The Impact of Paying Interest on Reserves in the Presence of Government Deficit Financing[1]

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Abstract

This paper re-examines the impact that paying interest on reserves has on price level indeterminacy, volatility, and economic well-being. Unlike the previous literature, this model includes an after-tax deficit that must be financed by assets (bonds and reserves) whose returns are linked. I show the number of steady state equilibria is equal to, or greater than, the number arising in the no-interest economy. Consequently, the level of indeterminacy is equal to, or greater than, in the no-interest economy. When the level of indeterminacy is the same, then economic volatility is reduced by paying interest. However, greater indeterminacy in the interest economy, results in greater volatility. Paying interest on reserves can enhance welfare and, under certain conditions, unpleasant monetarist arithmetic may also obtain.

JEL Classification: D6, E3, E5

1 Introduction

The issue of paying interest on reserves was introduced by Milton Friedman almost fifty years ago in *A Program for Monetary Stability.* Friedman's original motivation was to make the 100% reserve requirement of the "Chicago Plan" more palatable to a banking system subject to only a fractional reserve system. The goal of the Chicago Plan and the proposal to pay interest on reserves was to establish greater price level stability and to reduce excessive price level fluctuations.[1]

In the subsequent decades, there has been considerable research regarding the implications of paying interest on reserves.[2] Three studies, in particular Sargent and Wallace (1985), Smith (1991), and Freeman and Haslag (1996), have examined in detail whether Friedman's proposal would bring about the desired reductions in price level indeterminacy and volatility, as well as the welfare implications of switching from a system of not paying interest on reserves to one which did.[3] However, these works suffered from two specific limitations. First, they did not equate the interest paid on reserves to returns on assets of similar risk and duration. Second, they assumed that either the government ran a balanced budget or had a surplus.

By assuming the budget was not in deficit, these works side-stepped two important issues: (1) the impact that deficit financing has on the means for financing interest payments and (2) the complications that arise from simultaneously attempting to finance a deficit and set the real return on reserves. One of the key results of this previous literature was that how interest payments were financed was crucial to the likelihood of indeterminacy and volatility arising. However, if the sum

[1] The Federal Reserve currently does not have the authority to pay interest on reserves. However, in recent years Congress has introduced legislation which would allow interest payments, and this legislation has the support of the Federal Reserve, see Kohn (2004) and Meyer (2001). The Fed's objective is twofold: (1) to avoid potential volatility in the federal funds market and (2) to eliminate resources wasted due to reserve avoidance measures (for example, sweep accounts). Anderson and Rasche (2001) conclude that paying interest is unlikely to impact the use of sweep accounts as interest payments on reserves are unlikely to be sufficiently high to entice banks to hold greater reserves. Consequently, the primary benefit of paying interest would be to reduce volatility – the focus of this paper.

[2] The focus of these studies fall broadly into three categories: the use of interest payments on reserves strictly as a policy tool (see for example Woodford (2003), Goodfriend (2002), Hall (2002), and Goodhart (2000)), the role of paying interest on reserves on payment services policies (see for example Toma (1999) and Lacker (1997)), and the impact of interest payments on welfare, price level determinacy, and economic stability (see for example Sargent and Wallace (1985), Smith (1991), and Freeman and Haslag (1996)).

[3] Sargent and Wallace (1985) highlighted two key facts. First, paying interest on reserves, combined with a 100% reserve requirement, would not necessarily lead to a deterministic price level and less fluctuations. Second, the method of financing interest payments could lead to real differences in economic outcomes. Smith (1991) showed that if the rate of return on reserves were tied to productive investment technologies, then the indeterminacies described in Sargent and Wallace (1985) disappear. However, interest financed via taxes resulted in a series of oscillating equilibria, and thus, may lead to greater economic fluctuations. In addition, Smith (1991) showed that there was no welfare justification for paying interest on reserves. Finally, Freeman and Haslag (1996) explored means by which paying interest on reserves could be Pareto optimal. They showed that if an appropriate, accommodative open market operation was undertaken, then the initial old generation would be indifferent, while all future generations would be better-off. See Guzman (2004) for a more in-depth review of these three papers.

of government expenditures and interest payments (on bonds and reserves) exceeds tax revenue, then the issue of how interest payments on reserves are financed (via taxes or earnings on assets) is no longer relevant. Instead, the appropriate concern is whether the government can simultaneously finance the deficit (by issuing bonds or printing money), link the return on reserves to other assets (such as bonds), and maintain sufficient returns on bonds and reserves such that both assets are desired by consumers.

The objective of this paper is to re-examine, in the presence of an after-tax deficit, the impact of switching from a system where reserves earn no interest to one where they do. This is accomplished in the context of a two period overlapping generations model with multiple assets and an after-tax government deficit that must be financed by a combination of debt and seigniorage income. The primary goal is to compare the level of economic indeterminacy, economic volatility, and welfare gains in an economy where interest is paid on reserves to one where reserves earn no interest.

More specifically, this paper addresses the following three questions. First, in the presence of a government deficit and a return on storage that dominates all other rates of return, does paying interest on reserves reduce potential indeterminacy of equilibria? Second, under the same conditions does the amount of economic volatility increase or decrease? Third, are there any welfare justifications for switching to a system where reserves earn interest? In addition, given the presence of both debt and seigniorage in financing the deficit, the issue of unpleasant monetarist arithmetic is explored?

The key findings of this paper can be summarized as follows. When there exists an after-tax government deficit, and reserves are paid a rate of return equal to that of bonds (and less than the return on storage), the number of steady state equilibria (in terms of real money balances) are equal to, or greater than, the number that arise when no interest is paid on reserves. Thus, the level of economic indeterminacy is equal to, or greater than, in an economy without interest payments. This runs counter to what Friedman had envisioned and the results of Smith (1991). In addition, the steady state equilibrium associated with the highest level of real money balances is a source, while the steady state associated with the lowest level of balances is a sink. Any other steady states will alternate between being sinks and sources. For those steady states which are sinks, convergence is monotonic.

Second, when the number of steady state equilibria are the same in the interest and no-interest economies (i.e., the level of indeterminacy is the same), then economic volatility is reduced with the introduction of interest payments. However, when greater indeterminacy exists in the interest economy, there also exists greater volatility. Third, when multiple (generically two) steady state equilibria exist in both economies, then the equilibrium associated with low real money balances in the interest economy is welfare improving compared to the no-interest economy. In addition, if the economy is converging to the low real money balances steady state in the no-interest economy, then after switching to an interest paying regime, the economy will transition to the low real balances steady state in the interest economy. When at the unstable steady state in the no-interest economy, if the government begins to pay interest then a new equilibrium can be reached only if there is an accompanying expansionary open market operation. Finally, under a narrow set of conditions, unpleasant monetarist arithmetic may arise.

The key to the intuition behind these results is understanding the constraints that paying interest places on the means for financing the deficit.[4] In the no-interest economy, the government can price discriminate and choose whether to use more expensive (bonds) or less expensive (money) means to finance its deficit. In addition, it can decide whether to use a large seigniorage tax base (and small tax rate) or conversely a large seigniorage tax rate (and small tax base). Its decision regarding which instruments and what size tax rates to use are independent, to the extent that in the end, the deficit must be financed.

When the return on reserves is linked to the return on bonds, the government's ability to price discriminate in its financing options is limited. Thus, financing the deficit via seigniorage and bonds is more expensive relative to the no-interest economy. In addition, with the return on money linked to bonds, the government's ability to exercise a trade-off between the tax rate and the tax base is curtailed. As a result, paying interest on reserves shrinks the set of real money balances that is consistent with financing the fixed deficit.

When there exist multiple steady state equilibria in both the interest and no-interest economies, then the range of real money balances (the seigniorage tax base) will be smaller in the interest economy because the link between the returns on bonds and money reduces the government's options. This results in less volatility in the interest economy. However, because the government can still choose the initial level of real money balances (from an infinite set of possibilities), indeterminacy is unaffected.

[4] It is important to point out that the indeterminacy and volatility are not the result of any volatility present in the economic fundamentals (i.e. there are no stochastic aspects to this model). In this respect, the model presented in this paper differs from the ideas explored by Friedman in *A Program for Monetary Stability*. One of the issues with which Friedman was concerned was the existence of real world stochastic processes and their impact on equilibrium outcomes. In the end however, Friedman was focused on the volatility and indeterminacy of outcomes (prices, output, etc.). It is in this vein (outcomes) that this paper is consistent with Friedman's concerns regarding indeterminacy and volatility (despite a difference in the sources of this volatility).

The existence of multiple steady state equilibria is contingent on the deficit financing options (the quantity of bonds and reserves) being consistent with consumers wanting to hold all assets. Because the no-interest economy faces fewer constraints on financing its deficit, extreme values for tax bases and rates (for example, a very small seigniorage tax base and a very high tax rate, i.e. high inflation), are more likely to be consistent with financing its deficit. However, while financing the deficit may be possible, the associated tax base and tax rate may not be consistent with individuals wanting to hold all assets (i.e. if inflation is too high, individuals will not want to hold money). In this case, the low real money balances steady state in the no-interest economy is obviously not consistent with equilibrium. While there might be two candidate steady state levels of real money balances, only the larger one is consistent with financing the deficit without violating the requirement that all assets earn a non-negative return. Thus, under certain parameter settings, the interest economy will have two steady state equilibria while the no-interest economy only one. Obviously, in this case, both the level of indeterminacy and volatility will be greater on the interest bearing economy.

Finally, it is assumed that both economies are Samuelson-case economies, where savings (and hence consumption) are strictly increasing in the level of real money balances. Since the set of real balances consistent with equilibrium is larger in the no-interest economy, the steady state equilibrium values of real balances are higher at the high real balances steady state and lower at the low real balances steady state than those of the corresponding interest economy. Thus, when comparing the low real money balance steady states and dynamic equilibria converging to them, paying interest will be welfare improving.

The basic economic model used in this paper is a variation of Bhattacharya *et al.* (1998) and simply augments it with interest payments on reserves. The structure of the economy is as follows. The economy consists of an infinite sequence of two-period lived, overlapping generations, where individuals across generations are identical in all dimensions. Consumers are endowed each period with a given amount of a consumption good which they either consume or invest. Individuals may invest their saving in any of three different assets. There is a storage technology, which pays the highest rate of return, government bonds, and money, whose return is dominated by all other assets. It is assumed that individuals cannot invest directly in the storage technology and that all investment in storage must be intermediated and is subject to a reserve requirement. Required reserves pay a rate of return equal to that of government securities. Thus, individuals save by purchasing bonds and depositing their savings with intermediaries.

In addition, there exists a government which must finance a constant per capita after-tax deficit while also paying interest on bonds and reserves. This deficit and interest payments are funded by some combination of money creation and new debt offerings. Finally, it is assumed that the government conducts policy by choosing (once and for all in the first period) a ratio of bonds to currency. Variations in this ratio can be thought of as permanent open market operations.

The remainder of the paper proceeds as follows. Section 2 describes the model economy, while Section 3 states conditions necessary for steady state equilibrium to exist and examines the dynamic properties of the model. The propensity for unpleasant monetarist arithmetic to arise is also discussed in this section. Comparisons of steady state equilibria and the dynamic paths converging to them, of price level determinacy and volatility, and of economic welfare between the interest and no-interest economies are the topics of Section 4. Section 5 concludes and all proofs can be found in the Appendices.

2 The Model

The economy consists of an infinite sequence of two-period lived, overlapping generations, along with an initial old generation. Time is discrete and indexed by $t = 1, 2, ...$ At every date t a new generation, comprised of N identical members, is born. There exists a government that has a constant per capita real expenditure level of $g > 0$ in each period. The government levies no direct taxes, and so it must finance its deficit by issuing money and bonds.[5] Let M_t denote the per capita stock of money outstanding at the end of period t, and B_t denote the outstanding per capita supply of bonds (both in nominal terms). All bonds are of one-period maturity, and are default free.

2.1 Consumers

Individuals are endowed with some of a single, non-produced good, which can either be consumed or stored. The endowment of a representative individual is given by $\omega_1 > 0$ when young and by $\omega_2 \geq 0$ when old. In addition, members of the initial old are each endowed with $\omega_0 \geq 0$ units of consumption, and with $M_0 > 0$ units of fiat currency. Consumption of a representative agent born at t is denoted by c_t^t when young and c_{t+1}^t when old. All individuals have the identical utility

[5] Alternatively, one could imagine that the government levies some (fixed) lump-sum taxes. Then one would interpret the endowments received by individuals, $\omega_1$and ω_2, as after-tax endowments, and g as the after-tax deficit. In Freeman and Haslag (1996), Smith (1991), and Sargent and Wallace (1985), how interest payments were financed was important to the outcome. However, in this model a positive after-tax per capita deficit implies that the financing scheme will not be important.

function $U\left(c_t^t, c_{t+1}^t\right)$, where U is assumed to be strictly increasing in each argument, to be twice continuously differentiable, and to be strictly quasi-concave. The initial old value only old age consumption and desire as much of it as possible.

Young individuals can store their endowment, sell it to old individuals in exchange for money, or sell it to the government in exchange for either money or bonds. All individuals are assumed to have access to a non-stochastic, constant returns to scale technology for storing their endowment. In particular, one unit stored at date t returns $R > 1$ units of consumption at date $t+1$.[6] Let k_t denote the amount that an individual chooses to store at date t. In addition, let p_t denote the time t price level, let z_t denote the holdings of real balances by a young individual at t, and let b_t denote real bond holdings by a representative young agent at t. It is assumed that storage is subject to a reserve requirement,

$$z_t \geq \lambda k_t, \tag{1}$$

and that the government pays a gross rate of return x_{t+1} in period $t+1$ on the nominal balances which were obtained in period t. In addition to this reserve requirement, each young individual faces the following budget constraints at t:

$$c_t^t + z_t + k_t + b_t \leq \omega_1 \tag{2}$$

$$c_{t+1}^t \leq \omega_2 + Rk_t + \rho_{t+1}b_t + \left(\frac{p_t}{p_{t+1}}\right) x_{t+1}z_t, \tag{3}$$

where ρ_{t+1} is the gross real rate of return on government bonds between t and $t+1$.

The problem of a young individual at t is to maximize $U\left(c_t^t, c_{t+1}^t\right)$ subject to equations (1)-(3). If

$$R > x_{t+1}\frac{p_t}{p_{t+1}} \tag{4}$$

holds, then the reserve requirement is binding, and equation (1) holds as an equality.[7] This situation

[6] If population growth were allowed, then the condition that R exceeds one plus the rate of population growth would need to be imposed.

[7] The assumption that storage has a higher return, R, than that found on other assets is exogenous to the model. The return to storage in this model is analogous to the average long-run rate of return on capital in the real world. The assumptions made regarding R are motivated by, and consistent with, what is observed in the real world – namely returns to capital dominate the returns to government bonds which dominate the return to money.

is focused on throughout, in which case one can transform the young individuals' problem as follows. Let $d_t \equiv k_t + z_t = (1+\lambda)\, k_t$ denote storage plus reserves, which will be referred to as "deposits." In addition, let $\phi \equiv \frac{1}{1+\lambda}$, where ϕ denotes the fraction of deposits held in the form of storage, and $1-\phi$ can be thought of as the fraction of deposits required to be held as reserves. With this notation, the problem of a young individual at t can be rewritten as

$$\max U\left(c_t^t, c_{t+1}^t\right)$$

subject to

$$c_t^t + d_t + b_t \leq \omega_1 \tag{5}$$

$$c_{t+1}^t \leq \omega_2 + \left[\phi R + (1-\phi)\, x_{t+1}\left(\frac{p_t}{p_{t+1}}\right)\right] d_t + \rho_{t+1} b_t \tag{6}$$

Obviously, if bonds and deposits are both to be held,

$$\rho_{t+1} = \phi R + (1-\phi)\, x_{t+1}\left(\frac{p_t}{p_{t+1}}\right) \;\; ; \;\; t \geq 1 \tag{7}$$

must hold. The right hand side of equation (7) is simply the weighted return on a portfolio consisting of storage and currency, with $1-\phi$ being the portfolio weight attached to currency. Equation (7) then requires that the return on government bonds equal the appropriately weighted return on storage and currency, which is — in effect — the rate of return on deposits. Finally, in keeping with Friedman's original idea that the rate of return on reserves be equal to the short-term yield on government securities, it is assumed that $x_{t+1} = \rho_{t+1}$. Thus, equation (7) can be rewritten as

$$\rho_{t+1} = \frac{\phi R}{1-(1-\phi)\left(\frac{p_t}{p_{t+1}}\right)} \;\; ; \;\; t \geq 1. \tag{8}$$

When equation (8) holds, the problem confronting young individuals can be even further simplified. Let S_t denote total savings by a young individual at t : i.e., $S_t \equiv d_t + b_t$. Then this individual can be viewed as choosing S_t to maximize $U\left[\omega_1 - S_t, \omega_2 + \rho_{t+1} S_t\right]$. Let

$$S\left(\rho_{t+1}\right) \equiv \arg\max U\left[\omega_1 - S_t, \omega_2 + \rho_{t+1} S_t\right], \tag{9}$$

then the function S summarizes an individuals's optimal savings behavior. The following conditions on S are assumed to hold throughout the remainder of the paper.

Assumption 1 *For all dates $t \geq 1$, the function S satisfies*

$$S\left[\min\left\{\phi R, 1\right\}\right] \geq 0. \tag{A.1}$$

Assumption 2 *For all dates $t \geq 1$, the function S satisfies*

$$S'(\rho) > 0, \ \forall \rho > 0. \tag{A.2}$$

Assumption (A.1) implies that $S(1) \geq 0$ holds, rendering this a "Samuelson case" economy, and (A.2) asserts that savings are increasing in the rate of return, thereby ruling out "large" income effects.[8] Finally, Assumption (A.1) implies that $\phi \geq S^{-1}(0)/R$ must be satisfied; in effect this imposes an upper bound on the level of the reserve requirement. When this bound is in effect, individuals are willing to save non-negative amounts, regardless of the rate of return on reserves.

2.1.1 Remarks

With respect to how reserve requirements are modeled, equation (1) is meant to be interpreted as a conventional reserve requirement. Consistent with Bhattacharya *et al.* (1998), Espinosa-Vega and Russell (1998), and Wallace (1984), individuals can be thought of as not being allowed to store their own goods and hence require the services of an intermediary. Consumers decide what fraction of their savings to invest in bonds and what fraction to deposit in the bank, so that they can obtain the highest return – the return from the storage technology. However, intermediaries are required to hold a fraction of deposits — equal to $(1 - \phi)$ — in the form of cash reserves. Thus, all deposits are split between the storage technology and cash. If there is free entry into intermediation, intermediaries will earn zero profits and hold a portfolio maximizing the utility of a representative depositor. In this case, equations (1)-(3) simply represent the consolidated balance sheets of banks and individuals.[9,10]

The interpretation of the interest rate paid on reserves, x_{t+1} also deserves further attention.

[8] See Gale (1973).

[9] In this model, the reserve requirement is a means for ensuring that assets with dominated returns (such as money) are held in equilibrium. In other words, it is the governments way to ensure that it has a low cost option for financing its deficit. This has implications for how the government finances its deficit (which are discussed later in the paper). Understanding the impact of reserve requirements on government financing has been studied extensively before, for example see Dwyer and Saving (1986) for a steady state model that examines the role of reserve requirements in a model with money and deposits.

[10] It should also be noted that it has been assumed that bond-holders do not face a reserve requirement. See comments throughout Bhattacharya *et al.* (1998) about the impact on the basic model when interest is not paid on reserves.

While Friedman (1960) does not spend a great deal of time discussing how to set the interest rate on reserves, he does briefly suggest that a viable option would be to set the rate equal to the average yield on short-term government bonds from the previous few quarters.[11] This would correspond to setting $x_{t+1} = \rho_t$, as opposed to $x_{t+1} = \rho_{t+1}$ as done in this model. However, the key issue is linking the return on money to bonds, not whether the relationship is forward or backward looking. When interest is not paid on reserves, the government can price discriminate in its financing options (bonds being a more expensive option than money). However, when interest is paid and the returns on money and bonds are linked, then the government's ability to price discriminate is eliminated (or reduced). Consequently, when comparing the two economies what matters most is whether the ability to price discriminate is better – in the sense that it reduces indeterminacy and volatility in the economy.

2.1.2 The Government

The government must finance a real per capita deficit of g each period through the issue of money and bonds. The government's budget constraint is given by

$$g = \frac{M_t - M_{t-1}}{p_t} + b_t - \rho_t b_{t-1} - \frac{x_t M_{t-1}}{p_t} \;;\; \forall t \geq 1 \tag{10}$$

Equation (10) asserts that the real value of money created in period t, $\left(M_t - M_{t-1}\right)/p_t$, plus the real value of the bonds sold at that date, b_t, must equal the real value of the government budget deficit, g, plus the interest obligations on outstanding government debt, $\rho_t b_{t-1}$ and the interest obligations associated with reserves, $x_t M_{t-1}/p_t$. It is assume that the government conducts policy by choosing (once and for all in the first period), a ratio

$$\mu \equiv \frac{b_t}{z_t} \;;\; t \geq 1 \tag{11}$$

of bonds to currency. Variations in μ can be thought of as permanent open market operations.[12] In addition, the government sets the reserve requirement $1 - \phi$. The initial level of the money stock must satisfy $M_0 > 0$ and $B_0 = 0$ is assumed to be given as initial conditions.

[11] See Friedman (1961, chapter 3, p75) for the lone paragraph devoted to the appropriate choice of the rate of return to be paid on reserve holdings.

[12] Note that this definition of an open market operation differs from that in Freeman and Haslag (1996). Here it represents a shift in the composition of deficit financing instruments. In Freeman and Haslag (1996) it amounted to a purchase of an asset which was used to reduce the funds the bank needed to acquire to pay interest on reserves.

Substituting equations (7), (11), and $z_t \equiv M_t/p_t = \lambda k_t = (1-\phi)\,d_t$ in equation (10), it is possible to rewrite the government budget constraint as[13]

$$z_t\,(1+\mu) = g + z_{t-1}\left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)\,\rho_t}\right] + \rho_t z_{t-1}\left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)\,\rho_t} + \mu\right]\ ;\ t \geq 2. \tag{12}$$

Equation (12) can be interpreted as the government must issue enough liabilities at t, $z_t + b_t = (1+\mu)\,z_t$, to finance its current deficit plus the implied interest obligation on its inherited liabilities.

3 Equilibrium

Equilibrium requires that consumers maximize their utility and the government budget constraint holds. The first condition requires that the quantity of savings demanded must equal the quantity supplied. Given the definition of z_t, where $z_t \equiv M_t/p_t$ is the real value of the per capita money supply at t, $b_t + d_t = z_t\left(\mu + \frac{1}{1-\phi}\right)$ must hold in equilibrium. In addition, the supply of government bonds plus deposits must equal the savings of young individuals. Thus $b_t + d_t = S\left(\rho_{t+1}\right)$ must hold as well. Combining these two observations yields the following asset market clearing condition:

$$z_t\left(\mu + \frac{1}{1-\phi}\right) = S\left(\rho_{t+1}\right);\ t \geq 1. \tag{13}$$

Inverting equation (13) to obtain $\rho_{t+1} = S^{-1}\left\{z_t\left(\mu + \frac{1}{1-\phi}\right)\right\}$, and substituting the result into equation (12) yields the equilibrium law of motion for per capita real balances:

$$\begin{aligned} z_t\,(1+\mu) \;=\;& g + z_{t-1}\left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)\,S^{-1}\left\{z_{t-1}\left(\mu + \frac{1}{1-\phi}\right)\right\}}\right] + \\ & z_{t-1}S^{-1}\left\{z_{t-1}\left(\mu + \frac{1}{1-\phi}\right)\right\}\left[\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)\,S^{-1}\left\{z_{t-1}\left(\mu + \frac{1}{1-\phi}\right)\right\}} + \mu\right] \end{aligned} \tag{14}$$

It is now possible to solve for that equilibrium sequence of real balances, $\{z_t\}$. Once this is obtained, $S^{-1}\left\{z_t\left(\mu + \frac{1}{1-\phi}\right)\right\}$ gives the equilibrium rate of return on government bonds, while for a given ϕ,

$$(1-\phi)\,\frac{p_t}{p_{t+1}} = \frac{\rho_{t+1} - \phi R}{\rho_{t+1}} = 1 - \frac{\phi R}{S^{-1}\left\{z_t\left(\mu + \frac{1}{1-\phi}\right)\right\}} \tag{15}$$

[13] The initial, $t = 1$, government budget constraint is $(1+\mu)z_1 = g + (1 + x_1)\,M_0/p_1$. Once z_1 and x_1 are determined, then this government budget constraint gives us the initial price level.

describes the gross rate of return on real balances (the inverse of the gross rate of inflation.) It is straightforward to verify that the return on savings, $\rho_{t+1,}$ and the return on money, p_t/p_{t+1} , are both increasing functions of the level of real money balances. Consequently, an increase in the equilibrium level of real money holdings will result in an increase in utility for the consumer.

There are, of course, a number of conditions that an equilibrium sequence $\{z_t\}$ must satisfy. First, it must satisfy (14) at each date. Second, $z_t \geq 0$ for all dates $t \geq 1$ must also hold. Third, given the method of derivation, the reserve requirement must be binding at each date. And, finally, equation (15) must yield a non-negative gross return on real balances. These last two requirements can be written as

$$\phi R \leq S^{-1}\left\{z_t\left(\mu + \frac{1}{1-\phi}\right)\right\} < R;\ t \geq 1 \tag{16}$$

Equations (14), (16), and $z_t \geq 0$ constitute the equilibrium conditions.

3.1 Steady State Equilibria

Before examining the dynamic properties of this model, it will be useful to establish those conditions under which steady state equilibria exist. Setting $z_{t-1} = z_t = z$ in equation (14) and rearranging terms, one obtains the following steady state equilibrium condition:

$$g = z\left\{\left[1 + \mu - \left(\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)}\right)\right] - \left(\frac{1}{(1-\phi)} + \mu\right)S^{-1} + \frac{\phi R}{(1-\phi)\,S^{-1}}\right\}. \tag{17}$$

Define $H(z, \mu, \phi, R)$ by

$$H(z, \mu, \phi, R) \equiv z\left\{\left[1 + \mu + \frac{\phi R}{(1-\phi)} - \frac{1}{(1-\phi)}\right] - \left(\frac{1}{(1-\phi)} + \mu\right)S^{-1} + \frac{\phi R}{(1-\phi)\,S^{-1}}\right\} \tag{18}$$

The function $H(z, \mu, \phi, R)$ describes how much revenue, net of interest obligations, the government can raise in a steady state equilibrium if the per capita level of real balances is z, the bond-money ratio is μ, and the reserve requirement is $1-\phi$. In such an equilibrium, of course, the quantity of revenue raised must equal the government budget deficit g. However, in order for z to constitute a steady state equilibrium level of real balances, z must satisfy not only equation (17), but equation (16) as well.

To ascertain the conditions under which steady state equilibria exist, as well as their number, it will be useful to know more about the function $H(z, \mu, \phi, R)$. Its properties are stated in the following lemma.

Lemma 1 *(a) $H(z,\mu,\phi,R)=0$ holds iff $z=0$ or*

$$z = z^{\dagger} \equiv \frac{S\left\{\frac{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]+\left\{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]^2+4\left[\frac{1}{(1-\phi)}+\mu\right]\frac{\phi R}{(1-\phi)}\right\}^{1/2}}{2\left[\frac{1}{(1-\phi)}+\mu\right]}\right\}}{\left(\mu+\frac{1}{1-\phi}\right)} > 0.$$

(b) $H_1(0,\mu,\phi,R) > 0 > H_1\left(z^{\dagger},\mu,\phi,R\right)$ holds for all (μ,ϕ,R).

Lemma 1 is proved in Appendix A. For simplicity of exposition the following assumption is made.

Assumption 3 *For all values of μ, ϕ, R and $0 \le z \le z^{\dagger}$,*

$$H_{11}(z,\mu,\phi,R) < 0. \tag{A.3}$$

Thus, $H(z,\mu,\phi,R)$ is a concave function of z.

Under Assumptions (A.1)-(A.3), equation (18) has the configuration depicted in Figure 1. Any values of z satisfying equation (18) are candidate steady state equilibria. As shown in the figure, if there are any such candidates, there will generically be exactly two.[14] Let z^- denote the steady state with lower real balance holdings and z^+ the steady state with higher holdings. As is evident from Figure 1 the lower level of real balance holdings occurs on the "bad" side of the Laffer curve and the higher level on the "good" side: i.e., $H_1(z^-,\mu,\phi,R) > 0 > H_1(z^+,\mu,\phi,R)$ must hold.

In addition, any candidate steady state equilibria must also satisfy equation (16), which can be rewritten as

$$\frac{S(\phi R)}{\left(\mu+\frac{1}{1-\phi}\right)} \le z < \frac{S(R)}{\left(\mu+\frac{1}{1-\phi}\right)}. \tag{19}$$

The following result, and assumption on which it is based, will be useful in ascertaining when the right-hand side of equation (16) is binding. The lemma is proved in Appendix B.

Assumption 4 *Let μ, ϕ, R be such that*

$$1 \ge \mu(\phi R - 1) \tag{A.4}$$

[14] If Assumption (A.3) is relaxed, there can be more than two candidate steady states. In general, these equilibria will occur in pairs.

holds.[15]

Lemma 2 *Suppose that Assumption (A.4) holds, then* $z^{\dagger}$ *satisfies*

$$\frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)} \leq z^{\dagger} < \frac{S(R)}{\left(\mu + \frac{1}{1-\phi}\right)}.$$

For the remainder of the paper Assumption (A.4) is assumed to hold. Consequently, only the left-hand constraint in equation (16) can bind on the determination of a steady state equilibrium. There are three possibilities regarding whether z^- and z^+ constitute legitimate steady state equilibria.

Case 1 (Multiple Steady State) *If* $S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^-$, *then there are two genuine steady state equilibria.*

Case 2 (Unique Steady State) *If* $z^- < S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^+$, *then only* z^+ *constitutes a legitimate steady state equilibrium. Thus, there exists a unique steady state equilibrium.*

Case 3 (No Steady State) *If* $z^+ < S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right)$, *then no steady state equilibria exist.*

Obviously this last case is not of particular interest, and thus the remainder of the paper focuses on Cases 1 and 2, which are represented by Figure 1. An examination of these figures indicates that Case 1 is most likely to obtain for large values of g (given μ and ϕ), while Case 2 *must* obtain for sufficiently small values of g (again, for given choices of μ and ϕ). Thus, for sufficiently small but positive values of g (where small means relative to μ and ϕ), there will exist a *unique* steady state equilibrium. This is true even though the function $H(z, \mu, \phi, R)$ exhibits all of the standard properties that give rise to a "Laffer curve" phenomenon. The possibility that there is a unique steady state equilibrium, even in the presence of a Laffer curve, is a consequence of the fact that not all financing options consistent with supporting a given deficit, are also consistent with individuals wanting to hold all assets. In general, financing a given deficit can be achieved either by a small seigniorage tax base and high inflation or a large tax base and low inflation. When g is sufficiently small, it is not possible to finance the deficit with a small monetary base, maintain the link between the return on bonds and reserves, and have a positive return to money (i.e. equation (15) results in

[15] Assumption (A.4) holds for all values of μ if $\phi R \leq 1$. For an economy with a reserve requirement of 10 percent ($\phi = 0.9$), this condition will be satisfied if there is no asset with a safe, real rate of return in excess of 11.11 percent, which certainly seems empirically plausible. Of course if $\phi R > 1$ holds, then Assumption (A.4) places an upper bound on μ. In the third quarter of 2004, the outstanding gross public debt of the U.S. was \$7.38 trillion, while the monetary base was about \$749 billion. Thus, for the U.S., $\mu \approx 9.85$. In this case, Assumption (A.4) would hold so long as $\phi R < 1.102$.

a negative return to money). The key to this result is the fact that linking the return on reserves to the return on money restricts *how* the government can finance its deficit because its two options, using bonds and money, are not independent of each other anymore.

Finally, before investigating the dynamic properties of this economy, it will be useful to examine the impact of open market operations on steady state equilibrium values as well as to determine whether unpleasant monetarist arithmetic obtains.

3.2 Comparative Statics

Of particular interest is how changes in the bond-money ratio, μ, affect the steady state equilibrium level(s) of real balances, and the rate of inflation. An increase in μ corresponds to a (permanently) higher bond-money ratio, and hence to a contractionary open market operation, as conventionally defined.

Straightforward differentiation of equation (18) yields that, for any candidate steady state equilibrium,

$$H_1(z, \mu, \phi, R) \frac{\partial z}{\partial \mu} = -H_2(z, \mu, \phi.R). \tag{20}$$

The following lemma (which is proved in Appendix C) is now established.

Lemma 3 *Suppose that* $\phi R / [1 + \phi(1 - R)] > S^{-1}\left(z^{\dagger}\right)$ *holds, then* $H_2(z^{+}, \mu, \phi, R) < 0$ *and* $\partial z^{+} / \partial \mu < 0$.

Lemma 3 asserts that under the condition that the return on bonds not be too large relative to the return on storage (for a given reserve requirement), a contractionary open market operation necessarily reduces z^{+}.[16] Finally, what one would ultimately like to know is the effect of a change in μ on the steady state rate of return on real balances or, in other words, on the inverse inflation rate p_t / p_{t+1}. Appendix D establishes the following proposition.

Proposition 1 *The impact of a change in* μ *on* p_t / p_{t+1} *is given by*

$$(1 - \phi) \frac{\partial (p_t / p_{t+1})}{\partial \mu} = \frac{z \phi R S^{-1'}}{H_1 (S^{-1})^2} \left[\phi(R - 1) - 1 + \frac{\phi R}{S^{-1}}\right]. \tag{21}$$

If $\phi R / [1 + \phi(1 - R)] > S^{-1}\left(z^{\dagger}\right)$ *holds, as in Lemma 3, then* $\partial (p_t / p_{t+1}) / \partial \mu < 0$ *whenever* $H_1(z, \mu, \phi, R) < 0$. *In addition, when* $H_1(z, \mu, \phi, R) > 0$, *then* $\partial (p_t / p_{t+1}) / \partial \mu > 0$.

[16] Based on Lemma 3 nothing definitive can be said as to what happens to z^{-} : it may rise or fall. In addition, if $[1 + \phi(1 - R)] < 0$ or if $\phi R > 1$, then for any $z \in \left[0, z^{\dagger}\right]$, when $H_1(z, \mu, \phi, R) < 0$ it will be the case that $H_2(z, \mu, \phi, R) < 0$.

Proposition 1 states the familiar result about the "Laffer curve" and "unpleasant monetarist arithmetic." Under the conditions necessary for Lemma 3 to hold, $\partial\left(p_t / p_{t+1}\right) / \partial \mu$ increases on the upward sloping portion of $H(z, \mu, \phi, R)$, while decreasing on the downward sloping portion. Thus, a contractionary open market operation raises the steady state rate of inflation on the "good-side" of the Laffer curve and lowers inflation on the "bad-side." Consistent with Bhattacharya *et al.* (1998), this result does *not* require that ρ exceed the steady state rate of growth. All that is needed is for some asset whose real rate of return exceeds the economy's long-run rate of growth exist (in the model $R > 1$.) The unpleasant monetarist arithmetic is the result of $\rho > x_{t+1} p_t / p_{t+1}$ holding. In this case, an increase in the bond-money ratio substitutes a more expensive for a less expensive financing instrument. Consequently, heavier use must be made of the inflation tax.[17] Finally, it should also be noted that the set of interest rates, ρ, for which unpleasant monetarist arithmetic arises is potentially smaller when interest is paid on reserves than when it is not.[18] Thus, it is less likely to occur when the return on real balances is more closely tied to the real return on bonds.

Of course these remarks apply to *candidate* steady state equilibria [that is, those values of z satisfying equation (17)]. However, in this environment not all candidate steady state equilibria necessarily satisfy equation (16), and hence not all values of z satisfying equation (17) constitute legitimate steady states. There are two possibilities of interest regarding the number of steady states.

Multiple Steady States: *Let* $S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^-$, *both before and after the change in* μ.
Unique Steady State: *Let* $z^- < S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^+$, *both before and after the change in* μ.

In the multiple steady state case, z^- and z^+ are both legitimate steady state equilibria. In the alternative case, there is a *unique* steady state equilibrium, z^+. If Lemma 3 holds, then in both of these cases, unpleasant monetarist arithmetic prevails at the high real money balance equilibrium and *contractionary* open market activity will lead to a higher steady state inflation rate.

[17] See Bhattacharya *et al.* (1998) for a more in-depth explanation of this result and also a discussion of the likelihood that it applies to the United States.

[18] For example, if $S^{-1}\left(z^+\right) > \phi R / \left[1 + \phi(1-R)\right] > S^{-1}\left(z^-\right)$, then it would be the case that $\partial\left(p_t / p_{t+1}\right) / \partial \mu > 0$ at both steady state equilibria. In this case, unpleasant monetarist arithmetic would not arise. Finally, if $S^{-1}\left(z^-\right) > \phi R / \left[1 + \phi(1-R)\right]$ then the results of Proposition 1 are reversed. This is in contrast to the non-interest bearing economy where unpleasant monetarist arithmetic is always present at the high real money balance steady state.

3.3 Dynamic Equilibria

Before making a comparison between the interest and no-interest economies, it will be useful to understand the dynamic properties of the economy . Consistent with the previous analysis, the dynamic properties of the economy will depend largely on whether Case (1) or (2) prevails. Any potential equilibrium sequences of real money balances, $\{z_t\}$, must satisfy three conditions: equation (14), equation (16), and $z_t \geq 0$ for all t. Equation (14) is a simple, non-linear difference equation in terms of real money balances, z_t. Differentiating this equilibrium law of motion, one obtains

$$\begin{aligned}(1+\mu)\frac{dz_t}{dz_{t-1}} &= \left[\left(\frac{1}{(1-\phi)} - \frac{\phi R}{(1-\phi)}\right) + S^{-1}\left(\mu + \frac{1}{(1-\phi)}\right) - \frac{\phi R}{(1-\phi)\,S^{-1}}\right] \\ &\quad + z_{t-1}S^{-1'}\left(\mu + \frac{1}{1-\phi}\right)\left\{\frac{\phi R}{(1-\phi)\left(S^{-1}\right)^2} + \left(\mu + \frac{1}{1-\phi}\right)\right\}.\end{aligned} \tag{22}$$

An examination of equation (22) yields two general cases regarding the shape of equation (14): as depicted in Figures 2 and 3.

Case A1: *Suppose that*

$$\frac{\phi R}{(1-\phi)} \leq S^{-1}(0)\left(\mu + \frac{1}{(1-\phi)}\right),$$

then $dz_t/dz_{t-1} \geq 0$ for all $z \geq 0$. Thus, equation (14) defines an everywhere upward-sloping function, as depicted in Figure 2.

Case A2: *Suppose that*

$$\frac{\phi R}{(1-\phi)}\left(1 + \frac{1}{S^{-1}(0)}\right) > S^{-1}(0)\left(\mu + \frac{1}{(1-\phi)}\right) + \frac{1}{(1-\phi)}$$

and let $\underline{z}$ be such that

$$\begin{aligned}\frac{\phi R}{(1-\phi)}\left(1 + \frac{1}{S^{-1}(\underline{z})}\right) &= S^{-1}(\underline{z})\left(\mu + \frac{1}{(1-\phi)}\right) + \frac{1}{(1-\phi)} + \\ &\quad \underline{z}S^{-1'}(\underline{z})\left(\mu + \frac{1}{1-\phi}\right)\left\{\frac{\phi R}{(1-\phi)\left(S^{-1}(\underline{z})\right)^2} + \left(\mu + \frac{1}{1-\phi}\right)\right\}.\end{aligned}$$

For all $z > (<)\,\underline{z}$ then $dz_t/dz_{t-1} > (<)\,0$. If $\underline{z} < z^-$ then

$$\left.\frac{dz_t}{dz_{t-1}}\right|_{z_{t-1}=z^-} > 0$$

Thus, equation (14) defines a u-shaped function, as depicted in Figure 2.

In both of these cases, which are henceforth referred to as Case A, the equilibrium law of motion is upward sloping at the low real money balances steady state. In addition, the assumptions made previously regarding equation (14) imply that this equation is also positively sloped at the high real money balances steady state, i.e. equation (14) cannot be "backward" bending. The other possible case is the following.

Case B: *Suppose that* $\underline{z} > z^{-}$, *then*

$$\left.\frac{dz_t}{dz_{t-1}}\right|_{z_{t-1}=z^{-}} < 0$$

and equation (14) is u-shaped, as depicted by Figure 3.

These two cases (Case A and B) satisfy two of the three criteria necessary for a sequences of real money balances, $\{z_t\}$, to be an equilibrium path. As in the steady state analysis, it must also be the case that equation (16) holds, i.e. all assets must have a positive rate of return. Thus, for a sequence, $\{z_t\}$ to be an equilibrium path, it must be the case that every element in the sequence satisfies the constraints imposed by equation (16). The following proposition states conditions under which equation (16) will be violated.

Proposition 2 *Suppose that for some value of* z_{t-1}

$$\left.\frac{dz_t}{dz_{t-1}}\right|_{z_{t-1}} < 0.$$

Then this level of real money balances, z_{t-1}, *violates equation (16) and can not be an element in any equilibrium sequence,* $\{z_t\}$.

The proof of this proposition can be found in Appendix E. The implications of this proposition are that no element of an equilibrium path can lie on the downward sloping portion of equation (14). Thus, in the Case B economy, z^{-} cannot be a steady state equilibrium and any paths converging to this point also do not represent genuine equilibrium paths. In fact, in this particular case there exists a unique equilibrium at the high real money balances steady state, z^{+}.[19]

[19] It is important to point out that existence of at most two steady state equilibria is the result of Assumption A.3. If this assumption is relax, then multiple steady state equilibria may exist. In general there will be an even number of steady states. If more than two steady states exist, then a Case B economy will not have a unique equilibrium. In fact, the only difference between it and Case A economies will be that the potential steady state equilibrium with lowest level of real money holdings (and paths converging to it) will not be genuine equilibrium paths in a Case B economy.

In the Case A economy, the existence and nature of dynamic equilibria paths will depend on whether the economy is a Case 1 or 2 economy. Each is now consider in turn.

Case 1 (Multiple Steady States): In this case, $S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) \leq z^{-}$ holds, and there exist two genuine steady state equilibria as equation (14) is upward sloping at both steady state equilibria. It is easy to verify that for all initial levels of real money balances, z_0, that satisfy

$$z_0 \in \left[\max\left(\underline{z}, \frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)}\right), z^{+}\right],$$

then the resulting sequence of real money balances constitutes an equilibrium path. In this case the high real money balances steady state is a source while the low real money balances steady state is a sink. In addition, convergence to the low real money balances steady state is monotonic, as illustrated in Figure 2. Note that for initial values of real money holdings below the low real money balances steady state, $z_0 < z^{-}$, it must be the case that z_0 is "sufficiently close" to z^{-} [on the upward sloping portion of equation (14)] in order for a genuine equilibrium path to exist.

Case 2 (Unique Steady State): In this case, $S(\phi R) \Big/ \left(\mu + \frac{1}{1-\phi}\right) > z^{-}$ holds, and only z^{+} constitutes a legitimate steady state equilibrium. The point z^{-} cannot be an equilibrium because it violates equation (16), and hence this level of real balances can be achieved only if the returns to holding money are negative. Since z^{-} does not constitute a legitimate equilibrium value, any paths which converge to this point are also not viable equilibrium paths. Thus, as in the Case B economy, there exists a unique equilibrium value, the high real money balances steady state equilibrium z^{+}, as depicted in Figure 4.

4 Comparison to the No-Interest Economy

The objective of this paper is to compare of the results in the interest economy to an economy where interest is not paid on reserves. The no-interest economy is described in detail in Bhattacharya *et al.* (1998), and hence only the results are presented here.

When interest is not paid on reserves, then the consumer's budget constraints and the government's budget constraint are given by

$$c_t^t + d_t + b_t \leq \omega_1$$

$$c_{t+1}^{t} \leq \omega_2 + \left[\phi R + (1-\phi)\left(\frac{p_t}{p_{t+1}}\right)\right] d_t + \rho_{t+1} b_t$$

and

$$g = \frac{M_t - M_{t-1}}{p_t} + b_t - \rho_t b_{t-1} \text{ ; } \forall t \geq 1,$$

respectively. Solving for the equilibrium law of motion for per capita real balances yields

$$z_t (1+\mu) = g + z_{t-1}\left[S^{-1}\left(\mu + \frac{1}{(1-\phi)}\right) - \frac{\phi R}{(1-\phi)}\right] \tag{23}$$

which is very similar to equation (14). Equation (23) has the same basic shape(s) as equation (14), and thus it remains to establish their relative positions. The following proposition states the relationship between $z_t^I (z_{t-1}, \mu, \phi, R)$ and $z_t^{NI} (z_{t-1}, \mu, \phi, R)$, the interest and no-interest economies respectively.

Proposition 3 *Let $z_t^I (z_{t-1}, \mu, \phi, R)$ and $z_t^{NI} (z_{t-1}, \mu, \phi, R)$ define the equilibrium laws of motion for the interest and non interest economies respectively (i.e. equation (14) and (23) respectively).*
(i) For $z_{t-1} = 0$, then $z_t^I (0, \mu, \phi, R) = z_t^{NI} (0, \mu, \phi, R) = g/(1+\mu)$.
(ii) For all $z_{t-1} > 0$, then $z_t^I (z_{t-1}, \mu, \phi, R) > z_t^{NI} (z_{t-1}, \mu, \phi, R)$.
(iii) If $dz_t^I/dz_{t-1} < 0$ then $dz_t^{NI}/dz_{t-1} < 0$ and if $dz_t^{NI}/dz_{t-1} \geq 0$ then $dz_t^I/dz_{t-1} > 0$.

The proof of this proposition follows from a straightforward comparison of $z_t^I (z_{t-1}, \mu, \phi, R)$ and $z_t^{NI} (z_{t-1}, \mu, \phi, R)$, their derivatives, and equation (16), and hence the proof is omitted. Figure 5 provides a generalized illustration of the relative positions of $z_t^I (z_{t-1}, \mu, \phi, R)$ and $z_t^{NI} (z_{t-1}, \mu, \phi, R)$.[20]

The remainder of this section compares inflation, indeterminacy, volatility and welfare in the interest and no-interest economies. For the purposes of these comparisons, much of the remainder of the paper is restricted to the case where there exists two steady state equilibria in both economies. In this situation, it is easy to verify that if the economy is on an equilibrium path that converges to the low real money balances steady state equilibrium, $z^-(NI)$, in the no-interest economy and if at date t, $z_t < z^+(I)$, then when the government switches to paying interest on reserves, the economy will transition to the low real money balances steady state equilibrium $z^-(I)$. Conversely, if at date t, $z_t > z^+(I)$, then no equilibria exist after switching to interest payments on reserves

[20]Given the three general possibilities for the shapes of equations (14) and (23) (depicted in Figures 2 and 3), there are several possible combinations regarding their relative shapes. However, for all possible combinations Proposition 3 must hold. Thus, Figure 5 represents a single possibility of the relative shapes of the equilibrium laws of motion in the interest and no-interest economies (i.e. both upward sloping). However, the results discussed below generalize to any combination of equilibrium laws of motion one may draw.

unless a change in government policy, μ, can cause the equilibrium law of motion, equation (14) to shift to the right. Thus, Lemma 3 implies that paying interest combined with an expansionary open market operation can yield no change in the equilibrium level of real balances.[21]

4.1 Deficits and Inflation

As illustrated in Figure 5, for a given bond-money ratio, μ, and a given reserve requirement, ϕ, the set of sustainable government deficits is smaller when interest is paid on reserves. Let $\hat{g}$ be defined as the government deficit associated with the following condition

$$\left.\frac{dz_t\left(I\right)}{dz_{t-1}}\right|_{z_t=z_{t-1}=z} = 1.$$

It is easy to verify that for $g > \hat{g}$, no equilibria exist (i.e. equation (14) never intersects the 45^{o} line). Thus the range of government deficits consistent with the existence of equilibria is given by $g \in [0, \hat{g}]$. It is obvious from Figure 5 that $\hat{g}\left(NI\right) > \hat{g}\left(I\right)$, and thus, the set of sustainable government deficits is smaller when paying interest on reserves. This is not surprising considering that the total resources are the same in both economies and that paying interest on reserves results in fewer available resources to sustain the government deficit.

The inflation levels in the two economies can also be compared. In the no-interest economy, the inverse of the inflation rate is given by

$$\left(1-\phi\right)\frac{p_t}{p_{t+1}}\left(NI\right) = S^{-1}\left\{z_t\left(\mu + \frac{1}{1-\phi}\right)\right\} - \phi R \tag{24}$$

while for the interest economy it is given by equation (15). The following lemma states sufficient conditions under which the rate of inflation is greater, the lower the level of real money balances.

Lemma 4 *Suppose multiple steady state equilibria exist in both economies and let $\hat{z}$ be the value of real balances such that $S^{-1}\left(\hat{z}\right) = 1$. If $z^{-}\left(I\right) < \hat{z}$, then for all $z_{t-1} < \min\left(\hat{z}, z^{+}\left(I\right)\right)$,*

$$\frac{p_t}{p_{t+1}}\left(NI\right) < \frac{p_t}{p_{t+1}}\left(I\right).$$

The proof of this lemma follows from Assumption A.2, equation (16) and a comparison of equations (15) and (24). Lemma 4 states that for a given initial level of real money balances, the equilibrium

[21] In effect, if an appropriate expansionary monetary policy is undertaken when introducing interest payments on reserves, then equation (14) (I) in Figure 5 can be pivoted to the right and be identical to equation (23) (NI).

path converging to the low real money balances steady state will exhibit a lower rate of inflation at every date t in the economy where interest is paid on reserves.

The intuition for this follows from two facts. First, at every date t, $z_t^{NI} < z_t^I$ for all paths converging to the stable steady state. Second, at every date both economies must finance the same size after-tax deficit. For a given size deficit, the government can either have a large tax base and small tax rate or conversely a small tax base and relatively larger tax rate. Because $z_t^{NI} < z_t^I$ at every date t, the seigniorage tax base is always smaller in the no-interest economy. Hence, it must be the case that there is a larger tax rate (inflation) in the no-interest economy since the level of after-tax deficit is identical in the two economies.

4.2 Indeterminacy and Volatility

One of Friedman's primary concerns was eliminating price level indeterminacy and volatility. He felt that paying interest on reserves (and potentially combining it with a 100% reserve requirement) would achieve these goals. As is obvious from Figures 5 - 7, paying interest on reserves in the presence of a constant per capita government deficit at best maintains the level of price indeterminacy and at worst increases the indeterminacy when compared to an economy where reserves do not earn interest. In contrast, the impact on volatility, defined as the variance over equilibrium outcomes, may increase or decrease, depending on the number of steady state equilibria. The exact impact on both will depend on whether the lower bound on the interest rate paid on bonds is binding in equation (16).

Case I (Multiple Steady States – Both Economies): *Suppose that the following condition is satisfied*

$$\frac{S(\phi R)}{\left(\mu + \frac{1}{1-\phi}\right)} \leq z^{-}(NI).$$

In this case, the lower bound on real money balances does not bind in either the no-interest economy or the interest economy. Therefore, two steady state equilibria exist in both economies. As Figure 5 illustrates, because there is nothing to tie down the initial level of real money balances in either economies, there are an infinite number of equilibrium paths which converge to the low real money balances steady state, and thus, paying interest on reserves does not affect the determinacy of equilibria. In contrast, there will be less volatility in the case where interest is paid on reserves. The range of real money balances consistent with equilibria is given by $\left[S\left(\phi R\right)\Big/\left(\mu+\frac{1}{1-\phi}\right),z^{+}(I)\right]$ in the interest paying economy and $\left[S\left(\phi R\right)\Big/\left(\mu+\frac{1}{1-\phi}\right),z^{+}(NI)\right]$ in the no-interest economy. Since $z^{+}(I)<z^{+}(NI)$, the range of real money balances is greater when interest is not paid – consistent with Friedman's ideas.

The intuition for this relates to two key issues. First, both economies are financing the same size after-tax deficit. Second, the government's ability to finance this deficit is restricted in the interest paying economy and unrestricted in the no-interest economy. In the no-interest economy the government can use any mixture of bonds (a more expensive financing option) and money (a less expensive financing option) to finance the deficit. In addition, it is free to set any tax base and tax rate for both of these financing options, so long as sufficient revenue is raised to finance the deficit. In contrast, when interest is paid on reserves, the return on reserves is tied to bonds and therefore money is a more expensive means for financing the deficit relative to the no-interest economy. This also restricts the government's ability to exercise a trade-off between a larger seigniorage tax base and the seigniorage tax rate. Consequently the range of real money balances which can sustain the per capita deficit g, is smaller when interest is paid on reserves, i.e., $z^{+}(I)<z^{+}(NI)$. Thus, in this case interest payments have the desired effect of reducing economic volatility.

Case II (Multiple Steady States – Interest Economy): *Suppose that the following condition is satisfied*

$$z^{-}\left(NI\right)<\frac{S\left(\phi R\right)}{\left(\mu+\frac{1}{1-\phi}\right)}\leq z^{-}\left(I\right).$$

In this case there exist two steady state equilibria in the interest bearing economy and only one in the no-interest economy, as depicted in Figure 6. As a consequence, paying interest on reserves increases both indeterminacy and volatility dramatically. The explanation parallels that of the previous case just discussed. This result follows from the fact that in the no-interest economy, the government is attempting to finance its deficit with a mix of more (bonds) or less (money) expensive financing options. This can be achieved either by means of a large tax base and small tax or vice versa. However, the government faces a lower bound on the return it can offer on bonds (due to the presence of the storage technology), while still ensuring that individuals hold both money and bonds. This, in turn, results in a minimum level of the seigniorage tax base that must be maintained so that the fixed deficit, g, can be financed. However, in the no-interest economy, the level of real balances at the low real money balances steady state is less than the minimum level needed to ensure that all assets are held, and consequently is not consistent with existence of equilibrium. In addition, any dynamic paths which converge to this potential steady state are also not consistent with equilibrium. Thus, there exists a unique equilibrium level of real money balances in the no-interest economy.

This is not the case when interest is paid on reserves. The range of seigniorage tax base options is limited by the fact that the return on the less expensive financing option, money, is linked to the return on the more expensive financing option, bonds. Because the return on bonds and money are linked, this reduces the government's ability to choose the more or less expensive options to finance their deficit by limiting the trade-off between a large tax base and a large tax rate. Consequently, the small money balances equilibrium represented by $z^{-}(NI)$ is not an option in the interest economy, while $z^{-}(I)$is sufficiently large to be consistent with a binding reserve requirement and positive money and bond holdings. Hence, there exists an infinite number of equilibrium paths converging to the low real money balances steady state and the range of real money holdings consistent with equilibrium is again given by $\left[S(\phi R) \Big/ \left(\mu+\frac{1}{1-\phi}\right), z^{+}(I)\right]$. Thus, in this case paying interest on reserves increases both indeterminacy and volatility.

Case III (Unique Equilibrium – Both Economies): *Suppose that the following condition is satisfied*

$$z^{-}(I) < \frac{S(\phi R)}{\left(\mu+\frac{1}{1-\phi}\right)}.$$

In this case there exists a unique, high real money balance equilibrium in both the interest bearing and no-interest economies, as depicted in Figure 7. Consequently, there is no indeterminacy regardless of whether interest is paid on reserves. In addition, since equilibria consists on a singleton in both economies, the level of volatility is also the same. Thus paying interest on reserves neither helps nor hurts the economy in terms of reducing indeterminacy or volatility.

For Cases I – III, the level of indeterminacy and volatility depends on the number of steady state equilibria which exists in the respective economies. After examining the above three cases, one can conclude that either paying interest on reserves does not affect the number of steady state equilibria (and hence the level of indeterminacy is unaffected) or it increases the number of steady state equilibria (and also the level of indeterminacy). The latter case, a rise in the level of indeterminacy, is the opposite of what Friedman had envisioned.[22] With respect to the level of volatility, when there exist multiple steady states in both economies, paying interest reduces volatility, as Friedman had envisioned. In contrast, when interest payments increases the number of steady states, it also increases the level of volatility.

As mentioned in the introduction, one key difference between Friedman's ideas and this paper is the *source* of any volatility. Friedman had envisioned volatility coming from stochastic aspects of the economy. In this paper, any volatility or indeterminacy is a result of the tension which arises between the government trying to finance a given deficit while simultaneously trying to link together the rates of returns on its financing options.

4.3 Welfare

The final point of comparison is the impact of paying interest on reserves on consumer welfare. It is important to point out that the welfare analysis in this paper differs fundamentally from that found in both Smith (1991) and Freeman and Haslag (1996). In both of these previous papers, governments did not have after-tax deficits to finance, and consequently, neither did they have government bonds which paid a lower rate of return to that of capital.[23] Their analyses of welfare were, therefore, examining Pareto optimal solutions. In contrast, the examination of welfare conducted here is concerned with whether the transition to an interest paying regime is a Pareto improvement. Given the fact that after-tax deficits are financed via money and bonds (both of which pay a rate of return less than the storage technology), the solutions obtained in both economies are "second best" solutions.[24]

Given Assumptions (A.1) and (A.2), which require that savings be a strictly increasing function of the rate of return on savings, and the results of Lemma 4, it is straightforward to show that higher real money balances are Pareto superior to lower real balances. Since the issue of concern is the

[22] This result is consistent with Sargent and Wallace (1985) and runs counter to Smith (1991). The difference between Smith (1991) and this paper is two-fold. First, the real return to holding money balance in terms of date $t+1$ is not fixed to the return on storage, and second, the government must finance a deficit. If both of these conditions did not exist, the results of this paper would be consistent with those of Smith (1991).

[23] Although Freeman and Haslag (1996) did not have government bonds, they did have government capital, which can be thought of as negative quantities of bonds. However, in their model the return on "bonds" was the same as the return to storage.

[24] Although I am not ascertaining what the Pareto optimal solution is, the analysis conducted is useful from a purely "real world" policy perspective. In the case of the U.S. Federal Reserve System, for example, the option of whether to pay interest on reserves or not is being considered within the context of real world deficits which are being financed via instruments (bonds and seigniorage) whose rate of return is dominated by that of capital. Thus the policy consideration is one of "second best options" and the welfare analysis conducted in this paper sheds light on the likely impact of switching to an interest paying regime.

impact on welfare when the economy transitions to an interest paying regime, the welfare analysis is restricted to the case where there exist multiple steady state equilibria in both economies and the economies are on dynamic paths converging to the low real balances steady state equilibria.[25] This case is depicted in Figure 5. A comparison of low real money balance steady state equilibria (and the paths converging to them) yields $z_t(NI) < z_t(I)$ at all dates t. For these equilibria, welfare in the interest paying economy will be greater than when interest is not paid on reserves. In addition, it is obvious that switching from a no-interest regime to an interest paying regime will result in higher welfare. Thus, the transition will result in a Pareto improvement in welfare. However a caveat must be noted. These results are derived from Lemma 4, which in turn requires that, $\hat{z}$, the real money balances associate with the real gross return on bonds being equal to one, must be sufficiently large. In addition, the equilibrium paths of real money balances must be less than $\hat{z}$. If these conditions are not satisfied, then only under a more stringent set of conditions will the above results hold.

5 Conclusion

Although it is almost a half century since Milton Friedman first made his proposal to pay interest on reserves, the economic impact of this proposal is still not completely understood. Friedman was motivated by his desire to eliminate costly price level indeterminacy and volatility, and thereby improve overall economic well-being. Although several authors have examined the impact of paying interest on reserves on these issues, their models omitted two important issues: paying a rate of return on reserves equal to that of similar, short-term assets and accounting for how financing a government deficit would alter the implications of switching to an interest-on-reserves regime.

This paper has attempted to re-examine those issues of concern to Friedman in the context of a model where the rate of return on reserves equals that of government securities and where the government finances an after-tax deficit via debt and seigniorage. The model in this paper breaks from the previous literature by using a standard three asset model (storage, bonds and money), where the return to money is dominated by the return on other assets. Storage must be intermediated and is subject to a reserve requirement, where the return on reserves is equated to

[25] A comparison of welfare at all equilibria when there exist multiple or a unique steady state equilibria could also be undertaken. However, since the welfare analysis relies on the results of Lemma 4, this lemma would have to be expanded to include cases where $\hat{z}$ was greater than (or less than) the high and low real money balances steady states for both the interest and no-interest economies. This would require several subcases to be examined but would not shed light on the transition from one regime to another since in many cases a switch to paying interest would result in the non-existence of equilibria unless appropriate open market operation were simultaneously undertaken.

the return on bonds. Finally, I also include scope for a government that must finance an after-tax deficit while simultaneously paying interest on bonds and reserves.

As a result of adding these two features, which were absent from previous models, I am able to demonstrate results about indeterminacy, volatility, and welfare that differ from the previous literature. First, the level of indeterminacy is equal to or greater than the level when interest is not paid on reserves. Second, when the level of indeterminacy is the same in the two economies, then economic volatility is reduced with the introduction of interest payments. However, when greater indeterminacy in the interest-on-reserves economy exists, then there also exists greater volatility. Third, when the level of indeterminacy is the same in the two economies, then the equilibrium associated with low real money balances (and the paths converging to it) in the interest economy is welfare improving compared to the no-interest economy. Finally, under a narrow set of conditions, unpleasant monetarist arithmetic may apply to some of the steady state equilibrium. Many of these results run counter to what Friedman had envisioned and also to previous findings.

The key to these results is two-fold. First, by not pegging the return on reserves to the asset with the highest real return, the model allows for multiple equilibria (unlike in Smith (1991) and Freeman and Haslag (1996)). Multiple equilibria, and the possibility that the interest-on-reserves economy has a greater number of equilibria than the no-interest economy, generate indeterminacy, volatility, and welfare gains independent of the existence of sunspots, as in Smith (1991).

Second, the existence of an after-tax government deficit that must be financed via a combination of debt and seigniorage affects how a shift from a no-interest to interest-on-reserves regime impacts the overall economy. When the return on reserves is linked to the return on bonds, the options available to the government in terms of how it finances its deficit are limited. The government's ability to trade-off between higher cost funding (bonds) and lower cost funding (money) is diminished; as is the government's ability to make a trade-off between a large seigniorage tax base and a high seigniorage tax rate. Thus, paying interest on reserve reduces the set of real money balances which can support a given deficit while still making bonds and money an attractive investment option to individuals. As a result, multiple equilibria are more likely to occur, which implies a greater likelihood for indeterminacy and volatility when compared to the no-interest economy.

There is scope for extensions to this current work. Most obvious would be to analyze the impact of paying interest on reserves in the presence of both risky assets and returns and an after-tax government deficit. This paper set out to analyze only one of the two omissions found in the current literature: namely the lack of a government deficit. However, equally important in

Friedman's original ideas was that the returns to reserves should be equated to like assets – i.e., ones with similar risk and return structures. Only when a model includes both of these issues will we have a clearer understanding as to whether indeterminacy and volatility can be reduced by means of paying interest on reserves.

6 Appendix

A Proof of Lemma 1

a) That $H(z,\mu,\phi,R)=0$ iff $z=0$ or $z=z^{\dagger}$ follows immediately from the definition of $H(z,\mu,\phi,d)$, and the fact that $S(\rho)$ is an increasing function. Furthermore, $z^{\dagger}>0$ holds iff

$$\frac{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]+\left\{\left[1+\mu+\frac{\phi R-1}{(1-\phi)}\right]^{2}+4\left[\frac{1}{(1-\phi)}+\mu\right]\frac{\phi R}{(1-\phi)}\right\}^{1/2}}{2\left[\frac{1}{(1-\phi)}+\mu\right]}>S^{-1}(0).$$

However, Assumption (A.1).implies that $S^{-1}(0)<\min\{1,\phi R\}$ holds and thus this equation is satisfied.

(b) From the definition of $H(z,\mu,\phi,R)$, it follows that

$$\begin{aligned} H_{1} \;=\; & \left\{1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]-\left[\frac{1}{(1-\phi)}+\mu\right]S^{-1}+\frac{\phi R}{(1-\phi)\,S^{-1}}\right\} \\ & -zS^{-1'}\left(\mu+\frac{1}{1-\phi}\right)\left\{\frac{1}{(1-\phi)}+\mu+\frac{\phi R}{(1-\phi)\left(S^{-1}\right)^{2}}\right\}. \end{aligned} \tag{a.2}$$

However $H_{1}(0,\mu,\phi,R)>0$ holds iff

$$1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]-\left[\frac{1}{(1-\phi)}+\mu\right]S^{-1}(0)+\frac{\phi R}{(1-\phi)\,S^{-1}(0)}>0.$$

As in part (a) above, this is guaranteed by Assumption (A.1). It is easy to verify that $H_{1}\left(z^{\dagger},\mu,\phi,d\right)$ is given by

$$H_{1}|_{z=z^{\dagger}}=-z^{\dagger}S^{-1'}\left(z^{\dagger}\right)\left(\mu+\frac{1}{1-\phi}\right)\left\{\frac{1}{(1-\phi)}+\mu+\frac{\phi R}{(1-\phi)\left(S^{-1}\left(z^{\dagger}\right)\right)^{2}}\right\}<0.$$

Thus, it is the case that $H_{1}\left(z^{\dagger},\mu,\phi,R\right)<0<H_{1}(0,\mu,\phi,R)$. ■

B Proof of Lemma 2

From the definition of $z^{\dagger}$, the claim follows if

$$\phi R < \frac{\left[(1+\mu) - \frac{1}{(1-\phi)} + \frac{\phi R}{(1-\phi)}\right] + \left\{\left[(1+\mu) - \frac{1}{(1-\phi)} + \frac{\phi R}{(1-\phi)}\right]^2 + 4\left[\frac{1}{(1-\phi)} + \mu\right]\frac{\phi R}{(1-\phi)}\right\}^{\frac{1}{2}}}{2\left[\frac{1}{(1-\phi)} + \mu\right]} < R. \tag{b.1}$$

The left-hand inequality in equation (b.1) follows from Assumption (A.4). The right-hand inequality is implied by $R > 1$. ■

C Proof of Lemma 3

Straightforward differentiation yields

$$H_2 = z\left\{1 - S^{-1} - z\left[\frac{1}{(1-\phi)} + \mu\right]S^{-1'} - \frac{\phi R S^{-1'} z}{(1-\phi)\left(S^{-1}\right)^2}\right\}. \tag{c.1}$$

Re-writing equation (a.2), one obtains

$$\begin{aligned} H_1 \quad = \quad & \left(\frac{1}{(1-\phi)} + \mu\right)\left\{\frac{1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]}{\frac{1}{(1-\phi)}+\mu} - S^{-1} + \frac{\frac{\phi R}{(1-\phi)S^{-1}}}{\frac{1}{(1-\phi)}+\mu}\right\} \\ & -z\left(\frac{1}{(1-\phi)} + \mu\right)S^{-1'}\left\{\left[\frac{1}{(1-\phi)} + \mu\right] + \frac{\phi R}{(1-\phi)\left(S^{-1}\right)^2}\right\}. \end{aligned}$$

Thus, it is sufficient to show that

$$\frac{1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]}{\frac{1}{(1-\phi)}+\mu} + \frac{\frac{\phi R}{(1-\phi)S^{-1}}}{\frac{1}{(1-\phi)}+\mu} > 1. \tag{c.2}$$

If this equation holds, then $H_1\left(z^{+},\mu,\phi,R\right) < 0$ implies that $H_2\left(z^{+},\mu,\phi,R\right) < 0$ holds as well. However, equation (c.2) holds iff

$$\phi R > \left[1 + \phi\left(1 - R\right)\right]S^{-1}.$$

For $1 + \phi\left(1 - R\right) < 0$, this will obviously hold. In addition, if $1 + \phi\left(1 - R\right) > 0$ and $\phi R > 1$, then by equation (16) this will also always hold. If neither of these is the case, then given Assumptions

(A.1) and (A.2), a sufficient condition to guarantee the above is

$$\phi R > [1 + \phi(1 - R)]\, S^{-1}\left(z^{\dagger}\right).$$

It then follows from equation (20) that $\partial z^{+} / \partial\mu < 0$. ■

D Proof of Proposition 1

Differentiating equation (15) yields

$$(1-\phi)\frac{\partial \frac{p_t}{p_{t+1}}}{\partial \mu} = \frac{\phi R}{\left(S^{-1}\right)^2} S^{-1'}\left[\left(\mu + \frac{1}{1-\phi}\right)\frac{\partial z}{\partial \mu} + z\right]. \qquad \text{(d.1)}$$

In addition, equation (20), combined with equations (a.2) and (c.1) implies

$$\frac{\partial z}{\partial \mu} = -\frac{z\left\{1 - S^{-1} - z\left[\frac{1}{(1-\phi)} + \mu\right]S^{-1'} - \frac{\phi R S^{-1'} z}{(1-\phi)\left(S^{-1}\right)^2}\right\}}{\left[\frac{1}{(1-\phi)} + \mu\right]\left\{\frac{1+\mu-\left[\frac{1-\phi R}{(1-\phi)}\right]}{\frac{1}{(1-\phi)}+\mu} + \frac{\frac{\phi R}{(1-\phi)S^{-1}}}{\frac{1}{(1-\phi)}+\mu} - S^{-1} - zS^{-1'}\left\{\left[\frac{1}{(1-\phi)} + \mu\right] + \frac{\phi R}{(1-\phi)\left(S^{-1}\right)^2}\right\}\right\}}.$$

Substituting this equation into equation (d.1) and simplifying, one obtains

$$(1-\phi)\frac{\partial \frac{p_t}{p_{t+1}}}{\partial \mu} = \frac{z\phi R S^{-1'}}{H_1\left(S^{-1}\right)^2}\left[\phi(R-1) - 1 + \frac{\phi R}{S^{-1}}\right].$$

Thus, if $\phi(R-1) - 1 + \phi R / S^{-1} > 0$, then $\partial \frac{p_t}{p_{t+1}} \Big/ \partial \mu$ has the same sign as H_1. However, given Assumption (A.2), if $\phi R / [1 + \phi(1-R)] > S^{-1}\left(z^{\dagger}\right)$ holds, then $\phi(R-1) - 1 + \phi R / S^{-1} > 0$ for all $z \in \left[0, z^{\dagger}\right]$ and $\partial \frac{p_t}{p_{t+1}} \Big/ \partial \mu < 0$ when $H_1 < 0$. Conversely, if $H_1 > 0$ then so too must $\partial \frac{p_t}{p_{t+1}} \Big/ \partial \mu > 0$. ■

E Proof of Proposition 2

Equation (22) implies that $dz_t / dz_{t-1} < 0$ holds only if

$$S^{-1} < \frac{\phi R}{\mu(1-\phi) + 1} < \phi R.$$

However, this violates equation (16) and hence any z satisfying this condition cannot be part of an equilibrium path.

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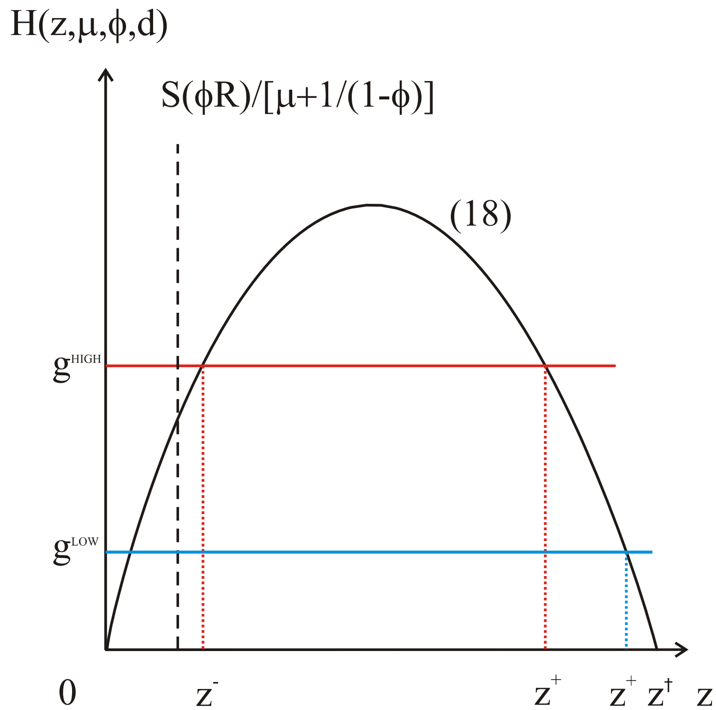

Figure 1: Steady State Equilibria

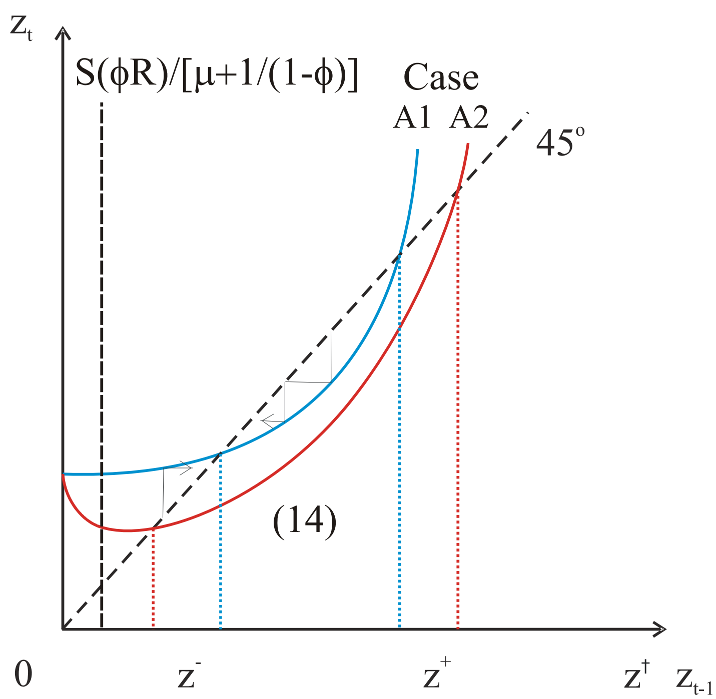

Figure 2: Upward Sloping Equilibrium Law of Motion with Multiple Steady States

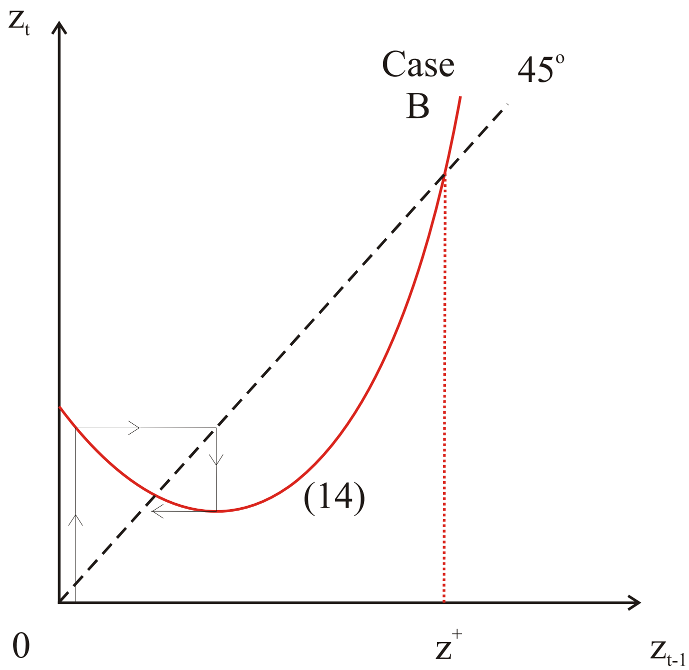

Figure 3: Unique Equilibrium

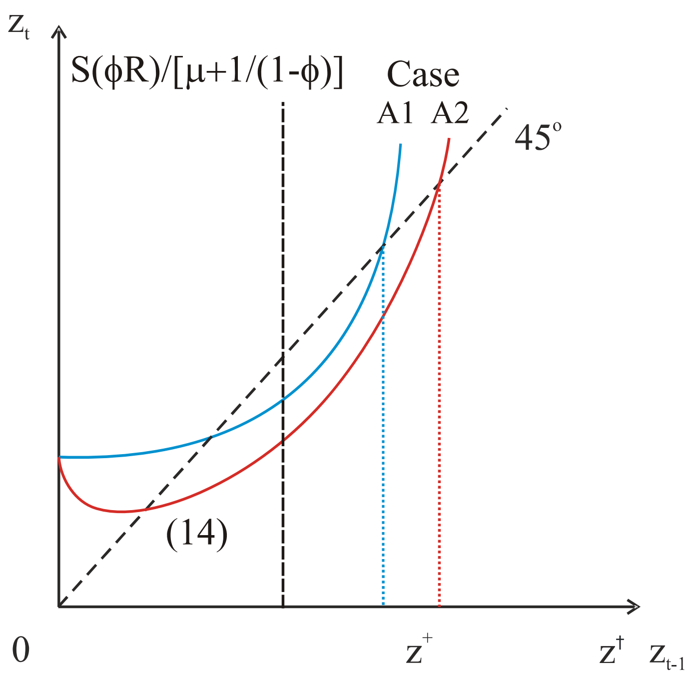

Figure 4: Upward Sloping Equilibrium Law of Motion with A Unique Equilibrium

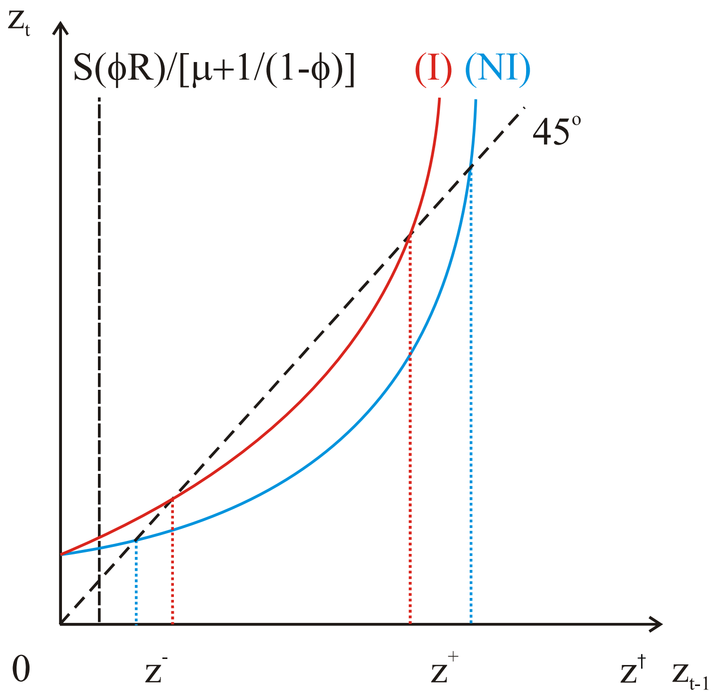

Figure 5: Multiple Steady State Equilibria in the Interest and No-Interest Economies

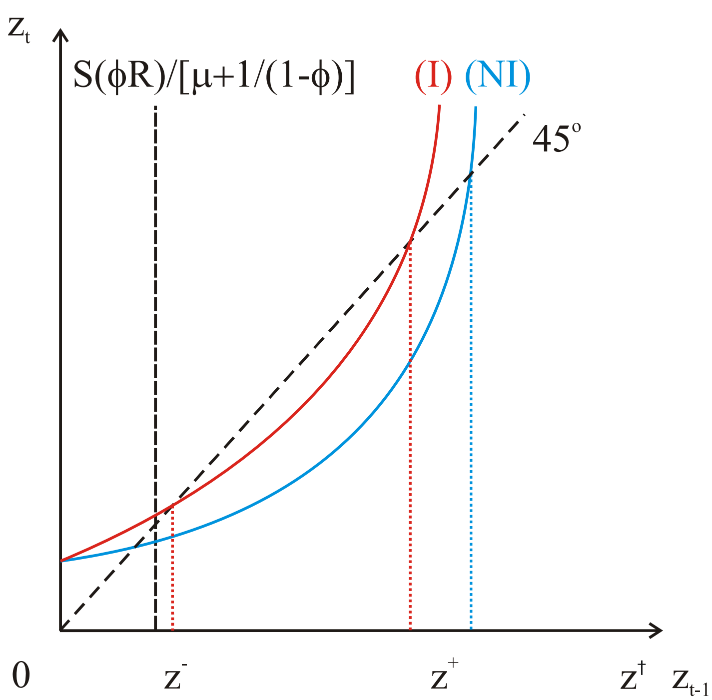

Figure 6: Multiple Steady State Equilibria in the Interest Economy Only

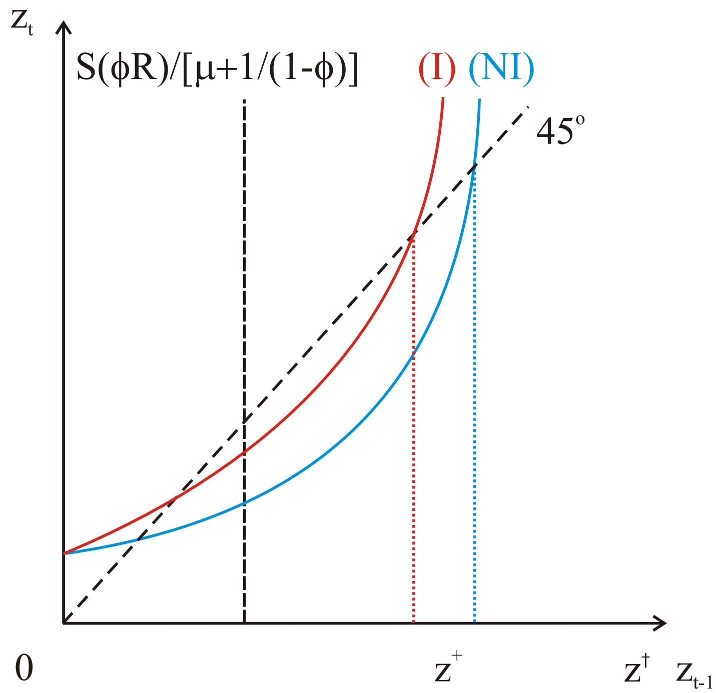

Figure 7: Unique Equilibrium in the Interest and No-Interest Economies