). The first column reports the estimates that result when the choice-specific constant regression is estimated via OLS; columns 2 and 3 report IV regression estimates that use the standard set of instruments (variables that describe land use and housing stock in the 3-5 mile ring), controlling for land use and housing stock within 1 mile and in a 1-3 mile ring, instrumenting for price and neighborhood sociodemographic characteristics, respectively. Columns 4 and 5 report IV estimates that use 'quasi-' optimal instruments. The final column is our preferred specification.

Appendix Table 3: OLS Crime and Education Production Functions

	Production Function	
Dependent Variable	crime index	average math score
Observations	200,000	200,000
R-Squared	0.33	0.41
Percent Black	0.285	-0.188
	0.005	0.005
Percent Hispanic	0.099	-0.074
	0.004	0.003
Percent Asian	0.088	-0.041
	0.003	0.003
Percent College Degree or More	0.017	0.127
	0.004	0.004
Average Income	-0.071	0.311
	0.046	0.043

Note: This table shows the results of the OLS estimation of simple crime and education

quality with changing neighborhood sociodemographic composition. We use these 'adjusted' .
results to provide a bound on the simulation results. Standard errors are provided below

Appendix Table 4: Eliminating Racial Differences in Education

A: Baseline: **Predictions of the Model**

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White
HH - Asian	**22.3%**	8.5%	9.5%	59.2%
HH - Black	11.5%	**31.9%**	13.6%	42.5%
HH - Hispanic Origin	9.9%	10.4%	**21.8%**	57.2%
HH - White	9.8%	5.2%	9.1%	**75.3%**
Overall	11.0%	8.8%	11.7%	68.5%

B: Simulation: **Eliminating Racial Differences in Education**
General Equilibrium

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage Increase in Own-Race 'Over-Exposure'**
HH - Asian	**32.0%**	7.8%	8.2%	51.6%	**85.5%**
HH - Black	10.6%	**42.8%**	11.6%	34.5%	**47.0%**
HH - Hispanic Origin	8.7%	8.7%	**24.7%**	57.2%	**28.7%**
HH - White	8.3%	4.1%	8.7%	**78.3%**	**44.4%**
Overall	11.0%	8.8%	11.7%	68.5%	

C: Simulation: **Eliminating Racial Differences in Income, Wealth, and Education**
General Equilibrium

Household's Race:	Percent Asian	Percent Black	Percent Hispanic	Percent White	**Percentage Increase in Own-Race 'Over-Exposure'**
HH - Asian	**33.3%**	8.0%	8.4%	50.0%	**97.0%**
HH - Black	10.8%	**39.9%**	10.0%	38.9%	**34.8%**
HH - Hispanic Origin	9.0%	7.5%	**24.0%**	58.8%	**21.9%**
HH - White	8.0%	4.6%	8.9%	**77.9%**	**37.6%**
Overall	11.0%	8.8%	11.7%	68.5%	

Note: Each entry in the table shows the average fraction of households of the race shown in the column heading that reside in the same neighborhood as households of the race shown in the row heading. Panel A repeats the exposure rates predicted by the model -- the pre-simulation equilibrium. Panel B reports the general equilibrium results of a simulation that randomizes education across households. Panel C reports the results for a simulation that randomizes, education, income, and wealth.