AJAE Appendix:

Deriving a Flexible Mixed Demand System: The Normalized Quadratic Model

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Flexibility Properties of the Normalized Quadratic Mixed Demand System

For the functional specification proposed here to be "flexible" according to Diewert's (1974) definition, it needs to be able to attain arbitrary values for the cost function and its first and second derivatives at a point $\bar{p},\bar{z},\bar{u}$. For simplicity, let's take this point to be unity for all variables involved (this is possible without loss of generality by appropriate choice of units of measurement). Also, because the a_i coefficients are predetermined, we will assume

(A.1) $$\sum_{i=1}^{n} a_i = 1 \ .$$

Furthermore, note that, because utility is ordinal, arbitrary levels of C and C_u could be attained by any linear transformation of u. Finally, as in Diewert and Wales (1988), we concern ourselves only with the class of cost functions satisfying the money-metric scaling, i.e., $C_{uu}\left(\bar{p},\bar{z},\bar{u}\right)=0$.

The arbitrary values that we need to consider, at the point $\bar{p}_i=1\ \forall i$, $\bar{z}_k=1\ \forall k$, and $\bar{u}=1$, are then:

(A.2) $$C_{p_i} = \delta_i + \beta_i + \sum_{j=1}^{n}\beta_{ij} + \sum_{k=1}^{m}\lambda_{ik} + a_i \cdot \left(\sum_{k=1}^{m}\mu_k + \sum_{k=1}^{m}\gamma_k - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} + \frac{1}{2}\sum_{s=1}^{m}\sum_{k=1}^{m}\gamma_{ks} \right), \quad i=1,\dots,n$$

(A.3) $$C_{z_k} = \mu_k + \gamma_k + \sum_{s=1}^{m}\gamma_{ks} + \sum_{j=1}^{n}\lambda_{jk} \quad , \quad k=1,\dots,m$$

(A.4) $$C_{p_i p_j} = \beta_{ij} - a_j \cdot \sum_{r=1}^{n}\beta_{ir} - a_i \cdot \sum_{r=1}^{n}\beta_{jr} + a_i a_j \cdot \sum_{r=1}^{n}\sum_{t=1}^{n}\beta_{rt} \quad , \quad i,j=1,\dots,n$$

(A.5) $$C_{z_k z_s} = \gamma_{ks} \quad , \quad k,s=1,\dots,m$$

(A.6) $$C_{p_i z_k} = \lambda_{ik} + a_i \left(\mu_k + \gamma_k + \sum_{s=1}^{m}\gamma_{ks} \right) \quad , \quad i=1,\dots n \text{ and } k=1,\dots,m$$

(A.7) $$C_{up_i} = \beta_i + \sum_{j=1}^{n}\beta_{ij} + \sum_{k=1}^{m}\lambda_{ik} + a_i \cdot \left(\sum_{k=1}^{m}\gamma_k - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} + \frac{1}{2}\sum_{k=1}^{m}\sum_{s=1}^{m}\gamma_{ks} \right) \quad , \quad i=1,\dots,n$$

(A.8) $$C_{uz_k} = \gamma_k + \sum_{s=1}^{m} \gamma_{ks} + \sum_{j=1}^{n} \lambda_{jk} \quad , \quad k = 1,\dots,m .$$

We can now ascertain the flexibility of our function. Arbitrary values for $C_{z_k z_s}$ are possible as long as the γ_{ks} parameters are unrestricted. Similarly, the free parameters β_{ij} and λ_{ik} allow the form for achieving arbitrary values for $C_{p_i p_j}$ and $C_{p_i z_k}$, respectively. The parameter γ_k allows arbitrary values for C_{uz_k}, and the parameter β_i allows arbitrary values for C_{up_i}. Finally, arbitrary values for C_{z_k} are possible because the parameters μ_k are unrestricted, and arbitrary values for C_{p_i} are possible because of the free parameters δ_i.

Of course, the parameters just mentioned can be subject to some restrictions without destroying the flexibility of the model for the problem at hand. Specifically, because of symmetry of second derivatives, flexibility is still guaranteed if we enforce the following restrictions:

(A.9) $$\beta_{ij} = \beta_{ji} \quad , \quad \forall i, j = 1,\dots,n$$

(A.10) $$\gamma_{ks} = \gamma_{sk} \quad , \quad \forall k, s = 1,\dots,m .$$

Furthermore, so far we have neglected the restrictions implied by the fact that $C(p,z,u)$ is homogeneous of degree one in p. Thus, the Euler theorem implies that the first derivatives C_{p_i} are subject to one restriction. Hence, we can write

(A.11) $$\sum_{i=1}^{n} \delta_i = 0 .$$

Because each first derivative C_{p_i} is homogeneous of degree zero in *p*, applying Euler's theorem once more we find that second derivatives $C_{p_i p_j}$ are subject to *n* restrictions, second derivatives

$C_{p_i z_k}$ are subject to m restrictions, and second derivatives C_{up_i} are subject to one restriction. These last three conditions allow, respectively,

(A.12) $$\sum_{j=1}^{n} \beta_{ij} = 0 \quad , \qquad i = 1,...,n$$

(A.13) $$\sum_{j=1}^{n} \lambda_{jk} = 0 \quad , \qquad k = 1,...,m$$

(A.14) $$\sum_{i=1}^{n} \beta_i = 1 .$$

This concludes the illustration of the flexibility of the model presented in the article, and provides a justification of the parametric restrictions of equations (21)-(27) in the text. ■

References

See the corresponding AJAE article.