Spillovers, Joint Ventures and Social Welfare in a Mixed Duopoly R&D Race

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1 Introduction

The past decade has witnessed an increasing interest in the performance of mixed research and development (R&D) oligopolies[1]. A contemporary and typical example of which is the research race to sequence the human genome between the private firm Celera and the public sector's Human Genome Project (HGP).[2] Of particular interest is whether the public sector can improve welfare by engaging in specific types of R&D (such as genome projects) when there are only a few private-sector firms conducting similar research. In their seminal contribution to this literature, Delbono and Denicolo (1993) (D&D, henceforth) examine a mixed duopoly in which a public-sector and a private-sector firm compete, by investing in R&D, to innovate and patent. In comparison to a pure duopoly (two private-sector firms), where competition between firms in a winner-take-all patent race leads to aggregate overinvestment in R&D (see also Arrow (1962) and Barzel (1968) for earlier formulations of this idea), D&D show that the presence of a welfare-maximizing, public-sector firm in the duopoly can alleviate the problem of overinvestment.[3]

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[1] Mixed oligopolies are characterized by the presence of both profit maximizing private firms and welfare maximizing public sector firms in the market. See de Fraja and Delbono (1990) for a review of the mixed oligopoly literature.

[2] In this race Craig Ventner, CEO of Celera and formerly part of the public-sector human genome project, announced in 1991 that his firm would complete the sequencing in half the time of the HGP and at approximately 10% of the cost. Nonetheless the HGP continued, and jointly with Celera announced and published a genetic sequence for the human genome in 2000 (Marshall, 2000; Roberts, 2001)

[3] The social planner's equilibrium defines the optimal solution; 'overinvestment', therefore, refers to R&D investment greater than the optimal solution.

However, the D&D result has not held up in further modeling. In particular, Poyago-Theotoky (1998) (P-T, henceforth) modifies the D&D model to consider the case in which public-sector R&D generates an innovation, which the private sector can costlessly imitate. In this case the private-sector firm has an incentive to free ride on the public-sector R&D, as the expected payoff in both the winning and losing states is equivalent. Welfare in the mixed duopoly can be higher or lower than in the pure duopoly, depending on the size of the reward to innovation.[4]

P-T's result that the private firm's reaction curve is negative is a direct consequence from assuming that the benefits of the public sector's innovation are fully appropriable by both firms. A state where the innovation can be perfectly imitated by a rival would arise if one assumes that firms are unable to protect their intellectual property or that they simply choose not to protect so as to encourage a wider dissemination of the innovation. The later case would seem to be consistent with the mission of the public-sector research firm, but as shown by P-T not necessarily socially optimal. Allowing for costless imitation of the public firm results in too little research by the private firm and under-investment in the economy. To counter the socially sub-optimal under-investment due to easy imitation, the public firm may limit access through IP protection. If we assume that there is perfect protection, such that no imitation occurs as in the D&D formulation, then the implication is that there is over-investment and the Nash equilibrium remains socially sub-optimal. The two extremes—of easy imitation and no imitation—would suggest that if the public-sector firm's innovation is imperfectly appropriable and the public

[4] As another example, in a somewhat different game Nett (1994) models private-sector firms as having more flexibility in their strategic behaviors than do public-sector firms. This flexibility, in some cases, implies that the social surplus generated by the pure duopoly will exceed that of the mixed duopoly.

sector firm could modulate the appropriability of its innovation, then the socially optimal level could be attained. We introduce an appropriability parameter and show this to be indeed the case.

The notion of imperfect appropriability of an innovation can also be characterized as spillovers that occur in some product market after the conclusion of a research race. As such they are referred to as output spillovers to differentiate them from input spillovers, which occur during the course of a research. Input spillovers are most often defined as externalities that arise from the research efforts of individuals or firms. Geroski (1995) identifies three modes through which input spillovers occur:

> "they routinely arise when different agents discuss subjects of mutual interest, or when research results are disseminated through publications and seminar presentations. Spillovers can also be created when one agent observes the actions of another and makes inferences about the thinking that lies behind those actions. Last but not least, spillovers occur when a researcher paid by one firm to generate new knowledge transfers to another firm (or creates a spin-off firm) without compensating his/her former employer for the full inventory of ideas that travels with him/her" (Geroski, 1995)

These kinds of spillovers are especially true for public R&D, where evidence suggests that public research, especially research on biotechnology, plays an important role in the innovation process of US industry (McMillan, Narin and Deeds, 2000).

Although the mechanism by which input and output spillovers affect R&D behavior is different, their impacts on overall research levels in a mixed oligopoly framework has not been studied. We therefore extend the basic modeling framework of D&D and P-T models to study the effect of input and output spillovers, and their impact on social welfare. The explicit modeling of input spillovers also allows us to consider the impact of joint ventures between private and public sectors firms where there is complete sharing of research among participants and hence complete spillovers.

The next section of the paper develops the model incorporating our ideas of R&D spillovers in a mixed oligopoly R&D race. The next two sections solve the model for the noncooperative and cooperative case with a view of discussing the properties of the two firm's reaction curves and the resulting equilibrium. Section five presents the social welfare criterion. Section six presents the simulation result, comparing the cooperative and noncooperative cases. The final section draws conclusions.

2 Model

Consider a one-shot non-cooperative game between a profit-maximizing private-sector firm (*P*) and welfare maximizing public-sector firm (*S*), where the firms invest in R&D with the aim of innovating. The firm that innovates first is awarded an exogenously determined prize (*W*), which is the same for the two firms, i.e., $W_S = W_P = W$ [5,6]. The prize is imperfectly appropriable such that the winning firm receives $(1-\alpha)W$ and the losing firm αW for $0 \leq \alpha \leq 0.5$. The interpretation of the appropriability parameter within the present context is the following. Most R&D race models have assumed that the returns to R&D for the winning firm is value of the prize (*W*), with the losing firm getting nothing (the winner-take-all assumption). However, if we assume that the neither firm is able to appropriate all of the returns to its research, than it is natural to assume that the rival firm will accrue some benefits even if it were to lose the race. By assuming that some benefits of the innovation are appropriated by the rival firm, so that $0 < \alpha \leq 0.5$, then the potential returns to the firm in the losing state are αW. The more

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[5] D&D justify the equivalence of prize assumption on the grounds that "when the private firm is a perfectly discriminating monopolist, whereas the public-sector firm maximizes social welfare also in the product market."

[6] Throughout the paper we use the subscript *P* to denote the private-sector firm and the subscript *S* to denote the public-sector firm

restrictive access to an innovation becomes, say due to increased patenting by the winning firm, the lower the opportunity for the rival firm to copy the winning firm's innovation and reap benefits. D&D implicitly assumed that the each firm can appropriate all the returns to its research, thus $\alpha = 0$ in their formulation. P-T considers the other extreme wherein the private-sector firm costlessly acquires any innovation generated from the public-sector research, so that in both states, the private firm receives W / r.[7]

As in D&D and P-T we characterize the public sector firm as maximizing welfare by accounting for the total R&D spent by both public and private firms, whereas the private sector only accounts for its (private) R&D cost in its profit maximizing calculus. Building on the existing literature on innovation races (Reinganum, 1989; Lee and Wilde, 1980), we model R&D as a Poisson process. That is the probability of a firm innovating in the time interval $(t, t+dt)$, and that no other firm has innovated by time t, is a function of the R&D intensity at t undertaken by that firm. Further the relationship between the probability of innovating and R&D is assumed to be independent and exponential.

Following Martin (2000), we model input spillovers as increasing a firm's effective research intensity, g_i defined as

$$g_i = x_i + s_j x_j \tag{1}$$

for $i, j = P, S$ and $j \neq i$ and with $0 \leq s_j \leq 1$. Here x_i is a firm's own research intensity and x_j is the research intensity of its rival. The spillover parameter determines how much of a rival firm's R&D becomes a part of own firm's effective research—with $s_i = 0$ implying no spillovers and

[7] In contrast to the P-T specification, we assume that the winning firm is guaranteed to receive at least half or more of its winnings. Hence, if the innovation is perfectly appropriable then $\alpha = 0$ and the winner keeps all of its proceeds. If the innovation is imperfectly appropriable ($0 < \alpha \leq 0.5$), the losing firm also benefits, as it is able to appropriate some of the winning firm's prize. In the extreme when the prize is not appropriable ($\alpha = 0.5$), both winning and losing receive an equal share of the prize.

$s_i = 1$ implying complete spillovers. Given the asymmetric objectives of the two firms, we assume that spillovers from the public firm will be greater than from the private firm: $s_P \leq s_S$. This is consistent with the observation that public sector firms are more open to disseminating their research results then private firms. To attain research intensity x_i, the instantaneous resource cost for both firms is increasing and given by the function $\gamma(x_i)$ having the following properties

1) $\gamma(0) = 0 = \gamma'(0)$

2) $\forall x \geq 0, \gamma'(x) \geq x, \gamma''(x) > x$

The payoff function of the private firm is specified as the present value of expected profits, net of R&D costs:

$$V_P = \int_0^\infty \exp\{-(g_P + g_S + r)t\}\left[\frac{(1-\alpha)Wg_P}{r} + \frac{(\alpha)Wg_S}{r} - \gamma(x_P)\right]dt \tag{2}$$

$$= \frac{(W/r)\left((1-\alpha)g_P + \alpha g_S\right) - \gamma(x_P)}{g_P + g_S + r} \tag{2b}$$

where *r* is the discount rate.

The exponential term consists of the discount factor e^{-rt} times the probability that no firm has innovated to time t, $e^{-(g_P+g_S)t}$. The numerator in equation (2b) is the expected value at time t of the net benefits from staying in the race from time *t* to time $t+dt$, conditional on the fact that no firm has yet innovated. These expected net benefits include the expected present value of winning, $((1-\alpha)W/r)g_P$, plus the expected present value of losing, $(\alpha W/r)g_S$, less the certain cost of staying in the race, x_P. Written this way, the private firm wins the race with probability $g_P dt$, and receives a flow of profits $(1-\alpha)W$. On the other hand, if the public-sector firm wins the race the flow of profits to the private-sector firm is reduced and becomes a function of the appropriability parameter α .

Next, we specify the payoff to the public-sector firm. The innovation entails a social benefit, which, as stated earlier, is assumed to be equivalent to the prize obtained by the private firm. Since only one innovation is in prospect, the public-sector firm is indifferent as to who wins the race; to the public-sector firm the expected date of innovation is what matters. Moreover, the public-sector firm takes into account the R&D costs of both firms. It is in these respects that the public-sector firm is considered a welfare-maximizing firm[8]. The public-sector firm's payoff is specified as

$$V_S = \int_0^\infty \exp\{-(g_P + g_S + r)t\} \left[\frac{W}{r} \left(\underbrace{\left((1-\alpha)g_P + \alpha g_S\right)}_{\text{P's probability}} + \underbrace{\left((1-\alpha)g_S + \alpha g_P\right)}_{\text{S's probability}} \right) - \gamma(x_P) - \gamma(x_S) \right] dt \qquad (3)$$

$$= \frac{(W/r)\left(g_P + g_S\right) - \gamma(x_P) - \gamma(x_S)}{g_P + g_S + r} \qquad (3b)$$

It is important to note that the appropriability parameter (α) does not appear into the public-sector firm's payoff. As a social-welfare maximizer, the public-sector firm is concerned only with generating benefits *W*, not with who obtains these benefits. Further, the fact that the private-sector firm can easily imitate public research firm when $\alpha = 0.5$, does not diminish the aggregate value of the prize. This is an important point and requires emphasizing. We have already assumed that the value of the innovation is the same across the two innovators and not related to who innovates. That is, the 'prize' to the public-sector firm is social welfare whereas to the private-sector firm it is private profits when it is a perfectly discriminating monopolist, the value of the prize is equal across the two firms. The notion of appropriability would suggest that if the

[8] The underlying assumption here is that a faster pace of innovation in the economy is welfare increasing.

benefits of a firm's innovation are appropriated among several users (due to large spillovers) then its *profits* should be lower relative to the case where it is able to appropriate all the benefits to itself. However, for the public-sector firm profits, and more crucially the distribution of profits among firms, are irrelevant, since the public-sector firm maximizes social welfare. Since the concept of social welfare constitutes individual firm profits, the public-sector firm is less concerned with whether it makes lower profits (and hence someone else more) due to decreased appropriability as total social welfare remains unchanged given our assumptions on equivalence of the prize. Appropriability would, however, matter to all firms if all were profit-maximizing firms. In this case, an increasing ability to appropriate returns by firm *i* (should it win) would imply decreasing post-innovation profits for firm *j* (the losing firm)[9].

3 The Noncooperative Case

Model Equilibrium

To characterize the Nash equilibrium in R&D space (x_P, x_S), we derive, for each firm, the best response function and its properties. We first examine the noncooperative case where the two firms compete in the research and product markets followed by analysis of the joint venture case, where firms compete in the research market but remain competitive in the output market. From the first order condition for a maximum, the reaction curve of the private-sector firm is

[9] That is if W is the value of the prize then, under conditions of imperfect appropriability, the winning firm in a pure duopoly will receive $(1-\alpha)W$ and the losing firm αW with $0 \le \alpha \le 0.5$ (the lower bound representing highest appropriability and the upper bound the lowest appropriability). Thus, the case where the winning firm is unable to appropriate any of the rents from winning, such that $\alpha = 0.5$, we get the result where prize in both winning and losing states are equal. In the mixed duopoly case, matters are different and the prize for the public sector firm in the two state will always be W.

$$R_P[x_P, x_S] \equiv \gamma(x_P) - \gamma'(x_P)(r + x_P + x_S) + W$$
$$+\frac{Wx_S}{r}\big((1 - s_P s_S)(1-2\alpha)\big) - W\alpha(1 - s_P) + s_P\gamma(x_P) - \gamma'(x_P)(s_P x_P + s_S x_S) = 0 \tag{4}$$

To facilitate interpretation, the reaction curve has been specified such that the effect of the spillover and appropriability parameters is separated. The term on the first line of the right hand side of equation (4) is the private firm's best response function without the spillover effect and under perfect appropriability. The terms on the second line represent the change to the best response function when the public sector firm's innovation is not perfectly appropriable and there are input spillovers present. Similarly the reaction curve of the public firm is implicitly defined by

$$R_S[x_S, x_P] \equiv \gamma(x_P) + \gamma(x_S) - \gamma'(x_S)(x_P + x_S + r) + W$$
$$s_S(W + \gamma(x_P) + \gamma(x_S)) - \gamma'(x_S)(s_P x_P + s_S x_S) = 0 \tag{5}$$

As with the reaction function of the private firm we specify the effect of the parameters separately. To help us identify the Nash equilibrium in the x_P, x_S space, we state the following lemmas to establish the shape of the reaction curves.

Lemma 1: Define $R_P[x_S, x_P] = 0$ as the private-sector firm's reaction function (equation (4)), and $R_S[x_S, x_P] = 0$ as the public firm's reaction function (equation (5)). Then

a) The reaction functions are continuous

b) For $s_P = s_S = \alpha = 0$, $x_P^0 = x_S^0$ --the stand alone incentives coincide

c) $\forall x_S,\ x_P > 0,\ \alpha \in |0, 0.5|,$ and $s_P, s_S \in |0,1|$

1. $dx_P[x_S]/dx_S > 0$ (private-sector firm's reaction curve positively sloped) iff $W(1 - s_P s_S)(1-2\alpha) > \gamma'(x_P)r(1 + s_S)$. For $s_P = s_S = \alpha = 0$, $dx_P[x_S]/dx_S$ is unambiguously positive.

2. $dx_P[x_S]/dx_S < 0$ (private-sector firm's reaction curve negatively sloped) iff $W(1-s_P s_S)(1-2\alpha) < \gamma'(x_P)r(1+s_S)$. For $s_P = s_S = 1$ and $\alpha = 0.5$, $dx_P[x_S]/dx_S$ is unambiguously negative

D. $\forall x_P, \alpha,\ x_S > 0$ and $s_P, s_S \in |0,1|$

1. $dx_S[x_P]/dx_P > 0$ (public-sector firm's reaction curve positively sloped) iff $(1+s_S)\gamma'[x_P] > (1+s_P)\gamma'[x_S]$. For $s_P = s_S$, $dx_S[x_P]/dx_P > 0$ is unambiguously positive for $x_P > x_S$.

2. $dx_S[x_P]/dx_P < 0$ (public-sector firm's reaction curve negatively sloped) iff $(1+s_S)\gamma'[x_P] < (1+s_P)\gamma'[x_S]$. For $s_P = s_S$, $dx_S[x_P]/dx_P < 0$ is unambiguously negative for $x_P < x_S$.

E. There exists a Nash equilibrium for this game.

All proofs are relegated to the Appendix.

The interpretation of Lemma 1 is as follows. Consider first the case of where all three parameter values are zero. The marginal cost of undertaking R&D is $\gamma'[x_i]$ and the prize value, or benefit, for the winning firm is *W/r*. Since the prize value is exogenous and does not change with research effort, the marginal benefit from R&D is equal to the total benefit, *W/r*. The private firm will increase its research effort in response to research by the public firm so long as the marginal benefit is greater than marginal cost. When there are no input spillovers and both firms can completely appropriate their research prize (losing firm gets nothing), then the private firm's reaction curve is always increasing as marginal benefit is always greate then marginal cost ($W/r > \gamma'[x_P]$). In the presence of imperfect appropriability, the marginal benefit decreases by

$(1-2\alpha)$. Likewise in the presence of input spillovers, marginal benefit decreases by $\frac{1-s_P s_S}{1+s_P}$. In the extreme case of complete spillovers and imperfect appropriability $s_P = s_S = 1$ and $\alpha = 0.5$, we get the case that marginal benefit $\left(\frac{W(1-s_P s_S)(1-2\alpha)}{r(1+s_P)}\right)$ reduces to zero and hence is always less than marginal cost, in which case the private firm's reaction curve is decreasing and the private firm will free ride off the research effort of the public firm.

The shape of the public firm can also be interpreted in a similar fashion. We have stated that from the perspective of the public firm, the timing of innovation is what matters not who wins the race or the value of the prize. As such, the appropriability parameter do not enter into the public firm's reaction function. The response of the public sector firm to research by the private firm depends on the relative marginal cost of undertaking research by the two firms. Since the public sector firm is concerned only that the innovation occur quickly, and assuming no spillovers, the public sector firm decreases its research effort if the private firm has a lower marginal cost (and thereby increasing the private firm's chances of winning) and increases its research if it (the public sector firm) has a lower marginal cost of doing research. Spillovers from the rival firm increase a firm's effective research intensity but the incoming spillovers does not lower the marginal cost of doing research. Rather the incoming spillovers, from the perspective of the public firm, effectively lowers the marginal cost of the R&D from where the spillovers originate as they not only increase the likelihood of the firm innovating, but also increases the likelihood of its rival innovating. In other words, spillovers increase the productivity of R&D in the economy. Increasing spillovers from the public-sector firm to the private firm decreases the effective marginal cost of doing research for the public firm $\left(\frac{\gamma(x_S)}{1+s_S}\right)$, and if it less than the effective marginal cost for the private firm $\left(\frac{\gamma(x_P)}{1+s_P}\right)$, the public firm responds by increasing its research effort.

Comparative Statics

The effect on research effort due to changes in appropriability and spillovers can be established by taking the partial derivatives of the reaction functions. For the private firm, research effort is decreasing in appropriability and input spillovers from the public sector, but is ambiguous for the for own input spillovers:

$$\frac{\partial x_P}{\partial \alpha} = -W(r(1-s_P) + 2x_S(1-s_P s_S)) < 0 \tag{6}$$

$$\frac{\partial x_P}{\partial s_S} = -x_S\left(\frac{W}{r} s_P(1-2\alpha) + \gamma'(x_P)\right) < 0 \tag{7}$$

$$\frac{\partial x_P}{\partial s_P} = W(r\alpha - x_S s_S(1-2\alpha)) + r\left(\gamma(x_P) - x_P\gamma'(x_P)\right) \tag{8}$$

The private firms decrease its research effort under decreasing appropriability as the public-sector firm finds it easy to imitate the private firm's innovation and thereby reducing the benefit of the innovation for the private firm. When there are input spillovers from the public sector firm, the free-rider effect works in favor of the private firm and consequently the private firm reduces its research effort (equation (7)). And although one would expect that the spillovers from the private firm to the public-sector firm would reduce the incentives for the private firm to undertake research, we need to account for the effect on incentives due to appropriability. Note that in equation (8) the second term $r\left(\gamma(x_P) - x_P\gamma'(x_P)\right)$ is always negative as marginal cost of undertaking research is greater than the average cost of research. This implies that the signage of

(8) depends on the first term, and critically on the value of α. When $\alpha = 0$ and the winner take all assumption is maintained, the private firm will look unfavorably on any research leakage as it stands not to benefit from its rival winning the race. However if the private firm stands to gain some benefit in the losing state—assume the extreme case of $\alpha = 0.5$—then it want its research effort to benefit the public-sector firm as well since the prize value when $\alpha = 0.5$ is equivalent in the losing and winning states.[10]

The public-sector firm is unaffected by appropriability and hence its research effort does not change with changes in appropriability-- $\partial x_S / \partial \alpha = 0$. Like the private sector, the public-sector firm's research effort is declining in spillovers from the private firm but ambiguous as to the effect of own spillovers. That is,

$$\frac{\partial x_S}{\partial s_P} = -x_P \gamma '(x_S) \tag{9}$$

$$\frac{\partial x_S}{\partial s_S} = W + \gamma(x_P) + \gamma(x_S) - x_S \gamma '(x_S) \tag{10}$$

Figure 1 is a graphical representation of how changes in the appropriability parameter affects the public and private reaction curves for fixed values of the spillover parameter. When $x_P = x_S = \alpha = 0$, the D&D result is obtained and the Nash equilibrium is represented by N^{DD}. As appropriability of the prize value decreases ($\alpha \to 0.5$), the reaction curve of the private firm shifts leftward and the slope becomes negative for high values of α. Since appropriability has no effect on the public-firm's reaction curve, it remains unchanged. Figure 2 show how changes an increase in the spillover from the rival firm affects the reaction curves. Increasing spillovers from the public-sector firm shifts the private firms' reaction curve leftward.

[10] When $\alpha = 0.5$ equation (8) reduces to $\partial x_P / \partial s_P = W / r(0.5) + \left(\gamma(x_P) - x_P \gamma '(x_P)\right)$ which is positive so long as $W / r(0.5) > \left(\gamma(x_P) - x_P \gamma '(x_P)\right)$

Increasing spillovers from the private firm shifts the public sector firm's reaction curve downward.

4. The Cooperative (Joint Venture) Case

In a joint venture, the two firms conduct independent research but equally share the results of their research and cost.[11] This means that there are complete R&D spillovers between the two firms ($s_P = s_S = 1$) and the cost of undertaking research for the each firms is $\left(\gamma(x_P)+\gamma(x_S)\right)/2$. With complete R&D spillovers, the effective research intensity for both firms becomes $g_i = g = x_i + x_j$ for $i = P, S$. The private firm's payoff then becomes

$$V_P^{JV} = \int_0^\infty \exp\{-(g_P + g_S + r)t\}\left[\frac{W}{2r}\left(g_P + g_S\right) - \left(\frac{\gamma(x_P)+\gamma(x_S)}{2}\right)\right]dt \qquad (11)$$

$$= \frac{(W/r)\left(g\right) - \left(\gamma(x_P)-\gamma(x_S)\right)/2}{2g+r} \qquad (11b)$$

and the public firm's payoff is

$$V_S^{JV} = \int_0^\infty \exp\{-(g_P + g_S + r)t\}\left[\left(\frac{W}{2r}\left(g_P + g_S\right) - \frac{\gamma(x_P)}{2}\right) + \left(\frac{W}{2r}\left(g_P + g_S\right) - \frac{\gamma(x_S)}{2}\right)\right]dt \qquad (12)$$

$$= \frac{(W/r)\left(2g\right) - \gamma(x_P) - \gamma(x_S)}{2g+r} \qquad (12b)$$

Note that equations (11) and (12) are equivalent which has some interesting implications. Firstly, appropriability in joint ventures no longer matters, as by combining their research effort, the two

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[11] This characterization of research is different from that of Martin (2001) who differentiates between and "operating entity [research joint venture] RJV" and a "secretariat RJV". Under an operating entity RJV the two firms establish a common R&D laboratory and share the cost, whereas under s secretariat RJV each firm conducts its own research but share the results, though not the costs.

firms have become one entity and the stakes in winning are the same for both of them. Second, by undertaking joint venture activities the private firm is forced to internalize the cost of research of the public sector firm. Since the public sector firm is welfare maximizing, it was already internalizing the cost (and benefits) of both firms. As with the noncooperative case, we characterize the Nash equilibrium of the game by first specifying the reaction curves for the two firms. From the first order condition for a maximum, the reaction curve of the private firm and the public sector firms is given by:

$$R_i[x_i, x_j] \equiv 2\left(W + \gamma(x_i) + \gamma(x_j)\right) - \gamma'(x_i)\left(r + 2(x_i + x_j)\right) = 0 \text{ for } i = P, S,\ i \neq j \qquad (13)$$

Given the symmetric nature of the reaction curves, the following lemma establishes the curvature properties of the reaction curves and the existence of the Nash equilibrium in a joint venture:

Lemma 2: Define $R_i[x_i, x_j] = 0$ for $i = P, S$ and $i \neq j$ as the *i*th firm's reaction function (equation (13), Then

A. $x_P^0 = x_S^0$ --the stand alone incentives coincide

B. The best-response function for each firm is downward sloping if $x_i > x_j$ and upward sloping if $x_i < x_j$

c) There exists a Nash equilibrium such that $x_P^N = x_S^N$

From lemma 2 observe that the behavior of the public firm remains unchanged relative to the noncooperative case (for $s_P = s_S$), but the private firm's reaction curve is now a direct mirror image of the public firm. Due to the symmetric nature of the reaction curves, the Nash equilibrium occurs where the two firms invest the same amount in R&D activities.

Figure 3 shows the reaction curves of the two firms in a joint venture.

5. Social Welfare

Following D&D and P-T, we define the social optimum as one that is an outcome of a Nash game between two welfare maximizing public-sector firms under the control of a social planner. Mathematically this means that the social welfare is given by equation (2), evaluated at $x_P = x_S = x$, where *x* is the common R&D investment rate. Since both firms are under the social planners' control, we assume that R&D spillovers between the two firms are equal ($s_P = s_S = s$), but not necessarily complete ($0 \le s \le 1$). By maintaining the parameterization of the spillover allows us to compare the noncooperative case with that of the social welfare for a given spillover level. Substituting *x* into equation (2b) we get

$$V = \frac{(W/r)\left(2x(1+s)\right) - 2\gamma(x)}{2x(1+s)+r} \tag{14}$$

From the first-order condition for a maximum, the social optimum is then given by the following condition:

$$W(1+s) - \gamma'(x)(2x(1+s)+r) + 2\gamma(x)(1+s) = 0 \tag{15}$$

Geometrically, the social optimum is the point where the social welfare reaction curve intersects the 45-degree line. This implies that in the mixed duopoly the social optimum is achieved if in the Nash equilibrium $x_P^* = x_S^* = x^*$.

Having established the social optimum point, we are now in a position to evaluate how the non-cooperative and cooperative Nash equilibrium compare to the social optimum. D&D show that for the case of no spillovers and perfect appropriability, the equilibrium point in the (noncooperative) mixed duopoly is closer to the social optimum then it is for the private duopoly. In the P-T case, where the appropriability of the prize is imperfect due to easy imitation, the social welfare performance of the mixed and private duopolies relative to each other is a function of the value of the prize. That is for small prize value, the welfare in the mixed duopoly exceeds welfare in the private duopoly; for large prizes the welfare in the private setting is greater.

In the presence of spillovers and imperfect appropriability the social optimum can be achieved by

Proposition 1: Within the mixed duopoly setting and assuming a noncooperative game, then, there exists $\tilde{\alpha}$ $(0 < \tilde{\alpha} < 0.5)$, $\tilde{s}_P$ $(0 < \tilde{s}_P < 1)$ and $\tilde{s}_S$ $(0 < \tilde{s}_S < 1)$ for which the social welfare is maximized.

What the above proposition says is that when input spillovers and appropriability (output spillovers) are exogenously determined and for any prize value, the amount of spillovers that the each firm allows in the strategic game can be regulated to achieve the social optimum. For example, consider a social planner that has prior knowledge of the payoffs for the private-sector firm and public-sector firms. Given any *W* and *r*, and the curvature properties of the best response functions, it is relatively straightforward to calculate for $\tilde{\alpha}$ that result in the Nash being the social optimum.

In a joint venture we have established that appropriability does not matter and input spillovers are predetermined to be at 1. Proposition 2 establishes that in such a state the equilibrium research of a joint venture is the social optimum.

Proposition 2: The Nash equilibrium attained in a joint venture is the social optimum when $s = 1$

6. Comparison of Nash Equilibrium: A Simulation

To further fix some of the ideas presented, we carry out a simulation exercise and examine how changes in the parameter values affect equilibrium R&D, firm payoffs and social welfare. Our approach is to assign values to *W* and *r*, as well as functional form to the cost function to simultaneously solve for the x_P and x_S using the derived reaction curves[12]. These Nash values for *x* are then used to calculate the equilibrium payoffs.

The simulation results are presented in Tables 1 to 3. For the joint venture case even though the social optimum is reached (97.7889), the private payoff is relatively small (48.8944). This suggests that the incentive for private firm to participate in a joint venture will be low under certain circumstances. That is private firms find it profitable to form a joint venture only if there are very high spillovers and low appropriability of the research prize in the noncooperative case (in which case the private firm has higher payoff by cooperating).

7. Conclusion

In this paper the theoretical contributions of the D&D and P-T were strengthened to include the nuances of public and private sector research interactions. Specifically our modeling takes into account the observation that research spillovers, ex-post, are seldom “all or nothing”, but can be modulated such that recipient of the spillovers (i.e. the private firm in a mixed duopoly) benefits only partly from the prize of the winning firm. We also incorporate the notion of input spillovers that occurs during the course of a research race. Our modeling points to instances where the private firm reduces its research effort in response to public sector research. However, given the social welfare maximizing objectives of the public sector firm, the loss of appropriability and spillovers from the public sector does not have same affect on the public firm. Furthermore, modeling and simulation results suggest that joint ventures between private and public sector firms may only occur in the presence of high spillovers and low appropiability.

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[12] For all of our simulation we used the following values: $W = 10$, $r = 0.1$ and $\gamma(x) = x^2$

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Proof of Lemma 1:

A) Following D&D, we note that both reaction functions (equations 4 and 5) are quasi-concave and, therefore, from the maximum theorem the functions are continous.

B) Setting $x_S = 0$ into (4) and $x_S = 0$ into (5) we get,

$$W - r\gamma'(x_i) + \left(\gamma(x_i) - x_i\gamma'(x_i)\right) = 0 \tag{16}$$

C) Implicitly differentiating equation (4), and assuming that the second order condition is satisfied, we get

$$sign\left[\frac{dx_P}{dx_S}\right] = sign\left[W(1-s_P s_S)(1-2\alpha) - \gamma'(x_P)r(1+s_S)\right] \tag{17}$$

Equation (17) breaks down the curvature of the private-sector firm reaction into two terms, $\left\{W(1-s_P s_S)(1-2\alpha)\right\}$ and $\left\{-\gamma'(x_P)r(1+s_S)\right\}$, with the signage depending on which value is greater. In the extreme case of $s_P = s_S = \alpha = 0$, $W > \gamma'(x_P)r$ as….

When $s_P = s_S = 1$ and $\alpha = 0.5$, $0 < \gamma'(x_P)r$.

D) Implicitly differentiating equation (5), and assuming that the second order condition is satisfied, we get

$$sign\left[\frac{dx_S}{dx_P}\right] = sign\left[(1+s_S)\gamma'[x_P] - (1+s_P)\gamma'[x_S]\right] \tag{18}$$

As in (C) above, Equation (17) breaks down the curvature of the public-sector firm reaction into two terms, $\left\{(1+s_S)\gamma'[x_P]\right\}$ and $\left\{-(1+s_P)\gamma'[x_S]\right\}$, with the signage depending on which value is greater. For the case of $s_P = s_S$, the curvature properties of the public firm are unambiguous and follow from the properties of the cost function.

E) Existence of a Nash equilibrium follows from the continuity and quasi-concavity properties of the reaction functions. (Mas-Collel, Whinston, and Green, pg .)

Proof of Lemma 2:

A) Setting $x_j = 0$ for $i = P, S$ into (13)

$$2\left(W + \gamma(x_i)\right) - \gamma'(x_i)\left(r + 2(x_i)\right) = 0 \text{ for } i = P, S \tag{19}$$

B) Implicitly differentiating equation (13) we obtain:

$$sign\left[\frac{dx_i}{dx_j}\right] = sign\left[\gamma'(x_j) - \gamma'(x_i)\right] \text{ for } i = P, S \tag{20}$$

which is positive for $x_j > x_i$ and negative for $x_i > x_j$

C) Existence of a Nash equilibrium follows from the continuity and quasi-concavity properties of the reaction functions.

Proof of Proposition 1:

Forthcoming

Proof of Proposition 2:

From Lemma 2 part C, we know that in the Nash equilibrium, both firms conduct the same amount of research. At the Nash the reaction curve of the each firm reduces to

$$2\left(W + 2\gamma(x)\right) - \gamma'(x)\left(r + 4x\right) = 0 \tag{21}$$

This is equal to the social welfare condition for $s = 1$.

Figure 1: Forthcoming

Figure 2: Forthcoming

Figure 3: Public and Private Sector Firm's Reaction Curves in a joint venture

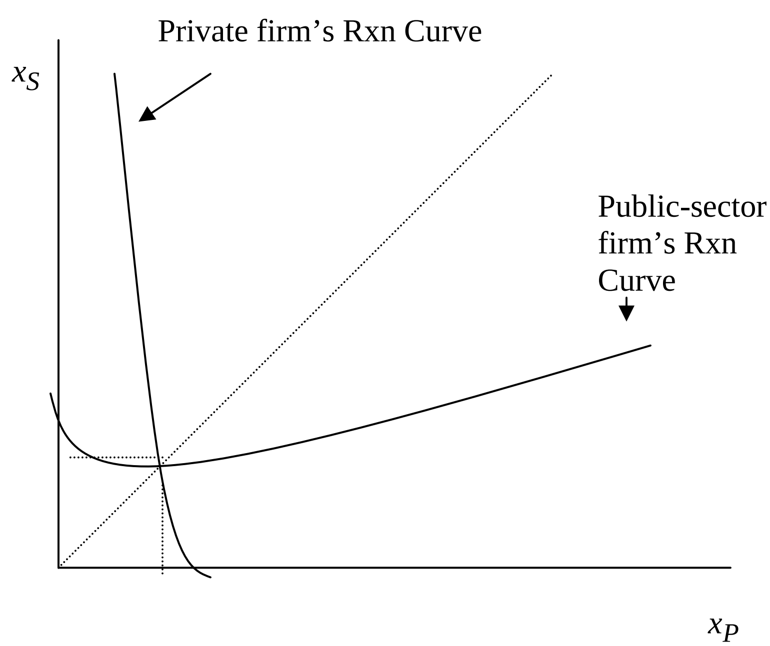

Table1: Simulation for the noncooperative case

		$\alpha = 0$				$\alpha = 0.25$				$\alpha = 0.5$			
s_P	s_S	x_S^*	x_P^*	V_P^*	V_S^*	x_S^*	x_P^*	V_P^*	V_S^*	x_S^*	x_P^*	V_P^*	V_S^*
0	1	11.3439	18.0040	63.9993	88.6539	6.1638	9.2550	56.4900	93.8362	2.9461	0.4036	49.1928	97.0539
0	.75	11.4272	19.3566	61.2868	86.9403	6.1757	9.9554	55.0893	92.9421	2.8901	0.4638	49.0723	96.6970
0	.5	11.2834	20.7664	58.4673	84.9554	6.0738	10.6897	53.6202	91.9016	2.8074	0.5454	48.9092	96.2568
0	.25	10.7584	22.0805	55.8390	82.7865	5.7813	11.3882	52.2236	90.7499	2.6784	0.6616	48.6768	95.7145
0	0	9.62761	22.9433	54.1134	80.7448	5.1864	11.824	51.2352	89.6271	2.4654	0.8377	48.3245	95.0693
.25	1	7.16234	12.3539	60.2338	92.8377	4.2430	6.5837	54.4661	95.7570	2.8556	0.5010	49.1843	97.144
.25	.75	7.5016	14.0671	57.4927	91.4267	4.3454	7.4642	53.0574	95.0339	2.77493	0.5871	49.0607	96.8286
.25	.5	7.6202	15.8058	54.7107	89.8125	4.3422	8.3647	51.6165	94.2105	2.6572	0.6917	48.8934	96.4570
.25	.25	7.37824	17.3811	52.1902	88.1948	4.1634	9.1966	50.2855	93.3386	2.4792	0.8393	48.6572	96.0333
.5	1	4.3719	7.7847	56.2871	95.6281	3.0376	4.4511	52.3986	96.9624	2.7490	0.6186	49.1752	97.2510
.5	.75	4.8834	9.8362	53.5517	94.4189	3.1710	5.5118	50.9843	96.3730	2.6414	0.7133	49.0490	96.9812
.5	.5	5.20015	11.8917	50.8111	93.0665	3.2232	6.5710	49.5720	95.7024	2.4882	0.8403	48.8796	96.6821
.75	1	2.7253	4.2247	52.3147	97.2743	2.3958	2.7078	50.4768	97.6042	2.6292	0.7296	49.1662	97.3708
.75	.75	3.2272	6.5737	49.6300	96.3118	2.4583	3.9971	49.0033	97.1905	2.4951	0.8410	49.0389	97.1484
1	1	2.5001	0.8415	47.5528	96.5528	2.5001	0.8415	49.0915	97.4900	2.5001	0.8415	49.1585	97.4999

Table 2: Simulation for the joint venture case

		$\forall \alpha$			
s_P	s_S	x_S	x_P	V_P	V_S
1	1	2.2112	2.2112	48.8944	97.7889

Table 3: Simulation for the social optimum under different spillover parameter values

s	x	V
1	2.2112	97.7889
.75	2.2077	97.4769
.5	2.2010	97.0627
.25	2.1964	96.4857
0	2.1866	95.6267