Monetary Policy in a Cash-in-Advance Economy: Employment, Capital Accumulation and the Term Structure of Interest Rates

Arman Mansoorian
Department of Economics
York University
North York, ON
Canada, M3J 1P3

and

Mohammed Mohsin
Department of Economics
University of Tennessee
Knoxville, TN 37996
U.S.A.

Abstract

This paper studies the effects of monetary policies on employment, capital accumulation, consumption, and the term structure of interest rates in a cash-in-advance economy, where money is required for consumption expenditures. Monetary policy involves targeting the inflation rate or the nominal interest rate. The detail dynamics of the model are fully worked out. As no numerical analysis is involved, we are able to identify very clearly the different channels through which monetary policy will impinge on the important macroeconomic variables.

JEL Classifications: E43, E52, E58, E62 and E63
Keywords: Employment, Capital Accumulation, Term Structure, Cash-in-Advance

----

We thank Sam Bucovetsky, Leif Danziger, Wia-Ming Ho, and Pauline Shum for their comments and suggestions. All remaining errors or omissions are the responsibility of the authors. Financial support from the Social Sciences and Humanities Research Council of Canada (grant # 410-97-1212) is gratefully acknowledged.

I. Introduction

The effects of monetary policies on employment, capital accumulation, consumption and the terms structure of interest rates have always been of interest to macroeconomists. The present paper attempts to fully work out the effects of monetary policies on these variables in an optimising framework. We consider two types of policies which have been of recent interest to policymakers; monetary policies which target the inflation rate, and policies which target the nominal interest rate.

The framework we use has an infinitely lived representative household with instantaneous utility a function of consumption and labor supply. Consumption expenditures are subject to cash in advance (CIA) constraints. There are three types of assets in the economy: short term government bonds, long term bonds, and equities (i.e., titles to productive capital).[1] We make two key assumptions that allow us to analyse the effects of policy changes using standard techniques involving phase diagrams. First, we employ a continuous time framework. This gives us a considerable degree of flexibility; and, clearly, without it we would not be able to use phase diagrams. Second, in accordance with the literature concerned with the time consistency of monetary policies (e.g., Kydland and Prescott (1977), Backus and Drifill (1985), and Walsh (1995)), we assume that the central bank targets the inflation rate (not the rate of growth of money *per se*).[2] This assumption precludes complicated, yet no so crucial, off steady

[1] The model is, therefore, an extension of the Stockman (1981) and Abel (1983) framework to incorporate endogenous labor supply. The model is also close to that used by Fisher and Turnovsky (1992), who abstract completely from money and monetary policies.

[2] Recently, Mishkin (2000) has argued that the central banks of most developed as well as emerging countries do indeed target the inflation rate rather than the rate of growth of money.

state effects, similar to those analysed by Fisher (1979). It, thus, reduces the dimensions of the dynamic system corresponding to the model, facilitating the use of simple phase diagrams.[3]

We show that a permanent unanticipated increase in the inflation rate increases the relative cost of consumption. This leads to a fall in labor supply, as the representative agents substitutes leisure for consumption. The fall in labor input reduces the marginal productivity of capital, and the return on equities. This reduces interest rates and capital accumulation. Hence, along the adjustment path to the new steady state capital will be falling over time. With falling capital there will be two competing effects on labor supply along the adjustment path. First, the fall in capital will reduce the wage rate, which will tend to reduce the labor supply. Second, the fall in capital will increase interest rates, which will tend to reduce labor supply, through the intertemporal substitution of leisure. Hence, along the adjustment path employment may be rising or falling. Nevertheless, employment in the new steady state will be lower than it was before the policy change.

On impact, after the increase in the inflation rate, consumption may rise or fall. On the one hand, the increase in the inflation rate increases the cost of consumption relative to leisure, which tends to reduce consumption. On the other hand, the instantaneous fall in interest rates tends to tilt the consumption profile to the present. Nevertheless, along the adjustment path to the new long run equilibrium consumption will be falling over time. As discussed above, along the adjustment path interest rates will be rising over time, while wages will be falling. Both of these effects will lead to consumption falling over time. The steady state level of consumption will be lower than it was before the policy change.

---

[3] When the central bank targets the rate of growth of money, instead, the inflation rate will be endogenous, and variable off the steady state. This will then increase the dimension of the dynamic system, and preclude the use of simple phase diagrams. In any case, it is well known that the steady state effects are the same regardless of whether the central bank is fixing the inflation rate or the rate of growth of money.

As no numerical analysis is involved, we are able to identify very clearly the different channels through which monetary policy will impinge on the important macroeconomic variables. Also, as the analysis is carried out using phase diagrams, one can also easily discuss the effects of anticipated permanent and unanticipated temporary policy changes.[4]

Using a standard arbitrage condition between assets with different maturities, we also work out the effects on the term structure of interest rates. There has long been an interest in the effects of government policies on the term structure of interest rates. Prominent papers in this literature which discuss the effects of *monetary* policies include Blanchard (1981), Turnovsky and Miller (1984), Turnovsky (1986, 1989) and McCafferty (1986). These papers use aggregative IS-LM type models. Fisher and Turnovsky (1992), on the other hand, discuss the effects of increases in government expenditures financed with lump sum taxes in a model similar to ours, but abstract completely from money and monetary policies. Our results, therefore, regarding the effects on the term structure, complement the results in these papers.

Finally, we discuss the effects of monetary policies which involve controlling the nominal interest rate. As emphasised by Friedman (2000), among other authors, the monetary policy followed by the U.S. Federal Reserve involves controlling the nominal interest rate. Recently, Carlstrom and Fraust (1995) and Rebelo and Xie (1999) discuss the efficiency of interest rate versus money growth targeting. Rebelo and Xie show that in a model without endogenous labour and with cash in advance on consumption a policy of interest rate targeting is optimal in the sense that the real part of the monetary economy coincides with the equilibrium with a frictionless barter economy. Carlstrom and Fraust consider a model with endogenous labour and cash in advance on consumption to identify which policy (interest rate targeting or

[4] The details of the results regarding unanticipated temporary and anticipated permanent policy changes are in an appendix available upon request.

money growth rate targeting) minimises the sizes of the various distortions in the economy generated by the cash in advance constraint. Our discussion of the effects of policy changes with nominal interest rate targeting complements these studies, by deriving the full dynamics of the effects of a change in the nominal interest rate on macroeconomic variables. It is shown that a change in monetary policy involving an increase in the nominal interest rate would be very similar to the effects of an increase in the inflation rate.

The paper is organized as follows. The model is presented in section II. The effects of monetary policies with inflation rate targeting are presented in section III. The effects of monetary polices with nominal interest rate targeting are worked out in section IV. Some concluding remarks are made in section V.

II. The Model

The model is in an infinite horizon setting with a representative household and a representative firm with perfect capital markets and perfect foresight. The household chooses his consumption, c, labor supply, l, and holding of real short-term government bond, b, in order to maximize the present value of lifetime utility:

$$\int_0^\infty u(c_t, l_t) e^{-\beta t} dt \tag{1}$$

where β is his fixed rate of time preference, $u(c,l) = U(c) + V(l)$, with $U'(c) > 0$, $U''(c) < 0$, $V'(l) < 0$ and $V''(l) < 0$.

Money is introduced through a cash in advance constraint, with the household requiring real money balances m to finance his consumption expenditures:

$$m_t \geq c_t. \tag{2}$$

The household owns all the firms, and therefore receives all their profits, π. He also receives from the government monetary transfer with real values of τ. There are two kinds of accumulable assets, money balances m and government bonds b.[5] The total real value of these assets held by the household is

$$a_t = b_t + m_t. \tag{3}$$

His flow budget constraint is

$$\dot{a}_t = \pi_t + w_t l_t + r_t b_t + \tau_t - c_t - \varepsilon_t m_t \tag{4}$$

where r is the short term interest rate, w the wage rate and ε the inflation rate. Hence, according to (4), the household's savings is equal to his net income $(\pi + wl + rb + \tau)$ less consumption expenditures and the inflation tax $(c + \varepsilon m)$.

The household also has the No Ponzi game condition

$$\lim_{t\to\infty} e^{\int_0^t r_v dv} a_t \geq 0. \tag{5}$$

The household's problem then is to maximize (1) subject to (2)–(5), and the initial condition given by his initial asset holdings a_0.

As money does not yield utility directly, and as the return on bonds completely dominates the return on money, (2) will always hold with strict equality. Hence, m_t is residually determined once c_t is chosen. Thus, setting $m_t = c_t$ in (2), we can write the Hamiltonian for the household's problem as

$$H^h = U(c) + V(l) + \lambda[\pi + wl + ra + \tau - c - (\varepsilon + r)c]. \tag{6}$$

The optimality conditions for this problem are:[6]

$$U'(c) = \lambda(1 + r + \varepsilon) = \lambda(1 + i) \tag{7}$$

---

[5] Long term bonds will be introduced later as the third asset.

[6] Note that in making his utility maximization decision, the household takes τ and r, as given.

$$V'(l) = -\lambda w \tag{8}$$

$$\dot{\lambda} = \lambda(\beta - r) \tag{9}$$

and the standard transversality condition

$$\lim_{t\to\infty} a_t \lambda_t e^{-\beta t} = 0. \tag{10}$$

Now consider the problem of the representative firm. Output Y is produced with a standard neoclassical constant return to scale production function $Y_t = F(K_t, l_t)$ with the following properties: $F_K > 0$, $F_l > 0$, $F_{KK} < 0$, $F_{ll} < 0$, $F_{Kl} > 0$, and $F_{ll}F_{KK} - F_{Kl}^2 = 0$. The profits of the firm are given by

$$\pi_t = F(K_t, l_t) - w_t l_t - I_t, \tag{11}$$

where I is investment expenditures.

The firm's problem is to maximize the present value of its profits

$$\text{Max} \int_0^\infty \pi_t e^{-\int_0^t r_v dv} dt = \int_0^\infty [F(K_t, l_t) - w_t l_t - I_t] e^{-\int_0^t r_v dv} dt \tag{12}$$

$$\text{subject to } \dot{K}_t = I_t \tag{13}$$

and the initial condition K_0.

The Hamiltonian for the firm's problem can be written as

$$H^f = [F(K, l) - wl - I] + \gamma I. \tag{14}$$

The optimality conditions are

$$F_l(K, l) = w \tag{15}$$

$$\gamma = 1 \tag{16}$$

$$\gamma r - F_K(K, l) = \dot{\gamma} \tag{17}$$

and the transversality condition

$$\lim_{t\to\infty} K_t \gamma_t e^{-\beta t} = 0. \tag{18}$$

The optimality conditions for the household's and the firm's problems imply

$$U'(c) = \lambda\left(1 + F_K(K,l) + \varepsilon\right) = \lambda(1+i) \tag{19}$$

$$V'(l) = -\lambda F_l(K,l) \tag{20}$$

$$\dot{\lambda} = \lambda\left(\beta - F_K(K,l)\right). \tag{21}$$

For further reference, note that (19) and (20) implicitly define the equilibrium c and l as

$$c = c(\overset{+}{K}, \overset{-}{\lambda}, \overset{-}{\varepsilon}) \tag{22}$$

$$l = l(\overset{+}{K}, \overset{+}{\lambda}) \tag{23}$$

where $c_K > 0$, $c_\lambda < 0$, $c_\varepsilon < 0$, $l_K > 0$ and $l_\lambda > 0$.

The government side of the model is kept as simple as possible. The government chooses it's real transfer τ, in order to satisfy its flow constraint

$$\dot{b}_t + \dot{m}_t + \varepsilon_t m_t = r_t b_t + \tau_t. \tag{24}$$

The right hand side of (24) is total government expenditures, while the left hand side is total government revenue from the bonds it issues ($\dot{b}$), and from seigniorage ($\dot{m} + \varepsilon m$). As stated in the introduction, we assume that the central bank is targeting the inflation rate ε (by adjusting τ) and not the rate of growth of money *per se*. This assumption is consistent with the assumptions made in the literature on the time consistency of monetary policies.

We are now in a position to work out the dynamics of the model. To this end, first note that from (24) and (4), we obtain the product market clearing condition:

$$F(K_t, l_t) = c_t + \dot{K}_t. \tag{25}$$

Next, substituting for c and l from (22) and (23) into (21) and (25), we obtain

$$\dot{K}_t = F(K_t, l(K_t, \lambda_t)) - c(K_t, \lambda_t, \varepsilon) \tag{26}$$

$$\dot{\lambda}_t = \lambda_t [\beta - F_K(K_t, l(K_t, \lambda_t)]. \tag{27}$$

These two equations jointly determine the dynamics of K and λ, which is presented in the phase diagram in Figure 1. In that figure the $\dot{K} = 0$ and $\dot{\lambda} = 0$ schedules are given, respectively, by

$$F\left(\tilde{K}, l(\tilde{K}, \tilde{\lambda})\right) = c(\tilde{K}, \tilde{\lambda}, \varepsilon) \tag{28}$$

and
$$F_K\left(\tilde{K}, l(\tilde{K}, \tilde{\lambda})\right) = \beta . \tag{29}$$

The slopes of these schedules are given, respectively, by

$$\left.\frac{d\lambda}{dK}\right|_{\dot{K}=0} = \frac{-\Phi_{11}}{\Phi_{12}} <,=,> 0 \quad \text{as } \Phi_{11} >,=,< 0. \tag{30}$$

$$\left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0} = \frac{-\Phi_{21}}{\Phi_{22}} > 0, \tag{31}$$

where, $\Phi_{11} = F_K + F_l l_K - c_K >,=,< 0$

$\Phi_{12} = F_l l_\lambda - c_\lambda > 0$

$\Phi_{21} = F_{KK} + F_{Kl} l_K < 0$

$\Phi_{22} = F_{Kl} l_\lambda > 0.$

From (26) and (27) we also obtain

$$\frac{\partial \dot{K}}{\partial \lambda} = F_l l_\lambda - c_\lambda > 0 \text{ and } \frac{\partial \dot{\lambda}}{\partial K} = -\lambda [F_{KK} + F_{Kl} l_K] > 0 \tag{32}$$

which determine the directions of the trajectories in Figure 1.

Clearly, as λ is a jump variable while K is predetermined, the model will exhibit saddle point stability if the $\dot{K}=0$ locus has a negative slope–that is, if $\Phi_{11}> 0$.[7] The saddle path XX is given by the following equations:

$$K_t = \widetilde{K} + (K_0 - \widetilde{K})e^{\mu t} \tag{33}$$

$$\lambda_t - \widetilde{\lambda} = -\left(\frac{\widetilde{\lambda}\Phi_{21}}{\mu + \widetilde{\lambda}\Phi_{22}}\right).\left(K_t - \widetilde{K}\right) = \left(\frac{\mu_1 - \Phi_{11}}{\Phi_{12}}\right).\left(K_t - \widetilde{K}\right). \tag{34}$$

At this point it is important to note the role played by the assumption that the central bank is targeting the inflation rate ε (by adjusting the lump sum transfers τ), and not the rate of growth of money. If ε were endogenous then we could not solve (26) and (27) for K and λ; as then there would be another differential equation for ε. In that case, for a given K, λ and ε would jump to place the equilibrium on the stable path. Then it would not be possible to use phase diagrams. The steady state effects would, nevertheless, be the same whether the central bank targets the rate of growth of money or the inflation rate. The reason for this is that, with $\dot{m} = 0$, the steady state inflation is equal to the rate of growth of money.

III. The Effects of Monetary Policies with Inflation Rate Targeting

In this section we assume monetary policy is targeted at keeping the inflation rate at a constant level. In this setting, we study the effects of changes in the inflation rate on employment, capital accumulation, output, consumption and the term structure of interest rates. The long run effects of an increase in ε can be obtained by totally differentiating (28) and (29):

---

[7] Throughout the paper will consider only the case in which $\Phi_{11}>0$, which gives us a negatively sloping $\dot{K}=0$ locus. If $\Phi_{11}<0$ then for saddlepoint stability the $\dot{K}=0$ locus should be shallower than the $\dot{\lambda}=0$ locus. If the $\dot{K}=0$ locus is steeper, then the dynamic system is either unstable or is indeterminate. As the issue of indeterminacy is beyond the scope of this paper, we will assume that if the $\dot{K}=0$ locus is upward sloping, then it is shallower than the $\dot{\lambda}=0$ locus. In this case, the saddlepath will still have a negative slope, and all the results we derive below will still hold. Numerical evaluation of the model, with reasonable functional forms and parameter values for the utility and production functions, give us a negatively sloping $\dot{K}=0$ locus.

$$\frac{d\tilde{K}}{d\varepsilon} = \frac{c_{\varepsilon}\Phi_{22}}{\Phi_{11}\Phi_{22} - \Phi_{21}\Phi_{12}} < 0 \tag{35}$$

$$\frac{d\tilde{\lambda}}{d\varepsilon} = \frac{-c_{\varepsilon}\Phi_{21}}{\Phi_{11}\Phi_{22} - \Phi_{21}\Phi_{12}} < 0. \tag{36}$$

In terms of the phase diagram in Figure 2, since the steady state capital stock and the marginal utility of wealth both decrease, it is obvious that the new steady state equilibrium will be somewhere south-west of the initial equilibrium point A, and the new stable path $X'X'$ will lie below XX. Immediately after the increase in ε the equilibrium will jump to B. Then it will adjust gradually from B to Q along $X'X'$.

Before providing the intuition for these results, it will be instructive to derive the effects of the increase in the inflation rate on employment. To derive the short run effects on employment, take derivative of equation (23) for time 0 to obtain

$$\frac{dl_0}{d\varepsilon} = l_K \frac{dK_0}{d\varepsilon} + l_\lambda \frac{d\lambda_0}{d\varepsilon} \tag{37}$$

Since $\frac{dK_0}{d\varepsilon} = 0$ and $\frac{d\lambda_0}{d\varepsilon} < 0$ (refer to Figure 2), it is clear that $\frac{dl_0}{d\varepsilon} < 0$. Hence, labor supply will decrease in the short run.

To obtain the long run effects on employment note that from (23)

$$\frac{d\tilde{l}}{d\varepsilon} = l_K \frac{d\tilde{K}}{d\varepsilon} + l_\lambda \frac{d\tilde{\lambda}}{d\varepsilon}\ . \tag{38}$$

It is clear that $\frac{d\tilde{l}}{d\varepsilon} < 0$ as from (35) and (36), $\frac{d\tilde{K}}{d\varepsilon} < 0$ and $\frac{d\tilde{\lambda}}{d\varepsilon} < 0$.

The intuition for these results is as follows. With CIA constraints on consumption expenditures alone, the increase in the inflation rate will increase the cost of consumption in terms of leisure. This will lead to a substitution of leisure for consumption, reducing labour supply. The fall in labour input will reduce the marginal productivity of capital, and therefore the marginal utility of wealth. Both of these effects will tend to reduce l_0. This explains the initial jump in the equilibrium from A to B.

The instantaneous fall in the marginal productivity of capital will reduce investment. Capital will be falling along the adjustment path to the new long run equilibrium. With K falling, there will be two competing effects on labour l along the adjustment path to the new long run equilibrium. This follows from (23), which gives us $\dot{l} = l_K \dot{K} + l_\lambda \dot{\lambda}$. On the one hand, the falling K along the adjustment path tends to reduce labor supply by reducing the wage rate. This is captured by $l_K \dot{K}$, which is negative. On the other hand, the falling K tends to increase the marginal productivity of capital, and, hence, the marginal utility of wealth. This will tend to increase labour supply by tilting the leisure profile towards the future. This is captured by $l_\lambda \dot{\lambda}$, which is positive. Thus, employment may be falling or rising along the adjustment path, although employment in the new long run equilibrium will be below what it was before the policy change.

Clearly, output will fall on impact, as K_0 is predetermined while l_0 falls. If both K and l fall along the adjustment path, output will continue to fall. On the other hand, if K falls while l rises along the adjustment path, then output may be rising or falling along the adjustment path. Nevertheless, as both K and l fall in the long run, output also falls in the long run.

One can also readily derive the effects on consumption. From (22), the short run effects are given by

$$\frac{dc_0}{d\varepsilon} = c_K \frac{dK_0}{d\varepsilon} + c_\lambda \frac{d\lambda_0}{d\varepsilon} + c_\varepsilon . \qquad (39)$$

Since $\frac{dK_0}{d\varepsilon}=0$ and $\frac{d\lambda_0}{d\varepsilon}<0$ (refer to Figure 2), the sign of $\frac{dc_0}{d\varepsilon}$ is ambiguous, and depends on the relative strength of the price effect c_ε vis-à-vis the wealth effect $c_\lambda \frac{d\lambda_0}{d\varepsilon}<0$.[8] However, after the initial jump, consumption will be falling along the adjustment path to the new steady state, as $\dot{c}=c_K\dot{K}+c_\lambda\dot{\lambda}<0$.[9] As may be clear from (25) the new steady state level of c will be lower than it was before the policy change.

We now turn to the effects on the term structure of interest rates. We will devote the rest of this subsection to this issue, because in the literature concerned with the term structure effects monetary policies are considered only in IS-LM type models.

To examine the behavior of short-term interest rate note first that from (16), (17) and (23)

$$r_t = F_K(K_t, l(K_t, \lambda_t)). \tag{40}$$

Thus the effect on the short term interest rate at time 0 is given by

$$\frac{dr_0}{d\varepsilon}=[F_{KK}+F_{Kl}l_K]\frac{dK_0}{d\varepsilon}+F_{Kl}l_\lambda\frac{d\lambda_0}{d\varepsilon}=\Phi_{21}\frac{dK_0}{d\varepsilon}+\Phi_{22}\frac{d\lambda_0}{d\varepsilon}. \tag{41}$$

We know that $\frac{dK_0}{d\varepsilon}=0$ and $\frac{d\lambda_0}{d\varepsilon}<0$. Hence, $\frac{dr_0}{d\varepsilon}<\dot{0}$. As the marginal utility of wealth (λ) jumps to a lower level at time 0 (Figure 2), it increases leisure in the short run (equation (23)). The short run decrease in labor supply decreases the marginal product of capital, lowering short-term interest rates.

---

[8] When ε increases the representative agent substitutes leisure for consumption. This is the price effect c_ε. On the other hand, the fall in employment reduces the marginal productivity of capital, reducing the marginal utility of wealth λ_0. This tends to reduce savings, and increase c_0. This is the wealth effect $c_\lambda \frac{d\lambda_0}{d\varepsilon}$.

[9] Along the adjustment path wages are falling (K is falling) and the marginal productivity of capital is rising (λ is rising). Both of these effects tend to reduce consumption along the adjustment path.

Next, note that from (40)

$$\dot{r}_t = (F_{KK} + F_{Kl} l_K)\dot{K}_t + (F_{Kl} l_\lambda)\dot{\lambda}_t = \Phi_{21}\dot{K}_t + \Phi_{22}\dot{\lambda}_t . \tag{42}$$

From Figure 2, it is obvious that after the initial jump in λ (to point B), $\dot{\lambda} > 0$ and $\dot{K} < 0$ along BQ. Hence, $\dot{r}$ in equation (42) is positive. After their initial jump to a lower level, interest rates will gradually increase to their new steady state level. The time path is drawn in Figure 3.

Now consider the effects on the term structure of interest rates. Introduce a long-term bond paying a constant real coupon of unity forever. Let the price of the bond and its yield be P and R respectively. Note that if the representative agent is willing to hold both short-term and long-term bonds, the equilibrium condition requires that, in the absence of risk and with efficient financial markets, the instantaneous rates of return from both bonds should be equal. Thus, we will have the following relationship between short-term and long-term interest rates:[10]

$$r_t = \frac{1 + \dot{P}_t}{P_t} = R_t - \frac{\dot{R}_t}{R_t} . \tag{43}$$

From (40) and (43) we obtain

$$\dot{R}_t = R_t\left[R_t - F_K(K_t, l(K_t, \lambda_t))\right]. \tag{44}$$

Solving either (43) or (44), we obtain

$$R_t = \frac{1}{\int_t^\infty e^{-\int_t^s r_v dv} ds} = \frac{1}{\int_t^\infty e^{-\int_t^s F_K(K_v, l(K_v, \lambda_v))dv} ds} . \tag{45}$$

---

[10] Although the above equation is written following the arbitrage condition, it could be derived as a first order optimal condition of the agent's maximization problem, should we introduce the long-term bond along with the short-term bond in the portfolio of the representative agent.

Hence, the current long term interest rate can be expressed in terms of future short term interest rates, which themselves are dependent upon future time paths of K and λ.

The above equation expresses the long term interest rate (R_t) as a weighted sum of the forecasted future short term rates.[11] The dynamics of both short term and long term interest rates can be approximated by linearizing equations (40) and (44) around the initial steady state equilibrium:

$$r_t = \tilde{r} + \Phi_{21}(K_t - \tilde{K}) + \Phi_{22}(\lambda_t - \tilde{\lambda}) \tag{46}$$

$$\dot{R}_t = \tilde{R}\left((R_t - \tilde{R}) - \Phi_{21}(K_t - \tilde{K}) - \Phi_{22}(\lambda_t - \tilde{\lambda})\right). \tag{47}$$

It should be noted that $\tilde{r} = \tilde{R} = \beta$. The solution for *r* can be obtained directly from (46) whereas *R* should be obtained by solving the first order differential equation (47).

Since the short term rate *r* is rising over time, and since long term rate *R* is discounting the expected future time path of *r*, it follows that the current long term rate must always exceed the current short term rate. The adjustment of short term and long term interest rates are shown in Figure 4. Note that the long term rates will be rising until they reach the steady state level.

Hence, from (43) it is clear that $\frac{\dot{R}_t}{R_t} = R_t - r_t > 0$.

IV. The Effects of Monetary Policies with Interest Rate Targeting

As emphasised by Friedman (2000), among other authors, the monetary policy followed by the U.S. Federal Reserve involves controlling the nominal interest rate. Hence, it is also of widespread current policy relevance to discuss the effects of monetary policies which involve

[11] Of course, with perfect foresight the forecasted future interest rates will coincide with the actual interest rates.

targeting the nominal interest rate. This can be done very easily using the framework developed in this paper.

Suppose the policy of the central bank is to control the nominal interest rate i $(= r + \varepsilon)$, by adjusting ε (through τ) continuously. This would not affect the maximisation problems faced by the representative firm or the representative household. Hence, equations (1)–(21) would remain intact, except that now in equations (7) and (19) the nominal interest rate $(r + \varepsilon$, or $F_K + \varepsilon)$ would be constant. Then solving equation (19) and (20) we obtain

$$c = c(\overset{-}{\lambda}, \overset{-}{i}) \tag{48}$$

$$l = l(\overset{+}{K}, \overset{+}{\lambda}). \tag{49}$$

Substituting these equations into (21) and (25) we obtain

$$\dot{K}_t = F(K_t, l(K_t, \lambda_t)) - c(\lambda_t, i) \tag{50}$$

$$\dot{\lambda}_t = \lambda_t [\beta - F_K(K_t, l(K_t, \lambda_t)]. \tag{51}$$

Comparing (50) and (51) with (26) and (27), respectively, it is clear that the dynamics of the model with inflation rate targeting will be very similar to the dynamics with interest rate targeting.

The long run effects of an increase in the nominal interest rate are obtained by differentiating (50) and (51) at the steady state:

$$\frac{d\tilde{K}}{di} = \frac{c_i \Phi_{22}}{\Phi^*_{11}\Phi_{22} - \Phi_{21}\Phi_{12}} < 0 \tag{52}$$

$$\frac{d\tilde{\lambda}}{di} = \frac{-c_i \Phi_{21}}{\Phi^*_{11}\Phi_{22} - \Phi_{21}\Phi_{12}} < 0 \tag{53}$$

where, $\Phi^*_{11} = F_K + F_l l_K > 0$.

In an appendix available upon request, we derive the full dynamics of the model with interest rate targeting and show that the effects of an increase in the nominal interest rate with this type of monetary policy are very similar to the effects of an increase in ε with inflation rate targeting.

Hence, an increase in i raises the cost of consumption relative to leisure. On impact, this leads to a fall in l, reducing the marginal productivity of capital, leading to capital decumulation. Along the adjustment path to the new long run equilibrium, K will be falling while the marginal utility of wealth λ will be rising. Hence, along the adjustment path employment may be rising or falling. The new long run level of employment will be lower than it was before the policy change. Also, as with inflation rate targeting, on impact c may rise or fall, depending on the relative strengths of the interest rate and wealth effects. But along the adjustment path c will be falling. The new steady state level of c will also be lower than it was before the policy change.

V. Conclusions

In this paper a utility maximizing framework is used in order to work out the effects of monetary policies on employment, consumption, capital accumulation and the term structure of interest rates. It is shown that a change in monetary policy, involving an increase in the inflation rate will lead to a reduction in employment, as with cash in advance constraint on consumption the higher inflation will induce households to substitute leisure for consumption. The fall in labour input will reduce the marginal productivity of capital, investment, and short and long term interest rates. Along the adjustment path to the new steady state capital will be falling over time, interest rates will be rising, while employment may be rising or falling. On impact, consumption may rise or fall, but it will definitely be falling over time along the adjustment path, and its new steady state level will be lower.

It is also shown that if monetary authorities target the nominal interest rate instead of the inflation rate then the effects of an increase in the nominal interest rate will be very similar to the effects of an increase in the inflation rate.

References

Abel, Andrew B. (1985): "Dynamic Behaviour of Capital Accumulation in a Cash-in-Advance Model." *Journal of Monetary Economics*, 16: 55-71.

Backus, D., and J. Driffill (1985): "Inflation and Reputation." *American Economic Review*: 530-538.

Blanchard, O. J. (1981): "Output, the Stock Market, and Interest Rates." *American Economic Review*: 132-43.

Blanchard, O., and S. Fischer (1989): *Lectures on Macroeconomics*. Cambridge, M.A.: MIT Press.

Carlstrom, Charles T. and Fraust, Tomothy S. (1995): "Interest Rate Rules vs. Money Growth Rules: A Welfare Comparison in a Cash-in-Advance Economy". *Journal of Monetary Economics*: 36, 247-267.

Dornbusch, R. (1976): "Expectations and Exchange Rate Dynamics." *Journal of Political Economy*, 84: 1161-1176.

Fischer, S. (1979): "Capital Accumulation on the Transition Path in a Monetary Optimizing Model." *Econometrica*, 47: 1433-1439.

Fisher, Walter H., and S. J. Turnovsky (1992): "Fiscal Policy and the Term Structure of Interest rates: An Intertemporal Optimising Analysis." *Journal of Money, Credit and Banking*, 24: 1-26

Friedman, B.M. (2000): "The Role of Interest Rates in Federal Reserve Policymaking." *National Bureau of Economic Research*, Working Paper 8047.

Kydland, F., and E. Prescott (1977): "Rules Rather than Discretion: the Inconsistency of Optimal Plans." *Journal of Political Economy*: 473-492.

McCafferty, S. A. (1986): "Aggregate Demand and Interest Rates: A Macroeconomic Approach to the Term Structure." *Economic Inquiry*, 24: 521-33.

Mishkin, Frederic S. (2000): "Inflation Targeting in Emerging-Market Countries." *American Economic Review Papers and Proceedings*, 90: 105-109.

Rebelo, Sergio, and Xie, Dangyang (1999): "On the Optimality of Interest Rate Smoothing". *Journal of Monetary Economics*: 43, 263-282.

Sidrauski, Miguel (1967): "Rational Choice and Patterns of Growth in a Monetary Economy". *American Economic Review Papers and Proceedings*, 30: 534-44.

Stockman, A.C. (1981): "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics*. 8: 387-393.

Turnovsky, Stephen J. (1986): "Short-Term and Long-Term Interest Rates in a Monetary Model of a Small Open Economy." *Journal of International Economics*, 20: 291-311.

Turnovsky, Stephen J. (1989): "The Term Structure of Interest Rates and the Effects of Macroeconomic Policy." *Journal of Money, Credit and Banking*, 21: 321-47.

Turnovsky, Stephen J. (2000): *Methods of Macroeconomic Dynamics*. MIT Press.

Turnovsky, Stephen J., and Marcus H. Miller (1984): "The Effects of Government Expenditure on the Term Structure of Interest Rates." *Journal of Money, Credit and Banking*, 16: 16-33.

Turnovsky, Stephen J., and P. Sen (1991): "Fiscal Policy, Capital Accumulation and Debt in an Open Economy." Oxford Economic Papers, 43:1-24.

Uzawa, Hirofumi (1968): "Time Preference, the Consumption Function, and Optimal Asset Holdings". In J.N. Wolfe (ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*. (Chicago: Aldine Publishing Company)

Walsh, C. (1995): "Optimal Contracts for Central Bankers." *American Economic Review*: 150-67.

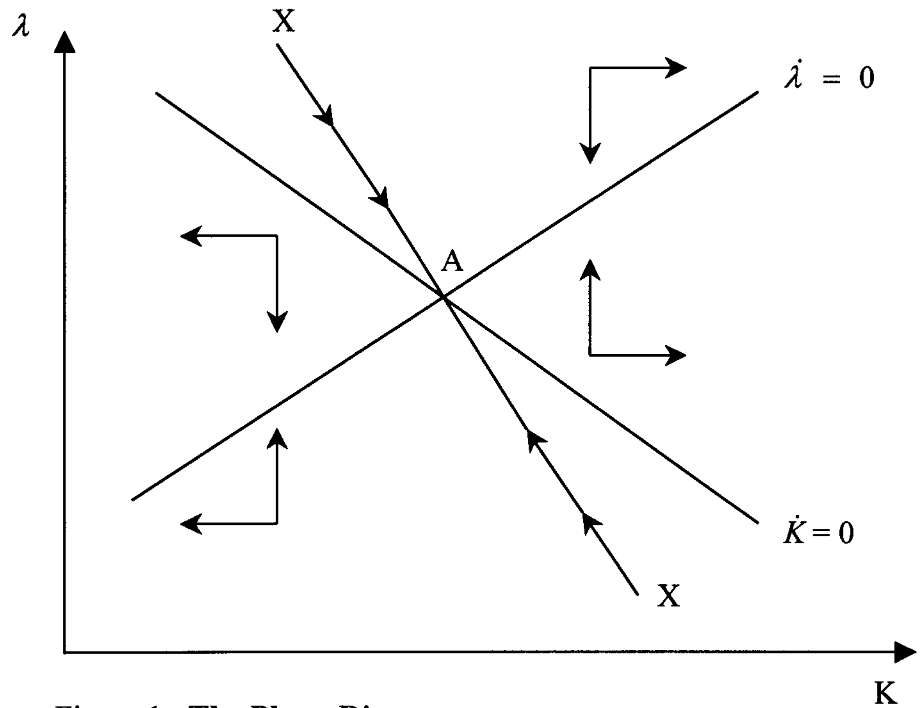

Figure 1 : **The Phase Diagram**

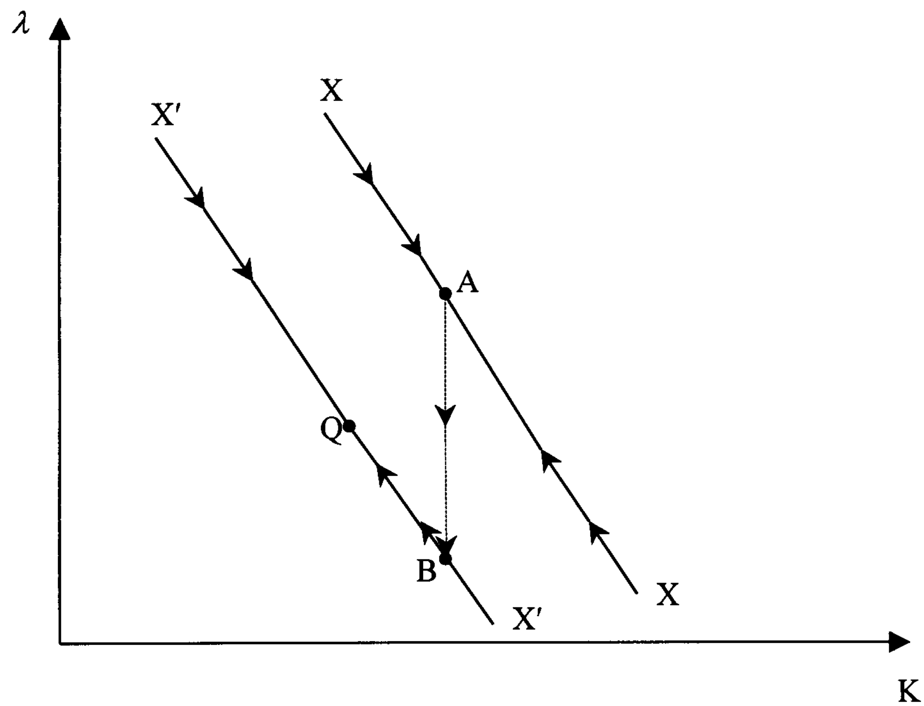

Figure 2 : **Unanticipated Permanent Increase in ε**

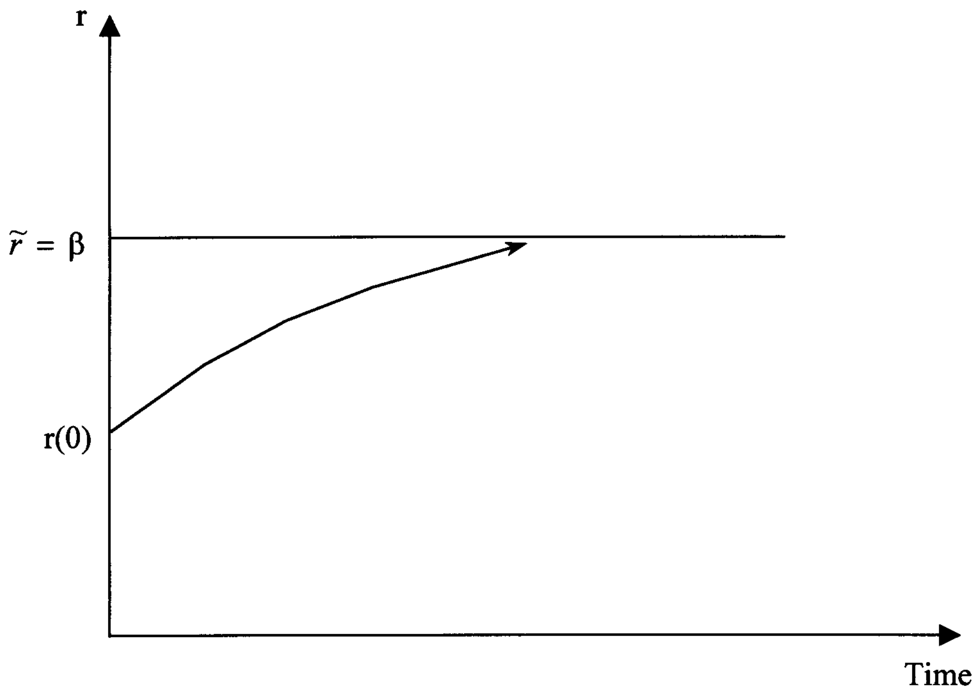

Figure 3: **Time Path of Short-term Interest Rate: Unanticipated Permanent Increase in ε**

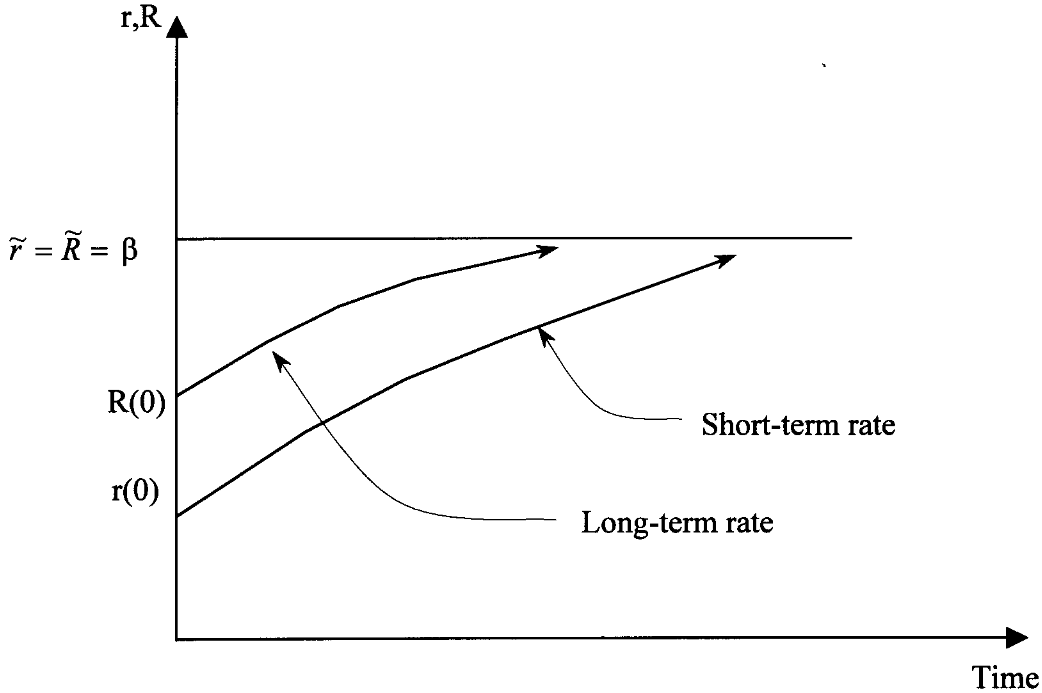

Figure 4 : **Time Path of Interest Rates: Unanticipated Permanent Increase in ε**