

**Examining the Functional Specification of Two-Parameter Model
Under Location and Scale Parameter Condition**

Takahiro Nakashima

National Agriculture and Food Research Organization
National Agricultural Research Center for Western Region, Japan
Email: tnakasim@affrc.go.jp

*Selected Paper prepared for presentation at the American Agricultural Economics
Association Annual Meeting, Long Beach, California, July 23-26, 2006*

*Copyright 2006 by Takahiro Nakashima. All rights reserved. Readers may make verbatim copies of
this document for non-commercial purposes by any means, provided that this copyright notice
appears on all such copies.*

Examining the Functional Specification of Two-Parameter Model Under Location and Scale Parameter Condition

Takahiro Nakashima

Abstract

The functional specification of mean-standard deviation approach is examined under location and scale parameter condition. Firstly, the full set of restrictions imposed on the mean-standard deviation function under the location and scale parameter condition are made clear. Secondly, the examination based on the restrictions mentioned in the previous sentence derives the new properties of the mean-standard deviation function on the applicability of additive separability and the curvature of expansion path which links the points that give the same slope of indifference curve. It reveals that attention has not been sufficiently paid to the restrictions in interpreting the linear mean-standard deviation model and the nonlinear mean-standard deviation model that have been used in previous research. Thirdly, the interpretation of the nonlinear mean-standard deviation model is reconsidered in detail and then an alternative nonlinear mean-standard deviation model is proposed. The implication of the two nonlinear mean-standard deviation models to the empirical approach “joint analysis of risk preference structure and technology” is discussed.

Keywords: mean-standard deviation approach, location and scale parameter condition, functional specification, risk aversion, uncertainty

Acknowledgement

This manuscript was written when the author was a visiting researcher at Department of Applied Economics and Management of Cornell University. This study was financially supported by National Agriculture and Food Research Organization (NARO). English language in this paper owes a great deal to Shoko Ishikawa.

Many studies have been conducted so far to examine farmer's production behavior under uncertainty and recently the approach called "joint analysis of risk preference structure and technology" has been employed to directly estimate structural parameters that indicate agent risk preference and production technology (Love and Buccola 1991; Saha, Shumway, and Talpaz. 1994; Chavas and Holt 1996; Saha 1997; Abdulkadri, Langemeier and Featherstone 2003; Nakashima 2006). In order to develop joint analysis models, the two distinctive decision-making criteria, expected utility (EU) theory and mean-standard deviation (MS) approach, have been particularly adopted. Needless to say, EU theory ranks random payoffs in accordance with expected value of suitably chosen utility function over payoff and MS approach evaluates random payoffs utilizing the objective function defined over the mean and standard deviation of payoff. The popularity of EU theory is in its axiomatic fundamentals (von Neumann and Morgenstern 1944) and analytical tools such as measures of risk aversion (Arrow 1974; Pratt 1964), while that of MS approach is in its simple and intuitively understandable framework (Markowitz 1952; Tobin 1958). Attention has been paid to the MS approach since Sinn (1983) and Meyer (1987) discovered that EU theory derives MS approach if random payoffs are restricted to the distribution class satisfying the location and scale parameter (LS) condition. Besides, they successfully translated under the LS condition the EU-based-behavioral hypothesis such as von Neumann-Morgenstern (vNM) utility function's curvature and Arrow-Pratt measure of risk aversion into appropriate analogues of the MS approach. They also pointed out that the LS condition is actually satisfied in a wide range of EU-based economic models since the random payoffs they analyzed is formulated as a linear function of random factor that is uniquely involved in their models (Feder, 1977).¹ For example, portfolio theory (Fishburn and Porter 1976), saving theory (Sandmo 1970), insurance demand theory (Ehrlich and Becker 1972), and production theory under uncertainty (Sandmo 1971; Ishii 1977; Feder 1980; Feder, Just, and Schmitz

1980) are all prominent models satisfying LS condition. The implication is that it is possible to translate many EU-based models into the MS framework with no loss of accuracy and that these transformed MS models can be analyzed on the basis of EU-based behavioral hypotheses. Thus, the MS approach is not only practical but is also flexible enough to provide full explanations for the LS class of EU-based economic models. Attempts have been made to take advantage of the practical and flexible framework of MS approach in developing joint analysis models from economic theories that satisfy the LS condition (e.g., Saha 1997; Abdulkadri, Langemeier and Featherstone 2003; Nakashima 2006). There is no doubt that employing MS approach instead of EU theory simplifies the models. Besides, theoretically, the models are free from ex ante assumptions on vNM utility function and distribution of random factor. For example, Hawawini (1978), Meyer and Robison (1988) and Leather and Quiggin (1991) enjoyed in their applied theoretical studies the MS framework that is flexible enough as well as tractable. However, when it comes to empirical applications, the theoretical flexibility of the MS-based empirical models is reduced, because MS functions have to be specified. Therefore, examining how the specification of MS functions restricts the theoretical flexibility is an important research area to empirically exploit the potential of MS approach under the LS condition.

It is well known that if some restrictions are imposed on vNM utility function and/or distribution of random payoff so that EU theory derives MS function, the derived MS function also needs to be restricted properly. For example, if vNM utility function is restricted to negative exponential utility function and random payoff is assumed to follow normal distribution, then the EU theory yields MS function that is specified as the linear mean-variance (LMV) model, supposedly one of the most frequently applied MS functions in the field of agricultural economics.² Likewise, if random payoffs are restricted to the distribution class that satisfies LS condition, then the derived MS function has to be properly

restricted as well. Compared with the case mentioned above, solving this specification problem is however somewhat complicated, because the LS condition does not specify a distribution of random payoffs but forms such a distribution family that nests a number of distribution, e.g. normal distribution and uniform distribution. Picking up a particular distribution from the LS distribution family loses generality of the condition. Therefore, the MS function under LS condition needs to be specified directly from MS framework, meeting the conditions that are imposed on MS function under the LS condition. Several attempts such as studies by Saha (1997) and Eggert and Tveteras (2004) were made to directly specify the MS function under the LS condition. However, nobody has pointed so far that the conditions have not been sufficiently fulfilled and that the conditions themselves have not been thoroughly discussed.

The objective of this study is to examine the functional specification of MS approach under the LS condition. The specification procedure adopted here is the one which directly specifies the MS function so that it fully meets the conditions imposed under the LS condition. Although the direct specification procedure relies upon a trial-and-error method that is far from mathematical elegance, it is suitable for exploring the possibility of specifying the MS function under LS condition. This study proceeds in the following order. Firstly, preliminary discussion is made on the conditions imposed on MS function under the LS condition. The conditions come from three sources, which are (a) cardinal property of vNM utility function, (b) behavioral hypotheses that are translated from EU theory into MS approach, and (c) relationship between Arrow-Pratt's risk aversion measures. Then, the imposed conditions are categorized by the type of risk aversion measures. Secondly, the direct specifications of MS function are applied to each type of risk aversion. In particular, it is examined whether the two functional properties, additive separability and homotheticity, are applicable to the functional specifications. Based on the examinations, MS functions

representing each type of risk aversion are considered and then some of MS functions that have been adopted such as the linear mean-variance (LMV) model and the linear mean-standard deviation (LMS) model are discussed in the context of LS condition. Thirdly, an attempt is made to specify MS function so that it nests several types of risk aversion under the LS condition. The flexible MS function proposed in this study provides an alternative interpretation of the nonlinear mean-standard deviation (NLMS) model. Finally, the implication to the empirical approach “joint analysis of risk preference structure and technology” is discussed.

Preliminary discussion

In this section, the full set of conditions that MS function has to satisfy under LS condition is prepared for the upcoming sections. The important thing that we have to be aware of when we consider the specification problem is that the MS framework established by Sinn (1983) and Meyer (1987) is by nature an EU theory (more precisely, a special case of EU theory) and utterly relies upon the EU-based analytical tools. For example, the definition of risk aversion and the degree of risk aversion are exactly those of EU theory. In other words, the theoretical fundamentals and the analytical tools of EU theory impose restrictions on the MS function. The sources of restrictions this study focuses on are categorized into three kinds, which are, (a) cardinal property of vNM utility function, (b) behavioral hypotheses that are translated from EU theory to MS approach, and (c) relationship between Arrow-Pratt's risk aversion measures.

Firstly, the existence of vNM utility, the core of EU theory, is guaranteed by von Neumann and Morgenstern's axioms, which implies that the utility is cardinal function that is transformable only by a positive linear function. This cardinal property is transformed into the MS approach in a straightforward manner. Suppose that under some condition on vNM

utility function and/or distribution of random payoff, EU theory derives a MS function such that $\int_{-\infty}^{+\infty} u(\pi) dF(\pi) = V(\sigma, \mu)$, where π denotes random payoff, $u(\pi)$ is a vNM utility function, $F(\pi)$ is a cumulative distribution function of π , $V(\sigma, \mu)$ is the derived MS function and μ and σ denote the mean and the standard deviation of π , respectively. A positive linear transformation of the vNM utility function derives the relationship,

$$\int_{-\infty}^{+\infty} \{au(\pi) + b\} dF(\pi) = aV(\sigma, \mu) + b \quad (a > 0),$$

which indicates the following result.

Proposition 1 (Cardinal property)

If MS approach is explained within EU theory, then the MS function is also cardinal that is transformable only by a positive linear function.

Secondly, Sinn (1983) and Meyer (1987) translated under LS condition the EU-based behavioral hypothesis such as vNM utility's curvature and Arrow-Pratt's measures of risk aversion into appropriate analogues of MS approach.

Proposition 2 (Behavioral hypothesis)

Property 1 $V_{\mu}(\sigma, \mu) > 0$ if and only if $U_{\pi}(\pi) > 0$.

Property 2 $V_{\sigma}(\sigma, \mu) < 0$ ($= 0$) if and only if $U_{\pi\pi}(\pi) < 0$ ($= 0$)

Property 3 The slope of the indifference curve of $V(\sigma, \mu)$, denoted as $S(\sigma, \mu) = -V_{\sigma}(\sigma, \mu)/V_{\mu}(\sigma, \mu)$, is positive (zero) if the agent is risk-averse (risk-neutral).

Property 4 $V(\sigma, \mu)$ is concave if and only if $U_{\pi}(\pi) > 0$ and $U_{\pi\pi}(\pi) \leq 0$.

Property 5 $S_{\mu}(\sigma, \mu) < 0$ ($= 0, > 0$) if and only if absolute risk aversion is decreasing (constant, increasing).

Property 6 $S_t(t\sigma, t\mu) < 0$ ($= 0, > 0$) if and only if relative risk aversion is decreasing (constant, increasing).

MS function is restricted by Proposition 2. For example, Properties 1 and 2 make it monotonously increasing in μ and decreasing in σ , respectively. Property 4 stipulates that the relevant Hessian matrix with respect to μ and σ is negative semi definitive. Properties 5 and 6 restrict the slope of the indifference curve when the individual is risk averter. Property 5 makes $S(\sigma, \mu)$ to be decreasing (constant, increasing) in μ when the individual's absolute risk aversion is decreasing (constant, increasing), while Property 6 makes it to be decreasing (constant, increasing) along rays through the origin when the individual's relative risk aversion is decreasing (constant, increasing).³ For later convenience, they are summarized as follows (This study focuses on risk averter's case).

$$(1) \quad V_{\mu}(\sigma, \mu) > 0,$$

$$(2) \quad V_{\sigma}(\sigma, \mu) < 0,$$

$$(3-i) \quad V_{\mu\mu}(\sigma, \mu) \leq 0,$$

$$(3-ii) \quad V_{\sigma\sigma}(\sigma, \mu) \leq 0,$$

$$(3-iii) \quad V_{\mu\mu}(\sigma, \mu)V_{\sigma\sigma}(\sigma, \mu) - V_{\mu\sigma}^2(\sigma, \mu) \geq 0,$$

$$(4-i: \text{DARA}) \quad -V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu) < 0,$$

$$(4-ii: \text{CARA}) \quad -V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu) = 0,$$

$$(4-iii: \text{IARA}) \quad -V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu) > 0,$$

$$(5-i: \text{DRRA})$$

$$\{-V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu)\}\mu + \{-V_{\sigma\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\sigma}(\sigma, \mu)V_{\sigma}(\sigma, \mu)\}\sigma < 0,$$

$$(5-ii: \text{CRRA})$$

$$\{-V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu)\}\mu + \{-V_{\sigma\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\sigma}(\sigma, \mu)V_{\sigma}(\sigma, \mu)\}\sigma = 0,$$

$$(5-iii: \text{IRRA})$$

$$\{-V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu)\}\mu + \{-V_{\sigma\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\sigma}(\sigma, \mu)V_{\sigma}(\sigma, \mu)\}\sigma > 0.$$

Conditions (1), (2), (3-i), (3-ii) and (3-iii) are derived from Properties 1, 2 and 4, respectively, and they are imposed at all times when individual is risk averse under LS condition.

Conditions (4-i), (4-ii) and (4-iii) could be expressed by Property 5 in terms of $V(\sigma, \mu)$.

Similarly, conditions (5-i), (5-ii) and (5-iii) could be expressed by Property 6 in terms of $V(\sigma, \mu)$ when $t = 1$. They are depending on types of risk aversion.

Thirdly, as discussed by Saha (1997), Arrow-Pratt risk aversion measures impose a certain restriction on the relationship between absolute risk aversion and relative risk aversion. Formally, let $A(\pi)$ and $R(\pi) = \pi A(\pi)$ respectively denote absolute risk aversion and relative risk aversion. Then the differentiation of $R(\pi)$ yields $R_{\pi}(\pi) = A(\pi) + \pi A_{\pi}(\pi)$. If absolute risk aversion is decreasing (DARA), i.e., $A(\pi) > 0$ and $A_{\pi}(\pi) < 0$, then the sign of $R_{\pi}(\pi)$ is not determined. In other words, DARA does not restrict the type of relative risk aversion. However, when absolute risk aversion measure is constant (CARA) or increasing (IARA), i.e., when $A(\pi) > 0$ and $A_{\pi}(\pi) \geq 0$, the sign of $R_{\pi}(\pi)$ is restricted to be positive, that is, increasing relative risk aversion (IRRA) is indicated and decreasing relative risk aversion (DRRA) and CRRA are ruled out. As shown in table 1, the combination of absolute and relative risk aversion is uniquely determined, except that relative risk aversion is not restricted under DARA and absolute risk aversion is not restricted under IRRA. Under the EU formulation, special attention needs not to be paid to the relationship, since it is automatically fulfilled in the specification of vNM utility. For example, if vNM utility is specified as a negative exponential function that indicates CARA, then IRRA automatically follows. However, its fulfillment is not guaranteed in the MS approach. Therefore, the relationship must be explicitly taken into consideration in the specification of the MS function in order that the relationship is maintained. Specifically, as Properties 5 and 6 in Proposition 2 implicitly assume the relationship, attention has to be paid to both of them. This imposes another restriction on MS function. For example, if MS function displays

CARA under LS condition, then $S(\sigma, \mu)$ has to be not only constant in μ but also increasing along rays through the origin. Thus, condition (5-iii) in addition to condition (4-ii) is imposed.

Combining the three kinds of restrictions discussed above, the complete set of conditions that MS function has to fully meet under LS condition is obtained. For example, when an individual is risk averse of CARA, then Proposition 2 and table 1 indicate that the MS function has to fully satisfy conditions (1), (2), (3-i), (3-ii), (3-iii), (4-ii) and (5-iii) (Other cases are summarized in table 2). Besides, Proposition 1 indicates that if the MS function is transformed, it needs to be linear transformation. In the following two sections, we consider the specification of MS function for each type of risk aversion, taking into full consideration the conditions shown in table 2.

Additive separability

In this section, the specification of MS function is examined for three types of absolute risk aversion. The examination proceeds in order of CARA, IARA, and DARA. If an individual is risk averter of CARA under LS condition, then the MS function must fully meet conditions (1), (2), (3-i), (3-ii), (3-iii), (4-ii) and (5-iii) as shown in table 2. Since condition (4-ii) indicates that the first term in left-hand-side of condition (5-iii) is zero, (5-iii) is reduced to:

$$(5-iii') \quad -V_{\sigma\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\mu\sigma}(\sigma, \mu)V_{\sigma}(\sigma, \mu) > 0.$$

Therefore, condition (5-iii) is replaceable to (5-iii'). The specification of the MS function may be carried out using the sign of its differential coefficients that fully satisfy these conditions. Although the procedure relies on a trial-and-error method, it allows the objective to be accomplished in the following three steps. The first step is to draw a rough outline of

the MS function using condition (1) and the signs of the derivatives, $V_{\mu\mu}(\sigma, \mu) = 0$, $V_{\mu\sigma}(\sigma, \mu) = 0$ and $V_{\sigma\sigma}(\sigma, \mu) < 0$, which satisfy conditions (3-i), (3-ii), (3-iii) (4-ii) and (5-iii'). Here, $V_{\mu\sigma}(\sigma, \mu) = 0$ indicates that the MS function is additively separable and the combination of condition (1) and $V_{\mu\mu}(\sigma, \mu) = 0$ shows that it is linearly increasing in μ . These inferences together imply the form, $V(\sigma, \mu) = \alpha\mu + g(\sigma)$, where α denotes a positive parameter and $g(\sigma)$ denotes some function of σ . The second step of specifying the MS function is to restrict the form so that it meets the remaining conditions, (2) and $V_{\sigma\sigma}(\sigma, \mu) < 0$. This step may be easily carried out by restricting function $g(\sigma)$ in such a way that it is monotonously decreasing and strictly concave. Thus, the additively separable and partial linear MS function with the restriction mentioned above,

$$(6) \quad V(\sigma, \mu) = \alpha\mu + g(\sigma)$$

$$(\alpha > 0, g_{\sigma}(\sigma) < 0 \text{ and } g_{\sigma\sigma}(\sigma) < 0),$$

fully meets the imposed conditions and therefore is one of functional forms that represents CARA under the LS condition. In order to apply form (6) to empirical work, the following third step is necessary that specifies function $g(\sigma)$ under the restrictions, $g_{\sigma}(\sigma) < 0$ and $g_{\sigma\sigma}(\sigma) < 0$. As one of candidates, let us consider specifying $g(\sigma)$ to be polynomial function. Now, expanding $g(\sigma)$ by the n-th order Taylor series approximation and evaluating σ at

$$\sigma^0 = 0, \text{ we obtain } g(\sigma) \cong g(\sigma^0) + \sum_{i=1}^n \frac{g^{(i)}(\sigma^0)}{i!} \sigma^i. \text{ Defining } g(\sigma^0) = \beta_0, g^{(i)}(\sigma^0) = -\beta_i \text{ (} i = 1, 2, \dots, n)$$

then yields

$$(7) \quad g(\sigma) = \beta_0 - \sum_{i=1}^n \frac{\beta_i}{i!} \sigma^i,$$

where β_i ($i = 0, 1, 2, \dots, n$) are parameters. Because, as has been already mentioned, function $g(\sigma)$ in form (6) must meet the restrictions, $g_\sigma(\sigma) < 0$ and $g_{\sigma\sigma}(\sigma) < 0$, the parametric restrictions, $\beta_i \geq 0$ ($i = 1, 2, 3, \dots, n$) and $\beta_i > 0$ for at least one i ($i = 2, 3, 4, \dots, n$), are imposed on form (7). Note also that since the parameter β_0 does not play an important role on the curvature of function $g(\sigma)$, $\beta_0 = 0$ is assumed a priori for the simplification. Substituting form (7) and $\beta_0 = 0$ for (6) yields the following MS function:

$$(8) \quad V(\sigma, \mu) = \alpha\mu - \sum_{i=1}^n \frac{\beta_i}{i!} \sigma^i$$

($\alpha > 0$, $\beta_i \geq 0$ ($i = 1, 2, 3, \dots, n$), $\beta_i > 0$ for at least one i ($i = 2, 3, 4, \dots, n$)).

When $\alpha = 1$ and $\beta_i = 0$ ($i = 1, 3, 4, \dots, n$), form (8) nests the linear mean-variance (LMV) model, probably one of most frequently applied MS functions in the field of agricultural economics. This is not, however, a surprising consequence in considering the assumptions imposed here. As originally demonstrated by Freund (1956), the LMV model is derived through EU theory assuming that vNM utility is a negative exponential function and random payoffs follow a normal distribution, and the negative exponential utility represents CARA preference and the normal distribution belongs to the LS family. Therefore, the specification procedure, adopted here, of MS function may be an alternative approach that can derive the LMV model. And undoubtedly, it is feasible to employ the LMV model under the assumptions of CARA preference and LS condition (an example is the recent study by Peterson and Ding 2005). On the other hand, form (8), because of the imposed parametric restrictions, does not nest the linear mean-standard deviation (LMS) model that has been recently applied by Eggert and Tveteras (2004) in the context of CARA and LS condition.

As is seen in the specification process of form (6), if the MS function is specified as being additively separable and linear in μ , then it has to be decreasing and strictly concave in σ . While the LMV model fully meets those properties, the LMS model fails to meet the strict concavity condition. Therefore, the LMS model is not a relevant form at least in this context. This arises because attention is not paid to the relationship between Arrow-Pratt's risk aversion measures in interpreting the LMS model under LS condition. As shown in table 1, CARA indicates IRRA that imposes condition (5-iii'). The condition in conjunction with condition (1) does not allow MS function to be linear in σ as long as it is additively separable.

Despite that the LMS model may not display CARA under LS condition, there is no doubt that additive separability is, if applicable, a useful property because it considerably simplifies the specification of MS function. The remaining part of this section considers whether the property applies to the case that an individual displays non-CARA preference. If an individual is risk averter of IARA under LS condition, then the MS function has to entirely fulfill conditions (1), (2), (3-i), (3-ii), (3-iii), (4-iii) and (5-iii). They also allow the MS function to be specified as being additively separable when it is increasing and strictly concave in μ and decreasing and concave in σ . This is easily shown through the following three-step procedure. Firstly, an outline of MS function is drawn using conditions (1), (2), (3-ii) and the signs of the derivatives, $V_{\mu\mu}(\sigma, \mu) < 0$, $V_{\mu\sigma}(\sigma, \mu) = 0$, which fulfill conditions (3-i), (3-iii), (4-iii) and (5-iii). Since $V_{\mu\sigma}(\sigma, \mu) = 0$ indicates that the MS function is additively separable and $V_{\mu\mu}(\sigma, \mu) < 0$ in conjunction with condition (1) indicates that it is increasing and strictly concave in μ , these together imply the expression, $V(\sigma, \mu) = h(\mu) + k(\sigma)$, where $h(\mu)$ denotes a function that is restricted to being $h_{\mu}(\mu) > 0$ and $h_{\mu\mu}(\mu) < 0$. Secondly, the remaining conditions (2) and (3-ii) restricts function $k(\sigma)$ to being $k_{\sigma}(\sigma) < 0$ and $k_{\sigma\sigma}(\sigma) \leq 0$. Thus, the additive separable MS function with the restrictions discussed here,

$$(9) \quad V(\sigma, \mu) = h(\mu) + k(\sigma)$$

$$(h_{\mu}(\mu) > 0, h_{\mu\mu}(\mu) < 0, k_{\sigma}(\sigma) < 0 \text{ and } k_{\sigma\sigma}(\sigma) \leq 0),$$

fully meets the imposed conditions, and therefore, represents IARA under LS condition. The third step is the specification of the functions $h(\mu)$ and $k(\sigma)$. It is omitted here, since it is easily carried out.

In contrast, MS function may not be specified as being additively separable when an individual is risk averter of DARA under LS condition. This can be demonstrated using conditions (1), (2), (3-i), (3-ii), (3-iii) and (4-i), which are imposed on MS function in this case. Suppose that the MS function is additively separable, i.e., $V_{\mu\sigma}(\sigma, \mu) = 0$. Then, condition (4-i) reduces to $V_{\mu\mu}(\sigma, \mu)V_{\sigma}(\sigma, \mu) < 0$. The condition reduces further to $V_{\mu\mu}(\sigma, \mu) > 0$, as a consequence of condition (2). The inequality however contradicts condition (3-i). Therefore, the MS function is non-additively separable. Besides, it is also derived that the MS function is nonlinear in both μ and σ , as follows. Suppose that the MS function is linear in either μ or σ , i.e., $V_{\mu\mu}(\sigma, \mu) = 0$ or $V_{\sigma\sigma}(\sigma, \mu) = 0$. Then condition (3-iii) reduces to $V_{\mu\sigma}^2(\sigma, \mu) \leq 0$, which implies $V_{\mu\sigma}(\sigma, \mu) = 0$. Thus, the MS function is additively separable. However, this contradicts the result established above. Therefore, the MS function is nonlinear in both μ and σ . These properties, non-additive separability and nonlinearity, do not facilitate the specification of MS function by means of the procedure used for proposing forms (6) and (9). Because they may not ‘decompose’ conditions (3-iii) and (4-i) into each derivation coefficient, $V_{\mu}(\sigma, \mu)$, $V_{\sigma}(\sigma, \mu)$, $V_{\mu\mu}(\sigma, \mu)$, $V_{\sigma\sigma}(\sigma, \mu)$ and $V_{\mu\sigma}(\sigma, \mu)$. The interaction between them may not be ignored here. Instead of using the specification approach, the next section considers specifying the DARA type’s MS function from different viewpoint.

In this section, we derived the following properties regarding MS function under LS condition.

Proposition 3 (applicability of additive separability)

- (1) If an individual is risk averter of DARA under LS condition, then the MS function is non-additively separable and nonlinear in both μ and σ .
- (2) If an individual is risk averter of CARA under LS condition, then the MS function may be additively separable as long as it is linearly increasing in μ and decreasing and strictly concave in σ .
- (3) If an individual is risk averter of IARA under LS condition, then the MS function may be additively separable as long as it is increasing and strictly concave in μ and decreasing and concave in σ .

Homotheticity

In this section, the specification of MS function is examined for three types of relative risk aversion. The examination starts from CRRA, a frequently adopted case in empirical study as well as CARA. If an individual is risk averter of CRRA under LS condition, then the MS function must fully satisfy conditions (1), (2), (3-i), (3-ii), (3-iii), (4-i) and (5-ii). In this case, the specification procedure used for forms (6) and (9) is no longer useful, because CRRA indicates DARA (table 1) and therefore Proposition 3 (1) holds. Instead, condition (5-ii) can play a significant role in specifying the MS function. The condition, or more apparently its alternative expression, $\partial/\partial t \{-V_{\sigma}(t\sigma, t\mu)/V_{\mu}(t\sigma, t\mu)\} = 0$, indicates that the MS function is homothetic and conversely a homothetic MS function satisfies the above condition (This directly follows from Lau's lemma (1969) that a function of two or more arguments is homothetic if and only if the ratio of the first derivatives of the function is homogeneous of

degree zero). Therefore, candidates of the MS function can be chosen from homothetic family that have been developed and exploited in economic analysis, and doing so fulfils condition (5-ii). For example, consider a constant elasticity of substitution (CES) type MS function,

$$(10) \quad V(\sigma, \mu) = (\mu^\delta - \sigma^\delta)^{\frac{1}{\delta}} \quad (\delta > 1; \mu > \sigma),$$

where δ denotes a parameter that is restricted to $\delta > 1$. Since form (10) is linear homogeneous, the homothetic property and therefore condition (5-ii) is met. Besides, it holds Proposition 3 (1), that is, it is non-additively separable and nonlinear in μ and σ . The remaining conditions (1), (2), (3-i), (3-ii), (3-iii) and (4-i) are also satisfied, as verified below.

$$\begin{aligned} V_\mu(\sigma, \mu) &= \mu^{\delta-1} (\mu^\delta - \sigma^\delta)^{\frac{1}{\delta}-1} > 0, \quad V_\sigma(\sigma, \mu) = -\sigma^{\delta-1} (\mu^\delta - \sigma^\delta)^{\frac{1}{\delta}-1} < 0, \\ V_{\mu\mu}(\sigma, \mu) &= (1-\delta)\mu^{\delta-2}\sigma^\delta (\mu^\delta - \sigma^\delta)^{\frac{1}{\delta}-2} < 0, \quad V_{\sigma\sigma}(\sigma, \mu) = (1-\delta)\mu^\delta\sigma^{\delta-2} (\mu^\delta - \sigma^\delta)^{\frac{1}{\delta}-2} < 0, \\ V_{\mu\mu}(\sigma, \mu)V_{\sigma\sigma}(\sigma, \mu) - V_{\mu\sigma}^2(\sigma, \mu) &= 0 \text{ and} \\ -V_{\mu\sigma}(\sigma, \mu)V_\mu(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu)V_\sigma(\sigma, \mu) &= (1-\delta)\mu^{\delta-2}\sigma^{\delta-1} (\mu^\delta - \sigma^\delta)^{\frac{2}{\delta}-2} < 0. \end{aligned}$$

Therefore, form (10) displays CRRA under LS condition. Recently, Nelson and Escalante (2004) proposed the following form,

$$(11) \quad V(\sigma, \mu) = -(\mu^2 - \phi\sigma^2)^{-1} (\phi > 0; \mu^2 - \phi\sigma^2 > 0),$$

where ϕ denotes a parameter, and showed that form (11) fully meet conditions (1), (2), (3-i), (3-ii), (3-iii), (4-i) and (5-ii). Therefore, it also displays CRRA under LS condition. Nelson and Escalante achieved the specification of form (11) by modifying the LMV model that displays CARA under LS condition. The modification consequently fits the LMV model

into Proposition 3 (1) and condition (5-ii). Thus, it transforms the additively separable, partial linear and non-homothetic function to the non-additively separable, nonlinear and homothetic function.

Although the homothetic property provides a useful clue to the specification of MS function that displays CRRA under LS condition, it is not applicable to the case of non-CRRA preference. The reason is evident from the conditions imposed on MS function under the preference. If an individual is risk averter of type DRRA under LS condition, then the MS function has to satisfy conditions (1), (2), (3-i), (3-ii), (3-iii), (4-i) and (5-i). On the other hand, if an individual is risk averter of the case of IRRA which indicates DARA under LS condition, then the MS function has to satisfy conditions (1), (2), (3-i), (3-ii), (3-iii), (4-i) and (5-iii) (Since the cases of IRRA which indicates CARA or IARA have been already discussed in the previous section, this section focuses on the combination of IRRA and DARA). Here, conditions (5-i) and (5-iii), or their alternative expressions, $\partial/\partial t \{-V_\sigma(t\sigma, t\mu)/V_\mu(t\sigma, t\mu)\} < 0$ and $\partial/\partial t \{-V_\sigma(t\sigma, t\mu)/V_\mu(t\sigma, t\mu)\} > 0$ indicate that the MS functions are nonhomothetic (Lau's lemma). Yet, Proposition 3 (1) still holds in both cases, as they show DARA. Thus, the MS functions are non-additively separable and nonlinear in μ and σ as well as nonhomothetic. In specifying the non-CRRA type's MS functions, an MS function displaying CRRA might help, because the conditions imposed on the non-CRRA type's MS functions and those on the CRRA type's MS function differ only one point. It is that condition (5-ii) is replaced by condition (5-i) or (5-iii). Therefore, the objective here is accomplished by modifying the CRRA type's MS function to fit condition (5-i) or (5-iii) with the remaining factors, conditions (1), (2), (3-i), (3-ii), (3-iii) and (4-i), still satisfied.

In order to do that, the first thing that we have to do is to realize the functional properties that reflect the difference between conditions (5-i) and (5-iii). As mentioned

above, conditions (5-i) and (5-iii) indicates that the MS functions are nonhomothetic. In other words, an expansion path, a locus which links the points that give the same slope of indifference curve in $\sigma - \mu$ axis, is nonlinear. Then, examining the curvature of the expansion path reveals the difference between conditions (5-i) and (5-iii). Formally, consider an expansion path, $\tilde{S} = -V_{\sigma}(\sigma, \mu)/V_{\mu}(\sigma, \mu)$, where \tilde{S} denotes an arbitral slope of indifference curve. Total differentiation of \tilde{S} yields

$$\left. \frac{d\mu}{d\sigma} \right|_{\tilde{S}} = - \frac{-V_{\sigma\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\sigma}(\sigma, \mu)V_{\mu\sigma}(\sigma, \mu)}{-V_{\mu\sigma}(\sigma, \mu)V_{\mu}(\sigma, \mu) + V_{\sigma}(\sigma, \mu)V_{\mu\mu}(\sigma, \mu)},$$

which expresses the slope of the expansion path. In the case of DRRA, this is less than $\frac{\mu}{\sigma}$ because of conditions (4-i) and (5-i). And in the case of IRRA and DARA, this is more than $\frac{\mu}{\sigma}$ because of conditions (4-i) and (5-iii).

Then, differentiating these relationships, $\left. \frac{d\mu}{d\sigma} \right|_{\tilde{S}} < \frac{\mu}{\sigma}$ and $\left. \frac{d\mu}{d\sigma} \right|_{\tilde{S}} > \frac{\mu}{\sigma}$, derive,

$$\left. \frac{d^2\mu}{d\sigma^2} \right|_{\tilde{S}} < \frac{1}{\sigma} \left(\left. \frac{d\mu}{d\sigma} \right|_{\tilde{S}} - \frac{\mu}{\sigma} \right) \text{ and } \left. \frac{d^2\mu}{d\sigma^2} \right|_{\tilde{S}} > \frac{1}{\sigma} \left(\left. \frac{d\mu}{d\sigma} \right|_{\tilde{S}} - \frac{\mu}{\sigma} \right),$$

respectively. These mean that the expansion path is strictly concave in the case of DRRA and strictly convex in the case of IRRA and DARA. Conversely, condition (4-i) and the strict concavity of expansion path derive condition (5-i) while condition (4-i) and the strict convexity of expansion path derive condition (5-iii). Therefore, conditions (5-i) and (5-iii) are replaced by each property of expansion path. If it is possible to modify MS function displaying CRRA in such a way that each property of the curvature of expansion path is satisfied with the remaining conditions (1), (2), (3-i), (3-ii), (3-iii) and (4-i), then the objective here is achieved. A successful example of the modification is obtained in the case of IRRA and DARA. Form (10) is modified by introducing a new parameter as follows.

$$(12) \quad V(\sigma, \mu) = (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}} \quad (1 < \delta < \eta; \mu^\delta - \sigma^\eta > 0),$$

where η is the newly introduced parameter that is restricted to $1 < \delta < \eta$. Form (12) is a nonhomothetic function whose curvature of expansion path is strictly convex, because of

$$\left. \frac{d^2 \mu}{d\sigma^2} \right|_S = \left(\tilde{S} \frac{\delta}{\eta} \right)^{1/\delta} \frac{1-\eta}{1-\delta} \frac{\delta-\eta}{1-\delta} \sigma^{\frac{\delta-\eta}{1-\delta}-1} > 0. \text{ Besides, it keeps satisfying the remaining conditions}$$

(1), (2), (3-i), (3-ii), (3-iii) and (4-i), as is obvious from the following derivation coefficients,

$$V_\mu(\sigma, \mu) = \mu^{\delta-1} (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}-1} > 0, \quad V_\sigma(\sigma, \mu) = -\frac{\eta}{\delta} \sigma^{\eta-1} (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}-1} < 0,$$

$$V_{\mu\mu}(\sigma, \mu) = (1-\delta) \mu^{\delta-2} \sigma^\eta (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}-2} < 0,$$

$$V_{\sigma\sigma}(\sigma, \mu) = \frac{\eta}{\delta} (1-\eta) \sigma^{\eta-2} (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}-1} + \frac{\eta^2}{\delta^2} (1-\delta) \sigma^{2\eta-2} (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}-2} < 0,$$

$$V_{\mu\mu}(\sigma, \mu) V_{\mu\mu}(\sigma, \mu) - V_{\mu\sigma}^2(\sigma, \mu) = \frac{\eta}{\delta^2} (1-\delta) (\delta-\eta) \mu^{\delta-2} \sigma^{2\eta-2} (\mu^\delta - \sigma^\eta)^{\frac{2}{\delta}-3} > 0 \text{ and}$$

$$-V_{\mu\sigma}(\sigma, \mu) V_\mu(\sigma, \mu) + V_{\mu\mu}(\sigma, \mu) V_\sigma(\sigma, \mu) = \frac{\eta}{\delta} (1-\delta) \mu^{\delta-2} \sigma^{\eta-1} (\mu^\delta - \sigma^\eta)^{\frac{2}{\delta}-2} < 0. \text{ Therefore, form (12)}$$

displays the case of IRRRA and DARA under LS condition.⁴ Unfortunately however, the author could not find an example that successfully modifies the MS function displaying CRRA to the one displaying DRRRA. Although the curvature condition is easy to meet, the remaining conditions do not seem to be so (For example, the strict concave condition of expansion path is satisfied if the restriction $1 < \eta < \delta$ instead of $1 < \delta < \eta$ is imposed on form (12), it does not fulfill condition (3-iii)). This case remains for further research.

In this section, the function properties we obtained regarding MS function is summarized as

follows.

Proposition 4 (homotheticity and nonhomotheticity)

- (1) If an individual is risk averter of DRRA under LS condition, then the MS function is non-homothetic function whose expansion path is strictly concave in $\sigma - \mu$ axis.
- (2) If an individual is risk averter of CRRA under LS condition, then the MS function is homothetic.
- (3) If an individual is risk averter that displays the combination of IRRA and DARA under LS condition, then the MS function is non-homothetic function whose expansion path is strictly convex in $\sigma - \mu$ axis.

A flexible specification

The specification of MS approach under LS condition has been considered for each type of risk aversion and then several MS functions have been proposed (see forms (6), (9), (10) and (12)). They can be applied to empirical analysis, assuming that the agent displays the corresponding type of risk aversion and the random payoffs it faces are restricted to the distribution class that satisfies the LS condition. However, as pointed out by Sinn (1983) and Meyer (1987), a wide range of EU-based economic models satisfies the LS condition owing to the theoretical structures themselves, and in such models, the EU theory is interchangeable with MS approach with no assumption imposed on vNM utility function. Therefore, a particular type of risk aversion needs not to be imposed a priori. In order to exploit the MS approach in empirical studies based on the LS class of economic models, we need to specify MS function flexible enough to nest as many types of risk aversion as possible. As far as the author knows, Saha (1997) is the first who tackled this flexible specification problem of the MS function. He proposed a nonlinear mean-standard deviation (NLMS) model,

$$(13) \quad V(\sigma, \mu) = \mu^\theta - \sigma^\gamma,$$

where θ and γ are parameters that are restricted to $\theta > 0$ and $\gamma > 0$,⁵ and then argued that it is capable of displaying any type of risk aversion as shown in table 3. The argument derives from the properties of its slope of indifference curve,

$$(14) \quad S(\sigma, \mu) = \frac{\gamma}{\theta} \mu^{1-\theta} \sigma^{\gamma-1}.$$

The Cobb-Douglas type's slope of indifference curve fully covers Properties 5 and 6 of Proposition 2 under the parametric range, $\theta > 0$ and $\gamma > 0$. For example, the slope is decreasing (constant, increasing) in μ if $\theta > 1$ ($\theta = 1, \theta < 1$), while it is decreasing (constant, increasing) along rays through the origin when $\theta > \gamma$ ($\theta = \gamma, \theta < \gamma$). In other words, it is compatible with conditions (4-i), (4-ii), (4-iii), (5-i), (5-ii) and (5-iii). Besides, it is tractable that the type of risk aversion is determined only by the parameters' value. That is quite attractive in empirical work, because statistical test on the parameters directly indicates the agent's type of risk aversion. The NLMS model has been applied in the field of production economics under uncertainty. For example, Saha (1997) applied the model to examine the Kansas wheat producers' behavior under price uncertainty during 1979 and 1982. He obtained the empirical results that the parameter θ is significantly more than 1 for both small and large producers and that the parameters γ was significantly larger than θ for small producer and that null hypothesis, $\gamma = \theta$, was not rejected for large producer, concluding that both producers exhibit DARA and relative risk aversion can vary by the firm size. On the other hand, Abdulkadri, Langemeier and Featherstone (2003), applying the NLMS model, investigated Kansas dryland wheat producers, irrigated corn producers and

milk producers under price uncertainty during 1993 and 1997. They obtained the empirical results supporting that dryland wheat and milk producers are risk averter of IARA and IRRA while irrigated corn farmers are risk averter of CARA and IRRA. The interpretation of the NLMS model about their empirical results follows table 3.

Although Saha (1997) and Abdulkadri, Langemeier and Featherstone (2003) argue from the comparison of table 3 with table 1 that the NLMS model provides a more generalized approach than EU theory, there is room for more careful consideration on the interpretation of the NLMS model. As discussed in the beginning of this article, the MS approach established by Sinn (1983) and Meyer (1987) is a special case of EU theory and therefore has to satisfy the restrictions imposed on EU theory. This study focused on the three restrictions, (a) cardinal property of vNM utility function, (b) behavioral hypotheses that are translated under LS condition from EU theory into MS approach, (c) relationship between Arrow-Pratt's risk aversion measures. Apart from the restriction (a) that will be discussed in the next section, it is easy to see that the interpretation of table 3 does not fully satisfy the restrictions (b) and (c). Firstly, as for the restrictions (c), there are certain combinations of risk aversion that are not feasible such as CARA & DRRA, CARA & CRRA, IARA & DRRA and IARA & CRRA (table 1). So long as the NLMS model is interpreted by the MS approach based on EU theory, it may not display those combinations either. This excludes from table 3 the parameters' combinations such as $\theta = 1 \ \& \ \theta > \gamma$, $\theta = 1 \ \& \ \theta = \gamma$, $\theta < 1 \ \& \ \theta > \gamma$ and $\theta < 1 \ \& \ \theta = \gamma$. Secondly, the restrictions (b) and (c) derive Propositions 3 and 4 for the feasible combinations of risk aversion measures. As the NLMS model is additively separable, Proposition 3 (1) indicates that it may not display DARA under LS condition. There should be still some restriction that is overlooked by the interpretation of table 3. If MS function displays all the feasible combinations of the risk aversion measures under LS condition, then it must entirely satisfy conditions (1), (2), (3-i), (3-ii) and (3-iii) and

be fully compatible with conditions (4-i), (4-ii), (4-iii), (5-i), (5-ii) and (5-iii) (see table 2). The compatibility with conditions (4-i), (4-ii), (4-iii), (5-i), (5-ii) and (5-iii) is maintained under the parameter restrictions, $\theta > 0$ and $\gamma > 0$, as discussed by Saha (1997). Besides, the conditions (1) and (2) are satisfied since $V_\mu(\sigma, \mu) = \theta\mu^{\theta-1}$ and $V_\sigma(\sigma, \mu) = -\gamma\sigma^{\gamma-1}$. However, conditions (3-i), (3-ii) and (3-iii) are not always fulfilled under the initial parametric range, because $V_{\mu\mu}(\sigma, \mu) = \theta(\theta-1)\mu^{\theta-2}$, $V_{\sigma\sigma}(\sigma, \mu) = -\gamma(\gamma-1)\sigma^{\gamma-2}$, and $V_{\mu\mu}(\sigma, \mu)V_{\sigma\sigma}(\sigma, \mu) - V_{\mu\sigma}^2(\sigma, \mu) = -\theta(\theta-1)\gamma(\gamma-1)\mu^{\theta-2}\sigma^{\gamma-2}$. In order to satisfy them, a stronger parametric restriction, $0 < \theta \leq 1$ and $\gamma \geq 1$, is necessary. Although conditions (1) and (2) as well as conditions (3-i), (3-ii) and (3-iii) are fully met under the new parametric range, the full compatibility with conditions (4-i), (4-ii), (4-iii), (5-i), (5-ii) and (5-iii) is lost, as conditions (4-i) and (5-i) are not satisfied. It means that the NLMS model is reduced to a model that is capable of displaying the two types of risk aversion, CARA and IARA, under LS condition. Actually, if $\theta = 1$ and $\gamma > 1$, then the NLMS model is categorized into form (6) that displays CARA under LS condition, and when $0 < \theta < 1$ and $\gamma \geq 1$, it is a member of form (9) that displays IARA under LS condition. The reconsideration of the NLMS model alters the interpretation from table 3 to table 4, implying the difficulty in explaining Saha's empirical result that the production agents are risk averter of DARA by means of the NLMS model. To incorporate this type of risk aversion, some modification would be needed.

In fact, the NLMS model can be easily modified so that it explains Saha's (1997) empirical result. It is carried out by combining the MS functions proposed in the previous sections. The MS function is derived as follows,

$$(15) \quad V(\sigma, \mu) = (\mu^\delta - \sigma^\eta)^{\frac{1}{\delta}} \quad (1 \leq \delta \leq \eta; \mu^\delta - \sigma^\eta > 0),$$

where δ and η are parameters that are restricted to $1 \leq \delta \leq \eta$. Apparently, if $1 < \delta < \eta$, form (15) corresponds to form (12) that displays the combination of DARA and IRRA under LS condition, and when $1 < \delta = \eta$, it corresponds to form (10) that displays CRRA under LS condition. Furthermore, when $1 = \delta < \eta$, it is a member of form (6) that displays CARA under LS condition. These types of risk aversion expressed by form (15) are shown in table 5. Here, it is observed that there is a relationship between the NLMS model and form (15). Specifically, form (15) is derived from transforming the NLMS model by the concave function, $W = \{V(\sigma, \mu)\}^{\frac{1}{\theta}}$, and imposing the restrictions, $1 \leq \theta \leq \gamma$ and $\mu^\theta - \sigma^\gamma > 0$. In the following section, we consider the meaning of this mathematical relationship from economic point of view, and then discuss the implications for an empirical approach called “joint analysis of risk preference structure and technology” that has been recently employed in the field of production economics under uncertainty.

Discussion

It is well known that a positive monotonous transformation of utility function has no essential meaning in the case of consumer choice without uncertainty. Since the traditional consumer theory relies upon ordinal utility theory, the utility function may be transformed by a positive monotone function and then the transformed utility function is considered to be essentially identical to the original one. However, the situation is different in the case of decision-making problems under uncertainty, especially those based on EU theory such as the MS approach established by Sinn (1983) and Meyer (1987). EU theory belongs to cardinal utility theory in which vNM utility function is transformable only by positive linear function. If MS approach is interpreted within EU theory, the transformation of the MS function also needs to be linear (Proposition 1). Nonlinear transformation of the MS function contradicts the assumption of interpreting MS approach within EU theory. Therefore, the

NLMS model and form (15), related to each other by a nonlinear transformation, should be clearly distinguished.

Despite this, it is impossible to make a distinction between the two MS functions in an empirical approach called “joint analysis of risk preference structure and technology” that has been recently employed in the field of production economics under uncertainty (e.g., Saha 1997; Abdulkadri, Langemeier and Featherstone 2003; Nakashima 2006). This is discussed below. Joint analysis utilizes the first-order conditions resulting from the optimization of production model to estimate the structural parameters that indicate agent risk preference and production technology. The first-order conditions based on MS approach are generally written as

$$(16) \quad \frac{\partial \mu}{\partial x_i} - S(\sigma, \mu) \frac{\partial \sigma}{\partial x_i} = 0 \quad (x_i = 1, 2, \dots, n),$$

where x_i ($i = 1, 2, \dots, n$) denote the endogenous variables of the underlying economic model.

Then, the specifications of $S(\sigma, \mu)$, $\frac{\partial \mu}{\partial x_i}$ and $\frac{\partial \sigma}{\partial x_i}$ follow. The specification of $S(\sigma, \mu)$ is, of

course, determined by the form of MS function that represents agent’s attitude toward

random payoff, while the specifications of $\frac{\partial \mu}{\partial x_i}$ and $\frac{\partial \sigma}{\partial x_i}$ depend on the remaining factors of

the model such as a random factor involved in the model (e.g., price uncertainty or yield

uncertainty) and a functional form chosen to represent technological constraint (e.g.,

production function or cost function). In the procedure for developing a joint analysis model,

special attention needs to be paid to the specification of $S(\sigma, \mu)$. Because MS function is

represented merely by the slope of indifference curve, the difference of MS functions such as

the NLMS model and form (15) that are related to each other by a functional transformation is cancelled. Eventually, the MS functions yield the same joint analysis model. Therefore, it is impossible to distinguish the NLMS model and form (15) in the empirical approach. This also implies that it is possible to provide more than one explanation for a joint analysis model. For example, regardless of the two types of MS functions, the NLMS model and form (15), the same joint analysis model arises, and therefore, there are at least two ways of explanation for Saha (1997) and Abdulkadri, Langemeier and Featherstone (2003)'s models, respectively. One is shown in table 4 that is derived from the NLMS model, and the other is table 5 that comes from form (15). The empirical result of Abdulkadri, Langemeier and Featherstone (2003) is interpreted by the former, while that of Saha (1997) is explained by the latter. Needless to say, more than one interpretation of a joint analysis model can cause confusion. These situations take place when the slope of indifference curve, $S(\sigma, \mu)$, covers several types of risk aversion described by Properties 5 and 6 of Proposition 2 and then is rationalized by more than one MS function in such a way that the coverage of the types of risk aversion is partial and different. As pointed out by Saha (1997), Cobb-Douglas type's slope of indifference curve (14) potentially covers the entire pattern of Properties 5 and 6 but the NLMS model and form (15) rationalize it only partially and differently (tables 4 and 5). That causes a multi-interpreting situation. In order to avoid this, we need to examine whether or not there exists such an MS function that fully rationalizes the types of risk aversion explained by the flexible slope of indifference curve. This remains for further research.

Conclusion

This study examined the functional specification of MS approach under LS condition. The contribution of this study can be summarized as three parts. Firstly, the conditions that MS function has to fully satisfy under LS condition were thoroughly discussed and then the full

set of conditions was made clear (Proposition 1 and table 2). Secondly, the examination based on the full set of conditions derived the properties of MS function on the applicability of additive separability (Proposition 3) and the curvature of expansion path which links the points that give the same slope of indifference curve in $\sigma - \mu$ axis (Proposition 4). It revealed that attention has not been sufficiently paid to the full set of conditions in interpreting the LMS model and the NLMS model. Thirdly, the interpretation of the NLMS model was reconsidered in detail (table 4) and then an alternative NLMS model (15) which also derives Cobb-Douglas type's slope of indifference curve (14) was proposed (table 5). The comparison of the two NLMS models and their implication to joint analysis approach might give us an idea as to the new direction of further research. If the slope of indifference curve, $S(\sigma, \mu)$, covers several types of risk aversion described by Properties 5 and 6 of Proposition 2, it is necessary to examine whether or not there exists such an MS function that rationalizes all the types of risk aversion. In tackling the unsolved problem, Cobb-Douglas type's slope of indifference curve (14), proposed by Saha (1997), seems to provide a good starting point, as it takes advantage of not only flexible but also tractable attribute of MS approach under LS condition.

Footnotes

¹ Because of this linearity of the payoff in random factor, Sinn (1983) referred to a set of random variables for which the LS condition holds as a linear distribution class.

² It is also known that EU theory derives MS functions under alternative restrictions such as quasi utility, normal distribution and the combination of semi-logarithmic utility and lognormal distribution. The derived MS functions have to be properly restricted (see, Tobin 1958, 1969; Chipman 1973; Feldstein 1969). Recently, it was established that the rank dependent expected utility theory, a generalized EU theory, also derives MS function under the monotone mean-preserving spread (Ormiston and Quiggin 1994).

³ As for the restriction imposed on MS function, Property 3 is not necessary, as it is automatically fulfilled when Properties 1 and 2 are satisfied.

⁴ Similarly, it is also shown that the MS function, $V(\sigma, \mu) = (\mu^\delta - \sigma^\eta)^{\frac{1}{\eta}}$ ($1 < \delta < \eta; \mu^\delta - \sigma^\eta > 0$)

exhibits the combination of IARA and DARA.

⁵ Unlike Saha (1997), this study limits the discussion to the case of $\gamma > 0$.

References

- Abdulkadri, A.O., M.R. Langemeier, and A.M. Featherstone. 2003. "Estimating Risk Aversion Coefficients for Dryland Wheat, Irrigated Corn and Dairy Producers in Kansas." *Applied Economics* 35: 825-834.
- Arrow, K. J. 1974. *Essays in the Theory of Risk-Bearing*. Amsterdam: North-Holland.
- Chavas, J.-P., and M. T. Holt. 1996. "Economic Behavior under Uncertainty: A Joint Analysis of Risk Preferences and Technology." *Review of Economics and Statistics* 78: 329-335.
- Chipman, J.S. 1973. "The Ordering of Portfolio in Terms of Mean and Variance." *Review of Economic Studies* 40: 167-190.
- Eggert, H., and R. Tveteras. 2004. "Stochastic Production and Heterogeneous Risk Preferences: Commercial Fishers' Gear Choices." *American Journal of Agricultural Economics* 86: 199-212.
- Ehrlich, I., and G. S. Becker. 1972. "Market Insurance, Self-Insurance, and Self-Protection." *Journal of Political Economy* 80: 623-648.
- Feder, G. 1977. "The Impact of Uncertainty in a Class of Objective Functions." *Journal of Economic Theory* 16: 504-512.
- Feder, G. 1980. "Farm Size, Risk Aversion and the Adoption of New Technology under Uncertainty." *Oxford Economic Papers* 32: 263-283.
- Feder, G., R. E. Just, and A. Schmitz. 1980. "Futures Markets and the Theory of the Firm under Price Uncertainty." *Quarterly Journal of Economics* 94: 317-328.
- Feldstein, M. S. 1969. "Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection." *Review of Economic Studies* 36: 5-12.
- Fishburn, P.C., and R. B. Porter. 1976. "Optimal Portfolios with One Safe and One Risky Asset: Effects of Changes in Rate of Return and Risk." *Management Science* 22:

- 1064-1073.
- Freund, R. J. 1956. "The Introduction of Risk into a Programming Model." *Econometrica* 24: 253-263.
- Hawawini, G. 1978. "A Mean-Standard Deviation Exposition of the Theory of the Firm under Uncertainty: A Pedagogical Note." *American Economic Review* 68: 194-202.
- Ishii, Y. 1977. "On the Theory of the Competitive Firm under Price Uncertainty: Note." *American Economic Review* 67: 768-769.
- Lau, L. J. 1969. "Duality and the Structure of Utility Functions." *Journal of Economic Theory* 1: 374-396.
- Leathers, H. D., and J. C. Quiggin. 1991. "Interactions between Agricultural and Resource Policy: The Importance of Attitudes toward Risk," *American Journal of Agricultural Economics* 73: 757-764.
- Love, H. A., and S. T. Buccola. 1991. "Joint Risk Preference-Technology Estimation with a Primal System." *American Journal of Agricultural Economics* 73: 765-774.
- Markowitz, H. 1952. "Portfolio Selection." *Journal of Finance* 7: 77-91.
- Meyer, J. 1987. "Two-Moment Decision Models and Expected Utility Maximization." *American Economic Review* 77: 421-430.
- Meyer, J., and L. J. Robison. 1988. "Hedging under Output Price Randomness." *American Journal of Agricultural Economics* 70: 268-272.
- Nakashima, T. 2006. "Linear Structural Models of Production under Price Uncertainty: A Mean-Standard Deviation Approach." *Japanese Journal of Rural Economics* 8: 1-11.
- Nelson, C. H., and C. Escalante. 2004. "Toward Exploring the Location-Scale Condition: a Constant Relative Risk Aversion Location-Scale Objective Function." *European Review of Agricultural Economics* 31: 273-287.
- Ormiston, M. B., and J. Quiggin. 1994. "Two-Parameter Decision Models and

- Rank-Dependent Expected Utility.” *Journal of Risk and Uncertainty* 8: 273-282.
- Peterson, J. M., and Y. Ding. 2005. “Economic Adjustment to Groundwater Depletion in the High Plains: Do Water-Saving Irrigation Systems Save Water?” *American Journal of Agricultural Economics* 81: 147-159.
- Pratt, J. W. 1964. “Risk Aversion in the Small and in the Large.” *Econometrica* 32: 122-136.
- Saha, A. 1997. “Risk Preference Estimation in the Nonlinear Mean Standard Deviation Approach.” *Economic Inquiry* 35: 770-782.
- Saha, A., C. R. Shumway, and H. Talpaz. 1994. “Joint Estimation of Risk Preference Structure and Technology Using Expo-Power Utility.” *American Journal of Agricultural Economics* 76: 173-184.
- Sandmo, A. 1970. “The Effect of Uncertainty on Saving Decisions.” *Review of Economic Studies* 37: 353-360
- Sandmo, A. 1971. “On the Theory of the Competitive Firm under Price Uncertainty.” *American Economic Review* 61: 65-73.
- Sinn, H.-W. 1983. *Economic Decisions under Uncertainty*. 2nd. English. ed. New York : North-Holland.
- Tobin, J. 1958. “Liquidity Preference as Behavior towards Risk.” *Review of Economic Studies* 25: 65-86.
- Tobin, J. 1969. “Comment on Borch and Feldstein.” *Review of Economic Studies* 36: 13-14.
- von Neumann, J., and O. Morgenstern. 1944. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.

Table 1. Relationships among Arrow-Pratt risk aversion measures

	DRRA	CRRA	IRRA
DARA	Feasible	Feasible	Feasible
CARA	Not feasible	Not feasible	Feasible
IARA	Not feasible	Not feasible	Feasible

Source: Saha (1997)

Table 2. The conditions imposed on MS function under LS condition

	DRRA	CRRA	IRRA
DARA	(1)(2)(3-i) (3-ii)(3-iii) (4-i)(5-i)	(1)(2)(3-i)(3-ii)(3-iii) (4-i)(5-ii)	(1)(2)(3-i)(3-ii)(3-iii) (4-i)(5-iii)
CARA	Not feasible	Not feasible	(1)(2)(3-i)(3-ii)(3-iii) (4-ii)(5-iii)
IARA	Not feasible	Not feasible	(1)(2)(3-i)(3-ii)(3-iii) (4-iii) (5-iii)

Table 3. Saha's argument on the NLMS model

	DRRA	CRRA	IRRA
DARA	$\theta > 1, \theta > \gamma$	$\theta > 1, \theta = \gamma$	$\theta > 1, \theta < \gamma$
CARA	$\theta = 1, \theta > \gamma$	$\theta = 1, \theta = \gamma$	$\theta = 1, \theta < \gamma$
IARA	$\theta < 1, \theta > \gamma$	$\theta < 1, \theta = \gamma$	$\theta < 1, \theta < \gamma$

Source: Saha (1997)

Table 4. Reconsideration of the NLMS model

	DRRA	CRRA	IRRA
DARA	Not applicable	Not applicable	Not applicable
CARA	Not feasible	Not feasible	$\theta = 1, \theta < \gamma$
IARA	Not feasible	Not feasible	$\theta < 1, 1 \leq \gamma$

Table 5. Form (15) and the corresponding types of risk aversion

	DRRA	CRRA	IRRA
DARA	Not applicable	$1 < \delta = \eta$	$1 < \delta < \eta$
CARA	Not feasible	Not feasible	$1 = \delta < \eta$
IARA	Not feasible	Not feasible	Not applicable