AJAE APPENDIX:

WILLINGNESS TO PAY VERSUS EXPECTED CONSUMPTION VALUE IN VICKREY AUCTIONS FOR NEW EXPERIENCE GOODS

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Date: January 15, 2007.

Note: The material contained herein is supplementary to the article named in the title and submitted to the American Journal of Agricultural Economics (AJAE).

Present value

In the calculations of the present value of the future consumer surplus from the given product class, we used the sum of an infinite geometric row starting *t* periods from now, A_t.

$$\text{(A1)}\quad \begin{aligned} A_0 &= \sum_{t=0}^{\infty} \frac{R}{(1+r)^t} = \frac{(1+r)R}{r} = R + A_1 = R + \frac{R}{1+r} + A_2 \\ A_1 &= \sum_{t=1}^{\infty} \frac{R}{(1+r)^t} = \frac{R}{r} = \frac{R}{1+r} + A_2 \\ A_2 &= \sum_{t=2}^{\infty} \frac{R}{(1+r)^t} = \frac{R}{r(1+r)} \end{aligned} .$$

Transforming $F(\pi, r, a, p_1, p_2)$

The expected payoff of trying the new brand in the market, $F(\pi, r, a, p_1, p_2)$, is the expected net value of trying the new brand, $\pi\left[v_2 - a - p_2 + (v_1 - p_1)/r\right] + (1-\pi)(v_2 + a - p_2)(1+r)/r$, minus the net value of continuing to purchase the incumbent brand, $(v_1 - p_1)(1+r)/r$.

$F(\pi, r, a, p_1, p_2)$ can also be expressed as the value of buying a high quality new brand instead of the incumbent brand, from this period on multiplied by the probability that the new brand is of high quality $(1-\pi)\left[(v_2 + a - p_2) - (v_1 - p_1)\right](1+r)/r$, minus the value of buying the incumbent brand instead of the new brand if the new brand is of low quality $\left[(v_1 - p_1) - (v_2 - a - p_2)\right]/(1+r)$, multiplied by the probability that the new brand is of low quality, π.

$$\text{(A2)}\quad \begin{aligned} F(\bullet) &= \pi\left[v_2 - a - p_2 + (v_1 - p_1)/r\right] + (1-\pi)(v_2 + a - p_2)(1+r)/r - (v_1 - p_1)(1+r)/r \\ &= \pi\left[v_2 - a - p_2 + (v_1 - p_1)/r\right] + (1-\pi)(v_2 + a - p_2)(1+r)/r - (1-\pi+\pi)(v_1 - p_1)(1+r)/r \\ &= \pi\left[v_2 - a - p_2 + (1-(1+r))(v_1 - p_1)/r\right] + (1-\pi)\left[(v_2 + a - p_2) - (v_1 - p_1)\right](1+r)/r \\ &= \pi\left[v_2 - a - p_2 - (v_1 - p_1)\right] + (1-\pi)\left[(v_2 + a - p_2) - (v_1 - p_1)\right](1+r)/r \end{aligned} .$$

Differentiating $F \;\; \pi, r, a, p_1, p_2$ with respect to its elements

We differentiate $F \;\; \pi, r, a, p_1, p_2$ with respect to its elements π, r, a, p_1, and p_2.

(A3)
$$\begin{aligned}\frac{\delta \mathrm{F}}{\delta \pi} &= v_2 - a - p_2 - \;\; v_2 + a - p_2 - \;\; v_1 - p_1 \quad 1 + r \;\; / r \\ &= v_2 - a - p_2 - v_2 + a - p_2 + \;\; v_1 - p_1 \;\; - \;\; v_2 + a - p_2 - \;\; v_1 - p_1 \quad / r, \\ &= -2a + \;\; v_1 - p_1 - \;\; v_2 + a - p_2 \quad / r < 0\end{aligned}$$

(A4) $$\frac{\delta \mathrm{F}}{\delta r} = \;\; 1 - \pi \quad v_1 - p_1 - \;\; v_2 + a - p_2 \quad / r^2 < 0,$$

(A5) $$\frac{\delta \mathrm{F}}{\delta a} = -\pi + \;\; 1 - \pi \quad 1 + r \;\; / r = \;\; 1 - \pi + r(1 - 2\pi) \;\; / r,$$

(A6) $$\frac{\delta \mathrm{F}}{\delta p_1} = \pi + \;\; 1 - \pi \quad 1 + r \;\; / r = \;\; 1 - \pi + r \;\; / r > 0,$$

(A7) $$\frac{\delta \mathrm{F}}{\delta p_2} = -\pi - \;\; 1 - \pi \quad 1 + r \;\; / r = - \;\; 1 - \pi + r \;\; / r < 0.$$

Conditions that give an optimal bid that equals the expected consumption value in an auction for a new experience good

It is straightforward to show that if $v_1 - p_1 = v_2 - a - p_2$, $v_1 - p_1 = v_2 + a - p_2$, $r = \infty$, $\pi = 0$, $\pi = 1$, or $a = 0$, there would not be any information value associated with trying the new brand in the auction, and the subgame perfect bidding strategy would be equal to the expected consumption value.

First, if $v_1 - p_1 = v_2 - a - p_2$, we have that $\mathrm{F} > 0$. This gives the following optimal bid from equation (14):

(A8) $Bid2=\pi \;\; v_2-a \;+\; 1-\pi \;\;\; v_2+a \;\;.$

Second, if $v_1-p_1=v_2+a-p_2$, we have that $\mathrm{F}<0$. This gives the following optimal bid from equation (15):

(A9) $Bid2=\pi \;\; v_2-a \;+\; 1-\pi \;\;\; v_2+a \;\;.$

Third, when *r* approaches ∞, we have that $\lim_{r\to\infty}\mathrm{F}$ can be both positive and negative. This gives the following optimal bid from equations (14) and (15):

(A10) $\lim_{r\to\infty} Bid2=\pi \;\; v_2-a \;+\; 1-\pi \;\;\; v_2+a \;\;.$

Fourth, if $\pi=0$, we have that $\mathrm{F}>0$. This gives the following optimal bid from equation (14):

(A11) $Bid2=\pi \;\; v_2-a \;+\; 1-\pi \;\;\; v_2+a \;\; =v_2+a\,.$

Fifth, if $\pi=1$, we have that $\mathrm{F}<0$. This gives the following optimal bid from equation (15):

(A12) $Bid2=\pi \;\; v_2-a \;+\; 1-\pi \;\;\; v_2+a \;\; =v_2-a\,.$

Sixth, if $a=0$, then either $\pi=0$ or $\pi=1$. In both cases, $Bid2=v_2$.