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Equilibrium Demand Elasticities across Quality Segments

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Abstract

Empirical studies find substantial differences in demand elasticities and associated markups among products of different quality. This paper analyzes the theoretical determinants of such variation. We present a simple model that allows for horizontal and vertical differentiation and accounts for endogenous entry. We find that most economic forces in our model, such as consumers' price sensitivity, the scope for product differentiation, and sunk costs of entry, are likely to induce lower equilibrium demand elasticities for higher quality products. In contrast, other economic forces, such as marginal cost of production and the distribution (across consumers) of the willingness to pay for quality, may induce the opposite pattern. These results provide an organizing framework through which empirical findings may be interpreted, and may also help to predict variation in demand elasticities for markets in which empirical estimates of elasticities are unavailable or infeasible to obtain.

KEYWORDS: demand elasticity, quality, vertical differentiation, horizontal differentiation

JEL classification: D5, L1

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1 Introduction

The importance that price elasticities of demand play in economics would be hard to overemphasize. The relationship among various economic variables in different fields of economics, as well as policymaking in these areas, is primarily driven by the magnitude of these elasticities. For example, price elasticities are a key determinant of the impact of trade restrictions on trade volumes, the extent of exchange-rate pass-through into export prices, the incidence of a tax increase or a merger, or the effect of wage differentials on labor demand.

Empirical studies estimating demand systems for oligopolistic industries (Bresnahan, 1987; Hausman, Leonard, and Zona, 1994; Berry, Levinsohn, and Pakes, 1995; Goldberg, 1995; Nevo, 2001; and Petrin, 2002) find substantial variation in estimated price elasticities of demand and associated markups across products. These studies attribute this variation to two key factors. First, consumers self-select into different quality segments within a market according to their price sensitivity. In particular, higher income consumers, who are less sensitive to price, are more likely to purchase high quality products. Therefore, price elasticities for high quality products are likely to be lower. Hausman, Leonard, and Zona (1994), for example, use this argument to explain why own-price elasticities for popular-priced beers are somewhat higher than those of premium beers. Second, different segments of the market may vary in market structure and density of products. In particular, products in more densely populated segments face more competition, and therefore higher own-price demand elasticities. Goldberg (1995), for example, finds low price elasticities for sports cars and luxury cars, higher for small (subcompact) cars, and highest for intermediate size cars. She argues that this pattern is driven by "the large number of models included in the intermediate and standard segments." Berry, Levinsohn, and Pakes (1995) find a similar pattern in the automobile market, and use a similar argument to explain it.

These two explanations, price sensitivity and density of products, are not independent of one another. If consumers of premium beers are less price sensitive, markups and profits of premium beer manufacturers will tend to be higher. Higher profits will induce entry and crowding of the product space in this segment, which will increase price elasticities. Similarly, highly dense segments in the automobile market might be such because of entry, which was induced by lower price sensitivity of consumers and higher markups. Therefore, an explanation of the observed differences in demand elasticities across quality segments based on one of these factors alone is not fully satisfactory. Furthermore, other forces might also influence demand elasticities, not only directly but also indirectly, through their effect on entry incentives. For example, high quality products tend to have more attributes along which they can be differentiated from other products. While more differentiation induces weaker competition and lower demand elasticities, the resulting higher markups provide incentives to enter, thus creating an opposing force. More generally, since demand elasticities are not a primitive but are endogenously determined in a market equilibrium, understanding how they vary across quality segments requires a framework that takes into account the relevant forces that determine the equilibrium, and in particular the endogenous determination of the number of competing products and the sorting of consumers. The goal of this paper is to provide such a framework.

We develop a stylized theoretical model of industry equilibrium that predicts variation in equilibrium demand elasticities across quality segments as a function of market primitives, such as price sensitivity of consumers, scope for product differentiation, and marginal costs of production. While including most of the relevant ingredients of markets with horizontal and vertical differentiation, the model is still sufficiently simple to deliver closed-form solutions for the equilibrium values of demand elasticities and markups. As such, it is appealing for organizing the various conceptual determinants of equilibrium demand elasticity differences across quality segments in a single framework.

Given the widespread relevance of demand elasticities in economics, the analysis of their theoretical determinants is not only relevant for understanding observed regularities, but also for predicting differences in elasticities across quality segments when empirical estimates of demand elasticities are not available, which is often the case. Furthermore, since quality varies systematically with characteristics of products, markets, or countries, theoretical results obtained from such analysis could also be powerful prediction tools even when direct information on quality is unavailable. For example, since rich countries tend to produce higher quality goods than poor countries, a negative relationship between quality and demand elasticities would imply a larger impact of a tariff on a country's exports if the country is poor rather than rich. This is so because the demand for the lower quality products of poor countries would be more sensitive to an increase in import price.

Our modeling framework of a two-dimensional product space, with both a vertical and a horizontal dimension, is similar to the frameworks used by Neven and Thisse (1990), Andersen, de Palma, and Thisse (1992, chapters 7.5 and 8.3.3), Economides (1993), and Brekke, Nuscheler, and Rune Straume (2006). All these papers, however, obtain equilibria with differentiation along only one (the horizontal) dimension. This property makes it difficult to address variation in equilibrium elasticities across vertical segments, which is the main focus of our paper.

Other papers study the provision of quality by multi-product firms and occasionally obtain results on the relationship between quality and markups. For example, the seminal work of Mussa and Rosen (1978) finds that the percentage markup charged by a multi-product monopolist decreases with quality, while Katz (1984), in a particular case with constant marginal costs, finds instead that markups increase with quality. More recent studies are specifically interested in understanding the theoretical determinants of the relationship between quality and markups. For example, Verboven (1999) proposes a duopoly model with imperfect information and asymmetric advertisement, and finds that percentage markups increase with quality. Canoy and Peitz (1997) develop a model that allows for limited entry (up to three firms), finding that absolute markups increase with quality.

In contrast to this multi-product quality-provision literature, we take the spotlight away from the interaction between vertically differentiated segments of a market, and instead place it on the role of entry, which we allow to respond freely to economic incentives and to be targeted to specific segments of the market where these incentives are present. While extremely stylized, our model is able to encompass the combined effect of most relevant forces that determine differences in equilibrium demand elasticities across quality segments of a product market. Its simplicity provides significant benefits in terms of tractability and transparency. The results are largely consistent with the common wisdom that higher quality goods have lower demand elasticities in equilibrium, but also point to cases where this might not be true. Economic forces such as higher sunk costs of entry and wider scope for product differentiation for higher quality goods, as well as lower price sensitivity of consumers who buy those goods, imply that high quality products are likely to face lower elasticities in equilibrium. In contrast, sufficiently higher marginal costs of production of high quality products and a large fraction of consumers willing to pay for such products may reverse this pattern.

The paper continues as follows. In Section 2 we describe the model. In Section 3 we characterize the equilibrium and derive most of the results discussed above. In Section 4, which is more technical in nature, we derive the conditions under which the equilibrium characterization of Section 3 holds. Section 5 concludes.

2 The model

Firms' decisions We consider a differentiated product market with both vertical and horizontal differentiation. The vertical component is characterized by the existence of two quality levels, each of which defines a "quality segment" of the market.[1] We denote the high quality level by δ_h and the low quality level by δ_l, with $\delta_h > \delta_l$. Within each segment, all products are of the same quality, but are horizontally differentiated. We use Salop's circular city framework (Salop, 1979) to model horizontal differentiation. The product space consists of two circles, one for high quality products and one for low quality products. We denote the circumferences of the circles for the two quality levels by ω_h and ω_l, respectively. The circumferences are allowed to be different across product categories. For example, high quality products may have more scope for product differentiation, in which case $\omega_h > \omega_l$.[2]

Within this framework, the supply side of the model is fairly standard. All firms in the model are single-product firms and are ex-ante symmetric. Firms compete in two stages. First, they decide whether to enter the high quality segment, the low quality segment, or not to enter the market. By entering the market, firms incur sunk costs of F_h and F_l, depending on which quality segment they enter. Once entry decisions have taken place, all firms within a segment are symmetrically (i.e. equidistantly) located along the circle, and compete in prices. Marginal costs of production may vary by quality segment, and are denoted by c_h and c_l.

Consumers' demand Consumers are of two types, high income and low income. High income consumers are assumed to have a lower sensitivity to price than low income consumers, $v_h < v_l$.[3] There is an exogenous number of consumers of each income level, S_h and S_l, who are free to choose from which quality segment to buy. The utility of consumer i with income level $z \in \{h, l\}$ from purchasing product j of quality segment $q \in \{h, l\}$ is given by

$$u_{ij} = \delta_q - v_z p_j - \left| r_j - r^*_{i,q} \right| \tag{1}$$

where p_j is the price of product j, and $|r_j - r^*_{i,q}|$ is the distance between the location of product j, r_j, and the location of consumer i's ideal variety in quality circle q, $r^*_{i,q}$.

The consideration of a product market with two segments (circles), as opposed to the standard circular city model, introduces the need to specify substitution patterns between products of different quality. In general, these patterns might be very rich but cumbersome to handle (see, for example, Ansari, Economides, and Steckel (1998) and the references therein). Therefore, we make assumptions that simplify substitution between products in different segments. First, we assume that consumers, as they enter the market, know the values of the quality levels, δ_h and δ_l, the number of equidistantly located firms in each circle, and the prices that they charge, but they do not know ex-ante the location of their ideal variety. They need to spend market research costs, R, to learn this location, drawn from a uniform distribution along the circle. Moreover, since products in different segments are differentiated along different attributes, consumers only learn their location in one segment. To learn their location in the other segment, they have

[1] As will become clear, this is essentially a vertical model, in which non-adjacent quality segments do not directly compete with each other. Therefore, it should be easy to extend the analysis to $N > 2$ quality segments. All the results of Section 3 would remain unchanged. The equilibrium conditions of Section 4 would be more complicated, as we would need to consider two (rather than one) adjacent quality segments.

[2] As an example, consider simple (low quality) t-shirts, which can only be different in their colors, vis-a-vis higher quality shirts, which can also be different in the shape of their buttons.

[3] Hummels and Lugovskyy (2005) derive a lower price sensitivity of high income consumers from their higher sensitivity to distance (to the ideal variety).

to incur an additional cost of R. For simplicity, we assume that consumer i's locations in both segments are independent of one another, so that learning i's location in, for example, the low quality segment is not informative about i's location in the high quality segment.[4] These assumptions drastically simplify substitution patterns. Conditional on R not being too low (so that it is not spent twice) or too high (so that it is spent at least once), a (risk neutral) consumer faces a nested decision. First, she chooses which quality segment to consider by taking the expected utility over her possible realized locations. Then, she learns her location and makes a product choice within the segment. Thus, consumers in our model have an indirect taste for variety, as higher variety increases the expected value of their closest match. In this sense, one can think of this approach as a discrete-choice version of a Dixit-Stiglitz utility function.

This simplified preference structure is tractable. In particular, the existence of a market research cost solves a fundamental problem: with two (or more) quality circles and endogenous entry, there is no obvious idea of "symmetry" that can be applied to the location of firms in such a product space.[5] This problem is solved once consumers are required to pay R to learn the location – independent across circles – of their ideal variety. Firms in one quality segment then compete "symmetrically" with all firms located in the other segment. The basic intuition and results should carry through to other cases, as long as substitution across segments is limited. In particular, the case we consider here can be thought of as a limiting case of a decreasing correlation between consumer i's location in the two quality segments. Our results will extend (with tighter restrictions on the parameters) as long as the correlation between these two locations is sufficiently small.[6]

3 Symmetric separating equilibrium

We focus on a symmetric equilibrium in which all entrants in a quality segment locate equidistantly from each other and charge the same price. In addition, we search for an equilibrium that is separating, such that high income consumers buy the high quality good and low income consumers buy the low quality good. Finally, we assume that all consumers are served in equilibrium. In the next section we derive parameter restrictions that make all these assumptions hold, and verify that such parameters exist.

Consider the profit maximization problem of firm j, which produces a product of quality q. Conditional on the rest of the firms in quality segment q charging (symmetric) price p_q and serving (in a separating equilibrium) only consumers of the corresponding income level, the residual demand for firm j is locally (i.e. for $p_{q,j}$ sufficiently close to p_q) given by

$$D_q(p_{q,j}) = \frac{S_q}{n_q} + v_q(p_q - p_{q,j})\frac{S_q}{\omega_q} \tag{2}$$

[4] While the assumption of independence is, of course, a simplification, one can imagine many markets in which it is a reasonable approximation. For example, many consumer products are differentiated by their color and size at the low-end of the product space, while at the high-end color and size can be customized, and differentiation takes place over various other features. Similar examples can arise in the service sector. Consider demand for accounting services. At the low-end, e.g. for tax preparation, location may be a primary factor. At the high-end, e.g. to prepare for an IPO, location is not a relevant factor.

[5] Note that firms in one quality segment would not only have direct competitors (i.e. adjacent firms) in their own segment, but they would also have direct competitors in the other segment. Generally, this will make the overall configuration of firms' locations asymmetric.

[6] Going back to the accounting services example, location could also be an attribute that is taken into account at the high-end. However, as long as the choice in the high quality segment is mostly driven by preferences over other attributes, our qualitative results should hold.

where $p_{q,j}$ is the price charged by firm j and n_q is the number of firms in quality segment q. This local demand function is the relevant function for the analysis of this section, which focuses on the intensive pricing margin. In the next section we consider pricing deviations at the extensive margin, for which we need to consider the demand function for prices further away from the equilibrium price p_q. For such deviations, the demand is a discontinuous step function; charging sufficiently low prices, a firm can steal all customers from either adjacent firms or from firms producing in the other quality segment, or both. This part of the analysis is deferred to the next section, which also derives parameter restrictions that make the local optimum found in this section a global optimum.

Profits for firm j in quality segment q are (locally) given by

$$\pi_q(p_{q,j}) = (p_{q,j} - c_q)D_q(p_{q,j}) - F_q \tag{3}$$

and the first order condition is

$$0 = \frac{d\pi_q}{dp_{q,j}} = D_q(p_{q,j}) + (p_{q,j} - c_q)\frac{d\left(D_q(p_{q,j})\right)}{dp_{q,j}} = \left[\frac{1}{n_q} - \frac{v_q}{\omega_q}(p_{q,j} - p_q)\right] - (p_{q,j} - c_q)\frac{v_q}{\omega_q} \tag{4}$$

In a symmetric equilibrium, $p_{q,j} = p_q$. Solving for the equilibrium price, we obtain

$$p_q = c_q + \frac{\omega_q}{n_q v_q} \tag{5}$$

so $\frac{\omega_q}{n_q v_q}$ is the absolute markup in equilibrium.[7] Own-price equilibrium demand elasticity is given by

$$\varepsilon_q(p_q) = -\frac{dD_q(p_q)}{dp_q}\frac{p_q}{D(p_q)} = 1 + c_q\frac{v_q n_q}{\omega_q} \tag{6}$$

Common wisdom about the effects of v and ω on equilibrium prices and demand elasticities is supported by equations (5) and (6). Lower price sensitivity of consumers who buy in the high quality segment ($v_h < v_l$) induces a higher markup and a lower demand elasticity in that segment, as does a larger scope for product differentiation ($\omega_h > \omega_l$). In contrast, higher quality products tend to have higher marginal costs ($c_h > c_l$), which increases the demand elasticity for products in this segment. The last effect is driven by the fact that markups in this example (as well as in other examples described below) are determined by competition, and are independent of the level of marginal costs. Thus, since elasticity is a percentage measure and prices increase linearly with marginal costs, the elasticity also increases.[8]

Even with available information on the actual values of v, ω, and c, we would not be able to predict differences in demand elasticities across quality segments without information on the number of entrants. This can be clearly seen in equation (6), where the pattern of demand elasticities can be reversed by conditioning on different values of n. However, we expect long-run forces to limit the scope of relevant variation in the number of entrants, even when this number might not correspond to a long-run equilibrium at a particular point in time. We focus here on equilibrium elasticities (across quality segments) in a long-run equilibrium. Therefore, the comparison of our theoretical predictions with empirical estimates of demand elasticities – which typically take the observed number of entrants as given – is appropriate to the extent that the number of entrants is sufficiently close to long-run equilibrium values.[9]

[7] Note that q, instead of z, is used in v_q to denote income class. As q also indexes quality segments, this is a slight abuse of notation. However, no confusion should arise since, in the separating equilibrium we consider, there is perfect sorting of consumers so that income and product class perfectly match.

[8] For the same reason, semi-elasticities and absolute (rather than percentage) markups are not affected by the level of prices, and are therefore invariant to marginal costs.

[9] Both in this model and in empirical studies, demand elasticities are calculated or estimated under the assumption that firms take the number of entrants as given. A referee suggested that we call these "short-run elasticities." Following this suggestion, they might be thought of as short-run elasticities evaluated at a long-run equilibrium.

In a long-run equilibrium with free entry, profits are zero. Thus, for firms in quality segment q:

$$\pi_q(p_q) = (p_q - c_q)D_q(p_q) - F_q = 0 \Rightarrow \frac{\omega_q}{v_q n_q}\frac{S_q}{n_q} - F_q = 0 \tag{7}$$

Solving for n_q, we obtain

$$n_q = \sqrt{\frac{\omega_q S_q}{v_q F_q}} \tag{8}$$

Substituting this equation into equation (6), we obtain the long-run equilibrium demand elasticities for each quality segment

$$\varepsilon_q(p_q) = 1 + c_q\sqrt{\frac{v_q S_q}{\omega_q F_q}} \tag{9}$$

Comparing equations (6) and (9), we can see that entry attenuates, but does not reverse, the effect that differences in v and ω impinge on equilibrium elasticities. The equilibrium elasticity still increases with price sensitivity and decreases with the width of the product space. It also increases with marginal costs. Equation (9) shows that, in a long-run equilibrium, demand elasticities also depend on the magnitude of sunk costs of entry and on market size, which thus become additional factors to take into account for the prediction of demand elasticity differences across quality segments. Sunk costs of entry (F) have a negative effect on equilibrium elasticities, as higher costs imply less entry and a more sparsely populated product space. Segment size (S) has a positive effect on equilibrium demand elasticities, as larger segments attract more firms.

These results, although derived from a very stylized model, constitute an organizing framework that allows us to predict systematic differences in equilibrium demand elasticities across quality segments. Consistent with what the empirical literature tends to find in many markets, most of the factors that affect demand elasticities are likely to induce lower demand elasticities and higher percentage markups for higher quality segments. First, we typically expect consumers who buy high quality to be less sensitive to price ($v_h < v_l$), which is true in the separating equilibrium of this model. Second, we expect the scope for product differentiation to be greater for high quality products ($\omega_h > \omega_l$), as they typically possess more attributes along which they can differentiate themselves from competing products. Third, we also expect higher sunk costs for higher quality products ($F_h > F_l$), as they often require higher R&D and advertising expenditures.

This prediction, however, need not always be true. A first offsetting force arises from the fact that higher quality goods tend to have higher marginal costs ($c_h > c_l$). Considering the opposite effects of marginal costs and sunk (or fixed) costs, the relationship between the two is important for predicting variation in elasticities across quality segments. In industries where high quality products require significantly larger sunk cost outlays relative to variable cost outlays, as might be the case for software or pharmaceuticals, we are more likely to find lower demand elasticities at the higher end of the quality spectrum. In contrast, in markets where the high quality nature of a product is primarily driven by the use of better intermediate materials in production or from other expenditure on variables cost as, for example, in the furniture industry, the prediction that high quality goods face lower demand elasticities and charge higher percentage markups might be overturned.[10] A second opposing force may arise from the segment size. Even though we typically associate a smaller segment size with higher quality, in many markets low quality is a small niche. In those cases, demand elasticities may be lower in those lower quality segments. This is consistent

[10] This is not the case for absolute markups, as marginal costs do not affect demand semi-elasticities.

with empirical findings for the automobile industry, where intermediate quality cars constitute the largest segment of the market, and have higher demand elasticities than low quality ones (Goldberg, 1995).

Because a separating equilibrium is not always sustainable, the next section characterizes parameter restrictions that must be imposed for such an equilibrium to exist. Conditional on those parameter restrictions, the optimal behavior of firms does not depend on the existence of potential consumers who buy from a different quality segment. Thus, the results of this section are equivalent to the results that would obtain if we assumed that quality segments were "isolated," an alternative environment in which low (high) income consumers would be "forced" to consume low (high) quality goods. This equivalence reflects our strong assumption on substitution patterns. As discussed earlier, however, similar results would be obtained if we allowed for limited substitution across quality segments.

Table 1 below provides results for two additional examples of alternative demand systems, comparing them to the circular city model presented above. For these alternative cases, we do not characterize the conditions for a separating equilibrium, as they cannot be expressed in closed form. However, similar qualitative parameter restrictions to those derived for our baseline case in the next section would be required. The first example is of a circular city with quadratic transportation costs, while the second example is of a logit demand model. For the logit model, we assume that all products in quality segment q have mean utility of δ_q, so the utility of consumer i with income level z from purchasing product j is given by $u_{ij} = \delta_q - v_z p_j + \varepsilon_{ij}$, where ε_{ij} is an i.i.d draw from an extreme value distribution with dispersion parameter ω_q (which is analogous to the same parameter in the circular city model). In these alternative models, the same qualitative results are obtained. It is interesting to note, however, that the impact of (endogenous) entry has considerable variation across models. In particular, since the product space is never exhausted in the logit model, entry increases at a faster rate with segment size, but has a smaller impact on the equilibrium elasticity. As emphasized and discussed by Andersen, de Palma, and Thisse (1992, chapter 6.4), these differences are also driven by the localized competition nature of the circular city model compared to the non-localized competition nature of the logit model.

Table 1: Key expressions from three models of product differentiation (subscript q is omitted in all cases)

	Circular (linear)	Circular (quadratic)	Logit
$D(p_j, p_{-j} = p)$	$\frac{1}{n} - \frac{v}{\omega}(p_j - p)$	$\frac{1}{n} - \frac{nv}{\omega^2}(p_j - p)$	$\frac{\exp(\frac{1}{\omega}\delta - \frac{v}{\omega}p_j)}{\exp(\frac{1}{\omega}\delta - \frac{v}{\omega}p_j) + (n-1)\exp(\frac{1}{\omega}\delta - \frac{v}{\omega}p)}$
n	$\sqrt{\frac{\omega S}{vF}}$	$\left(\frac{\omega^2 S}{vF}\right)^{\frac{1}{3}}$	$1 + \frac{\omega S}{vF}$
$\lvert\varepsilon(p)\rvert = -S\frac{dD(p)}{dp}\frac{p}{SD(p)}$	$1 + c\frac{vn}{\omega} = 1 + c\sqrt{\frac{vS}{\omega F}}$	$1 + c\frac{vn^2}{\omega^2} = 1 + c\left(\frac{\sqrt{v}S}{\omega F}\right)^{\frac{2}{3}}$	$1 + c\frac{v(n-1)}{\omega n} = 1 + c\left(\frac{F}{S} + \frac{\omega}{v}\right)^{-1}$
$\lvert s\varepsilon(p)\rvert = -S\frac{dD(p)}{dp}\frac{1}{SD(p)}$	$\frac{vn}{\omega} = \sqrt{\frac{vS}{\omega F}}$	$\frac{vn^2}{\omega^2} = \left(\frac{\sqrt{v}S}{\omega F}\right)^{\frac{2}{3}}$	$\frac{v(n-1)}{\omega n} = \left(\frac{F}{S} + \frac{\omega}{v}\right)^{-1}$

4 Conditions for equilibrium

In this section we characterize the conditions required to guarantee the existence of a symmetric separating equilibrium and the parameter restrictions that these conditions imply. In the end of the section we discuss these restrictions and their implications for the results.

Consumers' constraints

There are four types of consumer constraints that need to be satisfied in equilibrium: ex-ante and interim participation constraints, and ex-ante and interim incentive constraints. The participation constraints guarantee that every consumer buys a product (satisfying our assumption that the market is fully covered). The incentive constraints guarantee that the equilibrium is separating. Each constraint has to hold for both types of consumers. The ex-ante participation constraint requires that consumers prefer to invest the market research cost R to not investing them. The ex-ante incentive constraint requires that consumers prefer to invest the market research costs in the "correct" quality segment. Once initial market research costs have been invested, consumers learn the realization of their ideal variety. At this point, they also need to satisfy the two interim constraints. The interim participation constraint requires that, given their ideal variety, consumers prefer to purchase from the nearest firm to not purchasing at all. The interim incentive constraint requires that they are better off purchasing from the nearest firm than reinvesting market research cost in the other quality segment.

Ex-ante constraints Let $Eu^*_{q|z}$ denote the equilibrium expected utility of a consumer of income class z from entering quality segment q. Normalizing the outside option to zero, the participation constraint requires that[11]

$$Eu^*_{q|q} = \delta_q - v_q p_q - E[t_q] - R \geq 0 \quad \forall q \in \{l, h\} \tag{10}$$

where $E[t_q]$ is the expected transportation cost in segment q before knowing the location of the ideal variety (relative to the location of the firms). Since consumers' locations are drawn from a uniform distribution and firms are equidistantly located, expected transportation costs are $E[t_q] = \frac{\omega_q}{4n_q}$.

Similarly, the incentive constraints require that

$$Eu^*_{h|h} = \delta_h - v_h p_h - E[t_h] - R \geq \delta_l - v_h p_l - E[t_l] - R = Eu^*_{l|h} \tag{11}$$

and

$$Eu^*_{l|l} = \delta_l - v_l p_l - E[t_l] - R \geq \delta_h - v_l p_h - E[t_h] - R = Eu^*_{h|l} \tag{12}$$

Interim constraints It is sufficient to restrict attention to interim constraints for consumers with the worst location realization, namely those consumers whose ideal variety is just in the middle point between two adjacent firms. For such consumers in segment q, transportation costs are $\frac{\omega_q}{2n_q}$. Thus, the interim participation constraints are given by

$$\delta_q - v_q p_q - \frac{\omega_q}{2n_q} \geq 0 \quad \forall q \in \{l, h\} \tag{13}$$

Interim incentive constraints require that consumers do not prefer to reinvest market research cost R in the other quality segment. This implies that

$$\delta_h - v_h p_h - \frac{\omega_h}{2n_h} \geq \delta_l - v_h p_l - E[t_l] - R = Eu^*_{l|h} \tag{14}$$

and

$$\delta_l - v_l p_l - \frac{\omega_l}{2n_l} \geq \delta_h - v_l p_h - E[t_h] - R = Eu^*_{h|l} \tag{15}$$

[11] Again, we abuse notation by also using q to index income class.

Summary of consumers' constraints The above eight constraints do not impose independent restrictions, as the two participation constraints for high income consumers are redundant. In particular, it is easy to check that the ex-ante (interim) participation constraint for high income consumers is satisfied if the ex-ante (interim) incentive constraint is satisfied for these consumers and the ex-ante (interim) participation constraint is satisfied for low income consumers. The remaining six constraints can be collapsed into the following three inequalities:

$$\delta_l \geq v_l p_l + \frac{\omega_l}{4n_l} + \max\left(R, \frac{\omega_l}{4n_l}\right) \tag{16}$$

$$\delta_h - \delta_l \geq v_h(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \max\left(0, \frac{\omega_h}{4n_h} - R\right) \tag{17}$$

$$\delta_h - \delta_l \leq v_l(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \min\left(0, R - \frac{\omega_l}{4n_l}\right) \tag{18}$$

To summarize, given R, quality levels should be sufficiently high to justify the spending of market research and transportation cost. In addition, the difference in quality levels has to be large enough to attract high income consumers, but not too large so that low income consumers remain attracted to low quality products. The characterization above may seem incomplete, as some of the parameters – prices and the number of firms – are not primitives. One should note, however, that prices and the number of firms are given as a function of primitives in equations (5) and (8). Since each of these equations have a separate free parameter – marginal cost in the pricing equation, fixed cost and market size in the entry equation – the existence of a set of primitives that satisfy the consumers' constraints is still guaranteed. We return to this in the end of this section.

Firms' constraints

In Section 3 we solved for equilibrium prices assuming that the first order conditions of the profit maximization problem were sufficient. The profit function in this model, however, is not continuous, requiring us to make additional parametric restrictions to guarantee the sufficiency of the first order conditions. In particular, the discontinuities of the profit function are for prices below the equilibrium price. Therefore, we need to guard against profitable deviations downwards.

There are two sources of discontinuity. First, as is well known for linear/circular city models with linear transportation costs, once firm j sets its price sufficiently low to attract consumers located at the exact location of some other firm, firm k, all the rest of firm k's customers will discontinuously prefer to buy from firm j. The second source of discontinuity in the profit function is the possibility of "ex-ante" business stealing across quality levels. This source of discontinuity is less standard, and is driven by our assumption of discrete consumer types. When consumers decide in which quality circle to invest R, they only take into account the average price (or price index) in that circle, as they do not yet know the location of their ideal variety. If firm j drops its price, the average price in the segment only changes in the order of $1/n$, creating a public good problem that drastically reduces the firm's incentives to attempt such a deviation. If firm j is the only one to decrease its price, it needs to drop it substantially to create an impact, while all other firms in the segment enjoy the effect of this action almost just as much. Still, it is possible for firm j to set its price sufficiently low to induce quality switching. Since all consumers in the other quality segment have the same willingness to pay for quality, the price that induces quality switching for one consumer also induces quality switching for all consumers of the same type. However, firm j only gains some fraction of them, the size of which depends on how low it sets its price. The lower it is, the more of those consumers will prefer to buy from firm j.

Consider first the incentive for firm j to steal all business from an adjacent firm. To do so, firm j would have to compensate consumers located exactly at the neighbor's location by at least their transportation cost. Those consumers would then buy from firm j if and only if $v_q p_j + \frac{\omega_q}{n_q} \leq v_q p_q$, i.e. $p_j \leq p_q - \frac{\omega_q}{v_q n_q}$. But, by equation (5), this price cut is at least as large as the equilibrium markup, implying that such a deviation cannot generate positive profits, and therefore cannot be profitable. As stealing customers from other firms, which are further away, requires an even larger price cut, such deviations are not profitable either.

Thus, we only need to worry about profitable deviations that induce quality switching.[12] There are two cases to consider. In the first case, consumers who are induced to switch quality segments buy from all firms in the segment, although disproportionately from firm j, which charges a lower price. In the second case, the drop in the price of firm j is sufficiently low to prevent switching consumers from buying from adjacent firms. We consider each case in turn.

Consider the first case, and suppose that firm j produces high quality. Consider the case in which firm j's price is p_j while prices of all other high quality firms are p_h. After some algebra, we can write the expected utility for low income consumers who contemplate switching to the high quality segment as

$$\begin{aligned} Eu_{h|l} &= \delta_h - v_l E[p_h] - E[t_h] - R = \delta_h - v_l p_h - \frac{\omega_h}{4n_h} - R + \frac{v_l}{n_h}(p_h - p_j) + \frac{v_l^2}{2\omega_h}(p_h - p_j)^2 = \\ &= Eu^*_{h|l} + \frac{v_l}{n_h}(p_h - p_j) + \frac{v_l^2}{2\omega_h}(p_h - p_j)^2 \qquad (19) \end{aligned}$$

Intuitively, the high quality segment is now more attractive, as one of its firms charges a lower price. The first additional element is the direct price effect: with no substitution, with probability $1/n_h$ the price will be lower by $(p_h - p_j)$. The second element captures the substitution effect, as some of the consumers who are closer to neighboring firms will now buy from firm j.

In addition to ex-ante business stealing, firm j could also steal "interim" consumers from the other quality segment. In this case, the price is not sufficiently low to induce ex-ante quality switching, but it is sufficiently low to induce quality switching by consumers with bad location realizations.[13] The expected utility of entering the high quality segment for a low income interim consumer considering switching is also described by equation (19).

We can now derive a sufficient condition that rules out this kind of deviations. All we require is that even setting price equal to marginal cost, $p_j = c_h$, will not induce either ex-ante or interim quality switching by low income consumers.[14] Comparing the expected utility of switching quality (see equation (19)) with

[12] To the extent that, at least in the short-run, consumers may be less informed about product-specific prices in the "other" quality segment, inducing quality switching would be even more difficult. In such cases, the firm incentive constraints would be easier to satisfy, and supporting the equilibrium would be easier.

[13] Business stealing of "interim" consumers does not imply a discontinuity, but instead a kink in the profit function.

[14] We do not present necessary conditions for two reasons. First, they are algebraically much more complicated but offer no additional insights. Second, a complete set of closed form necessary conditions cannot be derived when research costs are below a minimum bound. If R is small enough, the interim incentive constraints may fail when a firm in the other quality level lowers its price. The ex-ante expected utility of the consumer's quality level is then affected in several ways. First, consumers know that if they get a bad draw, they may pay R again and switch quality levels. Therefore, the expected utility of entering the low quality segment also depends on the expected utility of buying high quality and on the probability of switching to that segment, both of which depend on the magnitude of the price deviation. In addition, the failure of the interim incentive constraint lowers the expected transportation costs conditional on getting a good draw. The interaction of all these effects makes the expected utility highly nonlinear in the price of the deviating firm, impeding a complete closed-form characterization of the necessary conditions. If R is large enough, however, the interim incentive constraints hold for all deviations of a firm in the other quality level as long as the ex-ante constraint also holds. In that case, it is possible to derive

the utility that these consumers obtain in the low quality segment, we can obtain the required constraint:

$$\delta_h - \delta_l \leq v_l(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \min\left(0, R - \frac{\omega_l}{4n_l}\right) - \frac{v_l}{n_h}(p_h - c_h) - \frac{v_l^2}{2\omega_h}(p_h - c_h)^2 \tag{20}$$

where equation (20) combines the constraints for both the ex-ante and the interim consumers. Since $p_h - c_h = \frac{\omega_h}{n_h v_h}$ in equilibrium, this inequality can be simplified to

$$\delta_h - \delta_l \leq v_l(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \min\left(0, R - \frac{\omega_l}{4n_l}\right) - \frac{v_l \omega_h}{n_h^2 v_h}\left(1 + \frac{v_l}{2v_h}\right) \tag{21}$$

An analogous analysis can be applied for a potential deviation of a low quality firm. In this case, the sufficient condition is

$$\delta_h - \delta_l \geq v_h(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \max\left(0, \frac{\omega_h}{4n_h} - R\right) + \frac{v_h \omega_l}{n_l^2 v_l}\left(1 + \frac{v_h}{2v_l}\right) \tag{22}$$

Consider now the second case. This is the case in which high quality firm j sets its price p_j sufficiently low, not only for inducing low income consumers to switch quality, but also for preventing some of the other firms in the high quality segment from selling to any low income consumers who switch. This case only applies for high quality firms.[15] Let $2k$ be the number of high quality firms who do not sell to low income consumers.[16] This is true when $p_j \in [p_h - (k+1)\frac{\omega_h}{n_h v_l}, p_h - k\frac{\omega_h}{n_h v_l}]$. Note also that k is bounded from above by $\frac{v_l}{v_h}$.[17] After some algebra, we can write the expected utility for low income consumers who contemplate switching to the high quality segment as

$$\begin{aligned} Eu_{h|l} &= \delta_h - v_l p_h - \frac{\omega_h}{4n_h} - R - \frac{\omega_h(k^2+k)}{2n_h^2} + \frac{v_l^2(p_h - p_j)^2}{2\omega_h} + \frac{v_l(p_h - p_j)(k+1)}{n_h} = \\ &= Eu^*_{h|l} - \frac{\omega_h(k^2+k)}{2n_h^2} + \frac{v_l^2(p_h - p_j)^2}{2\omega_h} + \frac{v_l(p_h - p_j)(k+1)}{n_h} \end{aligned} \tag{23}$$

It is straightforward to check that this expression reduces to equation (19) when $k = 0$ and that it is decreasing in p_j. Taking into account that k is bounded from above by $\frac{v_l}{v_h}$, and since $p_h - p_j$ cannot be higher than the equilibrium markup, $\frac{\omega_h}{n_h v_h}$, while still generating positive profits, we can simplify equation (23) to

$$Eu_{h|l} \leq Eu^*_{h|l} + \frac{v_l^2 \omega_h}{2v_h^2 n_h^2} + \frac{v_l \omega_h(\frac{v_l}{v_h} + 1)}{v_h n_h^2} = Eu^*_{h|l} + \frac{\omega_h}{n_h^2}\left(\frac{3v_l^2}{2v_h^2} + \frac{v_l}{v_h}\right) \tag{24}$$

Therefore, for such a price deviation not to attract low income consumers to switch (from both an ex-ante and an interim perspective), it is sufficient to require that

$$\delta_h - \delta_l \leq v_l(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \min\left(0, R - \frac{\omega_l}{4n_l}\right) - \frac{\omega_h}{n_h^2}\left(\frac{3}{2}\left(\frac{v_l}{v_h}\right)^2 + \frac{v_l}{v_h}\right) \tag{25}$$

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a set of conditions that guarantees the existence of an equilibrium. These conditions are qualitatively similar to equations (27) and (28), but they are much more contrived, without providing further insights.

[15] As shown earlier, stealing all low income consumers from adjacent firms is not profitable for a low quality firm, so stealing high income consumers from these firms would be even more costly (as high income consumers are less responsive to prices).

[16] This number is even since everything is symmetric on both sides of firm j. The number would be odd only if n_h is even and firm j is the only firm that sells to low income consumers. For simplicity, we ignore this case. All the conditions we derive would be sufficient for this case by setting $2k = n_h$.

[17] To see this, note that a consumer who is located exactly k firms away would be indifferent between firms j and k when $v_l p_h = v_l p_j + k\frac{\omega_h}{n_h}$. Since a profitable deviation for firm j can cut prices at most to marginal cost, c_h, this implies that $k < \frac{v_l(p_h - c_h)}{\omega_h / n_h} = \frac{v_l}{v_h}$.

It is easy to see that this condition is stricter than the condition derived in equation (21), which then becomes redundant. Therefore, sufficient conditions to prevent deviations by firms are given by equations (22) and (25), which bound the difference in quality levels from below and from above, respectively.

Summary and discussion

The firms' constraints (22) and (25) are very similar to the consumers' incentive constraints (17) and (18). This should not be surprising, as it is ultimately restrictions on consumers' willingness to switch quality that discourages firms from deviating. In fact, the firms' constraints imply the consumers' incentive constraints, rendering the latter redundant. We can therefore combine all relevant constraints (for both consumers and firms) to obtain a set of sufficient conditions for a separating equilibrium:

$$\delta_l \geq v_l p_l + \frac{\omega_l}{4n_l} + \max\left(R, \frac{\omega_l}{4n_l}\right) \tag{26}$$

$$\delta_h - \delta_l \geq v_h(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \max\left(0, \frac{\omega_h}{4n_h} - R\right) + \frac{v_h \omega_l}{n_l^2 v_l}\left(1 + \frac{v_h}{2v_l}\right) \tag{27}$$

$$\delta_h - \delta_l \leq v_l(p_h - p_l) + \left(\frac{\omega_h}{4n_h} - \frac{\omega_l}{4n_l}\right) + \min\left(0, R - \frac{\omega_l}{4n_l}\right) - \frac{\omega_h}{n_h^2}\left(\frac{3}{2}\left(\frac{v_l}{v_h}\right)^2 + \frac{v_l}{v_h}\right) \tag{28}$$

These three inequalities define the constraints on the parameters of the model that are sufficient to guarantee that the solution we derived in Section 3 is indeed an equilibrium. The constraints impose that quality levels are sufficiently high, while the difference in quality levels is not too low or not too high. For given parameters, it might be the case that the right hand side of equation (27) is larger than the right hand side of equation (28), in which case both inequalities cannot be simultaneously satisfied. The following proposition shows that the constraints can simultaneously hold for sufficiently large difference in marginal costs.

Proposition 1 *Let $\Delta c = c_h - c_l$. There exists a threshold value Δc^* such that, for all $\Delta c > \Delta c^*$, there exists a range of $\delta_h - \delta_l > 0$ that satisfies the constraints and a separating equilibrium can be sustained. If $\Delta c < \Delta c^*$, a separating equilibrium may not exist. Moreover, the range of $\delta_h - \delta_l > 0$ that satisfies the constraints is increasing in Δc.*

The proof is in the appendix. The intuition for this result is the following. First, equilibrium prices have to be sufficiently different in equilibrium, so that low income consumers cannot afford the prices that high income consumers pay for the high quality good. Conditional on other parameters of the model – in particular those that determine markups – the difference in equilibrium prices depends on the difference in marginal costs, which thus has to be sufficiently large. Second, the difference in quality levels also has to be sufficiently large, so that high income consumers find it worth paying a higher price to acquire the high quality good. The larger is the marginal cost difference, the larger is the difference in quality levels that can be supported in equilibrium, as the lure of a significantly higher quality is weighted by low income consumers against the cost of paying a significantly higher price.[18]

[18] It is interesting to briefly consider how the model would change if consumers' sensitivity to price were continuously distributed instead of concentrated on only two points. Let us focus on the case in which R is sufficiently high that no consumer pays this cost twice. In that case, a separating equilibrium is characterized by a threshold value v^*, such that all consumers with $v < v^*$ choose high quality and all consumers with $v \geq v^*$ choose low quality. Conditional on the existence of a separating equilibrium and on the implied market size for each segment, equilibrium demand elasticities would be determined by the same forces described in our simpler case. In the more general case, however, the distribution of v would play a key role

This result indicates that the existence of a separating equilibrium depends on the relationship between quality differences and marginal cost differences (and other parameters of the model such as differences in sunk costs). An underlying technology (not modeled in this paper) is likely to determine the relationship between quality and costs of production. It is natural to conjecture that, given this technology, firms would have incentives to occupy different segments of the quality spectrum when the available technology is consistent with the conditions required for a separating equilibrium.

5 Conclusions

The paper presents a stylized model that allows for vertical and horizontal differentiation and accounts for endogenous entry. The results shed light on the determinants of equilibrium demand elasticities across quality segments of a market. We show that most economic forces would typically induce lower demand elasticities for higher quality products, a pattern that is consistent with the findings of most of the empirical literature on demand system estimation. This prediction may be reversed, however, if marginal costs of production of high quality products are significantly higher, or if the low quality segment of a market attracts only a small fraction of market participants.

The model is highly stylized, as our main goal is to provide a simple framework encompassing several economic forces that simultaneously interact to determine equilibrium demand elasticities. The simplicity pays off in terms of the transparency of the results, which are obtained in closed-form. These results may provide an organizing framework to interpret empirical findings in the literature regarding the variation in estimated demand elasticities across quality levels. They may also be used to predict this variation based on limited information about product or market characteristics.

References

[1] Anderson, Simon P., Andre de Palma, and Jacques-Francois Thisse (1992), *Discrete Models of Product Differentiation*, Cambridge, MA: MIT Press.

[2] Ansari, Asim, Nicholas Economides, and Joel Steckel (1998),"The Max-Min-Min Principle of Product Differentiation," *Journal of Regional Science*, 38, 207-230.

[3] Berry, Steven, James Levinsohn, and Ariel Pakes (1995), "Automobile Prices in Market Equilibrium," *Econometrica*, 60(4), 889–917.

[4] Brekke, Kurt R., Robert Nuscheler, and Odd Rune Straume (2006), "Quality and Location Choices Under Price Regulation," *Journal of Economics and Management Strategy*, 15(1), 207-227.

[5] Bresnahan, Tim (1987), "Competition and Collusion in the American Automobile Market: The 1955 Price War," *Journal of Industrial Economics*, 35(4), 457-482.

in determining the size of each segment. With a continuum of price sensitivities, there is always a $\widetilde{v}$ such that the incentive constraints hold with equality. Consumers with that $\widetilde{v}$ are indifferent between buying high or low quality and can be easily induced to switch quality segments. Firms then take into account this source of new customers when setting prices. In a separating equilibrium, the marginal profits from new customers that can be attracted with a lower price has to be offset by the losses on infra-marginal customers. This implies that the value of v that satisfies this requirement, v^*, will be largely determined by the distribution of v. While our simple case, with exogenously given S_h and S_l, may closely characterize a case with a bimodal distribution of v, it does not characterize the more general relationship between the distribution of price sensitivities and segment size. In any event, the results of the simpler case are still relevant "conditional on observed segment sizes." Going back to the automobile example described earlier in the paper, the simple model does not characterize the distribution of consumers' price sensitivity that would induce a larger market for intermediate quality cars. However, conditional on observing that this is so in equilibrium, it predicts that demand elasticities will be higher in that segment.

[6] Canoy, Marcel, and Martin Peitz (1997), "The Differentiation Triangle," *Journal of Industrial Economics*, 45(3), 305-328.

[7] Economides, Nicholas (1993), "Quality Variations in the Circular Model of Variety-Differentiated Products," *Regional Science and Urban Economics*, 23, 235-257.

[8] Goldberg, Penny K. (1995), "Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry," *Econometrica*, 63, 891-952.

[9] Hausman, Jerry, G. Leonard, and J. D. Zona (1994), "Competitive Analysis with Differentiated Products," *Annales d'Economie et de Statistique*, 34, 159-80.

[10] Hummels, David, and Volodymyr Lugovskyy (2005), "Trade in Ideal Varieties: Theory and Evidence," mimeo, Purdue University.

[11] Johnson, Justin P., and David P. Myatt (2003), "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93(3),748-774.

[12] Katz, Michael (1984), "Firm-Specific Differentiation and Competition among Multi-Product Firms," *Journal of Business*, 57, 149-166.

[13] Neven, Damien, and Jacques-Francois Thisse (1990), "On Quality and Variety Competition," in *Economic Decision-Making: Games*, *Econometrics and Optimization*, eds. J.J. Gabszewicz, J.-F. Richard, and L.A. Wolsey, Amsterdam: North-Holland, 175-199.

[14] Nevo, Aviv (2001), "Measuring Market Power in the Ready-to-Eat Cereal Industry," *Econometrica*, 69, 307-42.

[15] Petrin, Amil (2002), "Quantifying The Benefits of New Products: The Case of The Minivan," *Journal of Political Economy*, 110, 705-729.

[16] Salop, Steven (1979), "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10, 141-156.

[17] Mussa, Michael, and Sherwin Rosen (1978), "Monopoly and Product Quality," *Journal of Economic Theory*, 18(2), 301-317.

[18] Verboven, Frank (1999), "Brand Rivalry and Market Segmentation, with an application to the Pricing of Optional Engine Power on Automobiles," *Journal of Industrial Economics*, 47(4), 399-425.

Appendix: proof of Proposition 1

Given the equilibrium conditions provided by equations (26), (27), and (28) we can now substitute for the endogenous variables. First we substitute for prices

$$\delta_l \geq v_l c_l + \frac{5\omega_l}{4n_l} + \max\left(R, \frac{\omega_l}{4n_l}\right) \tag{29}$$

$$\delta_h - \delta_l \geq v_h(c_h - c_l) + \left(\frac{5\omega_h}{4n_h} - \frac{5\omega_l}{4n_l}\right) + \max\left(0, \frac{\omega_h}{4n_h} - R\right) + \frac{v_h \omega_l}{n_l^2 v_l}\left(1 + \frac{v_h}{2v_l}\right) \tag{30}$$

$$\delta_h - \delta_l \leq v_l(c_h - c_l) + \left(\frac{5\omega_h}{4n_h} - \frac{5\omega_l}{4n_l}\right) + \min\left(0, R - \frac{\omega_l}{4n_l}\right) - \frac{\omega_h}{n_h^2}\left(\frac{3}{2}\left(\frac{v_l}{v_h}\right)^2 + \frac{v_l}{v_h}\right) \tag{31}$$

and then for the number of firms

$$\begin{aligned}
\delta_l &\geq v_l c_l + \frac{5}{4}\sqrt{\frac{v_l \omega_l F_l}{S_l}} + \max\left(R, \frac{1}{4}\sqrt{\frac{v_l \omega_l F_l}{S_l}}\right) && (32)\\
\delta_h - \delta_l &\geq v_h \Delta c + K_1 = \\
&= v_h \Delta c + \frac{5}{4}\left(\sqrt{\frac{v_h \omega_h F_h}{S_h}} - \sqrt{\frac{v_l \omega_l F_l}{S_l}}\right) + \max\left(0, \frac{1}{4}\sqrt{\frac{v_h \omega_h F_h}{S_h}} - R\right) + \frac{F_l v_h}{S_l}\left(1 + \frac{v_h}{2 v_l}\right) && (33)\\
\delta_h - \delta_l &\leq v_l \Delta c + K_2 = \\
&= v_l \Delta c + \frac{5}{4}\left(\sqrt{\frac{v_h \omega_h F_h}{S_h}} - \sqrt{\frac{v_l \omega_l F_l}{S_l}}\right) + \min\left(0, R - \frac{1}{4}\sqrt{\frac{v_l \omega_l F_l}{S_l}}\right) - \frac{F_h}{S_h}\left(\frac{3}{2}\frac{v_l^2}{v_h} + v_l\right) && (34)
\end{aligned}$$

where, as can be seen, K_1 and K_2 are functions of the primitives of the model, but not of marginal costs. These three inequalities are only functions of primitives. The first inequality would be satisfied for sufficiently high δ_l. The last two inequalities are satisfied for $\delta_h - \delta_l \in [v_h \Delta c + K_1, v_l \Delta c + K_2]$. Thus, we only need to guarantee that this interval is not empty. Since $v_l > v_h$ the interval is non-empty for sufficiently high Δc. In particular, for any $\Delta c > \Delta c^*$ where $\Delta c^* = \frac{K_1 - K_2}{v_l - v_h}$. Q.E.D