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Abstract

This paper presents a two-country sticky-price model that allows for capital and investment spending. It analyzes the conditions for equilibrium determinacy under alternative interest-rate rules that react to either domestic or consumer price inflation. It is shown that in the presence of investment, real indeterminacy is considerably easier to obtain once trade openness is permitted. Consequently we argue that sufficiently open economies should adopt a backward-looking rule and sufficiently closed economies should employ a current-looking rule, in order to minimize policy induced aggregate instability.

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1 Introduction

There is a large body of research that has considered the real indeterminacy implications of designing interest-rate rules in sticky-price monetary models.[1] The general conclusion that emerges from the literature is that in order to rule out real indeterminacy the monetary authority should follow the Taylor principle (i.e. an active policy stance), that is, a policy that aggressively targets either expected inflation (e.g. Bernanke and Woodford (1997), Clarida *et al.* (2000)) or current inflation (e.g. Kerr and King (1996)) by raising the nominal interest rate by proportionally more than the increase in inflation. However a number of recent studies have challenged the appropriateness of the Taylor principle in preventing multiplicity of equilibrium once the economic environment allows for capital and investment spending. Dupor (2001) introduces investment spending using a continuous- time framework and shows that a passive policy stance is required for equilibrium determinacy. In a discrete-time framework, Carlstrom and Fuerst (2005) show that with the addition of capital and investment spending equilibrium "determinacy is essentially impossible" under a forward-looking interest-rate rule.[2] For current-looking interest-rate rules, Carlstom and Fuerst (2005), Sveen and Weinke (2005) and Benhabib and Eusepi (2005) all find that the Taylor principle is not a sufficient condition for determinacy, although the range of indeterminacy generated is typically small.[3]

The purpose of this paper is to investigate the importance of investment spending for equilibrium determinacy for economies that are open to international trade in goods and assets. Using a discrete-time, money-in-the-utility function framework, this paper develops a two-country, sticky-price model that allows for capital and investment spending.[4] The conditions for real equilibrium determinacy are analyzed for forward, current and backward-looking versions of the interest rate rule. In addition, two alternative price

[1]By real indeterminacy we simply mean that there exists a continuum of equilibrium paths, starting from the same initial conditions, which converge to the steady state.

[2]Interestingly, Kurozumi and Van Zandweghe (2007) show that the range of determinacy can be significantly increased if the monetary authority implements an interest rate policy that responds to both expected inflation and current output.

[3]Sveen and Weinke (2005) show that the range of indeterminacy is higher if firm-specific capital is assumed, relative to the more common assumption of a competitive rental market for capital. Benhabib and Eusepi (2005) show that the range of parameter values that guarantee local determinacy do not necessarily guarantee global determinacy.

[4]To facilitate comparison with the existing literature, this paper adopts the traditional convention that end-of-period money balances enter the utility function. Assuming an alternative timing-assumption would have important consequences for equilibrium determinacy, as discussed by Carlstrom and Fuerst (2001), Kurozumi (2006) and McKnight (2007).

indexes, which can be chosen as the policy indicator, are considered: domestic price inflation and consumer price inflation. By allowing for trade openness, it is shown that the indeterminacy problem is more severe under a forward-looking interest-rate rule and gets increasingly worse as the degree of openness increases. This result is robust for both possible indexes of inflation that can enter the feedback rule. However, unlike closed-economy models, this indeterminacy problem can no longer be dramatically reduced by adopting an active current-looking rule. For both indexes of inflation, indeterminacy is induced provided the degree of trade openness is sufficiently high.

The intuition behind these results rests with how the degree of trade openness exacerbates the cost-channel of monetary policy which arises in sticky-price models with capital. As discussed by Kurozumi and Van Zandweghe (2007) for a closed-economy, under an active policy an increase in the real interest rate puts upward pressure on the expected future rental price of capital, which raises expected marginal cost. Consequently indeterminacy can arise if the upward pressure on inflation generated through this cost channel outweighs the downward pressure on inflation generated through the standard aggregate demand channel of monetary policy. Allowing for trade openness increases the effect of the aggregate demand channel on the dynamics of inflation. This arises since an active policy stance results in a improvement in the terms of trade, thereby generating additional downward pressure on marginal cost. However trade openness also raises expected marginal cost as a future expected deterioration in the terms of trade is anticipated. Consequently if the degree of trade openness is sufficiently large then this cost channel strictly dominates the demand channel and inflation expectations become self-fulfilling.

This paper contributes to the growing body of literature that focuses on the real indeterminacy implications of designing interest-rate rules in the presence of trade openness. A general conclusion arising from the existing literature is that the degree of trade openness is only important for aggregate stability if monetary policy responds to expected future consumer price inflation. Reacting to expected future domestic price inflation or implementing a current-looking rule guarantees equilibrium determinacy if the Taylor principle is adhered to.[5] Our analysis suggests that with the addition of capital and investment

[5]See, for example, Zanna (2002), De Fiore and Liu (2005), Batini *et al.* (2004), Llosa and Tuesta (2006), Linnemann and Schabert (2002, 2006) and McKnight (2007).

spending, the degree of trade openness increases the prominence of aggregate instability with the violation of the Taylor principle. This is robust not only to the index of inflation targeted, but also to the timing of the interest-rate rule employed. The failure of the Taylor principle for open-economies therefore suggests that monetary authorities face much greater challenges in the design of interest-rate rules. Specifically, in a sense to be made precise below, we argue that in order to minimize aggregate instability sufficiently open economies should react to backward-looking consumer price inflation, whereas sufficiently closed-economies should target current-looking consumer price inflation.

The remainder of the paper is organized as follows. Section 2 develops the two-country model. Section 3 examines the conditions for real equilibrium determinacy when current-looking interest-rate rules are employed. Section 4 considers the implications of alternative interest rate rules that react to forward-looking or backward-looking inflation. Finally Section 5 concludes.

2 The Model

Consider a global economy that consists of two countries denoted *home* and *foreign*, where an asterisk denotes *foreign* variables. Within each country there exists a representative infinitely-lived agent, a representative final good producer, a continuum of intermediate good producing firms, and a monetary authority. The representative agent owns all domestic intermediate good producing firms and supplies labor and capital to the production process. Intermediate firms operate under monopolistic competition and use domestic labor and capital as inputs to produce tradeable goods which are sold to the *home* and *foreign* final good producers. The labor and rental capital markets are both assumed to be competitive. Each representative final good producer is a competitive firm that bundles domestic and imported intermediate goods into non-tradeable final goods, which are consumed by the domestic agent. Preferences and technologies are symmetric across the two countries. The following presents the features of the model for the *home* country on the understanding that the *foreign* case can be analogously derived.

2.1 Final Good Producers

The *home* final good (Z) is produced by a competitive firm that uses Z_H and Z_F as inputs according to the following CES aggregation technology index:

$$Z_t = \left[a^{\frac{1}{\theta}} Z_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} Z_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the relative share of domestic and imported intermediate inputs used in the production process is $0 < a < 1$ and the constant elasticity of substitution between aggregate *home* and *foreign* intermediate goods is $\theta > 0$. The inputs Z_H and Z_F are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$Z_{H,t} = \left[\int_0^1 z_{H,t}(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}, \qquad Z_{F,t} = \left[\int_0^1 z_{F,t}(j)^{\frac{\lambda-1}{\lambda}} dj \right]^{\frac{\lambda}{\lambda-1}},$$

where the elasticity of substitution across domestic (*foreign*) intermediate goods is $\lambda > 1$, and $z_H(i)$ and $z_f(j)$ are the respective quantities of the domestic and imported type i and j intermediate goods. Let $p_H(i)$ and $p_F(j)$ represent the respective prices of these goods in *home* currency. Cost minimization in final good production yields the aggregate demand conditions for *home* and *foreign* goods:

$$Z_{H,t} = a \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} Z_t, \qquad Z_{F,t} = (1-a) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} Z_t, \tag{2}$$

where the demand for individual goods is given by

$$z_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\lambda} Z_{H,t}, \qquad z_{F,t}(j) = \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\lambda} Z_{F,t}. \tag{3}$$

Furthermore, since the final good producer is competitive it sets its price equal to marginal cost

$$P_t = \left[a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{4}$$

where P is the consumer price index and P_H and P_F are the respective price indices of *home* and *foreign* intermediate goods, all denominated in *home* currency:

$$P_{H,t} = \left[\int_0^1 p_{H,t}(i)^{1-\lambda} di\right]^{\frac{1}{1-\lambda}}, \qquad P_{F,t} = \left[\int_0^1 p_{F,t}(j)^{1-\lambda} dj\right]^{\frac{1}{1-\lambda}}.$$

We assume that there are no costs to trade between the two countries and the law of one price holds, which implies that

$$P_{Ht} = e_t P^*_{Ht}, \qquad P^*_{Ft} = \frac{P_{Ft}}{e_t}, \tag{5}$$

where e is the nominal exchange rate. Letting $Q = \frac{eP^*}{P}$ denote the real exchange rate, under the law of one price, the CPI index (4) and its *foreign* equivalent imply:

$$\left(\frac{1}{Q_t}\right)^{1-\theta} = \left(\frac{P_t}{e_t P^*_t}\right)^{1-\theta} = \frac{aP_{H,t}^{1-\theta} + (1-a)\left(e_t P^*_{F,t}\right)^{1-\theta}}{a\left(e_t P^*_{F,t}\right)^{1-\theta} + (1-a)P_{H,t}^{1-\theta}} \tag{6}$$

and hence the purchasing power parity condition is satisfied only in the absence of any bias between *home* and *foreign* intermediate goods (i.e. $a = 0.5$). The relative price T, the terms of trade, is defined as $T \equiv \frac{eP^*_F}{P_H}$.

2.2 Intermediate Goods Producers

Intermediate firms hire labor and rent capital to produce output given a (real) wage rate w_t and capital rental cost rr_t. A firm of type i has a production technology:

$$y_t(i) = K_t(i)^{\alpha} L_t(i)^{1-\alpha}, \tag{7}$$

where K and L represent capital and labor usage respectively, and the input share is $0 < \alpha < 1$. Given competitive prices of labor and capital, cost-minimization yields:

$$w_t = mc_t(1-\alpha)\left(\frac{P_t}{P_{H,t}}\right)\left(\frac{K_t(i)}{L_t(i)}\right)^{\alpha} \tag{8}$$

$$rr_t = mc_t\alpha\left(\frac{P_t}{P_{H,t}}\right)\left(\frac{L_t(i)}{K_t(i)}\right)^{1-\alpha}, \tag{9}$$

where $mc_t \equiv \frac{MC_t}{P_{H,t}}$ is real marginal cost.

Firms set prices according to Calvo (1983), where in each period there is a constant probability $1-\varphi$ that a firm will be randomly selected to adjust its price, which is drawn independently of past history. A domestic firm i, faced with changing its price at time t, has to choose $p_{H,t}(i)$ to maximize its expected discounted value of profits, taking as given the indexes P, P_H, P_F, Z and Z^*:[6]

$$\max_{p_{H,t}(i)} E_t \sum_{s=0}^{\infty} (\beta\varphi)^s X_{t,t+s} \left\{ \left[p_{H,t}(i) - MC_{t+s}(i) \right] \left[z_{H,t+s}(i) + z^*_{H,t+s}(i) \right] \right\}, \tag{10}$$

where

$$z_{H,t+s}(i) + z^*_{H,t+s}(i) \equiv \left(\frac{p_{H,t}(i)}{P_{H,t+s}} \right)^{-\lambda} \left[Z_{H,t+s} + Z^*_{H,t+s} \right]$$

and the firm's discount factor is $\beta^s X_{t,t+s} = [U_c(C_{t+s})/U_c(C_t)][P_t/P_{t+s}]$.[7] Firms that are given the opportunity to change their price, at a particular time, all behave in an identical manner. The first-order condition to the firm's maximization problem yields

$$\widetilde{P}_{H,t} = \frac{\lambda}{\lambda - 1} E_t \sum_{s=0}^{\infty} q_{t,t+s} MC_{t+s}. \tag{11}$$

The optimal price set by a domestic *home* firm $\widetilde{P}_{H,t}$ is a mark-up $\frac{\lambda}{\lambda-1}$ over a weighted average of future nominal marginal costs, where the weight $q_{t,t+s}$ is given by

$$q_{t,t+s} = \frac{(\beta\varphi)^s X_{t,t+s} P^{\lambda}_{H,t+s} \left(Z_{H,t+s} + Z^*_{H,t+s} \right)}{E_t \sum_{s=0}^{\infty} (\beta\varphi)^s X_{t,t+s} P^{\lambda}_{H,t+s} \left(Z_{H,t+s} + Z^*_{H,t+s} \right)}.$$

Since all prices have the same probability of being changed, with a large number of firms, the evolution of the price sub-indexes is given by

$$P^{1-\lambda}_{H,t} = \varphi P^{1-\lambda}_{H,t-1} + (1-\varphi) \widetilde{P}^{1-\lambda}_{H,t} \tag{12}$$

since the law of large numbers implies that $1-\varphi$ is also the proportion of firms that adjust their price each period.

[6] While the demand for a firm's good is affected by its pricing decision $p_{H,t}(i)$, each producer is small with respect to the overall market.

[7] Under the assumption that all firms are owned by the representative agent, this implies that the firm's discount factor is equivalent to the individual's intertemporal marginal rate of substitution.

2.3 Representative Agent

The representative agent chooses consumption C, domestic real money balances M/P, and leisure $1-L$, to maximize utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t, \frac{M_t}{P_t}, 1-L_t\right] \tag{13}$$

where the discount factor is $0<\beta<1$, subject to the period budget constraint

$$E_t \Gamma_{t,t+1} B_{t+1} + M_t + P_t\left(C_t + I_t\right) \leq B_t + M_{t-1} + P_t w_t L_t + P_t rr_t K_t + \int_0^1 \Pi_t d(h) - \Upsilon_t. \tag{14}$$

The agent carries M_{t-1} units of money, B_t nominal bonds and K_t units of capital into period t. Before proceeding to the goods market, the agent visits the financial market where a state contingent nominal bond B_{t+1} can be purchased that pays one unit of domestic currency in period $t+1$ when a specific state is realized at a period t price $\Gamma_{t,t+1}$. During period t the agent supplies labor and capital to the intermediate good producing firms, receiving real income from wages w_t, a rental return on capital rr_t, nominal profits from the ownership of domestic intermediate firms Π_t and a lump-sum nominal transfer Υ_t from the monetary authority. The agent then uses these resources to purchase the final good, dividing purchases between consumption C_t and investment I_t. The purchase of an investment good forms next period's capital according to the law of motion

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{15}$$

where $0<\delta<1$ is the depreciation rate of capital.

For analytical simplicity we assume that the period utility function is separable among its three arguments and the labor supply elasticity is infinite.[8] The first-order conditions from the *home* agent's maximization problem yield:

$$\beta R_t E_t \left\{ \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}} \right\} = 1 \tag{16}$$

[8]Both Dupor (2001) and Carlstrom and Fuerst (2005) use the same functional form for their respective closed-economy studies.

$$\frac{U_L(L_t)}{U_c(C_t)} = w_t \tag{17}$$

$$U_c(C_t) = \beta E_t U_c(C_{t+1}) \left[E_t rr_{t+1} + (1-\delta) \right] \tag{18}$$

$$\frac{U_m(m_t)}{U_c(C_t)} = \frac{R_t - 1}{R_t} \tag{19}$$

where R_t denotes the gross nominal yield on a one-period discount bond defined as $R_t^{-1} \equiv E_t\{\Gamma_{t,t+1}\}$. Equation (16) is the consumption Euler equation for the holdings of domestic bonds and the money demand equation is given by (19). Equations (17) and (18) are the respective labor supply and optimal investment conditions. Optimizing behavior implies that the budget constraint (14) holds with equality in each period and the appropriate transversality condition is satisfied. Analogous conditions apply to the *foreign* agent.

From the first-order conditions for the *home* and *foreign* agent, the following risk-sharing conditions can be derived:

$$R_t = R_t^* E_t \left\{ \frac{e_{t+1}}{e_t} \right\} \tag{20}$$

$$Q_t = q_0 \frac{U_c(C_t^*)}{U_c(C_t)} \tag{21}$$

where the constant $q_0 = Q_0 \left[\frac{U_c(C_0)}{U_c(C_0^*)} \right]$. Equation (20) is the standard uncovered interest rate parity condition and equation (21) is the risk sharing condition associated with complete asset markets, which equates the real exchange rate Q with the marginal utilities of consumption.

2.4 Monetary Authority

The monetary authority can adjust the nominal interest rate in response to changes in domestic price inflation π^h_{t+v} or to changes in consumer price inflation π_{t+v}, according to the rules:

$$R_t = \mu\left(\pi^h_{t+v}\right) = \overline{R} \left(\frac{\pi^h_{t+v}}{\overline{\pi}^h} \right)^{\mu}, \tag{22}$$

$$R_t = \mu\left(\pi_{t+v}\right) = \overline{R} \left(\frac{\pi_{t+v}}{\overline{\pi}} \right)^{\mu}, \tag{23}$$

where $\overline{R} > 1$ and the timing-index v represents the inflation-targeting behavior of the monetary authority. If $v = 0$, the monetary authority targets current inflation. If $v = -1$ the policy rule is backward-looking, whereas $v = 1$ corresponds to forward-looking inflation targeting. The parameter μ determines whether monetary policy is active or passive. An active monetary policy corresponds to $\mu > 1$, where the real interest rate rises in response to higher inflation, as the monetary authority increases the nominal interest rate by more than the increase in inflation. A passive monetary policy on the other hand corresponds to $0 \leq \mu < 1$, where the real interest rate falls in response to higher inflation.

2.5 Market Clearing and Equilibrium

Market clearing for the *home* goods market requires

$$Z_{H,t} + Z^*_{H,t} = Y_t. \tag{24}$$

Total *home* demand must equal the supply of the final good,

$$Z_t = C_t + I_t, \tag{25}$$

and the labor, capital, money and bond markets all clear:

$$\Upsilon_t = M_t - M_{t-1} \qquad B_t + B^*_t = 0. \tag{26}$$

Definition 1 (Rational Expectations Equilibrium): *Given an initial allocation of* B_{t_0}, $B^*_{t_0}$, K_{t_0}, $K^*_{t_0}$, *and* M_{t_0-1}, $M^*_{t_0-1}$, *a rational expectations equilibrium is a set of sequences* $\{C_t,\, C^*_t,\, M_t,\, M^*_t,\, L_t,\, L^*_t,\, K_t,\, K^*_t,\, B_t,\, B_t,\, R_t,\, R^*_t,\, MC_t,\, MC^*_t,\, w_t,\, w^*_t,\, rr_t,\, rr^*_t,\, Y_t,\, Y^*_t,\, e_t,\, Q_t,\, P_t,\, P^*_t,\, P_{H,t},\, \widetilde{P_{H,t}},\, \widetilde{P^*_{F,t}},\, P^*_{H,t},\, P_{F,t},\, P^*_{F,t},\, Z_t,\, Z^*_t,\, Z_{H,t},\, Z_{F,t},\, Z^*_{H,t},\, Z^*_{F,t}\}$ *for all* $t \geq t_0$ *characterized by:* (i) the optimality conditions of the representative agent, (16) to (19), and the capital accumulation equation (15); (ii) the intermediate firms' first-order conditions (8) and (9), price-setting rules, (11) and (12), and the aggregate version of the production function (7); (iii) the final good producer's optimality conditions, (2), and (4); (iv) all markets clear, (24) to (26); (v) the representative agent's budget constraint

(14) is satisfied and the transversality conditions hold; (*vi*) the monetary policy rule is satisfied, (22) or (23); along with the foreign counterparts for (*i*)-(*vi*) and conditions (5), (6), (20) and (21).

2.6 Local Equilibrium Dynamics

In order to analyze the equilibrium dynamics of the model, a first-order Taylor approximation is taken around a steady state to replace the non-linear equilibrium system with an approximation which is linear.[9] We employ the Aoki (1981) decomposition which decomposes the two-country model into two decoupled dynamic systems: the aggregate system that captures the properties of the closed world economy[10] and the difference system that portrays the open-economy dimension. Consequently for the equilibrium to be determinate it must be the case that there is a unique solution both for cross-country differences and world aggregates. The complete linearized system of equations is summarized in Table 1. Note that since money balances are separable, the money demand equation and its foreign equivalent are irrelevant for equilibrium determinacy and are subsequently ignored. In what follows below it will also be convenient to define $x \equiv \frac{L}{K}$. The parameters of the model are as follows: $\sigma > 0$ measures the intertemporal substitution elasticity of consumption; $\phi > 0$ measures the inverse of the Frisch labor supply elasticity; $0 < \alpha < 1$ measures the production input share of intermediate firms; $\theta > 0$ measures the elasticity of substitution between aggregate *home* and *foreign* goods; $\lambda > 1$ is the degree of monopolistic competition in the intermediate firm sector; $\Lambda_1 \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi} > 0$ is the real marginal cost elasticity of inflation, where $0 < \beta < 1$ is the discount factor and $0 < \psi < 1$ is the degree of price stickiness; and $a \in \{(0, 0.5) \cup (0.5, 1)\}$ is the degree of trade openness measured by the relative share of intermediate imports used in final good production $(1 - a)$.[11]

[9] To be precise the model is linearized around a symmetric steady state in which prices in the two countries are equal and constant ($\overline{P}_H = \overline{P}_F = \overline{P} = \overline{P}^* = \overline{P}^*_H = \overline{P}^*_F$). Then by definition inflation is zero ($\overline{\pi} = \overline{\pi}^* = 1$), and the steady state terms of trade and nominal and real exchange rates are $\overline{T} = \overline{e} = \overline{Q} = 1$.

[10] The choice of which index of inflation each monetary authority targets is irrelevant for the aggregate system. This follows since world aggregate inflation (π^W) is given by

$$\pi^W = \frac{\pi + \pi^*}{2} = \frac{\pi^h + \pi^{*f}}{2}.$$

[11] The analysis does not consider the case when $a = 0.5$ since this would imply that purchasing power parity (PPP) is satisfied and consequently the linearized inflation equation $\widehat{\pi}_t^R = (2a-1)\widehat{\pi}_t^{R(h-f^*)} + 2(1-a)\Delta\widehat{e}_t$

Table 1: Linearized system of equations

Cross-Country Differences	
$E_t\left\{\widehat{C}^R_{t+1}\right\} = \widehat{C}^R_t + \sigma\widehat{R}^R_t - \sigma E_t\left\{\widehat{\pi}^R_{t+1}\right\}$	IS^R
$\widehat{R}^R_t = E_t\left\{\Delta\widehat{e}_{t+1}\right\}$	UIP
$\widehat{\pi}^{R(h-f^*)}_t = \Lambda_1\widehat{mc}^R_t + \beta E_t\left\{\widehat{\pi}^{R(h-f^*)}_{t+1}\right\}$	AS^R
$\widehat{mc}^R_t = \frac{\widehat{C}^R_t}{\sigma} + \alpha\widehat{x}^R_t + 2(1-a)\widehat{T}_t$	Marginal cost
$\widehat{C}^R_{t+1} = \widehat{C}^R_t + \sigma\Lambda_2\left[\widehat{mc}^R_{t+1} + (1-\alpha)\widehat{x}^R_{t+1} - 2(1-a)\widehat{T}_{t+1}\right]$	Investment
$\widehat{K}^R_{t+1} = \frac{(1-\alpha)}{(2a-1)}\left(\frac{\overline{Z}}{\overline{K}}\right)\widehat{x}^R_t - \left(\frac{\overline{C}}{\overline{K}}\right)\widehat{C}^R_t - \frac{4\theta a(1-a)}{(2a-1)}\left(\frac{\overline{Z}}{\overline{K}}\right)\widehat{T}_t + \Lambda_3\widehat{K}^R_t$	Resource constraint
$\widehat{R}^R_t = \mu E_t\left\{\widehat{\pi}^R_{t+v}\right\}$ *or* $\widehat{R}^R_t = \mu E_t\left\{\widehat{\pi}^{R(h-f^*)}_{t+v}\right\}$	Taylor rule
$\widehat{\pi}^R_t = (2a-1)\widehat{\pi}^{R(h-f^*)}_t + 2(1-a)\Delta\widehat{e}_t$	Inflation
$\widehat{Q}_t = \frac{1}{\sigma}\widehat{C}^R_t = (2a-1)\widehat{T}_t$	RER
World Aggregates	
$E_t\left\{\widehat{C}^W_{t+1}\right\} = \widehat{C}^W_t + \sigma\widehat{R}^W_t - \sigma E_t\left\{\widehat{\pi}^W_{t+1}\right\}$	IS^W
$\widehat{\pi}^W_t = \Lambda_1\widehat{mc}^W_t + \beta E_t\left\{\widehat{\pi}^W_{t+1}\right\}$	AS^W
$\widehat{mc}^W_t = \frac{\widehat{C}^W_t}{\sigma} + \alpha\widehat{x}^W_t$	Marginal cost
$\widehat{C}^W_{t+1} = \widehat{C}^W_t + \sigma\Lambda_2\left[\widehat{mc}^W_{t+1} + (1-\alpha)\widehat{x}^W_{t+1}\right]$	Investment
$\widehat{K}^W_{t+1} = (1-\alpha)\left(\frac{\overline{Z}}{\overline{K}}\right)\widehat{x}^W_t - \left(\frac{\overline{C}}{\overline{K}}\right)\widehat{C}^W_t + \left[\frac{\overline{Z}}{\overline{K}} + 1 - \delta\right]\widehat{K}^W_t$	Resource constraint
$\widehat{R}^W_t = \mu E_t\left\{\widehat{\pi}^W_{t+v}\right\}$	Taylor rule

Notes: The index R refers to the difference between *home* and *foreign* variables e.g. $\widehat{C}^R_t \equiv \left(\widehat{C}_t - \widehat{C}^*_t\right)$, $\widehat{\pi}^{R(h-f^*)}_t \equiv \left(\widehat{\pi}^h_t - \widehat{\pi}^{*f}_t\right)$. The index W refers to world aggregates where $\pi^W = \frac{\pi+\pi^*}{2} = \frac{\pi^h+\pi^{*f}}{2}$ and $\Delta\widehat{e}_t \equiv \widehat{e}_t - \widehat{e}_{t-1}$. The parameters are defined as: $\Lambda_1 \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi}$, $\Lambda_2 \equiv 1-\beta(1-\delta)$ and $\Lambda_3 = \left[\frac{\overline{Z}}{\overline{K}}\frac{1}{(2a-1)} + 1 - \delta\right]$ where the steady state levels are given by $\frac{\overline{Z}}{\overline{K}} = \frac{\overline{C}}{\overline{K}} + \delta$ and $\frac{\overline{C}}{\overline{K}} = \frac{1}{\alpha}\left[\frac{1}{\beta} - (1-\delta)\right]\frac{\lambda}{\lambda-1} - \delta$.

3 Equilibrium Determinacy

We start by examining the conditions for equilibrium determinacy when monetary policy is characterized by a current-looking rule. First note that for a labor-only version of this open economy model, in which production is linear in labor ($\alpha = 0$), under a current-looking rule the Taylor principle ($\mu > 1$) prevents the emergence of real indeterminacy for both the aggregate and difference systems.[12] As this section shows the conditions for

reduces to the relative PPP condition for consumer price inflation i.e. $\widehat{\pi}^R_t = \Delta\widehat{e}_t$.

[12] For example, as shown by McKnight (2007), the Taylor principle is both a necessary and sufficient condition for local determinacy of the aggregate system and the difference system when domestic inflation is targeted. If CPI inflation is targeted then the Taylor principle is a necessary condition for determinacy

determinacy alter substantially with the inclusion of capital.

3.1 Aggregate System

The set of linearized equations for the world aggregates, given in Table 1, can be reduced to the following four-dimensional system:

$$E_t \mathbf{b}_{t+1}^W = \mathbf{A}^W \mathbf{b}_t^W, \qquad \mathbf{b}_t^W = \left[\widehat{mc}_t^W \ \widehat{x}_t^W \ \widehat{\pi}_t^W \ \widehat{K}_t^W\right]',$$

$$\mathbf{A}^W \equiv \begin{bmatrix} (1-\alpha)+\frac{\Lambda_1}{\beta}\left[1+\frac{\alpha(1-\Lambda_2)}{\Lambda_2}\right] & -\alpha(1-\alpha) & \left(\mu-\frac{1}{\beta}\right)\left[1+\frac{\alpha(1-\Lambda_2)}{\Lambda_2}\right] & 0 \\ \frac{\Lambda_1(1-\Lambda_2)}{\Lambda_2\beta}-1 & \alpha & \left(\mu-\frac{1}{\beta}\right)\left[\frac{1-\Lambda_2}{\Lambda_2}\right] & 0 \\ -\frac{\Lambda_1}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left(\frac{\overline{C}}{\overline{K}}\right)\sigma & \frac{(1-\alpha)\overline{Z}}{\overline{K}}+\sigma\alpha\frac{\overline{C}}{\overline{K}} & 0 & 1+\frac{\overline{C}}{\overline{K}} \end{bmatrix}.$$

Since the dynamics of mc, x, and π are independent of the capital stock dynamics, one eigenvalue of the system is $1+\frac{\overline{C}}{\overline{K}} > 1$. Consequently, given that capital is the only predetermined variable in the column vector $\mathbf{b}^W$, equilibrium determinacy requires that two of the remaining eigenvalues of $\mathbf{A}^W$ are outside the unit circle and one eigenvalue is inside the unit circle. Then by Proposition C.2 of Woodford (2003) the following result is obtained:

Proposition 1 *Suppose that monetary policy is characterized by a current-looking interest rate rule. Then a necessary and sufficient condition for determinacy of the aggregate system is* $\mu > 1$ *and either*

$$(2\beta-1)\Lambda_2 < \Lambda_1\left[1-\beta(1-\delta)(1-\alpha)\right] \tag{27}$$

or

$$\frac{\mu\Lambda_1\alpha}{\Lambda_2\beta}\left[\frac{\Lambda_1\alpha(\mu-1)}{\Lambda_2}-1-\Lambda_1(1-\alpha)\right]+1-\beta+\Lambda_1\mu(1-\alpha)+\frac{\Lambda_1\alpha}{\Lambda_2}>0, \tag{28}$$

where $\Lambda_1 = \frac{(1-\psi)(1-\beta\psi)}{\psi}$ *and* $\Lambda_2 = 1-\beta(1-\delta)$.

The determinacy conditions summarized in Proposition 1 are isomorphic to the conditions obtained by Carlstrom and Fuerst (2005) for the closed-economy. Suppose $\alpha = 0.36$, $\beta = 0.99$ and $\delta = 0.025$. For these parameter values (27) is violated only if $\psi \geq 0.75$. Thus if prices are sufficiently sticky, indeterminacy (of order two) can arise for some values

of the difference system.

of $\mu > 1$ provided condition (28) is violated. However the region of indeterminacy is small. For example if $\psi = 0.8$ then indeterminacy arises provided $1.1 < \mu < 1.71$, whereas if $\psi = 0.75$ condition (28) is satisfied $\forall \mu > 1$ and thus indeterminacy is not possible.

3.2 Difference System

3.2.1 Domestic Price Inflation

If domestic price inflation is the policy indicator, then the set of linearized conditions for cross-country differences yields a system of the form:

$$E_t \mathbf{b}_{t+1}^R = \mathbf{A}_{PPI}^R \mathbf{b}_t^R, \quad \mathbf{b}_t^R = \left[\widehat{mc}_t^R \ \widehat{x}_t^R \ \widehat{\pi}_t^{R(h-f^*)} \ \widehat{K}_t^R \right]',$$

$$\mathbf{A}_{PPI}^R \equiv \begin{bmatrix} 1-\alpha(2a-1)+\frac{\Lambda_1}{\beta}J_1 & \alpha^2(2a-1)-\alpha & \left(\mu-\frac{1}{\beta}\right)J_1 & 0 \\ -(2a-1)+\frac{\Lambda_1}{\beta}J_2 & \alpha(2a-1) & \left(\mu-\frac{1}{\beta}\right)J_2 & 0 \\ -\frac{\Lambda_1}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left[\sigma(2a-1)\frac{\overline{C}}{\overline{K}}+\frac{4\theta a(1-a)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\right] & J_3 & 0 & 1+\frac{\frac{\overline{C}}{\overline{K}}+\delta 2(1-a)}{(2a-1)} \end{bmatrix},$$

where $J_1 = \left[1+\frac{\alpha(1-\Lambda_2)(2a-1)}{\Lambda_2}\right]$ $J_2 = \frac{(1-\Lambda_2)(2a-1)}{\Lambda_2}$ and $J_3 = \frac{(1-\alpha)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\left[1+\alpha 4\theta a(1-a)\right] + \sigma\alpha(2a-1)\frac{\overline{C}}{\overline{K}}$. As before, the capital stock dynamics can be decoupled from the rest of the system. However, the eigenvalue associated with the capital stock dynamics now depends on the degree of trade openness. Consequently this eigenvalue can be either inside or outside the unit circle depending on the value of a. The Appendix proves the following:[13]

Proposition 2 *Suppose that monetary policy reacts to current-looking domestic price inflation. Then for an active monetary policy ($\mu > 1$), the necessary and sufficient conditions for determinacy of the difference system are:*

(Case I) $a > 0.5$ *and either*

$$(2\beta-1)\Lambda_2 < \Lambda_1\left[1-\beta(1-\delta)(1-(2a-1)\alpha)\right] \qquad (29)$$

or

$$\frac{\mu\Lambda_4}{\Lambda_2\beta}\left[\frac{\Lambda_4(\mu-1)}{\Lambda_2}+\Lambda_4-(1+\Lambda_1+\Lambda_2\beta)\right]+(1-\beta)+\Lambda_1\mu+\frac{\Lambda_4}{\Lambda_2} > 0; \qquad (30)$$

[13] While determinacy of the difference system can also be achieved under a passive monetary policy ($\mu < 1$), such conditions are not reported since the aggregate system is always indeterminate (from Proposition 1).

$$\textbf{\textit{(Case II)}} \quad 0.5 > a > \frac{1}{2-\delta}\left[1-\delta-\frac{\overline{C}}{\overline{K}}\frac{1}{2}\right]$$

$$\textit{and} \qquad 1+\mu < \frac{2(1+\beta)\Lambda_2}{\Lambda_1\left[(1-2a)\alpha(2-\Lambda_2)-\Lambda_2\right]} \quad \textit{if} \quad \frac{2\alpha(1-2a)}{1+\alpha(1-2a)} > \Lambda_2;$$

$$\textbf{\textit{(Case III)}} \quad a < \frac{1}{2-\delta}\left[1-\delta-\frac{\overline{C}}{\overline{K}}\frac{1}{2}\right] < 0.5 \quad \textit{and} \tag{31}$$

$$\textit{(i)} \quad 1+\mu > \frac{2(1+\beta)\Lambda_2}{\Lambda_1\left[(1-2a)\alpha(2-\Lambda_2)-\Lambda_2\right]} \quad \textit{and} \quad \textit{(ii)} \quad \frac{2\alpha(1-2a)}{1+\alpha(1-2a)} > \Lambda_2. \tag{32}$$

where $\Lambda_1 = \frac{(1-\psi)(1-\beta\psi)}{\psi}$, $\Lambda_2 = 1-\beta(1-\delta)$ *and* $\Lambda_4 = \Lambda_1\alpha(2a-1)$.

Proof. See Appendix A.1. □

Cases I and II of Proposition 2 show the regions of determinacy when the root associated with the capital stock dynamics is unstable, whereas Case III shows the regions of determinacy when this root is stable. We illustrate these determinacy conditions using the following parameter values. Suppose $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$, $\lambda = 7.66$ and $\psi = 0.75$. Figure 1 depicts the regions in the parameter space (a, μ) that are associated with determinacy (D), first-order indeterminacy (I^1) and second-order indeterminacy (I^2) around the neighborhood of the steady state.[14] If the degree of trade openness is sufficiently low ($a > 0.5$) then second-order indeterminacy can arise in the open-economy. This follows from the violation of conditions (29) and (30) of Case I in Proposition 2. Indeed comparing (29) with condition (27) of Proposition 1 yields

$$(2\beta-1)\Lambda_2 < \Lambda_1\left[1-\beta(1-\delta)(1-(2a-1)\alpha)\right] < \Lambda_1\left[1-\beta(1-\delta)(1-\alpha)\right],$$

which by inspection implies that a higher degree of price stickiness is required in the open economy to prevent the emergence of second-order indeterminacy. Furthermore, as depicted in Figure 1, if the degree of trade openness is sufficiently high then first-order indeterminacy can also exist. This arises from the violation of condition (32) of Case III in Proposition 2. First consider condition (31). Under the assigned parameter values,

[14] Recall that with these parameter values indeterminacy is not possible in the closed-economy (aggregate system) $\forall\mu > 1$.

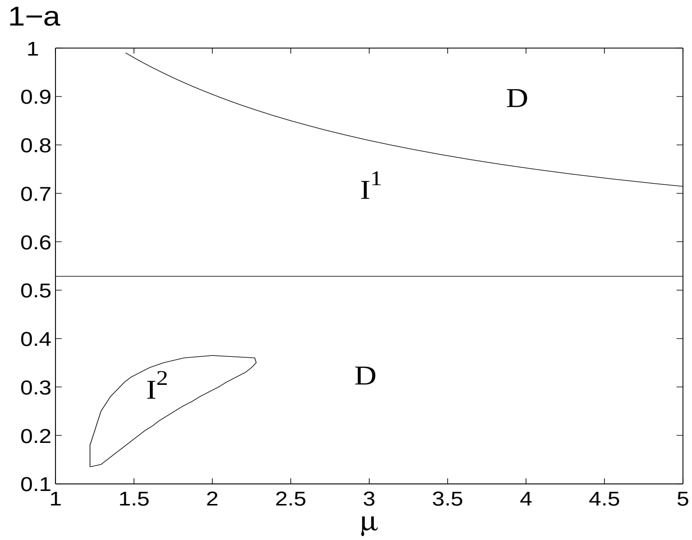

Figure 1: Regions of determinacy under a current-looking domestic price inflation rule

Case III is relevant for any $a < 0.4716$[15] and condition (ii) of (32) is always satisfied.[16] Thus from condition (i) of (32) first-order indeterminacy can arise provided the inflation coefficient μ is sufficiently low. It is straightforward to show that this lower bound on μ is decreasing with respect to a:

$$\frac{\partial(32)(i)}{\partial a} = -\frac{4\alpha\Lambda_2(2-\Lambda_2)(1+\beta)}{\left[(1-2a)\alpha(2-\Lambda_2)-\Lambda_2\right]^2} < 0$$

and thus as the degree of trade openness is reduced, the higher the inflation coefficient that is required to prevent indeterminacy.

3.2.2 Consumer Price Inflation

If consumer price inflation is the policy indicator, then the set of linearized conditions for cross-country differences yields the five-dimensional system of the form:

[15] Note that this threshold level for a is independent of the degree of price stickiness (ψ). Furthermore it is remarkably robust to variations in α and λ. For example setting $\lambda = 4$ requires $a < 0.467$ whereas setting $\alpha = 0.25$ requires $a < 0.459$.

[16] Given the benchmark values for β and δ, condition (ii) of (32) is always satisfied for any $a \leq 0.47$ provided $\alpha > 0.295$.

$$E_t \mathbf{b}_{t+1}^R = \mathbf{A}_{CPI}^R \mathbf{b}_t^R, \quad \mathbf{b}_t^R = \left[\widehat{mc}_t^R \ \widehat{x}_t^R \ \widehat{\pi}_{t-1}^R \ \widehat{\pi}_t^R \ \widehat{K}_t^R\right]', \text{ and } \mathbf{A}_{CPI}^R \equiv$$

$$\begin{bmatrix} 1-\alpha(2a-1)+\frac{\Lambda_1(1+\alpha J_1)}{\beta} & \alpha^2(2a-1)-\alpha & J_2(1+\alpha J_1) & J_3(1+\alpha J_1) & 0 \\ -(2a-1)+\frac{\Lambda_1}{\beta}J_1 & \alpha(2a-1) & J_2 J_1 & J_3 J_1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{\Lambda_1(2a-1)}{\beta} & 0 & -\frac{2(1-a)\mu}{\beta} & \frac{1+2(1-a)\beta\mu}{\beta} & 0 \\ -\left[\frac{\sigma(2a-1)\overline{C}}{\overline{K}}+\frac{4\theta a(1-a)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\right] & J_4 & 0 & 0 & 1+\frac{\frac{\overline{C}}{\overline{K}}+\delta 2(1-a)}{(2a-1)} \end{bmatrix},$$

where $J_1 = \frac{(1-\Lambda_2)(2a-1)}{\Lambda_2}$, $J_2 = \frac{2(1-a)\mu}{(2a-1)\beta}$, $J_3 = \mu - \frac{1}{\beta(2a-1)}$ and $J_4 = \frac{(1-\alpha)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}[1+\alpha 4\theta a(1-a)] + \sigma\alpha(2a-1)\frac{\overline{C}}{\overline{K}}$. Now there are two predetermined variables $\widehat{K}_t^R$ and $\widehat{\pi}_{t-1}^R$. Note that the eigenvalue associated with the capital stock dynamics is the same regardless of the index of inflation targeted. The Appendix proves the following:

Proposition 3 *Suppose that monetary policy reacts to current-looking consumer price inflation. Then for an active monetary policy ($\mu > 1$), the necessary and sufficient conditions for determinacy of the difference system are:*

(Case I) $a > 0.5$ *and at least one of (33) and (34) is satisfied;*

$$\mu > \frac{(2\beta-1)\Lambda_2 - \Lambda_1[\Lambda_2 + \alpha(1-\Lambda_2)(2a-1)]}{\Lambda_2\beta 2(1-a)}, \tag{33}$$

$$\frac{\mu}{\beta}\left[2(1-a)+\frac{\Lambda_4}{\Lambda_2}\right]\left[2(1-a)(1-\beta)\mu + \frac{\Lambda_4[\mu-(1-\Lambda_2)]}{\Lambda_2} - (1+\Lambda_1)\right] + (1-\beta) + \mu[\Lambda_1 - \Lambda_4] + 2(1-a)\mu\beta + \frac{\Lambda_4}{\Lambda_2} > 0; \tag{34}$$

(Case II) $0.5 > a > \frac{1}{2-\delta}\left[1-\delta-\frac{\overline{C}}{\overline{K}}\frac{1}{2}\right]$, *and*

$$\Lambda_1 < \frac{2\Lambda_2(1+\beta)[1+2\mu(1-a)]}{(1+\mu)[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2]} \quad \textit{if} \quad \frac{2\alpha(1-2a)}{1+\alpha(1-2a)} > \Lambda_2, \tag{35}$$

and at least one of (33) and (34) is satisfied;

$$\textbf{\textit{(Case III)}} \quad a < \frac{1}{2-\delta}\left[1-\delta-\frac{\overline{C}}{\overline{K}}\frac{1}{2}\right] < 0.5, \qquad \textit{and}$$

$$\Lambda_1 > \frac{2\Lambda_2(1+\beta)[1+2\mu(1-a)]}{(1+\mu)\left[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2\right]} \qquad \textit{and} \tag{36}$$

$$\Lambda_2 < \min\left\{\frac{2\alpha(1-2a)}{1+\alpha(1-2a)}, \frac{\Lambda_1\alpha(1-2a)}{2(1-a)}\right\}; \tag{37}$$

where $\Lambda_1 = \frac{(1-\psi)(1-\beta\psi)}{\psi}$, $\Lambda_2 = 1-\beta(1-\delta)$ *and* $\Lambda_4 = \Lambda_1\alpha(2a-1)$.

Proof. See Appendix A.2. □

Comparison of Propositions 2 and 3 highlight important qualitative differences between reacting to consumer and domestic price inflation. First, the range of second-order indeterminacy is relatively lower if consumer price inflation is targeted. This follows from direct comparison of conditions (29) and (33):

$$(2\beta-1)\Lambda_2 < \Gamma_1 < 2\beta\Lambda_2(1-a)\mu + \Gamma_1,$$

where $\Gamma_1 = \Lambda_1\left[1-\beta(1-\delta)(1-(2a-1)\alpha)\right]$. Secondly, the range of first-order indeterminacy is relatively greater under consumer price inflation targeting. By comparing the Case III conditions of Propositions 2 and 3, it is straightforward to show that (36) is a stronger requirement for determinacy than condition (i) of (32).[17] Furthermore by comparing condition (ii) of (32) with (37), reacting to consumer price inflation introduces an additional determinacy condition given by $\Lambda_2 < \frac{\Lambda_1\alpha(1-2a)}{2(1-a)}$. Figure 2 depicts the regions in the parameter space (a, μ) that are associated with determinacy (D) and (first-order) indeterminacy (I) around the neighborhood of the steady state given $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$, $\lambda = 7.66$ and $\psi = 0.75$. First observe that second-order indeterminacy does not arise under these parameter values. Furthermore determinacy exists only if the degree of trade openness is sufficiently large $a \geq 0.4716$, otherwise first-order indeterminacy prevails for any value of $\mu > 1$.

3.3 Discussion

In the absence of capital the Taylor principle holds under a current-looking rule regardless of whether the economy is open or closed. However as shown in the previous section,

[17] This follows since $\Lambda_1 > \frac{2\Lambda_2(1+\beta)[1+2\mu(1-a)]}{(1+\mu)[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2]} > \frac{2\Lambda_2(1+\beta)}{(1+\mu)[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2]}$.

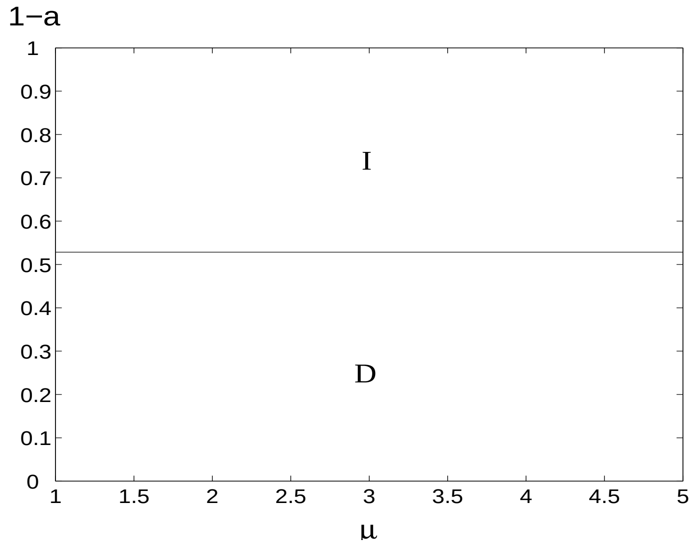

Figure 2: Regions of determinacy under a current-looking consumer price inflation rule

the inclusion of capital generates substantial differences for equilibrium determinacy not only between closed and open-economies, but also between the index of inflation targeted. What is the economic intuition behind these results? First consider a labor-only economy. Suppose that in response to a non-fundamental shock agents believe (CPI) inflation will increase. In a closed-economy an active monetary policy ($\mu > 1$) increases the real interest rate. From the aggregate demand channel of monetary policy this reduces (real) marginal cost such that current inflation rises by less than expected inflation via the Phillips curve. In an open economy the CPI inflation rate depends on both the domestic inflation rate and the terms of trade:

$$\widehat{\pi}_t = \widehat{\pi}_t^h + (1-a)\left(\widehat{T}_t - \widehat{T}_{t-1}\right) \tag{38}$$

where $\widehat{T}_{t-1}$ is predetermined. An increase in the real interest rate not only reduces domestic inflation via a fall in marginal cost but in addition results in an improvement in the terms of trade ($\widehat{T}_t \downarrow$). From (38) this trade channel of monetary policy generates additional downward pressure on CPI inflation. Consequently for both closed and open-economies the initial belief cannot be self-fulfilling. Now consider an economy with capital accumulation. Here the initial increase in inflationary expectations can be self-fulfilling provided that there is a further rise in expected inflation. This can occur since an increase in the real interest rate puts upward pressure on the expected future rental price of capital (from the investment condition (18)) which leads to an increase in expected future marginal cost and thus an additional rise in expected future inflation. Therefore indeterminacy is generated if the effect of this cost channel of monetary policy is sufficiently strong to counteract the downward pressure on inflation arising from the aggregate demand channel. In the open-economy the increase in expected future marginal cost is exacerbated as the degree of trade openness increases. This strengthens the effect of the cost channel making a rise in domestic price inflation more likely. From (38) there are two opposing effects on CPI inflation, the strength of both are increasing as the degree of trade openness increases ($a \downarrow$). Therefore depending on the size of a this determines which effect dominates and thus whether the initial inflationary belief is validated.

4 The Timing of Interest-rate Rules

So far the analysis has focused on interest-rate rules that target contemporaneous inflation. In this section we consider interest-rate rules that react to either forward-looking or backward-looking inflation.

4.1 Forward-looking rules

We start by examining the conditions for equilibrium determinacy under forward-looking inflation rules. The aggregate and difference systems are both four-dimensional generating a zero eigenvalue in each case. Since capital is the only predetermined variable, equilibrium determinacy requires the remaining three eigenvalues to lie outside the unit circle. This automatically suggests that for determinacy of the difference system, the root associated with the capital stock dynamics must be unstable, which in turn implies that determinacy is impossible for sufficiently open economies.

4.1.1 Aggregate System

Under a forward-looking rule, the set of linearized equations for the world aggregates, given in Table 1, yields a system of the form:

$$E_t \mathbf{b}_{t+1}^W = \mathbf{B}^W \mathbf{b}_t^W, \qquad \mathbf{b}_t^W = \left[\widehat{mc}_t^W \ \widehat{x}_t^W \ \widehat{\pi}_t^W \ \widehat{K}_t^W \right]',$$

$$\mathbf{B}^W \equiv \begin{bmatrix} (1-\alpha) - \frac{\Lambda_1(\mu-1)[\alpha+\Lambda_2(1-\alpha)]}{\beta\Lambda_2} & -\alpha(1-\alpha) & \frac{(\mu-1)[\alpha+\Lambda_2(1-\alpha)]}{\beta\Lambda_2} & 0 \\ -1 - \frac{(\mu-1)\Lambda_1(1-\Lambda_2)}{\beta\Lambda_2} & \alpha & \frac{(\mu-1)(1-\Lambda_2)}{\beta\Lambda_2} & 0 \\ -\frac{\Lambda_1}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left(\frac{\overline{C}}{\overline{K}}\right)\sigma & (1-\alpha)\frac{\overline{Z}}{\overline{K}} + \sigma\alpha\frac{\overline{C}}{\overline{K}} & 0 & 1+\frac{\overline{C}}{\overline{K}} \end{bmatrix}.$$

One eigenvalue of the system is given by $1 + \frac{\overline{C}}{\overline{K}} > 1$, while another eigenvalue is zero. Consequently, equilibrium determinacy requires that the two remaining eigenvalues of $\mathbf{B}^W$ are outside the unit circle. Then by Proposition C.1 of Woodford (2003) the following result is obtained:

Proposition 4 *Suppose that monetary policy is characterized by a forward-looking interest rate rule. Then a necessary and sufficient condition for determinacy of the aggregate system is*

$$1 < \mu < 1 + \min\left\{ \Gamma_1^A \equiv \frac{(1-\beta)\Lambda_2}{\alpha\Lambda_1}, \Gamma_2^A \equiv \frac{2(1+\beta)\Lambda_2}{\Lambda_1\left[\alpha(2-\Lambda_2)+\Lambda_2\right]} \right\} \tag{39}$$

where $\Lambda_1 = \frac{(1-\psi)(1-\beta\psi)}{\psi}$ *and* $\Lambda_2 = 1-\beta(1-\delta)$.

As discussed by Carlstrom and Fuerst (2005) the regions of determinacy for a closed-economy are remarkably narrow under a forward-looking rule with capital.[18] Again suppose that $\alpha = 0.36$, $\beta = 0.99$ and $\delta = 0.025$. Then the upper bound for determinacy is $1.01124 = \Gamma_1^A < \Gamma_2^A$. Propositions 5 and 6 below show that in an open-economy the range of determinacy is even smaller.

4.1.2 Difference System

The set of linearized conditions for cross-country differences yields a system of the form:

[18] Carlstrom and Fuerst (2005) present a necessary condition for determinacy, whereas Proposition 4 provides a necessary and sufficient condition for determinacy.

$$E_t \mathbf{b}^R_{t+1} = \mathbf{B}^R \mathbf{b}^R_t, \quad \mathbf{b}_t = \left[\widehat{mc}^R_t \ \widehat{x}^R_t \ \widehat{\pi}^{R(h-f^*)}_t \ \widehat{K}^R_t \right]',$$

$$\mathbf{B}^R \equiv \begin{bmatrix} 1-\alpha(2a-1)-\Lambda_1 J_1 & \alpha^2(2a-1)-\alpha & J_1 & 0 \\ -(2a-1)-\Lambda_1 J_2 & \alpha(2a-1) & J_2 & 0 \\ -\frac{\Lambda_1}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left[\sigma(2a-1)\frac{\overline{C}}{\overline{K}} + \frac{4\theta a(1-a)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\right] & J_3 & 0 & 1+\frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + \delta 2(1-a)\right] \end{bmatrix},$$

where $J_1 = \frac{(\mu-1)}{\beta}\left[1+\frac{\alpha(1-\Lambda_2)(2a-1)}{\Lambda_2}\right]$ and $J_2 = \frac{(\mu-1)(1-\Lambda_2)(2a-1)}{\beta\Lambda_2}$ under domestic inflation targeting; whereas $J_1 = \frac{(\mu-1)(2a-1)\left[\alpha(1-\Lambda_2)+\frac{\Lambda_2}{(2a-1)}\right]}{\beta\Lambda_2[1-2(1-a)\mu]}$ and $J_2 = \frac{(\mu-1)(1-\Lambda_2)(2a-1)}{\beta\Lambda_2[1-2(1-a)\mu]}$ under CPI inflation targeting. Finally $J_3 = \frac{(1-\alpha)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\left[1+\alpha 4\theta a(1-a)\right] + \sigma\alpha(2a-1)\frac{\overline{C}}{\overline{K}}$. Analogous to the aggregate system, one eigenvalue of the system is zero. Therefore determinacy requires the eigenvalue $1+\frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + \delta 2(1-a)\right]$ to have a modulus greater than one, and the two remaining eigenvalues of $\mathbf{B}^R$ are also outside the unit circle. By Proposition C.1 of Woodford (2003) the following results are obtained:

Proposition 5 *Suppose that monetary policy reacts to forward-looking domestic price inflation. Then for an active monetary policy ($\mu > 1$), the necessary and sufficient conditions for determinacy of the difference system are*

(Case I) $a > 0.5$ *and*

$$1 < \mu < 1 + \min\left\{\Gamma_1^B \equiv \frac{(1-\beta)\Lambda_2}{\alpha\Lambda_1(2a-1)} \Gamma_2^A \equiv \frac{2(1+\beta)\Lambda_2}{\Lambda_1\alpha(2-\Lambda_2)(2a-1)}\right\}; \quad (40)$$

(Case II) $$0.5 > a > \frac{1}{2-\delta}\left[1-\delta-\frac{\overline{C}}{\overline{K}}\frac{1}{2}\right]; \quad (41)$$

where $\Lambda_1 = \frac{(1-\psi)(1-\beta\psi)}{\psi}$ *and* $\Lambda_2 = 1-\beta(1-\delta)$.

Proposition 6 *Suppose that monetary policy reacts to forward-looking consumer price inflation. Then for an active monetary policy ($\mu > 1$), the necessary and sufficient con-*

ditions for determinacy of the difference system are

$$\textbf{\textit{(Case I)}} \quad a > 0.5 \quad \textit{and} \quad 1 < \mu < \min\left\{\Gamma_1^C, \Gamma_2^C, \Gamma_3^C\right\}; \tag{42}$$

$$\textbf{\textit{(Case II)}} \quad 0.5 > a > \frac{1}{2-\delta}\left[1-\delta-\frac{\overline{C}}{\overline{K}}\frac{1}{2}\right] \quad \textit{and} \tag{43}$$

$$2(1+\beta)+\frac{(\mu-1)\Lambda_1}{\Lambda_2\left(2\mu(1-a)-1\right)}\left[\Lambda_2-2\alpha(1-2a)\left(2-\Lambda_2\right)\right]<0;$$

where $\Gamma_1^C \equiv \frac{1}{2(1-a)}$, $\Gamma_2^C \equiv \frac{(1-\beta)\Lambda_2+\Lambda_1\alpha(2a-1)}{\Lambda_1\alpha(2a-1)+(1-\beta)\Lambda_2 2(1-a)}$, $\Gamma_3^C \equiv \frac{2(1+\beta)\Lambda_2+\Lambda_1[\Lambda_2+(2a-1)\alpha(2-\Lambda_2)]}{4(1+\beta)(1-a)\Lambda_2+\Lambda_1[\Lambda_2+(2a-1)\alpha(2-\Lambda_2)]}$.

Suppose $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$ and $\lambda = 7.66$. Given the assigned parameter values conditions (41) and (43) of Propositions 5 and 6 are violated if $a < 0.4716$ and thus (first-order) indeterminacy arises $\forall \mu > 1$. If $a > 0.5$ then under domestic price inflation targeting the open-economy introduces no additional requirements for determinacy. This follows by direct comparison of the upper bounds on μ given by conditions (39) and (40): $\Gamma_1^A < \Gamma_1^B$ and $\Gamma_2^A < \Gamma_2^B$. However if $a > 0.5$ and consumer price inflation is targeted then comparing (39) with (42) yields $\Gamma_2^C < \Gamma_1^A$ and $\Gamma_3^C < \Gamma_2^A$. Since $\partial\Gamma_i^C/\partial a > 0$ for $i = 1, 2, 3$, the inflation coefficient μ is constrained by these upper bounds, all of which are increasing with respect to a. Thus the range of indeterminacy is potentially greater the higher the degree of trade openness (i.e. the lower is a).

As discussed by Kurozumi and Van Zandweghe (2007) in a closed economy an active forward-looking policy makes inflation expectations self-fulfilling entirely because of the cost channel of monetary policy. However in the open-economy indeterminacy is more severe because of the additional impact the trade channel has on inflation. Under an active forward-looking policy, the increase in the real interest rate results in a future expected deterioration in the terms of trade ($\widehat{T}_{t+1}$ increases relative to $\widehat{T}_t$). Thus the trade effect puts upward

$$E_t\widehat{\pi}_{t+1} = E_t\widehat{\pi}_{t+1}^h + (1-a)\left(E_t\widehat{T}_{t+1} - \widehat{T}_t\right)$$

pressure on inflation, the effect of which is stronger the higher the degree of trade openness ($a \downarrow$).

4.2 Backward-looking rules

We now turn our attention to backward-looking interest-rate rules. The determinacy analysis proceeds as before except now the aggregate system is five-dimensional and determinacy requires two eigenvalues to lie inside the unit circle and the remaining three eigenvalues be outside the unit circle. The difference system is six-dimensional under consumer price inflation targeting and determinacy therefore requires that there are exactly three eigenvalues inside the unit circle and three eigenvalues outside the unit circle. As before the capital dynamics eigenvalue can lie inside or outside the unit circle depending on the size of a. Since responding to backward inflation makes the analytical conditions for determinacy more complex to derive, we will simply report some numerical results. Suppose $\alpha = 0.36$, $\beta = 0.99$ and $\delta = 0.025$. Then determinacy of the aggregate system requires that $1 < \mu < 3.171$ otherwise no equilibrium exists. Figures 3 and 4 depict the regions in the parameter space (a, μ) that are associated with determinacy (D), (first-order) indeterminacy (I) and an explosive solution (N) around the neighborhood of the steady state, for both possible indexes of inflation. First consider the case when the capital dynamics root is unstable ($a \geq 0.4716$). Inspection of figure 3 suggests that if domestic price inflation is targeted the open-economy places no additional restrictions for equilibrium determinacy. However if consumer price inflation is targeted, figure 4 suggests that the upper bound on the inflation coefficient is more severe in the open-economy. Furthermore the range of determinacy decreases as the degree of trade openness increases therefore implying that this upper bound is increasing with respect to a. Next consider the case where the eigenvalue associated with the capital dynamics lies inside the unit circle ($a < 0.4716$). If domestic price inflation is targeted then determinacy is not possible, whereas if consumer price inflation is targeted determinacy prevails.

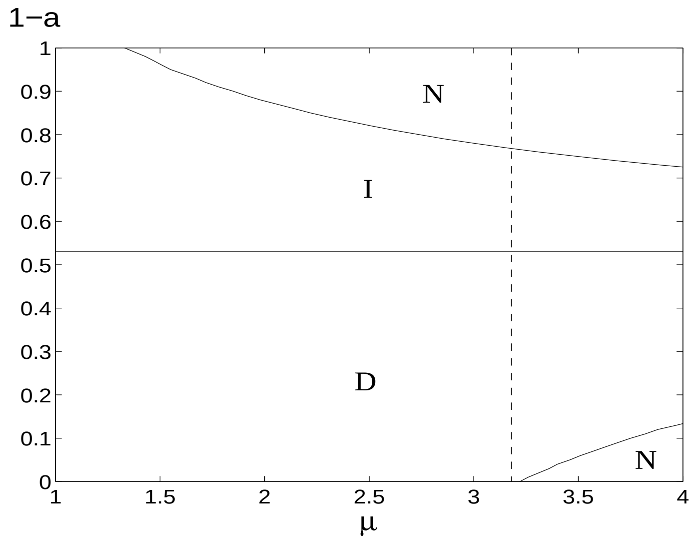

Figure 3: Regions of determinacy under a backward-looking domestic price inflation rule

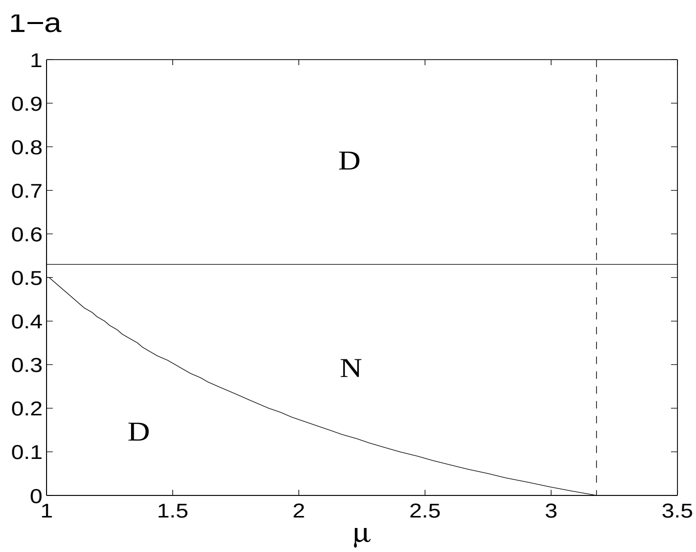

Figure 4: Regions of determinacy under a backward-looking consumer price inflation rule

The above analysis suggests that in the presence of capital there are important implications for equilibrium determinacy depending on whether the economy is open or closed. In a closed-economy, indeterminacy can easily be prevented by avoiding forward-looking interest-rate rules. However in open-economies implementing current-looking or backward-looking policies is contingent upon the degree of trade openness of the economy in question. For sufficiently closed economies, the monetary authority should adopt a current-looking CPI interest-rate rule to minimize policy-induced aggregate instability. However for economies that are sufficiently open to trade a backward-looking CPI interest-rate rule is a more appropriate policy.

5 Conclusion

This paper has considered the importance of trade openness for equilibrium determinacy in the presence of capital and investment spending. It has been shown that policy induced indeterminacy is considerably easier to obtain using a sticky-price open-economy model compared to its closed-economy variant. This conclusion is robust to the index of inflation chosen as the policy indicator and the timing of the interest-rate rule employed. The analysis suggests that considerable care needs to be taken when designing interest-rate rules for economies that actively engage in cross-country trade. Adopting policies advocated for closed-economies, may not be sufficient to prevent the emergence of real indeterminacy.

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A Appendix

A.1 Proof of Proposition 2

As shown in the main text, if monetary policy targets current-looking domestic price inflation this results in a four dimensional system $\left[\widehat{mc}_t^R \ \widehat{x}_t^R \ \widehat{\pi}_t^{R(h-f^*)} \ \widehat{K}_t^R\right]'$, where $\widehat{K}^R$ is the only predetermined endogenous variable. Determinacy of the difference system thus requires one eigenvalue to lie inside the unit circle and the other three eigenvalues to lie outside the unit circle. One eigenvalue of the coefficient matrix $\mathbf{A}_{PPI}^R$ is given by $e_K \equiv 1 + \frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + 2\delta(1-a)\right]$, which modulus can be greater or less than one. The remaining three eigenvalues are determined by the upper left 3×3 submatrix of $\mathbf{A}_{PPI}^R$, denoted by $\overline{\mathbf{A}}_{PPI}^R$. Then the characteristic equation of $\overline{\mathbf{A}}_{PPI}^R$ is

$$r^3 + a_2 r^2 + a_1 r + a_0 = 0$$

where

$$\begin{aligned}
a_2 &= -1 - \frac{1}{\beta} - \frac{\Lambda_1}{\beta}\left[1 + \frac{\alpha(1-\Lambda_2)(2a-1)}{\Lambda_2}\right] \\
a_1 &= \frac{1}{\beta} + \frac{\Lambda_1 \mu}{\beta}\left[1 + \frac{\alpha(1-\Lambda_2)(2a-1)}{\Lambda_2}\right] + \frac{\alpha\Lambda_1(2a-1)}{\Lambda_2\beta} \\
a_0 &= -\frac{\Lambda_1\mu\alpha(2a-1)}{\Lambda_2\beta}.
\end{aligned}$$

First suppose the eigenvalue e_K is unstable, $|e_K| > 1$, which requires either $a > 0.5$ or $0.5 > a > \frac{1}{2-\delta}\left[1 - \delta - \frac{1}{2}\frac{\overline{C}}{\overline{K}}\right]$. Then using Proposition C.2 of Woodford (2003), two of the remaining three eigenvalues are outside the unit circle and one eigenvalue is inside the unit circle if and only if: (Case I)

$$1 + a_2 + a_1 + a_0 > 0 \Leftrightarrow \frac{(\mu-1)\Lambda_1}{\beta}\left[1 - (2a-1)\alpha\right] < 0, \tag{A1}$$

$$-1 + a_2 - a_1 + a_0 > 0 \Leftrightarrow -2(1+\beta) - (\mu+1)\Lambda_1\left[1 + \frac{(2a-1)\alpha(2-\Lambda_2)}{\Lambda_2}\right] > 0, \tag{A2}$$

or (Case II)

$$1 + a_2 + a_1 + a_0 > 0 \Leftrightarrow \frac{(\mu-1)\Lambda_1}{\beta}\left[1 - (2a-1)\alpha\right] > 0, \tag{A3}$$

$$-1 + a_2 - a_1 + a_0 > 0 \Leftrightarrow -2(1+\beta) - (\mu+1)\Lambda_1 \left[1 + \frac{(2a-1)\alpha(2-\Lambda_2)}{\Lambda_2}\right] < 0, \quad \text{(A4)}$$

and either

$$a_0^2 - a_0 a_2 + a_1 - 1 > 0, \quad \text{(A5a)}$$

or

$$|a_2| < -3. \quad \text{(A5b)}$$

Assume $\mu > 1$, since otherwise the aggregate system would be indeterminate. Then Case I is not relevant since condition (A1) is violated by assumption. For Case II, condition (A3) is satisfied $\forall\ \mu > 1$ since $1 - \alpha(2a-1) > 0$. If $a > 0.5$ then by inspection condition (A4) is automatically satisfied and either (A5a) or (A5b) is required for determinacy. If $a < 0.5$ then condition (A5a) can be derived as:

$$\frac{\Lambda_1\alpha(1-2a)\mu}{\Lambda_2\beta}\left[\frac{\Lambda_1(1-2a)(\mu-1)}{\Lambda_2} + \Lambda_1 + \Lambda_2\beta + (1-2a)\alpha\Lambda_1\right] \\ +(1-\beta) + \Lambda_1\mu + \frac{\Lambda_1\alpha(1-2a)\mu(1-\beta)}{\Lambda_2\beta} > 0,$$

which is always satisfied by inspection and thus condition (A5b) does not apply. Finally condition (A4) is automatically satisfied provided $\frac{(1-2a)\alpha(2-\Lambda_2)}{\Lambda_2} < 1$. Otherwise the following upper bound on μ is required: $\mu < \frac{2(1+\beta)\Lambda_2}{\Lambda_1[(1-2a)\alpha(2-\Lambda_2)-\Lambda_2]} - 1$.

Now suppose that the eigenvalue e_K is stable, $|e_K| < 1$ which requires $a < \frac{1}{2-\delta}\left[1 - \delta - \frac{1}{2}\frac{\overline{C}}{\overline{K}}\right] < 0.5$. Determinacy then requires that the remaining three eigenvalues be outside the unit circle. From the characteristic equation of $\overline{\mathbf{A}}^R_{PPI}$ this implies that $r(0) = \frac{\Lambda_1\mu\alpha(1-2a)}{\Lambda_2\beta} > 0$. If $\mu > 1$ then $r(1) = \frac{(\mu-1)\Lambda_1}{\beta}[1 + \alpha(1-2a)] > 0$. Therefore if $r(-1) > 0$ then the three roots, either real or complex, are outside the unit circle. Since $r(-1) = -2(1+\beta) - \Lambda_1(\mu+1)\left[1 - \frac{\alpha(1-2a)(2-\Lambda_2)}{\Lambda_2}\right]$, then $r(-1) > 0$ provided $\alpha(1-2a)(2-\Lambda_2) > \Lambda_2$ and $1 + \mu > \frac{2(1+\beta)\Lambda_2}{\Lambda_1[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2]}$. This completes the proof. ■

A.2 Proof of Proposition 3

If monetary policy targets current-looking consumer price inflation then one eigenvalue of the coefficient matrix $\mathbf{A}^R_{CPI}$ is given by $e_K \equiv 1 + \frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + 2\delta(1-a)\right]$. The remaining four eigenvalues are determined by the upper left 4×4 submatrix of $\mathbf{A}_{CPI}^R$, denoted by $\overline{\mathbf{A}}_{CPI}^R$. Then the characteristic equation of $\overline{\mathbf{A}}_{CPI}^R$ is

$$r\left(r^3 + a_3 r^2 + a_2 r + a_1\right) = 0$$

where

$$\begin{aligned}
a_3 &= -1 - \frac{1}{\beta} - \frac{\Lambda_1}{\beta}\left[1 + \frac{\alpha(1-\Lambda_2)(2a-1)}{\Lambda_2}\right] - 2(1-a)\mu \\
a_2 &= \frac{1}{\beta} + \frac{\Lambda_1\mu}{\beta}\left[1 + \frac{\alpha(1-\Lambda_2)(2a-1)}{\Lambda_2}\right] + \frac{\alpha\Lambda_1(2a-1)}{\Lambda_2\beta} + \frac{2(1-a)\mu(1+\beta)}{\beta} \\
a_1 &= -\frac{\mu}{\beta}\left[2(1-a) + \frac{\Lambda_1\alpha(2a-1)}{\Lambda_2}\right].
\end{aligned}$$

Hence one eigenvalue is zero and the three remaining eigenvalues are the solutions to the cubic equation $r^3 + a_3 r^2 + a_2 r + a_1 = 0$. Determinacy requires two eigenvalues to lie inside the unit circle and the other three eigenvalues to lie outside the unit circle. First suppose the eigenvalue e_K is outside the unit circle $|e_K| > 1$, which requires either $a > 0.5$ or $0.5 > a > \frac{1}{2-\delta}\left[1 - \delta - \frac{1}{2}\frac{\overline{C}}{\overline{K}}\right]$. Then using Proposition C.2 of Woodford (2003), two of the remaining three eigenvalues are outside the unit circle and one eigenvalue is inside the unit circle if and only if: (Case I)

$$1 + a_3 + a_2 + a_1 > 0 \Leftrightarrow \frac{(\mu-1)\Lambda_1}{\beta}\left[1 - (2a-1)\alpha\right] < 0, \tag{B1}$$

$$-1+a_3-a_2+a_1 > 0 \Leftrightarrow -2(1+\beta)-4\mu(1-a)(1+\beta)-(\mu+1)\Lambda_1\left[1 + \frac{(2a-1)\alpha(2-\Lambda_2)}{\Lambda_2}\right] > 0, \tag{B2}$$

or (Case II)

$$1 + a_3 + a_2 + a_1 > 0 \Leftrightarrow \frac{(\mu-1)\Lambda_1}{\beta}\left[1 - (2a-1)\alpha\right] > 0, \tag{B3}$$

$$-1+a_3-a_2+a_1 > 0 \Leftrightarrow -2(1+\beta)-4\mu(1-a)(1+\beta)-(\mu+1)\Lambda_1\left[1 + \frac{(2a-1)\alpha(2-\Lambda_2)}{\Lambda_2}\right] < 0, \tag{B4}$$

and either

$$a_1^2 - a_1 a_3 + a_2 - 1 > 0, \tag{B5a}$$

or

$$|a_3| < -3. \tag{B5b}$$

Assume $\mu > 1$, or otherwise the aggregate system would be indeterminate, then Case I is not relevant since condition (B1) is violated by assumption. For Case II, condition (B3) is satisfied $\forall \mu > 1$. If $a > 0.5$ then by inspection condition (B4) is automatically satisfied and either (B5a) or (B5b) is required for determinacy. If $a < 0.5$ condition (B4) is automatically satisfied provided $\frac{(1-2a)\alpha(2-\Lambda_2)}{\Lambda_2} < 1$. Otherwise the following condition is required: $\Lambda_1(1+\mu)\left[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2\right] < 2\Lambda_2(1+\beta)[1+2\mu(1-a)]$. In addition either (B5a) or (B5b) is required for determinacy.

Now suppose that the eigenvalue e_K is inside the unit circle $|e_K| < 1$, which requires $a < \frac{1}{2-\delta}\left[1-\delta-\frac{1}{2}\frac{\overline{C}}{\overline{K}}\right] < 0.5$. Determinacy then requires that the remaining three eigenvalues be outside the unit circle. From the characteristic equation of $\overline{\mathbf{A}}_{CPI}^{R}$ this implies that $r(1) = \frac{(\mu-1)\Lambda_1}{\beta}[1+\alpha(1-2a)] > 0$ given the assumption that $\mu > 1$ and $r(0) = -\frac{\mu}{\beta}\left[2(1-a)-\frac{\Lambda_1\alpha(1-2a)}{\Lambda_2}\right]$. This has to be positive $r(0) > 0$ otherwise there would be (at least) one stable root, which requires $\Lambda_1\alpha(1-2a) > 2(1-a)\Lambda_2$. Therefore if $r(-1) > 0$ then the three roots, either real or complex, are outside the unit circle. Since $r(-1) = -2(1+\beta)4\mu(1-a)(1+\beta)-\Lambda_1(\mu+1)\left[1-\frac{\alpha(1-2a)(2-\Lambda_2)}{\Lambda_2}\right]$, then $r(-1) > 0$ provided $\alpha(1-2a)(2-\Lambda_2) > \Lambda_2$ and $\Lambda_1(1+\mu)\left[\alpha(1-2a)(2-\Lambda_2)-\Lambda_2\right] > 2\Lambda_2(1+\beta)[1+2\mu(1-a)]$. This completes the proof. ■