A Closer Look at Long Run Money Demand*

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Abstract: We study annual United States data from 1869 or 1900 to 1999. We find evidence for a well-specified and stable model of money demand with data from 1946 to 1999. We carry out diagnostic and stability tests, including nonlinearity tests. A linear cointegration model with the monetary base performs better than a model with M1. A specification with M2 is not supported. We use real GNP as the scale variable and a short term interest rate as the opportunity cost measure. We estimate an income elasticity of .86 and an interest rate elasticity of -.44 for the monetary base.

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1. Introduction

The stability of money demand has been a longstanding issue.[1] Lucas (1988) informally analyses the stability of a log-linear M1 money demand function and concludes that real money demand is a stable function, and he supports earlier findings of Meltzer (1963) with an income (or wealth) elasticity of around unity.[2] With the development of cointegration methods (Engle and Granger, 1987), numerous empirical studies have applied these techniques to long run U.S. money demand. Examples are Hafer and Jansen (1991), Hoffman and Rasche (1991), Miller (1991), and Baba, Hendry and Starr (1992), who find support for a cointegrated money demand model with either M1 or M2.[3] King, Plosser, Stock, and Watson (1991) also find support for cointegration with M2 and a short term interest rate.

In contrast to these studies, others have argued against a stable US money demand function in the postwar period. For example, Friedman and Kuttner (1992) do not find support for cointegration, especially when the period 1970:3 to 1990:4 is considered. Similarly, Miyao (1996) studies the behavior of the M2 money demand function over the period from 1959 to 1993 with quarterly data and various interest rates, in levels and in logs. Miyao concludes that M2 is not a useful intermediate target for monetary policy in the 1990s.[4] However, Carlson, Hoffman, Keen, and Rasche (2000) reach the conclusion that M2 does have predictive content for nominal economic activity, and that the real M2 money demand function is stable but only after accounting for financial innovation in the first half of the 1990s.

Two recent papers, Stock and Watson (1993) and Ball (2001), focus on particularly long data sets for the US and M1. Stock and Watson (1993) analyse various estimation methods for cointegrating relationships and illustrate their performance for money demand functions with annual data from 1900 to 1987. They pay particular attention to the postwar period and use monthly data in addition to annual data for this period. They employ a semi-log linear real M1 money demand model with real NNP and the level of the 6-month commercial paper rate. For the postwar period, they also consider the 90-day Treasury bill

[1] The seminal paper by Goldfeld (1976) that found "missing money" sparked an array of empirical investigations of possible causes of the instability in money demand. These studies are surveyed by Judd and Scadding (1982). See also Friedman and Schwartz (1982) on using phase-averaged data.
[2] See in addition Lucas (2000) for similar conclusions and also for theoretical models.
[3] Hoffman and Rasche (1991) and Miller (1991) also analyse a specification with the monetary base as the money measure and the former study finds empirical support for it.
[4] See also Estrella and Mishkin (1997).

rate and the 10 year Treasury bond rate. They compare and contrast their results to those of various others. They also carry out recursive estimations to check their results for stability.

Stock and Watson (1993) conclude that a long span of data is needed to estimate long run money demand functions precisely. Their overall evidence is in favour of a stable long run money demand function with an income elasticity near one and an interest rate semi-elasticity of near -.10. However, the postwar data is found to be not very informative. A lack of low frequency variation in the postwar data, as suggested by Lucas (1988), does not allow a disentangling of the effects of output and interest rates on money demand for this time period. Estimates are imprecise and sensitive to sub-sample specifications due to substantial multicollinearity.

Ball (2001) revisits the M1 money demand model of Stock and Watson (1993), using the same data set but extended through 1996. He applies the same array of estimation techniques as Stock and Watson did. Ball obtains precise estimates over the postwar period for his extended data set: an income elasticity of approximately .5 and a semi-elasticity of -.05 for the interest rate. The M1 model is not stable over the period 1900 to 1996. Also, he does not find support for a velocity specification.

Lütkepohl, Teräsvirta, and Wolters (1999) apply a nonlinear error-correction model with smooth transition adjustment to capture instabilities in the linear cointegration model. They analyze German M1 money demand. Granger and Teräsvirta (1993), Teräsvirta (1998), and van Dijk, Teräsvirta, and Franses (2001) provide details on the econometric theory for estimating and testing such nonlinear models.

In this paper, we extend the study of Stock and Watson (1993) and Ball (2001) to M0 (base money) and M2. In addition, using data up to 1999, we subject the linear cointegration models for M0, M1, and M2 to various new specification and diagnostic tests and we explore, as an attractive alternative to the conventional linear specification, a nonlinear error-correction model with smooth transition of the exponential or logistic form. The structural break tests that we apply allow for various forms of continuous as well as an abrupt structural change. It is of interest to find out whether a nonlinear model over the period from 1900 to 1999 fits the data for M1 and is stable. It is also of interest to find out how well Ball's postwar model holds up to rigorous diagnostic tests and whether a nonlinear model can provide an improved fit. Furthermore, the question arises whether M0 or M2 would possibly perform better than M1.

Section 2 outlines the tests of the linear against the nonlinear error-correction model and the diagnostic tests for both models. Section 3 briefly discusses the data used. In Section

4, we study the long span of data, 1869 to 1999 for M0 and M2, and 1900 to 1999 for M1. Section 5 discusses the empirical findings for the postwar period from 1946 to 1999. Section 6 summarizes our results.

2. Econometric Methodology

We analyse the linear cointegration models with standard unit root and cointegration techniques. A plausible alternative specification to the linear model is that of a nonlinear error-correction model with smooth transition. Lütkepohl et al. (1999) propose to apply a nonlinear error-correction model of the Granger and Teräsvirta (1993) type to money demand. The nonlinear model allows for smooth as well as for sudden changes in parameters if the changes exhibit some regularity. These features make such models attractive for money demand modelling because they may capture what shows up in tests of linear models as structural breaks.

The nonlinear smooth transition regression (STR) model takes the following form:

$$\Delta y_t = \alpha' X_t + (\beta' X_t) F(\tau_t; \gamma, k) + e_t ,$$

where $\Delta y_t = \alpha' X_t + e_t$ represents a conventional linear single-equation error-correction model, and

$$X'_t = (1, \Delta y_{t-1},\ \Delta y_{t-2},\ \ldots,\ \Delta y_{t-p},\ \Delta w_{t-1},\ \Delta w_{t-2},\ \ldots,\ \Delta w_{t-p}, (y_{t-1} - \hat{\alpha}_0 - \hat{\alpha}_1 w_{t-1})) ,$$

with w_t a vector of regressors and the last expression in parentheses the error-correction term. F(·) is the transition function that describes the transition from one regime to another. It is generally bounded between 0 and 1. γ is a positive slope parameter to indicate how rapidly the transition from one regime to another takes place, and k locates where the transition occurs in time. τ_t is the transition variable. Granger and Teräsvirta (1993), and Teräsvirta (1998), among others, suggest to use a functional form for F(·) of the exponential (ESTR) or logistic (LSTR) type. The ESTR function is non-monotonic in τ_t and symmetric around k: $F(\tau_t;\gamma,k) = 1 - \exp[-\gamma(\tau_t - k)^2]$. In the LSTR model, the transition function is monotonically increasing in τ_t and it allows for asymmetric transition: $F(\tau_t;\gamma,k) = \{1 + \exp[-\gamma(\tau_t - k)]\}^{-1}$. The parameters γ and k can be estimated via non-linear least squares. The nonlinear models are quite general formulations that allow for nonlinearities in the error-correction term as well as in the short term error-correction dynamics.

When testing linearity, the null hypothesis is that γ=0. However, the model is not identified under the null. Teräsvirta (1998) suggests estimating an auxiliary regression:

$$\Delta y_t = \beta' X_t + \delta_0' X_t \tau_t + \delta_1' X_t \tau_t^2 + \delta_2' X_t \tau_t^3 + e_t,$$

where the null hypothesis of linearity is $\delta'_0 = \delta'_1 = \delta'_2 = 0$. It is important that τ_t is moment stationary up to a certain order, except when it is dominated by a polynomial in time (see Lin and Teräsvirta, 1994). The linearity tests can be interpreted more generally as misspecification tests of the linear model.

The non-linear STR models must pass four groups of model adequacy tests. This requires regressions involving the gradient vectors from the non-linear maximum likelihood function (see Teräsvirta, 1998, pp. 518-525; we implemented step 1' on p. 520; see also Eitrheim and Teräsvirta, 1996). The tests are all Lagrange multiplier or LM tests with an asymptotic χ^2 distribution. However, an F statistic in connection with the F distribution was found to have better size properties and about the same power in finite samples and we will therefore use this approximation. We test for ARCH-like behavior in the residuals and for the degree of autocorrelation in these same residuals. Probably the two most important diagnostic tests are whether there is any remaining non-linearity and whether the estimated parameters are constant. The former test involves testing whether non-linear terms are statistically significant when added to the STR model (see Teräsvirta 1998, pp. 520-22). We denote the tests for parameter constancy by LM1, LM2, and LM3. LM1 is a test for smooth monotonic changes in the parameters of the STR model. As the speed of adjustment goes to infinity, the limiting case is a single structural break. The LM2 test is a test for a smooth non-monotonic change in the STR parameters that is symmetric about t-k, with the limiting case being two structural breaks. LM3 modifies the transition function to permit non-monotonic, as well as monotonic, and non-symmetric changes in the STR model parameters (see Lin and Teräsvirta 1994, and Teräsvirta 1998, pp. 522-24). We apply the same type of tests to the linear models. Lin and Teräsvirta find good power properties of their tests in simulations and they compared favourably to the CUSUM and fluctuations tests.

3. Data

All data are for the U.S. The relevant variables to be considered in this paper are: three measures of money balances, M0 (base or high powered money), M1, and M2; a scale measure in the form of real income; and a measure of the opportunity cost of money.

We discuss below the specific choice for each variable of interest. M0, M1 and M2 are used as alternative measures of money. Due to financial innovations the definitions of M1 and M2 in particular changed over time. Rasche (1987) discusses some of the problems with the construction of a long time series for M1 back to 1947. In addition, M0, M1 and M2 are affected by the introduction of deposit-sweep accounts in 1994.[5] However, appropriately adjusted series for the full postwar period are not available.

The revised 1929-1999 U.S. National Income and Product Accounts figures, released in 2000 by the Bureau of Economic Analysis, are used for real GNP as the measure of income, and for the GNP deflator as a measure of the price level. Various measures of the opportunity cost of money are considered. A short term interest rate in the form of a 6 month commercial paper rate is used, as well as a long term interest rate in the form of the yield on corporate bonds.

All data, apart from high powered money and M1, were obtained from Balke and Gordon (1986) and updated appropriately. Data on high powered money were obtained from Friedman and Schwartz (1982), and updated appropriately. M1 between 1900 and 1958 was obtained from Stock and Watson (1993), and then spliced to the Fed's measure of M1 from 1959 onwards. For further details of the data sources and updates, see the Appendix.

4. Empirical Results for the Long Span of Data

In this section, we analyze the money demand model with M0 and M2 over the period 1869 to 1999 and with M1 over the period 1900 to 1999. We first carry out Dickey-Fuller and Phillips-Perron (Z_t) tests for a unit root. These tests support an I(1) specification for all variables considered in modeling empirical money demand, over the long span, as well as over the postwar period.

Next, we use Johansen's (1995) method to carry out cointegration tests. The Schwarz Bayesian information criterion is applied to choose the lag length for the vector error-correction model (VECM) but we also explored the sensitivity of the results to the chosen lag lengths. Asymptotic and reasonably accurate P-values are calculated with a program of MacKinnon, Haug, and Michelis (1999). We follow Carlson, Hoffman, Keen, and Rasche (2000), among many others, and choose a VECM specification that allows for linear deterministic time trends in the data but no trend in the cointegrating relationship. In other words, the cointegrating equilibrium relation, if it exists, eliminates all stochastic and deterministic trends in the data. The normalized cointegrating relation takes the following form:

$$\ln(\text{real money balances}) - \theta_y \ln(\text{real GNP}) - \theta_r (\text{interest rate}) - \mu = \varepsilon_t, \qquad (1)$$

[5] See Anderson and Rasche (2001).

where μ is a constant term and ε_t is a stationary Gaussian error correction term. θ_y is the income elasticity and θ_r the interest rate elasticity or semi-elasticity of money demand. We start with M0 as the measure of money balances and use the GNP deflator to arrive at real quantities. We consider for the interest rate in turn the short-term rate, ln (short rate), the long rate, and ln (long rate). We look next at the same specifications but replace M0 by M1 and then by M2.

Table 1 reports results for the trace test of cointegration. For M0 and M2, we find cointegration only when the opportunity costs of holding money are measured by the short-term interest rate, either in levels or logarithms. For M1, only ln (short rate) produces a cointegrating relation. The lag length chosen for the VECMs is 0 in all cases. Akaike's information criterion chooses more lags but these specifications no longer lead to cointegration. However, Akaike's criterion may overestimate the lag length.

In Table 2, we report the elasticity and semi-elasticity estimates for the cointegrated cases, using the Johansen method. The income elasticity, θ_y, is quite precisely estimated and close to 1. The interest rate elasticities are precisely estimated as well and the semi-elasticities are in the range of -.2 to -.1 and the other elasticities are in the range of -.5 to -.3. Stock and Watson (1993) obtain very similar estimates. However, the crucial question is whether the linear money demand relation is misspecified.

An attractive alternative specification is the nonlinear error-correction model of the ESTR or LSTR type with smooth transition adjustment back to equilibrium. This nonlinear model may capture changes in parameters that are reflected in the linear model as structural breaks. We consider two plausible alternative transition variables: the error-correction term from the cointegrating vector and the interest rate spread. The transition variable triggers the change from one regime to another. The error-correction term measures the deviation from the long run equilibrium and the spread the deviation of our short term from the long term interest rates, i.e., the slope of the yield curve. We therefore carry out an LM-type test of the null hypothesis of a linear error-correction model against the alternative of a non-linear error-correction model of the ESTR or LSTR type for the two transition variables. This test can also be interpreted as a specification test of the linear model. Due to the dimensionality of the nonlinear model, we focus on a single equation, which may cause inefficiencies in estimating the nonlinear model but is otherwise not crucial. The null hypothesis cannot be rejected for M0 when the short rate, and for M1 and M2 when ln (short rate) is the measure of the opportunity costs. However, the linear models are not well specified in these cases and all display substantial residual autocorrelation and non-normality. The results for testing the linear model for M1 are reported in Table 3. For M0 and M2, two LM tests are borderline cases with P-values around .06 in Table 2, and we therefore explored also the ESTR and LSTR models (not reported) but without success.

The null hypothesis of a linear model for M0 with ln (short rate) and for M2 with the short rate is clearly rejected in favor of the nonlinear model. Table 3 therefore explores how well such a specification fits the data. For M0, the LSTR model, and for M2, the ESTR and LSTR models show significant error-autocorrelation and remaining nonlinearity and are therefore rejected by the data. For M0, the ESTR model does not show these problems, however, the LM3 test clearly indicates parameter instability.

The analysis of the long span of data does not point towards the existence of a stable and well-specified model for money demand. The linear models show substantial autocorrelation that cannot be resolved by increasing the lag length of the VECM. Where the linear model is rejected, the nonlinear STR models provide a better fit to the data but the nonlinear models are rejected by formal tests because the residuals do not behave well either.

Our results for the long span support those of Ball (2001), after adding an additional three years of data and using the revised GNP figures. The way interest rates enter the model does have some effect on the results, with short rates clearly favoring cointegration and long rates leading to no cointegration. Ball considered only M1. We explored M0 and M2 but were unable to find a well-specified money demand model with any of the three measures of money.

5. Empirical Results for the Postwar Data

We analyze in this section the post WWII period from 1946 to 1999. We use again annual data. Ball (2001) shows that monthly data and annual data lead to very similar results for M1. One would expect this result as the frequency of observation does not add much information when the span of data is the same because cointegration is a long run concept. In addition, using annual data avoids complications that arise from seasonal factors, like cointegration at seasonal frequencies, and finding appropriate seasonal adjustment methods.

Stock and Watson (1993) demonstrate for the postwar period, using equation (1) with M1 and the short rate, that cointegration parameter estimates are very sensitive to the estimation method used. The literature has suggested several asymptotically equivalent methods for estimating cointegrating vectors. Stock and Watson recommend a dynamic OLS

(DOLS) estimator over the Johansen method. This problem does not arise over the long span of data where DOLS and the Johansen method produce very similar estimates. In addition, Stock and Watson find that parameter estimates fluctuated widely depending on the starting date of the postwar period. Furthermore, they also find that parameters are imprecisely estimated. They conclude that the overall empirical evidence for M1 suggests that a stable money demand relation over the period 1900 to 1987 is consistent with the data. Their postwar data contain very limited information and individual elasticities are not well determined, only θ_y / θ_r is. In contrast, Ball (2001) shows that extending the same data set to 1996 leads to precise estimates for M1 that allow to disentangle the separate effects of θ_y and θ_r. However, Ball rejects that the M1 relation is stable over the long span from 1900 to 1999. He finds smaller elasticity estimates for the postwar period as compared to the prewar period.

Guided by the findings of Stock and Watson, we explore the performance of our money measures over the period 1946 to 1999. In addition to M1, we also use M0 and M2. Besides extending the sample from 1996 to 1999, we apply several new tests to judge model adequacy. It is of interest to see whether the results of Ball stand up to more scrutiny and whether M0 and M2 might be more suitable measures of money balances. Carlson et al. (2000) study an M2 measure of money and find qualified support for a cointegrating relation that fits the data but only after accounting for a break with a permanent upward trend in velocity during the period 1990-1994, using monthly data from 1964-1998.

5.1 Results for M0

We start with the analysis of M0 and consider all possible starting dates from 1946 to 1956. The short term interest rate, the long rate, and ln (long rate) lead to varying cointegrating parameter estimates, confirming the findings in Stock and Watson. The latter interest rate shows the least variation with θ_y ranging from .91 to 1.13 and θ_r from -.66 to -.46, using the Johansen method. On the other hand, the specification with ln (short rate) leads to very similar parameter estimates regardless of the starting date in the period 1946 to 1956. We will therefore analyze this specification in more detail.

Table 4 reports results in the first row for an M0 specification with ln (short rate) and without a deterministic time trend in the cointegrating relationship. The Schwarz and Akaike criteria choose 1 lag for the VECM. The null hypothesis of no cointegration is strongly rejected with a P-value of .002. We find 1 cointegrating vector in the VECM system. The LM test does not lead to a rejection of the linear model when it is tested against a nonlinear model of the ESTR or LSTR type. Table 4 also reports the cointegrating vector estimates from the VECM. The estimate for θ_y is .86 and for θ_r it is -.44. We get very precise estimates for these elasticities with asymptotic standard errors of .101 and .063 respectively. This is consistent with the findings of Ball that the data beyond the 1987 end date of Stock and Watson's data set give more precise estimates. Because of the recommendation of DOLS by Stock and Watson, we apply in addition this estimator. The Schwarz criterion picks 2 leads and lags but there is also support for a specification with 0 leads and lags. We use a quadratic kernel with an associated, data based, automatic bandwidth estimator with pre-whitening in order to estimate the long run variances. The DOLS parameter estimates are somewhat lower than the ones obtained with the Johansen VECM-based method. The estimates are quite precise with 2 leads and lags, however, the standard error estimates are sensitive to the number of lags and leads chosen and 0 leads and lags produce much larger standard errors. The VECM-based method does not show this sensitivity.

The next step is to subject the linear model with M0 and ln (short rate) to various specification tests. We base these on the VECM estimates and report results in Table 5. We find no ARCH effects in the residuals of the money demand equation, nor any autocorrelation. Furthermore, the Jarque-Bera test does not reject normality and the LM1, LM2, and LM3 tests cannot detect any significant parameter changes over the postwar period. From this evidence, we conclude that the M0 model with ln (short rate) provides a good fit to the data over the postwar period from 1946 to 1999.

Lastly, we turn to two more specification issues. One is to test whether the hypothesis that $\theta_y = 1$ is supported by the data. If it is supported by the data, this would allow a velocity specification. Using the VECM method, a likelihood ratio test, with or without a so-called Bartlett correction (see Haug, 2001), strongly suggests that $\theta_y = 1$ with a P-value of .909. However, this result does not carry over to the DOLS estimates with the 2 leads and lags. The null hypothesis that $\theta_y = 1$ is rejected with a P-value of .001. On the other hand, a DOLS specification with 0 leads and lags (not reported) leads to much less precise estimates and the hypothesis that $\theta_y = 1$ is no longer rejected at the 5% significance level (the P-value is .089).

A second issue is the role of a time trend. Equation (1) does not allow for a deterministic time trend in the cointegrating relationship. Ball (2001) points to the possibility that money demand may have trended downwards due to new transaction technologies and the creation of near monies. We therefore consider a specification that allows for a deterministic time trend in the cointegrating vector and in the data.[6] Using the Schwarz criterion to compare the cointegrated model with and without this trend leads to choosing the model with trend. Table 4 reports results in the second row. We find one cointegrating vector. The trend is significant for the VECM estimates but less so for the DOLS estimates. In addition, θ_y and θ_r are precisely determined, in contrast to the findings for M1 below. The VECM estimate for θ_y is somewhat large taking on a value of 2.78. It is significantly larger than 1. The P-value (LR test with Bartlett correction) for this hypothesis is .0003. On the other hand, the DOLS estimate for θ_y is not significantly different from 1 and the relevant P-value is .150. However, DOLS estimates of the standard errors prove to be again sensitive to the number of leads and lags chosen, whereas VECM estimates are precise regardless of whether 0, 1, or 2 lags are used. The VECM parameter estimates fall with the lag length and 0 lags produce a value for θ_y of 2.04 with a standard error of .599 that is not significantly different from 1.

Because of these mixed results, we consider a different approach to specify the most suitable VECM. Chao and Phillips (1999) suggest using an information criterion to determine the cointegration rank and lag length simultaneously. We use the Schwarz criterion to evaluate the VECMs across all possible ranks and from 0 to 4 lags, with and without a deterministic time trend in equation (1). The Schwarz criterion is minimized for the specification with one cointegrating vector, a trend in equation (1), and 1 lag in the VECM. This is the same specification that we chose above when we separated the choice of the lag length from the choice of the cointegration rank. The specification without a trend has the second lowest value for the Schwarz criterion.

Table 5 reports results for the diagnostic tests for the M0 model with a time trend in the cointegrating vector. The trend has the expected negative sign and enters significantly. The model passes all tests at the 5% significance level, however, the tests for parameter constancy are borderline cases with P-values of .057, .059 and .78 for LM1, LM2, and LM3. In contrast, the P-values for the model without the time trend were much larger and the null hypothesis could not be rejected at the 10% level. This leads us to the conclusion that the M0 model with ln (short rate) and no time trend in the cointegrating vector provides a much better fit to the postwar data than the model with a trend. The empirical evidence suggests that neither a trend nor a nonlinear model adequately captures changes in money demand

[6] This is case 2* in the terminology of Osterwald-Lenum (1992). We also considered case 2 (it allows for a quadratic trend in the data and a linear trend in the cointegrating vectors) in all trend regressions but results did not improve.

over time and that a model without a trend fits the data best. We experimented in addition with a specification that imposes θ_y=1, however, a linear model was rejected and a nonlinear model did not provide a satisfactory fit either.

5.2 Results for M1

We focus on an M1 specification of equation (1) used by Ball (2001) with the short rate as the measure of the opportunity cost of holding money. We consider again all possible starting dates over the period 1946 to 1956. The parameter estimates do not vary much for this specification. Table 4 reports results in the third row without a time trend in the cointegrating vector. The Schwarz and Akaike criteria choose 1 lag for the VECM. The null hypothesis of no cointegration is rejected at the 5% level of significance. The income elasticity, θ_y, is .45 and the interest semi-elasticity, θ_r, is -.05. DOLS estimation leads to very similar estimates of .46 and -.04, using 0 leads and lags as chosen by the Schwarz criterion. The estimates are very precise and close to those of Ball, who obtained values of .5 and -.05.

Extending the sample of Ball by three years confirms his findings. Data after 1987 provide sufficient information to determine the separate effects of θ_y and θ_r on money demand. The estimate of θ_y is significantly different from 1. A velocity specification is therefore not supported for M1. The estimates for both parameters are precise, in contrast to the findings of Stock and Watson (1993) for a postwar sample that ended in 1987.

Including a time trend in the cointegrating vector has the same bad effects on standard errors as in Ball's study. Table 4 gives results. VECM-based and DOLS standard errors for θ_y increase substantially. The trend and θ_y are not precisely estimated at all. Furthermore, the trend does not have the expected negative sign. The DOLS estimate has in addition the wrong sign for the income elasticity. Moreover, the relationship with a trend is no longer cointegrated and we will therefore not analyze this specification any further. A trend specification for M1 does not provide us with a reasonable empirical model.

Ball (2001) does not carry out diagnostic tests on the postwar M1 model. We subject the linear M1 money demand relation without a trend to various diagnostic tests. Results are given in Table 5. This M1 specification passes all autocorrelation, normality and parameter constancy tests. It only reveals in the residuals some ARCH effects (with 1 lag). The overall fit is quite good and the empirical evidence otherwise supports the specification of the M1 model with the short interest rate and no time trend.

5.3 Results for M2

We find evidence for cointegration for equation (1) with M2 as the money measure when the interest rate is specified as either the short rate, ln (short rate), the long rate, or ln (long rate). However, no matter what interest rate is used, the estimate of θ_r has the wrong sign. We consider all possible starting dates from 1946 to 1956 and get quite some variation of the parameter estimates, but all with the wrong sign of the interest rate elasticity.[7] Table 4 reports results for the short rate. Including a deterministic time trend in the cointegrating vector does not solve the problem either.

In contrast to M0 and M1 specifications, M2 does not lead to a reasonable money demand specification for the postwar periods considered. This is consistent with the empirical evidence of Mayo (1996) and also of Carlson et al. (2000) who found with monthly data a permanent upward shift in velocity from 1990 to 1994. Our interest here is to find a stable relation over the postwar period without considering shifts.

6. Conclusion

In this paper we searched for a well-specified model of money demand. We considered real M0, M1, and M2, with real GNP as the scale variable, and a short or a long term interest rates (in levels and in logs) as the measure of opportunity costs. For the long span of data from 1869 to 1999 for M0 and M2, and from 1900 to 1999 for M1, we did find linear cointegration with a short term interest rate, however, the linear model was either rejected in favor of a nonlinear model, or the linear model did not pass our diagnostic tests. A nonlinear error-correction model with smooth exponential or logistic transition did not provide an acceptable fit to the long span of data either.

We considered next the postwar period 1946-1999. A specification with M2 is very sensitive to the starting date of the sample as far as elasticity estimates are concerned. We find evidence for cointegration but the interest rate coefficient has the wrong sign for all M2 specifications considered.

The postwar results for M1 are consistent with those of Ball (2001). We first analyze a linear specification with no deterministic time trend in the cointegrating vector. There is cointegration with the short term interest rate. The income elasticity (.45) and the interest rate semi-elasticity (-.05) are very precisely estimated. The income elasticity is significantly different from 1 and a velocity specification is therefore rejected. The M1 model holds up quite well to various diagnostic tests. Linearity is not rejected, parameters are constant, and residuals are Gaussian and show no autocorrelation. The only problem is that there are ARCH effects present.

In addition, we studied a specification with a linear deterministic time trend in the cointegrating vector. Ball (2001) argues for such a trend to account for new transaction technologies and the creation of near monies. Our results are the same as in Ball: the trend and income elasticity are very imprecisely estimated and the two effects cannot be separated out when M1 is the measure of money balances.

A specification with M0 and the logarithm of the short term interest rate provides a better fit to the postwar data than M1. The evidence in favor of linear cointegration is very strong. We first consider the case with no time trend in the cointegrating vector. The income elasticity (.86) and the interest elasticity (-.44) are very precisely estimated. The income elasticity is not significantly different from 1 for VECM estimates, however, it is significantly different from 1 for DOLS estimates. We prefer the VECM estimates because the results for DOLS are sensitive to the number of leads and lags included in the regressions. The linear M0 model passes all diagnostic tests without any problems. All P-values are above .15.

We also consider a specification for M0 with a linear deterministic time trend in the cointegrating vector. The trend coefficient has the predicted negative sign and is precisely estimated. The same holds true for the income elasticity. This specification passes all diagnostic tests at the 5% significance level. At a 10% level, we find evidence for parameter changes. Instead, the M0 model without a time trend leads to much stronger results.

Our empirical results show that a linear specification with M0 provides a better fit to postwar U.S. data than M1. M0 has been much less affected by changes in the financial sector in recent decades than M1 or M2 that changed in definition several times. A time trend for the M0 specification picks up some of the effects of technological change but seems to be a rather crude measure because the model becomes less stable with it included. Future research should try to find a refined measure of technological change. In addition, the effect on M0 of the introduction of sweep accounts in 1994 should be quantified and a consistent adjusted M0 series constructed for the entire postwar period.

[7] Including an own rate of return for M2 in equation (1) should not affect results as long as the own rate and the interest rate we considered are cointegrated, which is suggested by the theory of the term structure.

Appendix: Data Sources

Nominal GNP
(Billions of dollars)
1869-1928: Balke and Gordon (1986) pp. 781-782.
1929-1999: Table 1.9, of *National Income and Product Accounts of the United States*, from the Bureau of Economic Analysis.

Real GNP
(Billions of 1996 dollars)
1869-1928: Balke and Gordon (1986) pp. 781-782. Spliced by a factor of 2.63 to be consistent with data from 1929 onwards.
1929-1999: Table 1.10, of *National Income and Product Accounts of the United States*, from the Bureau of Economic Analysis.

Commercial Paper rate
1869-1969: 6 month commercial paper rate from Balke and Gordon (1986) pp.781-783.
1970-97: 6 month prime commercial paper rate (CP6M) from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED) database. Annual averages of monthly data were taken. This series was discontinued in 1997.08.
1998-1999: A proxy was calculated as follows. Monthly data on the three-month AA financial commercial paper rate (CPF3M) and the three-month AA non-financial commercial paper rate (CPN3M) was obtained from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED) database, each series starting in 1997.01. For each month, the average of these two series was calculated. This average series was then compared to the 6 month prime commercial paper rate between 1997.01-1997.08. The average monthly differential was calculated. This differential was added onto the averaged series of the three month commercial paper rates for 1998-1999, to obtain a monthly proxy for the 6 month commercial paper rate. Then, annual averages were taken.

Yield on corporate bonds
1869-1918: Balke and Gordon (1986) pp. 781-782.
1919-1999: Moody's seasoned Baa corporate bond yield (BAA) from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED) database. Annual averages of monthly data were taken.

Money supply, base
(Billions of dollars)
1869-1958: Friedman and Schwartz (1982), Table 4.8, pp. 122-25.
1959-1999: Board of Governors monetary base, not adjusted for changes in reserve requirements, not seasonally adjusted, (BOGUMBNS) from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED) database.

The above measures of the monetary base have not been adjusted for the effects of changes in statutory reserve requirements on the quantity of base money held by depositories. The unadjusted series were used because an alternative adjusted series that was consistent over the entire time span was not available.

The monthly data was seasonally adjusted (as prior to 1959 the data is seasonally adjusted) using the ratio to moving average (multiplicative) method in EViews Version 3.1, and then annual averages were calculated.

Money supply, M1
(Billions of dollars)
1900-1958: Stock and Watson (1993), data available at: http://www.wws.princeton.edu/~mwatson/publi.html. Spliced by a factor of 1.03 to be consistent with data from 1959 onwards.
1959-1999: M1 Money Stock, seasonally adjusted, (M1SL) from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED) database. Annual averages of monthly data were taken.

Money supply, M2
(Billions of dollars)
1869-1958: Balke and Gordon (1986) pp. 784-785
1959-1999: M2 Money Stock, seasonally adjusted, (M2SL) from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED) database. Annual averages of monthly data were taken.

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Table 1

P-Values for the Trace Test for Cointegration Using Alternative Interest Rates

Money measure	**short rate**	**ln (short rate)**	**long rate**	**ln (long rate)**	**Period**
ln (real M0)	**.008 (0)**	**.0001 (0)**	.139 (1)	.097 (1)	1869-1999
ln (real M1)	.138 (0)	**.005 (0)**	.375 (0)	.111 (0)	1900-1999
ln (real M2)	**.012 (0)**	**.020 (0)**	.175 (0)	.175 (0)	1869-1999

Note: The money demand model involves the variables ln (real money), ln (real GNP), and the interest rate as stated. The null hypothesis tested is that of no cointegration. A VECM that involves a measure of the natural logarithm of real money balances, the natural logarithm of real GNP, and an interest rate as stated, is estimated. In all cases the tests indicated at most one cointegrating vector. The number of lags for first differences in the VECM is given in parentheses. The Schwarz Bayesian information criterion is used for lag selection. Bold faced numbers mark cases where cointegration is supported at the 5% level of significance.

Table 2

Elasticity Estimates from the Cointegrating Vectors and P-Values for Non-Linearity Tests

Money measure	**short rate**			**ln (short rate)**			**Period**
	θ_y	θ_r	*LM test*	θ_y	θ_r	*LM test*	
ln (real M0)	.85 (.036)	-.16 (.024)	.069	.84 (.018)	-.49 (.035)	**.010**	1869-1999
ln (real M1)	--	--	--	.76 (.032)	-.40 (.049)	.242	1900-1999
ln (real M2)	1.06 (.048)	-.10 (.022)	**.045**	1.03 (.047)	-.29 (.068)	.064	1869-1999

Note: See Note to Table 1. Asymptotic standard errors are given in parentheses. The LM-test P-values refer to testing the linear error-correction model (null hypothesis) against a non-linear exponential and logistic model with smooth transition. The interest rate spread is used as the transition variable. Bold faces indicate rejection of the null at the 5% level.

Table 3

P-Values for Diagnostic Tests: Long Sample

	ln (real M0)		**ln (real M1)**	**ln (real M2)**	
Interest Rate	**ln (short rate)**		**ln (short rate)**	**short rate**	
Type of model	ESTR	LSTR	linear ECM	ESTR	LSTR
Type of Test					
No ARCH	**.000**	**.000**	.675	.185	**.000**
No error auto-correlation	.177 (1) .175 (2) .280 (3)	**.017 (1)** **.023 (2)** **.006 (3)**	**.009 (1)** **.006 (2)** **.013 (3)**	**.0001** **.001** **.004**	**.002 (1)** **.009 (2)** **.025 (3)**
No *remaining non-linearity*	.060	**.002**	**.000** (Jarque-Bera test for normality)	**.030**	**.042**
Parameter Constancy *LM1* *LM2* *LM3*	 .746 .795 **.004**	 .416 .326 .219	 .766 .789 .905	 .265 .087 .119	 .173 .201 .258

Note: ESTR refers to the exponential model with smooth transition and LSTR to the logistic model. The interest rate spread is the transition variable. One lag is chosen for the ARCH tests. The number of lags for the autocorrelation tests is given in parentheses. See Teräsvirta (1998) for details on the tests. Bold faces mark the cases where the null hypothesis (as stated at the beginning of each row) is rejected at the 5% level of significance.

Table 4

P-Values for Cointegration (Trace) and Non-Linearity (LM) Tests and Elasticity Estimates: 1946-1999

Variables in the Model	Trace	LM	VECM			DOLS		
			θ_y	θ_r	trend	θ_y	θ_r	trend
ln (real M0), ln (real GNP), and ln (short rate)	**.002** (1)	.379	.86 (.101)	-.44 (.063)	--	.62 (2) (.082)	-.32 (2) (.076)	--
	.004 (1)	.206	2.78 (.76)	-.56 (.086)	-.061 (.022)	1.58 (.559)	-.40 (.136)	-.03 (.015)
ln (real M1), ln (real GNP), and short rate	**.043** (1)	.619	.45 (.031)	-.05 (.005)	--	.46 (0) (.059)	-.04 (0) (.007)	--
	.171 (1)	--	.11 (.318)	-.04 (.006)	.011 (.010)	-.09 .385	-.04 .006	.018 .014
ln (real M2), ln (real GNP), and short rate	**.041** (1)	--	.79 (.035)	.02 (.008)	--	.83 (0) (.080)	.01(0) (.006)	--
	.047 (1)	--	1.30 (.387)	.02 (.008)	-.017 (.013)	1.04 (.232)	.001 (.004)	-.007 (.009)

Note: See Note to Table 2. For DOLS, the first number in parentheses gives the number of leads and lags used, the second one gives standard errors. "Trend" refers to a deterministic linear time trend in the cointegrating vector.

Table 5

P-Values for Diagnostic Tests of the Linear Model: 1946-1999

Variables in the Model	**No ARCH**	**No auto-correlation**	**Normality (Jarque-Bera)**	**Parameter Constancy**		
				LM1	*LM2*	*LM3*
ln (real M0), ln (real GNP), and ln (short rate)	.445	.281 (1) .375 (2) .568 (3)	.549	.410	.246	.154
ln (real M0), ln (real GNP), ln(short rate), and trend	.360	.342 (1) .576 (2) .533 (3)	.403	.057	.059	.078
ln (real M1), ln (real GNP), and short rate	**.019**	.485 (1) .783 (2) .908 (3)	.508	.278	.198	.335

Note: See Note to Table 3.