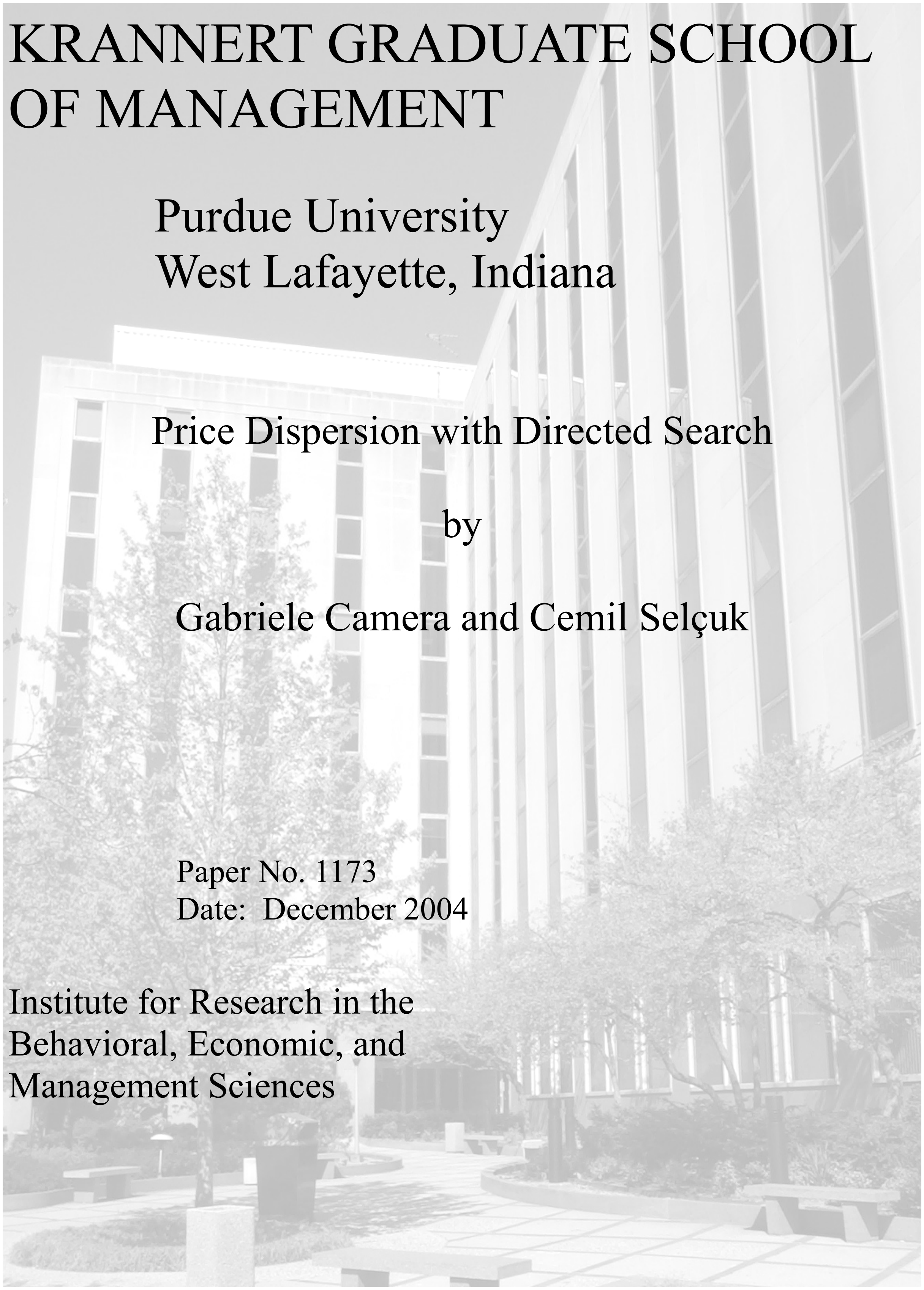

# KRANNERT GRADUATE SCHOOL OF MANAGEMENT

Purdue University
West Lafayette, Indiana

## Price Dispersion with Directed Search

by

Gabriele Camera and Cemil Selçuk

Paper No. 1173
Date: December 2004

Institute for Research in the Behavioral, Economic, and Management Sciences

# Price Dispersion with Directed Search[1]

Gabriele Camera — Purdue University
Cemil Selçuk — Purdue University

December 28, 2004

**Abstract**

We study a market where identical capacity-constrained sellers compete to attract identical buyers, via price advertisements. Once buyers reach a store, prices might be renegotiable in a manner that is responsive to excess demand. We focus on strongly symmetric equilibria, proving their existence and providing explicit solutions for the distributions of advertised and sale prices as functions of market characteristics. Since variations in the posted price can affect the store's attractiveness and the incidence of haggling, the model endogenizes the 'pricing convention' prevailing in the market and generates several empirically testable predictions on market behavior.

Keywords: Directed Search, Endogenous Trading Mechanisms, Market Frictions, Price Dispersion
JEL: C780, D390, D490, E390,

[1] We thank for comments seminar participants at Purdue (particularly J. Abrevaya and D. Kovenock), the Midwest Theory Meetings and Macro Meetings in the Spring 2004, and the Midwest Theory Meetings in the Fall 2004. The NSF grant DMS-0437210 partly supported G. Camera's research.

## 1 Introduction

The existence of price disparities for homogeneous products is empirically well documented (e.g. see Pratt et al., 1979 or Baye et al., 2004). In providing a rationale for such observations, the theoretical literature has stressed the importance of market frictions, for instance information heterogeneity as in Varian (1980) or costly search as in Carlson and McAfee (1983). Several studies have relied on frameworks where frictions are made *explicit* by means of a trade process based on random search. This friction limits the information available on prices in Burdett and Judd (1983), for example, and impairs the buyers' ability to match to the cheapest sellers in Camera and Corbae (1999).

Our work broadly contributes to this research discourse. We study the theoretical underpinnings of equilibrium price dispersion in a market for a homogeneous good or service. We do not impose ex-ante heterogeneity elements or information frictions and make trade frictions explicit by assuming a process of exchange that is decentralized and subject to spatial and capacity constraints. We also set buyers free to *direct* their search: trade matches do not follow the rather disorganized process of stochastic encounters so common in matching frameworks. Rather, buyers are free to go where they prefer—much as the "informed customers" in the pricing game studied by Baye et al. (1992)—and stores compete to attract buyers by costlessly advertising (or posting) a price.

Our model is based on that of Burdett, Shi and Wright (2001). There, a countable number of buyers and capacity-constrained sellers makes uncoordinated pricing and trading choices. Every buyer likes equally the indivisible good owned by each seller. Every seller advertises a price at which she commits to sell and, given this information, buyers independently select to approach a seller. In a symmetric equilibrium, sellers compete for customers by listing a price below the buyer's reserve value and buyers are indifferent across stores. Although this creates a non-degenerate distribution of demand, every sale occurs at the listed price and stores that realize excess demand simply ration their good. Hence, the price distribution is degenerate, unless heterogeneity is introduced (e.g. rationing-wary buyers may prefer 'larger' stores, which thus advertise higher prices).

In reality, capacity-constrained sellers have incentives to sell above the advertised price when demand is unusually high or to attract customers by committing to price reductions if business is slow. This is perhaps why houses tend to sell above their list price in the densely populated L.A. county (unlike Tippecanoe county), why new hot car models trade above their MSRP (unlike older ones), why hotels rent rooms below the advertised prices when demand is slow, or why airlines list fares 'subject to change without notice.' In short, it is desirable to account for the fact that sale prices tend to respond to excess demand, even if advertised prices do not.

To do so we generalize Burdett et al. relaxing the assumption that the listing of a price *necessarily* precludes further negotiations at the store.[2] For instance, we consider markets where sellers can exclude discounts but are free to suddenly raise the sale price, or markets with better consumer protection, where stores can offer price reductions but cannot sell above the advertised price. Of course, when the posted price is not binding one must indicate how sale prices are determined. For generality, we do not impose a specific price-formation protocol; instead, we provide results valid under any protocol generating prices that increase in the store's excess demand.

In this framework we prove a theorem of existence of strongly symmetric subgame perfect equilibria, i.e., equilibria in which agents take the same action both on and off the equilibrium path. Symmetry offers enough tractability to fully characterize the distributions of advertised and sale prices by means of explicit solutions. Hence, the model can be used to make predictions on market behavior that can be empirically tested. For example, we find that haggling over price discounts—as car dealers tend to do—is generally optimal from the seller's point of view only if the expected demand matches the store's capacity. Otherwise, trading at a fixed price is more profitable.

Equilibrium prices respond intuitively to commitment and market composition. Generally speaking, sellers advertise low prices in markets that have a small customer base. In such a 'buyer's market' average sale prices are low but dispersion in sale prices is considerable. As the customer base expands we move into a seller's market where advertised and average sale prices grow, but their dispersion drops. What is the intuition? When the customer base is small, a store's distribution of demand—hence expected profit—impinges heavily on the distribution of demand at other stores. Realizing this, stores compete aggressively for customers by listing low prices. As the customer base expands the covariance of demand across stores falls, lessening the need to compete.

Our analysis also contributes to a growing literature on endogenous selection of pricing mechanisms (see Camera and Delacroix, 2004, for references). Indeed, the 'pricing convention' adopted in the market, i.e. the incidence of fixed-price trading versus negotiations, is determined endogenously. We find that if sellers can commit to avoid price reductions,

[2] Coles and Eeckhout (2003), take a similar viewpoint letting stores run an auction in Burdett et al. A two-point price distribution arises but sale prices are invariant to excess demand (be it two or two-million buyers), to market size and composition. Arbatskaya (2004) studies the distribution of prices when stores commit to a posted price but are sampled in a predetermined order by consumers differing in search costs. Those with higher cost search less and spend more.

then haggling is pervasive in a buyer's market but very rare in a seller's market. The opposite occurs when prices cannot exceed what advertised. To the extent that haggling involves a resource cost (say, time or personnel) the model provides a rationale as to why fixed-price sales seem the convention in large but not in smaller markets.

## 2 The Model

The environment builds on the directed search model of Burdett, Shi and Wright (2001). It is a static economy with one indivisible good type, capacity constraints, and a finite number of spatially separated agents; $S \geq 2$ identical sellers with one good each, and $B \geq 2$ identical buyers without endowment. The good generates consumption utility linear in consumption and normalized to zero, for a seller, and one, for a buyer. Utility is also transferable so there are gains from trade.

Sellers compete for buyers. They advertise by posting a publicly observable price $r \in [0, 1]$, the *reference* price (alternatively, advertised or posted price). In this context the act of "posting" $r$ simply makes it costlessly observable to every market participant. Once reference prices have been posted, buyers simultaneously and independently select to visit a single seller (search for a second store is assumed very costly). Since buyers' choices are uncoordinated, different stores might be visited by different number of buyers. To account for this possibility we let $n = 0, 1, 2..., B$ denote the *realized demand* at the store, i.e. the number of buyers who end up visiting the store.

We also define the variable $\lambda = \frac{B}{S}$ we call the "customer base" of a store, to capture the notion of market composition. If there are many sellers but only few buyers, then stores have a small customer base $\lambda$ and we are in a *buyer's market*. When $\lambda$ is large we are in a *seller's market* with few stores serving a large customer base. Any $n > 1$ results in excess demand due to capacity constraints. In this case the seller selects a buyer to trade with, at random. Thus, the existence of unit-capacity constraints contributes to make trade frictions explicit. In equilibrium, market participants may experience idiosyncratic trading risk; seller can experience demand shocks, while buyers can experience rationing.

Regarding sale price determination we relax Burdett et al.'s assumption that sellers commit to charge $r$. Precisely, consider a store with $n$ customers. As in Burdett et al. we assume that the seller chooses a buyer at random, with equal probability among all $n$ present at the store. However, we consider the possibility that the transaction price might differ from $r$, being determined via some endogenous price-formation mechanism—referred to as "negotiations" for short—that is taken as given. The available *commitment technology* specifies whether negotiations can take place and who can initiate them.

Specifically—once the seller has chosen a customer—the technology exogenously gives to either the seller or the buyer (or both) the option to initiate negotiations. We denote by $\boldsymbol{\theta} = (\theta_b, \theta_s)$ such a technology, where $\theta_i = 0, 1$ is the probability that agent $i = b, s$ ($b$ for buyer and $s$ for seller) is given the option to initiate negotiations. We clarify how this formalization captures four broad notions of price commitments, in what follows.

## 3 The Determination of Sale Prices

The economic interactions can be thought of as proceeding in three stages. First sellers choose and 'post' $r$ simultaneously and independently. These selections are observed by every agent. In the second stage, buyers choose which store to visit, simultaneously and independently. Following these selections, every buyer reaches some store and the demand realized at that store is observed by everyone present. In the third stage, at each store the seller selects a buyer at random, among those present. Following this selection a sale may take place at a price that is different from what originally posted, depending on the seller's ability to commit to it.

We study the strongly symmetric subgame perfect equilibria of this economy. These are outcomes in which agents take the same action both on and off the equilibrium path (see Abreu, 1986). In particular, in equilibrium every store optimally selects the same reference price $r$ and every buyer optimally visits any store with equal probability. We move backwards in the analysis starting by determining the optimal sale price at a store that has posted $r$ and is visited by $n$ buyers. Then, we study the optimal choice of store, for the representative buyer, and the optimal choice of price $r$ for the representative store. We start by deriving the optimal sale price, given some $\boldsymbol{\theta}$, at a store that posted $r$ and has $n$ customers.

### 3.1 The Outcome of Negotiations at the Store

Consider a match between a seller $i = 1, 2, ..., S$ and $n = 1, ..., B$ buyers; $n$ is observed by everyone in the match. Abstract (for a moment) from $r$ and suppose seller and buyers were free to determine the transaction price via some endogenous price-formation mechanism that is taken as given by everyone.[3] Let $q_n$ be the price arising from such a negotiations process, denoting $q_0 = 0$ and $q_{B+1} = 1$. For generality we do not impose a specific mechanism but simply make three assumptions on the properties of its outcome. First, the price-formation mechanism leads to a trade without delay at a unique price $q_n$.

[3] For example, the seller can auction the good (as in Coles and Eeckhout, 2003, or Benoit, Kennes and King, 2000) or there can be a negotation process whereby the seller makes an initial proposal to some buyer, calling onto other buyers if the initial offer is rejected, as in Camera and Selcuk (2004).

Second, no party gets the entire surplus. Third, the resulting price responds positively to excess demand at the store, but is unaffected by factors outside the store (e.g. distribution of demand elsewhere). Formally, assume

$$0 < q_n < q_{n+1} < 1 \text{ and } \frac{\partial q_n}{\partial B} = \frac{\partial q_n}{\partial S} = 0 \text{ for all } n = 1, ..., B. \tag{1}$$

A negotiation framework that generates (1) is studied by Camera and Selcuk (2004). There, one seller and $n$ buyers randomly alternate in making offers, discounting future utility by $\beta \in (0, 1)$. If $\gamma \in (0, 1)$ is the seller's probability of making an offer then

$$q_n = \frac{(n-\beta)[1-\beta(1-\gamma)]}{n(1-\beta)+\beta\gamma(n-1)}. \tag{2}$$

This captures the notion that—when demand is strong—stores can negotiate prices up by playing buyers against each other. Our numerical analysis will assume $q_n$ satisfies (2) with $\gamma = 0.3$ and $\beta = 0.9$.

### 3.2 The Sale Price at a Store Visited by $n$ Customers

Now that we know the outcome of possible negotiations, we can discuss the *sale price*. Denote $p_n$ the sale price at a store visited by $n \geq 1$ buyers, letting $p_0 = 0$. Linearity in preferences implies that the seller enjoys utility $p_n$ and the buyer $1 - p_n$.

The interaction between seller and the $n$ customers proceeds as follows. The seller chooses a buyer at random, with equal probability among all $n$ present at the store. Then, depending on the commitment technology $\boldsymbol{\theta} = (\theta_b, \theta_s)$, someone may get the option to start the negotiations process. Given this option, denote $\eta_i$ the (conditional) probability to start negotiations, for $i = b, s$. It is assumed that if no-one selects negotiations trade takes place at the listed price, so $p_n = r$. Otherwise, negotiations take place and result in the sale price $p_n = q_n$. If we focus on pure strategies, the individually optimal choices are

$$\eta_s = \begin{cases} 1 & \text{if} \quad r < q_n \\ 0 & \text{if} \quad r \geq q_n \end{cases} \quad \text{and } \eta_b = \begin{cases} 0 & \text{if} \quad r < q_n \\ 1 & \text{if} \quad r \geq q_n. \end{cases} \tag{3}$$

It follows

**Lemma 1.** *Let $r \in [0, 1]$ be the price posted by a store visited by $n = 1, ..., B$ buyers and let $q_n$ be the price expected to arise from negotiations. Then trade takes place at price*

$$p_n = \begin{cases} q_n & \textit{if} \quad \boldsymbol{\theta} = \boldsymbol{\theta}_N \equiv (1, 1) \\ \min(q_n, r) & \textit{if} \quad \boldsymbol{\theta} = \boldsymbol{\theta}_C \equiv (1, 0) \\ \max(q_n, r) & \textit{if} \quad \boldsymbol{\theta} = \boldsymbol{\theta}_F \equiv (0, 1) \\ r & \textit{if} \quad \boldsymbol{\theta} = \boldsymbol{\theta}_X \equiv (0, 0). \end{cases} \tag{4}$$

The commitment technology $\boldsymbol{\theta} = \boldsymbol{\theta}_k$, for $k = C, F, N, X$, delimits the set of sale prices, relative to $r$, allowing us to formalize four basic notions of pricing conventions:

| | $\theta_s = 0$ | $\theta_s = 1$ |
|---|---|---|
| $\theta_b = 0$ | Fixed Prices | Price Floors |
| $\theta_b = 1$ | Price Ceilings | Negotiations |

If $\boldsymbol{\theta} = \boldsymbol{\theta}_N$ we say that there is no commitment to the posted price, while sales necessarily occur at that price when there is full commitment, i.e. $\boldsymbol{\theta} = \boldsymbol{\theta}_X$ (as assumed in Burdett et al.). We can then have two cases with limited commitment: the 'price floor' $\boldsymbol{\theta} = \boldsymbol{\theta}_F$, when sellers charge at least $r$, and the 'price ceiling' $\boldsymbol{\theta} = \boldsymbol{\theta}_C$ when sale prices cannot exceed $r$.[4]

The proof of the Lemma is obvious. Once the seller selects a buyer to trade with—among the $n$ identical buyers present at the store—they both know that negotiations will generate $q_n$. Thus, the buyer will negotiate—if given the option—only if $q_n \leq r$. The converse is true for the seller. Thus, $p_n \leq r$ under price ceilings and $p_n \geq r$ under price floors. Using (4) we let

$$P = \{r\} \cup \{q_n\}_{n=1}^{B} \tag{5}$$

denote the ordered set of possible equilibrium sale prices, for any given $\boldsymbol{\theta}$ and any possible $n$. We will let $p$ denote a generic element of $P$.

When $q_n$ satisfies (1), then $\{q_n - q_{n-1}\}_{n=1}^{B}$ is a positive decreasing sequence, so $\{p_n\}_{n=1}^{B}$ and $\{p_n - p_{n-1}\}_{n=1}^{B}$ are non-decreasing and non-increasing. In short, the sale price $p_n$ tends to increase in the excess demand. From (4) we see that $p_n$ is defined by functions that are continuous and non-decreasing in $r$, bounded above by one and below by zero. Also, $\frac{\partial p_n}{\partial r} = 0, 1$ depending on both $r$ and $\boldsymbol{\theta}$. Clearly, $\frac{\partial p_n}{\partial r} = 0$ if $p_n = q_n$ and $\frac{\partial p_n}{\partial r} = 1$ if $p_n = r$. However, $\frac{\partial p_n}{\partial r} = 1$ either (i) if $r < q_n$ when $p_n = \min(q_n, r)$ or (ii) if $r \geq q_n$ when $p_n = \max(q_n, r)$; it is zero otherwise. Thus, we say that sale prices at store $i$ are *responsive* to the price posted by the store if $\frac{\partial p_n}{\partial r} = 1$ for some $n \geq 1$. Of course, sale prices will generally responsive to $r$ *only if* some commitment is available.

There are two key consequences. First, variations in the posted price $r$ will impinge on the store's "attractiveness", i.e., the buyers' choice to visit it, whenever the posted price affects the set of feasible sale prices. Second, a weakening of the commitment to a fixed

[4] A practical way to commit to a certain price (or price range) is by means of hiring sale representatives. For example, the commitment to fixed prices is credible if reps trading at non-authorized prices are fired, and the commitment to charging at least $r$ is credible is sale reps are compensated by $p_n - r$.

price $r$ implies that sale prices will not *solely* hinge on the posted price (as in Burdett et al.) but also on the excess demand experienced by the store. Thus, in our framework a change in the reference price $r$ has a strategic impact not only because it may modify the distribution of buyers at the store, but also because it may affect the distribution of sale prices at that establishment. This is formalized in the following section.

## 4 Individually Optimal Selection of Prices and Stores

We now study the visiting and pricing choices of a representative agent. Recall that we are focusing on strongly symmetric equilibria and that agents make choices in isolation taking as given the strategies of others. Thus, we differentiate the strategy of everyone else from that of a representative buyer or seller, by using a superscript '*'.

### 4.1 The buyer's problem

Let $v_i$ denote the probability that the representative buyer visits store $i = 1, 2, ..., S$, and define $\mathbf{v} = (v_i)_{i=1}^S$ . Thus, $\mathbf{v} \in \Delta^S$, i.e. $\mathbf{v}$ is a vector in the $S-$dimensional unit simplex $\Delta^S = \{\mathbf{v} \in \mathbb{R}^S : \mathbf{v} \geq 0 \text{ and } \sum_{i=1}^{S} v_i = 1\}$. To find the optimal $\mathbf{v}$ we must examine how the buyer's expected payoff, conditional on being at store $i$, compares to the expected payoff from being at any other store.

To do so suppose that store $i$ posts $r$ and every other store posts $r^*$, possibly different than $r$. Denote by $f_n(B, v_i^*)$ the probability that a specific seller $i$ is visited exactly by $n$ out of $B$ possible buyers, given that every buyer visits this store with probability $v_i^*$. Since buyers choose stores in an uncoordinated manner the probability of visiting any store is independent across buyers. It follows that, under symmetry, the distribution of buyers *at a store* is given by $\text{bin}(B, v_i^*)$, i.e.

$$f_n(B, v_i^*) = \binom{B}{n} (v_i^*)^n (1 - v_i^*)^{B-n} \quad \text{for} \quad n = 0, 1, ..., B. \tag{6}$$

It follows that, conditional on being at store $i$, the representative buyer faces probability $f_n(B - 1, v_i^*)$ that there are $n = 0, 1, ..., B - 1$ other customers. In that contingency the expected payoff for the buyer is $\frac{1-p_{n+1}}{n+1}$. Given the expected sale price $p_{n+1}$, then $1 - p_{n+1}$ is the buyer's payoff if he gets to buy, which occurs with probability $\frac{1}{1+n}$.

Letting $U_i$ denote the buyer's expected payoff, conditional on being at store $i$, we have

$$U_i = \sum_{n=0}^{B-1} \frac{f_n(B - 1, v_i^*)(1 - p_{n+1})}{n + 1}. \tag{7}$$

Clearly, the buyer prefers to visit the store where $U_i$ is the highest. It is easy to demonstrate (see also the proof of Lemma 3, later) that, all else equal, $U_i$ is lower at more 'popular stores', i.e. $\frac{\partial U_i}{\partial v_i^*} < 0$. This result hinges on the higher expected demand generated by a larger $v_i^*$. First, there is always an extensive margin effect: the buyer's ability to make a purchase falls as the number of customers increases, because stores are capacity constrained. Second, there *may* be an intensive margin effect: higher expected demand means higher prices since $\{p_n\}$ is a non-decreasing sequence.

Expression (7) tells us that, all else equal, the representative buyer will prefer to be at stores expected to be less crowded or with lower sale prices. It follows that a buyer may be indifferent between a cheap and an expensive store, if the latter is also likely to have less customers. Of course, from our earlier discussion we know that—given $n$—the sale price *may* differ across stores only if the stores advertised different prices (see (4)).

Now consider the representative buyer's selection of store $i$ versus other stores, given that everybody else is playing an identical strategy (strong symmetry). He can visit store $i$ that has posted $r$ and sells at prices $\{p_n\}$, or *any* other store $h \neq i$ that has posted $r^*$ and sells at prices $\{p_n^*\}$. Given that $\mathbf{v}^*$ is the selection of every other buyer, then we have $v_h^* = v^*$ for all $h \neq i$ where

$$v^* = \frac{1 - v_i^*}{S - 1}, \tag{8}$$

a decreasing function of $v_i^*$. Denote by $U$ the representative buyer's expected payoff from being at any store $h \neq i$,

$$U = \sum_{n=0}^{B-1} \frac{f_n(B-1, v^*)(1 - p_{n+1}^*)}{n+1}. \tag{9}$$

It follows that if $v_i$ satisfies

$$v_i = \begin{cases} 1 & \text{if} \quad U_i > U \\ [0,1] & \text{if} \quad U_i = U \\ 0 & \text{if} \quad U_i < U \end{cases} \tag{10}$$

then we say that $v_i$ is individually optimal or, equivalently, that it is a best response of the representative buyer. Clearly,

$$v = \frac{1 - v_i}{S - 1}$$

where $v_i$ depends on posted prices only if these affect sale prices, and it depends on the choices $v_i^*$ of other buyers, as these affect the representative buyer's trading risk.

### 4.2 The seller's problem

Given some commitment technology $\boldsymbol{\theta}$, here we discuss the choice $r$ of a representative seller $i$ when every other seller selects $r^*$ and every other buyer plays $\mathbf{v}^*$. Clearly, the optimal $r$ must maximize the seller's expected profit from trading that depends on expected demand, i.e. the probability of visits $v_i^*$, and on the possible sale prices $\{p_n\}$. Since $\mathbf{v}^*$ generally depends on $r$ and $r^*$, we let $W(r, r^*)$ denote the seller's expected profit, i.e.

$$W(r, r^*) = \sum_{n=0}^{B} f_n(B, v_i^*) p_n \tag{11}$$

Since $r$ is chosen while taking as given $r^*$, we define the seller's choice set by letting $\varphi : [0,1] \twoheadrightarrow [0,1]$ define a continuous correspondence with nonempty compact values such that $\varphi(r^*) = [0,1]$ for $r^* \in [0,1]$. Thus, we can define the "value function" $\hat{W} : [0,1]^2 \rightarrow [0,1]$ by

$$\hat{W}(r) = \max_{r \in \varphi(r^*)} W(r, r^*)$$

and the correspondence $\mu : [0,1] \twoheadrightarrow [0,1]$ of maximizers

$$\mu(r^*) = \left\{ r \in \varphi(r^*) : W(r, r^*) = \hat{W}(r) \right\}.$$

Therefore, if $\mu$ is nonempty valued, we say that $r$ is individually optimal or, equivalently, it is a best response of seller $i$, if

$$r \in \mu(r^*). \tag{12}$$

Of course, in a symmetric equilibrium sellers must post identical prices, or

$$r = r^*. \tag{13}$$

Buyers must also select identical strategies. It is easy to prove that this implies each buyer must visit every store with an identical probability.[5] Specifically,

$$v_i = v_i^* = \frac{1}{S} \textit{ for all } i = 1, 2, ..., S. \tag{14}$$

We can now provide the following definition of symmetric equilibrium.

**Definition 2**. *Given a commitment technology $\theta$, a strongly symmetric subgame perfect equilibrium (SSE) is a reference price $r$, a vector of sale prices $\{p_n\}_{n=1}^{B}$ and a vector of probabilities $\mathbf{v}$ that satisfy (2)-(4), (6)-(10) and (11)-(14).*

Before proving existence of equilibrium, it may be helpful to remark on some of our modeling choices. The assumption that buyers cannot visit several stores in sequence captures the notion of existence of search costs that, in the short run, 'lock-in' consumers at

[5] Of course indifference across stores implies that $v_i = \frac{1}{S}$ for every $i$. In the appendix we formalize why $\mathbf{v} = \mathbf{v}^*$ necessarily implies that each element of $\mathbf{v}^*$ must have value $\frac{1}{S}$.

a store. Relative to richer dynamic analyses, this formulation allows a precise identification of the effect that market composition and commitment have on advertised and sale prices. The focus on SSE (where strategies are mixed) captures the notion that traders make uncoordinated decisions, which seems a natural description of several market settings. From a technical standpoint, this also allows us to clearly characterize the distribution of sale prices across different markets; we can focus on sellers' pricing behavior while 'controlling' for the equilibrium distribution of demand—invariant to the posted prices—without assuming it exogenous (as, say, in a random search model).

## 5 Existence of Equilibrium

To discuss existence we move in steps. First, given a pair $(r, r^*)$, we prove existence of a unique symmetric best response $v_i^* = v(r, r^*)$ and we characterize it relative to $r$ and $r^*$. Then, given $v_i^*$, we prove existence of a symmetric best response $r^*$ and we characterize it relative to the parameters that define the market.

### 5.1 Directing Search in Equilibrium

Suppose store $i$ posts $r$ and every other store posts $r^*$. The following is proved:

**Lemma 3.** *Consider a commitment technology $\boldsymbol{\theta}$. Suppose that store $i$ posts $r \in [0,1]$ while every other store posts $r^* \in [0,1]$. Then,*

*(i) there is a unique value of $v_i^*$, denoted $\hat{v}_i^* \in [0,1]$, that satisfies $U_i = U$ and it is such that $\frac{\partial \hat{v}_i^*}{\partial r} \leq 0 \leq \frac{\partial \hat{v}_i^*}{\partial r^*}$;*

*(ii) there is a unique symmetric best response $v_i = v_i^*$ that satisfies (10) and $v = v^*$ satisfies (8). In particular,*

$$v_i^* = v(r, r^*) = \begin{cases} 1 & \text{if} \quad r \in [0, \underline{r}^*) \\ \hat{v}_i^* & \text{if} \quad r \in [\underline{r}^*, \bar{r}^*] \\ 0 & \text{if} \quad r \in (\bar{r}^*, 1] \end{cases} \tag{15}$$

*where $r^* \in [\underline{r}^*, \bar{r}^*] \subseteq [0,1]$ and $\underline{r}^*$ and $\bar{r}^*$ are non-decreasing functions of $r^*$.*

Even if the price $r$ posted by store $i$ differs from the price $r^*$ of every other store, buyers can still be indifferent across stores as long as such price differences are moderate, i.e. if $r \in [\underline{r}^*, \bar{r}^*]$. In fact, if store $i$ posts a really low or a really high price, i.e., $r \in [0, \underline{r}^*)$ or $r \in (\bar{r}^*, 1]$, then buyers either select store $i$ or avoid it entirely.

The main implication is that buyers can be indifferent across stores posting unequal prices, although they generally prefer to visit the store posting the lowest price. Indeed, (15) indicates $v_i^* = \hat{v}_i^* \in (0,1)$ can hold under $r \neq r^*$ and generally[6]

$$v_i^* \begin{cases} \geq \frac{1}{S} \geq v^* & \text{if} \quad r < r^* \\ = v^* = \frac{1}{S} & \text{if} \quad r = r^* \\ \leq \frac{1}{S} \leq v^* & \text{if} \quad r > r^*. \end{cases}$$

Why? In deciding whether to visit store $i$, the buyer considers not only (i) the expected sale price but also (ii) the expected trading risk, relative to every other store. The first element hinges on the posted price and—unlike Burdett el al.—on the demand expected at the store, as both may influence the sale price. The second element depends entirely on the demand expected at the store.

Expression (4) and Table 1 indicate that if store $i$ posts a price $r$ above every other store, then the average sale price at store $i$ might also be higher than elsewhere. This intensive margin consideration reduces the incentive to visit store $i$ in favor of other stores. Unless the difference in posted prices is enormous, however, this does not lead to a corner solution because extensive margin effects also exist. Indeed, greater expected demand at stores with lower posted prices reduces the payoff expected by potential customers; every customer is more likely to end up empty handed—due to capacity constraints—but also the expected sale price can be higher if sellers cannot commit to the posted price. As a result, (i) cheaper stores tend to attract more buyers on average, i.e., $v_i^*$ tends to fall while $v^*$ grows as $r$ rises above $r^*$, and (ii) buyers can be indifferent across establishments with greater disparities in posted prices, when sellers cannot fully commit to $r$.

Figures 1$a$ and 1$b$ depict the set $[\underline{r}^*, \bar{r}^*]$ across $r^*$ for baseline parameters and $S = B = 2$. Panel $a$ focuses on full commitment (i.e., fixed prices) and panel $b$ on weaker commitment (price floors, ceilings and negotiations). Draw a vertical line through some $r^*$ to identify the set $[\underline{r}^*, \bar{r}^*]$ of $r$ values leaving buyers indifferent between store $i$ and every other store, i.e., $v_i^* \in (0,1)$. Start by observing that if every store sell at a negotiated price, $\boldsymbol{\theta} = \boldsymbol{\theta}_N$, then a mixed strategy is always feasible, i.e., $[\underline{r}^*, \bar{r}^*] = [0,1]$ for all $r^*$. This is also true under price ceilings when $S = B = 2$, as here too sale prices are unresponsive to posted prices. However, in all other pricing scenarios we have $v_i^* \in (0,1)$ only if $r \in (\underline{r}^*, \bar{r}^*) \subset [0,1]$, i.e., the difference between $r$ and $r^*$ cannot be extreme or buyers would simply avoid the store advertising the highest price. Indeed, if store $i$ is charges

[6] Of course, strict inequalities require sale prices that are responsive to posted prices. For example, setting $r > r^*$ does not lead to $v_i^* < v^*$ if, say, $p_n = \min(q_n, r)$ and $r^* > q_B$ as here $p_n = p_n^* = q_n$.

much higher prices than any other store, then it is best to avoid it even if this would lead to a certain trade, i.e. $v_i^* = 0 < v^* = \frac{1}{S-1}$ when $r \in (\bar{r}^*, 1]$.

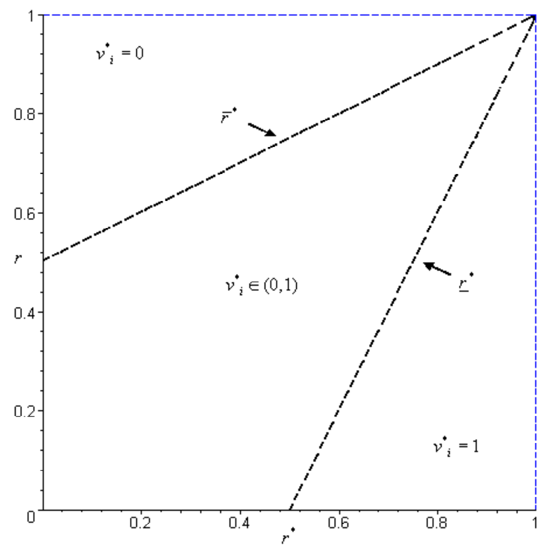


Figure 1a

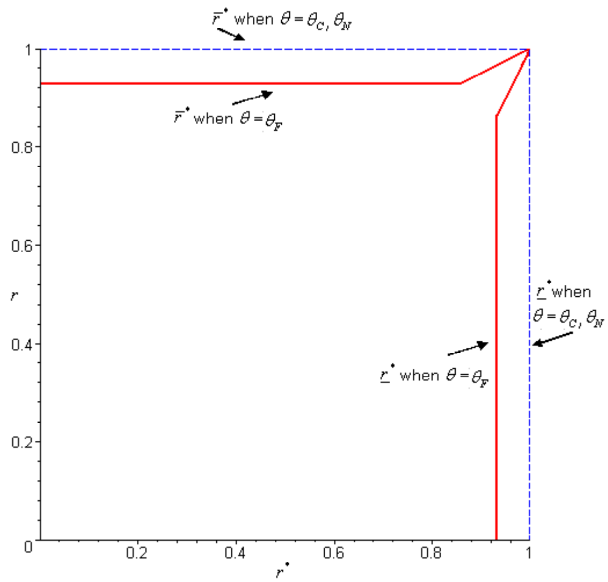


Figure 1b

As indicated in Lemma 3, the difference $\bar{r}^* - \underline{r}^*$ is function of $r^*$. Under fixed prices $\bar{r}^* - \underline{r}^*$ is the smallest and vanishes as $r^* \to 1$ (when store $i$ is always preferred). Under price floors, $\bar{r}^* - \underline{r}^*$ shrinks for high values of $r^*$, i.e. when every other store essentially sells at fixed prices ($p_n^* = r^*$ for $r^* \geq q_n$). We also emphasize that $\underline{r}^*$ and $\bar{r}^*$ depend on market composition, $\lambda$, because of capacity constraints. For example, it is not necessarily optimal to visit only a store that gives a good for free, when other stores don't. If the store is mobbed by customers then it may be nearly impossible to obtain the good.

### 5.2 Attracting Buyers in Equilibrium

It should now be obvious that when the buyer's strategy is as in (15) then the representative seller faces a trade-off in competing for customers. For example, suppose the store posts $r > r^*$. This can have two opposing effects: it may generate higher revenue per sale but it may also discourage customer visits. The optimal $r$ maximizes $W$ by balancing these intensive and extensive margins.

Figure 2 (benchmark with $S = B = 2$) traces $W$ under price floors, given $\mathbf{v}^*$ as in Lemma 3 and $r^* = 0.5$. Due to price floors, sale prices are responsive to $r$ only if $r \geq q_1$, which is when the seller can trade-off some revenue per sale against the expected demand. This trade-off is favorable until $r$ reaches 0.5, which is why $r = r^*$ is optimal (indeed, it is a symmetric equilibrium).

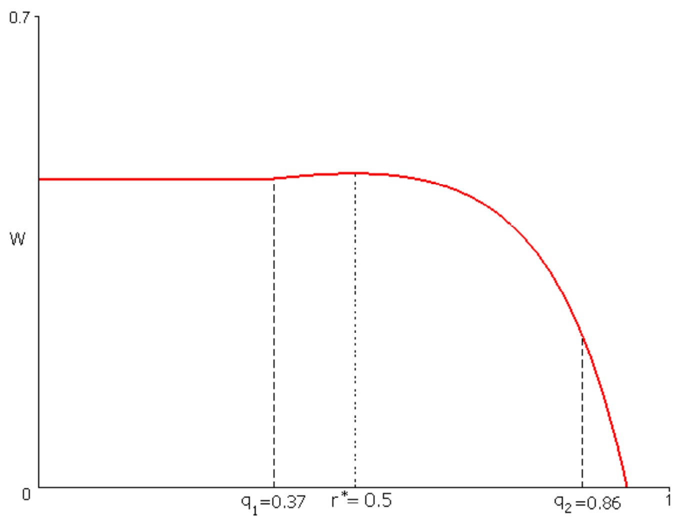


Figure 2

The next lemma proves general existence of a symmetric best response $r = r^*$.

**Lemma 4**. *Consider a commitment technology $\boldsymbol{\theta}$. Let $v_i^*$ and $v^*$ satisfy (8) and (15). Then a symmetric profit-maximizing price $r = r^* \in [0, 1]$ always exists.*

The lemma is an application of Kakutani's fixed point theorem. While it does not establish uniqueness of $r^*$, it is easy to see that there may be payoff-equivalent cases in which it is not. For example, if $\boldsymbol{\theta} = \boldsymbol{\theta}_N$ then posted prices do not affect sale prices (hence the buyers' payoffs and strategies) so any $r^* \in [0, 1]$ is a profit-maximizing candidate. Indeed, multiple values $r^*$ can arise also with some commitment, since sellers may *choose* to always negotiate by posting either a very low price ($r \leq q_1$, under price floors) or a very high price ($r \geq q_B$ in price ceilings). We explore these possibilities, next.

### 5.3 Pricing in Equilibrium

Here, we present a theorem formalizing existence of equilibrium and characterizing $r^*$ in terms of the market parameters. Start by defining $R_k$ as the set of equilibrium $r^*$, given some $\boldsymbol{\theta} = \boldsymbol{\theta}_k$. Also, define the constant

$$\omega \equiv \frac{\sum_{n=1}^{B} M_n q_n - A}{S-1}$$

where $A > 0$ but $M_n$ can be negative; specifically recalling that $\lambda = B/S$ we have

$$M_n = S^2 f_n(B, \tfrac{1}{S}) \left(1 - \tfrac{n-\lambda}{S-1}\right)$$

$$A = S^2 \left[1 - \left(\tfrac{S-1}{S}\right)^B - \lambda \left(\tfrac{S-1}{S}\right)^{B-1}\right].$$

Put simply, $\omega$ measures the strength of extensive margin effects, i.e. the change in profit due to a small increase in the probability of visits, all else equal (see the proof of Theorem 5). Such effects are strong when $\omega > 0$ and weak, otherwise. Then we have

**Theorem 5**. *Let $\boldsymbol{\theta} = \boldsymbol{\theta}_k$ and let $q_n$ satisfy (1) for all $n$. An SSE always exists such that for all $i = 1, 2, ..., S$ we have $v_i^* = v^* = \frac{1}{S}$ and $r = r^* \in R_k \subseteq [0, 1]$, with*

$$R_C = \begin{cases} \{r_C\} & \text{if } \omega > 0 \\ [q_B, 1] & \text{if } \omega \leq 0 \end{cases}, R_F = \begin{cases} [0, q_1] & \text{if } \omega \geq 0 \\ \{r_F\} & \text{if } \omega < 0 \end{cases}, R_N = [0, 1],\ R_X = \{r_X\}$$

*where*

$$r_X = \frac{A}{\sum_{n=1}^{B} M_n}, \quad r_F = \frac{A + M_j q_j - \sum_{n=j}^{B} M_n q_n}{\sum_{n=1}^{j} M_n} \quad \text{and } r_C = \frac{A + M_h q_h - \sum_{n=1}^{h} M_n q_n}{\sum_{n=h}^{B} M_n}. \tag{16}$$

*Here, $r_X \in (0, 1)$, $r_F \in (q_j, q_{j+1})$, $r_C \in (q_{h-1}, q_h)$ and $j$ and $h$ are unique values such that $1 \leq j, h \leq B$. In particular, (i) $r_C = r_X$ iff $r_X \leq q_1$ and $r_C > r_X$ otherwise and (ii) $r_F = r_X$ iff $r_X \geq q_B$, and $r_F \neq r_X$ otherwise.*

There are three main findings. First, there always exists a symmetric equilibrium in which the posted price reflects market conditions and available commitment. Second, in the absence of full commitment the equilibrium posted price may be indeterminate, with a continuum of $r$ supporting payoff-equivalent outcomes. Third, under limited commitment, the model determines endogenously the equilibrium trading mechanism, i.e. the incidence of transactions occurring at the posted price. We offer an intuitive interpretation of these results, first, followed by a technical explanation.

Under limited commitment—price floors or ceilings—sellers have discretion over the trading mechanism. In these scenarios choosing $r$ is akin to selecting the probability of haggling, because negotiated prices are positively correlated with realized demand. Indeed, all else equal, a higher $r$ corresponds to (i) a lower probability of haggling for top prices, under price floors, and (ii) a higher probability of haggling for a worse deal, under price ceilings. In any other scenario—negotiations or fixed prices—the sellers' hands are tied to either always or never negotiating, and the trade-off between $r$ and sale prices cannot be exploited. Hence, posted prices will generally vary with the available commitment. We discuss such differences aided by Figure 3, reporting the equilibrium $r$ in economies with $S = 6$ and $B$ varying from 2 to 40.[7]

[7] Moving left to right both market size and $\lambda$ vary. The resuls are similar if the market size is large but fixed, since in large markets increments in $B$ impact $\lambda$ more than the market size. In small markets $r^*$ may vary non-monotonically with $\lambda$, for low values of $\lambda$, since the relative strength of the intensive and extensive margin effect may change non-monotonically.

A first observation is that we expect higher posted prices in those markets where sellers can be prevented from trading above the advertised price. Indeed, the price ceiling $r_C$ can exceed $r_X$ while the price floor $r_F$ is generally below $r_X$. Consider price ceilings, when consumer are protected against price-hikes. Here stores can remain competitive even when advertising a price above $r_X$ because they can give discounts. If demand is scarce ($B$ is small), however, there is heightened competition that drives $r_C$ below the minimum negotiable price $q_1$; effectively, sellers trade at fixed prices, which is why $r_C = r_X$. With very large demand competition is minimal, which pushes $r$ above the maximum negotiable price $q_B$. Indeterminacy arises because any $r \geq q_B$ is a payoff-equivalent posting.

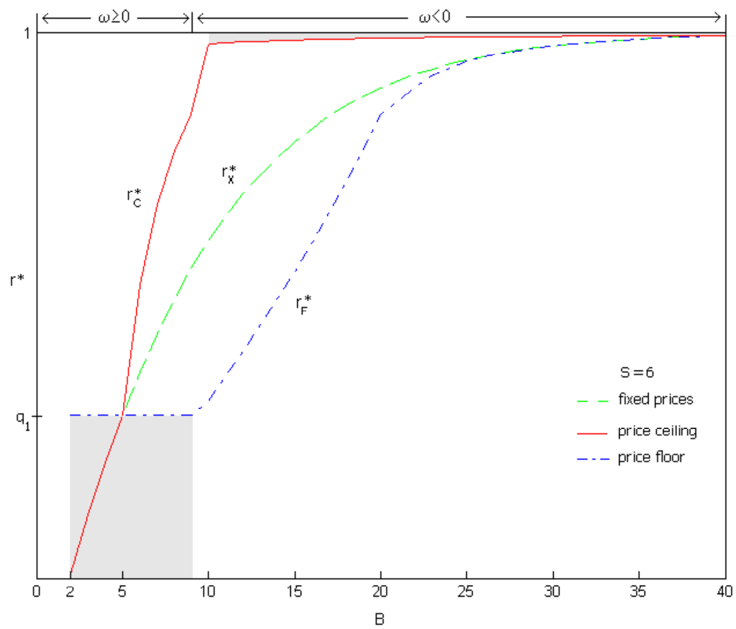


Figure 3

The reverse explanation applies to price floors: if buyers are unprotected against price increases, sellers must remain competitive by advertising below $r_X$.[8] Very large demand pushes $r_F$ above $q_B$, hence $r_F = r_X$, and indeterminacy arises when demand is so low that $r$ falls below the minimum negotiable price. Hence, markets with different compositions and commitment can generate identical price advertisements.

A second observation is that in sellers' markets stores advertise higher prices than in buyers' markets. Figure 3 indicates that $r^*$ generally rises in $B$, hence in $\lambda$. This is

[8]This is not always the case. It can be proved that if $\lambda > 1$ and $r_X$ is sufficiently close (without exceeding) to $q_B$, then $r_X < r_F < q_B$. This is a rare occurrence in which sellers barely need to compete for customers. Being able to commit to a minimum price can only benefit sellers so $r_F$ can exceed $r_X$. As an example, set $q_{B-1} < r_X < q_B$ and observe that for $j = B-1$ we have $q_{B-1} < r_X < r_F < q_B$.

because stores compete *solely* by means of price advertisements instead of, say, product differentiation. When the market is awash in buyers there is little incentive to compete aggressively. The converse holds when the customer base is small, which is when advertising low prices can substantially improve the expected demand (hence profits). For instance, consider $r_F$ in Figure 3. For $B < 9$ demand is so scarce that price reductions sort strong extensive margin effects ($\omega \geq 0$); thus, in equilibrium sellers commit to always negotiate with every customer setting $r^* \leq q_1$ (the shaded area). As $B$ passes 9, then $r^*$ grows because sellers do not need to compete as aggressively for customers.

As for the technical side of the story, denoting $f_n$ the equilibrium probability of $n$ visits, we report (from the theorem's proof) the seller's first order condition:

$$\sum_{n=1}^{B} f_n p_n = 1 - f_0 - f_1 + \frac{\sum_{n=1}^{B} f_n(n-\lambda)p_n}{S-1}.$$

Recalling that $p_0 = 0$ is the seller's profit under zero demand, the above expression says that $r^*$ must be such that the expected profit (left hand side) equals the probability of having excess demand $(1 - f_0 - f_1)$ plus an additional term. Since—as we explain in the next section—$\lambda$ is the equilibrium expected demand, then the numerator of the last term is simply the covariance of profits with demand, which is positive and finite. In a market with many sellers this term is negligible, so an approximate solution for $r^*$ must satisfy

$$\sum_{n=1}^{B} f_n p_n = 1 - f_0 - f_1.$$

Note that $p_n$ equals $r$ for all $n$ under fixed prices, can exceed $r$ only under price floors and can trail $r$ only under price ceilings. It follows that $r_F \leq r_X$ and $r_C \geq r_X$, in general, with strict inequalities when $q_1 < r_X < q_B$. It is also obvious that since excess demand is more likely as the customer base grows, then expected profit must grow with $\lambda$. Since profits and average sale prices generally depend on posted prices, then the equilibrium $r$ and the average sale price must be non-decreasing in the customer base. To expand on this, however, we must study the equilibrium distribution of demand and sale prices.

## 6 Equilibrium Price Dispersion

Here, we start by calculating the distribution of demand at a representative store. Knowing the advertised price from the prior section, we can then find the distribution of sale prices as a function of the model's parameters.

Start by observing that in a symmetric equilibrium every buyer visits every store with identical probability $\frac{1}{S}$, independently of $\boldsymbol{\theta}$ and the value of $r^*$. Letting $\Pr[n] = f_n(B, \frac{1}{S})$

denote the equilibrium probability that a store $i$ is visited by $n$ buyers, (6) implies

$$\Pr[n] = \quad \binom{B}{n}\left(\tfrac{S-1}{S}\right)^B \tfrac{1}{(S-1)^n} \text{ for } n = 0, 1, ..., B, \tag{17}$$

so average demand is $\lambda = \frac{B}{S}$ and its variance is $\lambda(\frac{S-1}{S})$. In a buyer's market $\lambda$ is small while the opposite occurs in a seller's market (where variance is the largest).

Note that demand is not independently distributed across stores since the overall number of customers must add up to $B$. Thus, if a store has many buyers other sellers are likely to have few. That is, the covariance between the demand at any two stores is negative. Let $n_i = 0, 1, ..., B$ be the demand at store $i = 1, .., S$, where $\sum_{i=1}^{S} n_i = B$. Since the distribution of demand *at a store* is $bin(B, \frac{1}{S})$ then the distribution of demand *in the market* is multinomial with parameters $B$ and $\frac{1}{S}$. Thus, $cov(n_i, n_j) = -\frac{\lambda}{S} < 0$ for any two stores $i \neq j$, clearly tiny in a large market or in a sellers' market.

Now consider the representative store, given some $\boldsymbol{\theta} = \boldsymbol{\theta}_k$. The support $P$ of the sale price distribution is the discrete set defined in (5). Let $\Pr[p|\boldsymbol{\theta}_k]$ be the equilibrium probability of a sale at price $p \in P$ and define the mean sale price at a store by

$$\bar{p}_k = \sum_{p \in P} p \Pr[p|\boldsymbol{\theta}_k]. \tag{18}$$

Theorem 5 tells us that the equilibrium distribution of sale prices generally depends not only on the distribution of buyers but also on the equilibrium posted price. A store always makes a sale when there is at least one buyer so, from (17), the probability of observing a sale price is $\Pr[n \neq 0] = 1 - \left(\frac{S-1}{S}\right)^B$. Now let $\Pr[n|n \neq 0]$ define the probability that a sale takes place when there are $n \geq 1$ buyers at the store, i.e.

$$\Pr[n|n \neq 0] = \tfrac{\Pr[n]}{\Pr[n \neq 0]} = \binom{B}{n} \tfrac{(S-1)^{B-n}}{S^B - (S-1)^B} \quad \text{for } n = 1, 2, ..., B. \tag{19}$$

If in equilibrium $r \in [q_j, q_{j+1})$ then we say $r = r_j$ for a unique $j = 0, 1, ..., B$; hence, $r_j$ denotes a generic element of the following sets partitioning $[0, 1]$:

$$r_j \in \begin{cases} [0, q_1) & \text{for } j = 0 \\ [q_j, q_{j+1}) & \text{for } \ 1 \leq j \leq B-1 \\ [q_j, 1] & \text{for } j = B. \end{cases} \tag{20}$$

To characterize the distribution of prices at the representative store we start with the simplest case of an economy with full commitment. Here, although demand is random, sale prices are fixed (deterministic) so we have a degenerate distribution with unit mass at $p = r$. To study economies with weaker commitment, we start with the case of no commitment at all, as the remaining cases hinge on this.

### 6.1 Sale Price Distribution Under Negotiations

Lemma 1 indicates $p_n = q_n$ for $n \neq 0$, which means that the sale price $p$ is always random as it depends on the realized demand $n$, i.e.

$$\Pr[p|\boldsymbol{\theta}_N] = \begin{cases} \Pr[n|n \neq 0] & \text{if } p = q_n \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

so sale prices are distributed as a $\text{bin}\big(B, \frac{1}{S}\big)$ conditional on $n \neq 0$.

Expression (18) easily indicates that average sale prices increase in the average demand $\lambda$, since $\{q_n\}$ is an increasing sequence. To calculate a measure of price dispersion we consider the coefficient of variation, which is hump-shaped. This reflects the coefficient of variation of the demand distribution (conditional on $n \neq 0$) which is hump-shaped. Indeed, for $\lambda$ small, sellers have a small customer base so most trades are likely to occur at low prices. When $\lambda$ is large, instead, seller is likely to trade at high prices and their dispersion is low. Thus dispersion is highest for moderate values of $\lambda$.

Finding the distribution of prices in the market is more laborious, as we must calculate the marginal probability of each possible demand realization. For instance, if $S = 3$ and $B = 4$ then there can be four possible sale prices. The probability of observing any price depends on the number of sales, hence on the distribution of buyers in the market. The distribution of prices in the market is thus (see the Technical Appendix):

$$\Pr[q_1] = \tfrac{12}{27}, \ \ \Pr[q_2] = \tfrac{10}{27}, \ \ \Pr[q_3] = \tfrac{4}{27}, \ \ \Pr[q_4] = \tfrac{1}{27}.$$

Thus, the average sale prices is low since average demand at a store is low, $\lambda \approx 1.3$, hence trade at price $q_1$ is the most likely.

### 6.2 Price Distribution Under Price Floors or Ceilings

When traders cannot fully commit to $r$, the distribution of sale prices still hinges on (21). Suppose $r = r_j$ for a unique $j = 0, 1, ..., B$. In a price floor $p_n = r_j$ if $n \leq j$ and $p_n = q_n$ if $n > j$. Clearly, if $r_j \geq q_B$ then $p_n = r$ for all $n$ (i.e. fixed prices) and if $r_j \leq q_1$ we have $p_n = q_n$ for all $n$ (i.e. negotiated prices). It follows that

$$\Pr[p|\boldsymbol{\theta}_F] = \begin{cases} \Pr[n \leq j|n \neq 0] = & \sum_{n=1}^{j} \binom{B}{n} \frac{(S-1)^{B-n}}{S^B - (S-1)^B} & \text{if } p = r = r_j \\ \Pr[n|n \neq 0] = & \binom{B}{n} \frac{(S-1)^{B-n}}{S^B - (S-1)^B} & \text{if } p = q_n > r \\ 0 & & \text{otherwise.} \end{cases} \tag{22}$$

The probability to sell at the posted price $r_j$ is the probability of being visited by at most $j$ customers. Of course, the seller never charges $q_n > r$ if $n > j$ and never charges $p < r$.

Under a price ceiling $p_n = q_n$ if $n \leq j$ and $p_n = r_j$ if $n > j$. Thus, if $r_j \leq q_1$ then $p = r$ for all $n$ while if $r_j \geq q_B$ then $p_n = q_n$ for all $n$, so that

$$\Pr[p|\boldsymbol{\theta}_C] = \begin{cases} \Pr[n|n \neq 0] = & \binom{B}{n}\frac{(S-1)^{B-n}}{S^B-(S-1)^B} & \text{if } p = q_n < r \\ \Pr[n \geq j+1|n \neq 0] = & \sum_{n=j+1}^{B}\binom{B}{n}\frac{(S-1)^{B-n}}{S^B-(S-1)^B} & \text{if } p = r = r_j \\ 0 & & \text{otherwise.} \end{cases} \tag{23}$$

Here, there is trade at the posted price when the demand is at least $j$. Otherwise, the prices is bargained. In any event, the seller never charges $p > r$. The key conclusion is

**Lemma 6.** *In an SSE we have*

$$\bar{p}_C \leq \bar{p}_N \leq \bar{p}_F, \tag{24}$$

*with strict inequality if comparing equilibria with unique $r^*$, while $\bar{p}_X$ cannot be ranked.*

The lesson here is that the commitment technology affects in intuitive ways the average sale price, hence the sellers' profit and the buyers' surplus. We emphasize that this is not due to changes in the endogenous distribution of demand, which is invariant to $\boldsymbol{\theta}$. Indeed, differences in average sale prices hinge on differences in $r$ and the ability to depart from it once customers arrive at the store.

Under price floors sale prices can only surpass what had been initially advertised so average sale prices are the highest, for a given parameterization. The opposite occurs under price ceilings, when sellers may end up giving discounts. Comparisons under fixed prices are less clear-cut since they hinge on disparities between the price posted under different $\boldsymbol{\theta}$ but also the shape of the sequence $\{q_n\}$. However, it is obvious that if the commitment technology has a limited effect on the posted price, then $\bar{p}_C \leq \bar{p}_X \leq \bar{p}_F$; in this case any differences in the equilibrium $r$ are of a lesser significance than the ability to sell at a price above or below what posted (we present some examples, later).

Of course, the dispersion in sale prices is also affected by commitment. It is highest under negotiations, as sale prices are completely independent of $r$, and zero under fixed prices. In between these two extremes we have the coefficient of variation for price floors and ceilings. To provide further analytical results, however, it is useful to consider an approximation, which is the subject of the next subsection.

### 6.3 Price Distribution in Large Markets

Consider economies with identical customer base $\lambda = B/S$ but different populations $\lambda S + S$. We obtain the following result

**Lemma 7.** *Fix $\lambda \in \mathbb{R}_{++}$ and let $B = \lambda S$. Then, in an SSE as $S \to \infty$ demand is identically and independently distributed across stores according to a Poisson with parameter $\lambda$. Thus, (17) becomes*

$$\Pr[n] = \tfrac{e^{-\lambda}\lambda^n}{n!} \qquad \text{for } n = 0, 1, ..., B \tag{25}$$

*Thus, in large markets the distribution of demand $n$ at a store has approximately mean $\lambda$ and coefficient of variation$\sqrt{\frac{1}{\lambda}}$.*

The result hinges on the fact that demand co-varies little across stores in a large market so the distribution of demand is approximately independent across stores. To see why fix $\lambda$ and let the market grows in size. The demand at any store is less and less affected by the demand present at any other store. Hence, the distribution of demand at a store approaches the marginal distribution, which is a Poisson with parameter $\lambda$.

The main implication of this approximation, is that we can easily characterize the distribution of demand in a large market, via $\lambda$. Expected demand is higher in a sellers' market and it is more dispersed in a buyer's market (when measured by the coefficient of variation) since there are many stores to choose from. We can then approximate the price distribution at the representative store in a large market, using (25).

Since prices are observed only if a transaction takes place, the sale price distribution is the Poisson $\lambda$, conditional on $n \neq 0$. The probability that a seller trades is $1 - e^{-\lambda}$ (i.e., the probability of $n \geq 1$) and the seller has $n$ customers with probability $\frac{e^{-\lambda}\lambda^n}{n!(1-e^{-\lambda})}$, thus a sale price $p_n$ is observed with probability $\frac{\lambda^n}{n!(e^{\lambda}-1)}$,for $n = 1, ..., B$. This expression can then be substituted into (21), (22) and (23).

The main implication of Lemma 8 is that we can use (**??**) to approximate the distribution of sale prices in a large market. This allows us to easily study how changes in market structure affect not only the distribution of sale prices but also the equilibrium posted price. Especially, we find the following

**Lemma 8.** *Fix $\lambda \in \mathbb{R}_{++}$ and let $B = \lambda S$. Then, in an SSE as $S \to \infty$ we have*

$$\begin{aligned} r_X = &\quad 1 - \tfrac{\lambda}{e^{\lambda}-1} \\ r_F = &\quad \frac{r_X(e^{\lambda}-1) - \sum_{n=j+1}^{B} \lambda^n q_n/n!}{\sum_{n=1}^{j} \lambda^n/n!} \text{ for } 1 \leq j \leq B-1 \\ r_C = &\quad \frac{r_X(e^{\lambda}-1) - \sum_{n=1}^{h-1} \lambda^n q_n/n!}{\sum_{n=h}^{B} \lambda^n/n!} \quad \text{for } 2 \leq h \leq B. \end{aligned} \tag{26}$$

*Here, $\bar{p}_k$ and $r_k$ increase in $\lambda$, $r_F < r_X$ if $q_1 < r_X < q_B$, while $r_C > r_X$ if $r_X > q_1$.*

The lemma is useful because it allows us to approximate the values of prices posted in a large market with the expressions in (26). This allows us to establish the important result that average prices and posted prices grow in markets with higher expected demand, for any commitment technology.

Average sale prices grow in $\lambda$ because stores (i) not only can compete less for customers—so they can afford to post higher prices—but (ii) stores expect a greater incidence of high demand, which is when sale prices can be higher. To discuss the first element, consider a large market under $\boldsymbol{\theta} = \boldsymbol{\theta}_X$, when every trade occurs at the posted price. Here

$$\bar{p} = \bar{p}_X \approx r_X \sum_{n=1}^{B} \frac{\lambda^n}{n!\,(e^\lambda - 1)} = r_X = 1 - \frac{\lambda}{e^\lambda - 1}$$

that increases in $\lambda$ solely because the posted price $r_X$ increases. To illustrate the second element, consider $\boldsymbol{\theta} = \boldsymbol{\theta}_N$ when every sale is negotiated. Here

$$\bar{p} = \bar{p}_N \approx \sum_{n=1}^{B} \frac{\lambda^n q_n}{n!\,(e^\lambda - 1)}$$

that grows with $\lambda$ because low-demand shocks are less likely and $\{q_n\}$ is an increasing sequence. Finally, notice that under price floors and ceilings we have that sale prices are more strongly correlated with demand (whose distribution is unchanged relative to fixed prices or negotiations) while $r_F$ and $r_C$ increase in $\lambda$ . This explains why $\bar{p}_C$ and $\bar{p}_F$ respond positively to increases in $\lambda$.

## 7 Predictions on Market Behavior

We now simulate several economies to expand on our analytical results and to build intuition on how equilibrium prices hinge on market structure and the pricing 'convention,' be it price floors or ceilings, fixed prices or negotiations. We start with $S = B = 2$ and then study richer environments.

### 7.1 Equilibrium Posted Prices

Let $B = S = 2$. If $\{q_n\}$ satisfies (2) then

$$q_1 = 1 - \beta(1-\gamma) \quad \text{and} \quad q_2 = \tfrac{(2-\beta)[1-\beta(1-\gamma)]}{2-\beta(2-\gamma)} > \tfrac{1}{2},$$

so that $q_1 \geq \frac{1}{2}$ if $\gamma \geq \bar{\gamma}(\beta) = 1 - \frac{1}{2\beta}$. As $\gamma$ capture the notion of seller's bargaining power, this simply means that sellers can get the greatest share of surplus only if they are skilled negotiators. Note also that $\omega \leq 0$ if $\gamma \in [0, \bar{\gamma}(\beta)]$, and $\omega > 0$ otherwise.

Under negotiations $(p_1, p_2) = (q_1, q_2)$, and $r^*$ can be anything in the unit interval, while if prices are fixed we are back into Burdett et al. (2001) and in an SSE

$$p_n = r_X = \frac{1}{2} \text{ for } n = 1, 2.$$

Now suppose sellers can commit to a price floor, i.e., $p \geq q_1$. The choice of posted price can be one of two types. Sellers may advertise a high price $r^* \geq q_2$, which is equivalent to choosing to always trade at the posted price. Alternatively, sellers may charge prices that are progressively higher depending on the realized demand, setting $q_1 \leq r^* < q_2$. We find that for all parameters, sellers will never choose to charge fixed prices, i.e. equilibrium sale prices are positively correlated with the realized demand. Specifically, if $\boldsymbol{\theta} = \boldsymbol{\theta}_F$ then

$$(p_1, p_2) = \begin{cases} (r^*, q_2) & \text{if } \gamma \in [0, \bar{\gamma}(\beta)] \\ (q_1, q_2) & \text{otherwise} \end{cases} \quad r^* = \begin{cases} r_F = \frac{1}{2} \in [q_1, q_2) & \text{if } \gamma \in [0, \bar{\gamma}(\beta)] \\ [0, q_1] \text{ with } q_1 > \frac{1}{2} & \text{otherwise.} \end{cases}$$

Since sellers can commit to charge at least $q_1$, they can trade off improvements in their store's attractiveness (relative to other stores) versus the expected loss from doing so. Thus, sellers will tend to exploit their ability to commit to a minimum price only if their proficiency in negotiations is weak, i.e. when $q_1$ is small. Indeed, $r^* = r_F = \frac{1}{2} \geq q_1$ only if $\gamma \leq \bar{\gamma}(\beta)$. While $r^*$ is unresponsive to further decreases in $q_1$, if $q_1$ raises above $\frac{1}{2}$ then sellers will compete aggressively by advertising a low price $r^* \leq q_1$. In this case every sale is negotiated so there is indeterminacy in posted price, as any $r^* \in [0, q_1]$ conveys identical information to the market.

When sellers commit to a price ceiling, then $p \leq q_2$. Here, too, seller may either choose to charge a low fixed price $r^* \leq q_1$ or can sell at prices that grow with the demand $n$. Once again, selling at fixed prices is not an equilibrium, since for $\boldsymbol{\theta} = \boldsymbol{\theta}_C$ then

$$(p_1, p_2) = \begin{cases} (r^*, q_2) & \text{if } \gamma \in (\bar{\gamma}(\beta), 1] \\ (q_1, q_2) & \text{otherwise} \end{cases} \quad r^* = \begin{cases} r_C = \frac{1}{2} \in [q_1, q_2) & \text{if } \gamma \in (\bar{\gamma}(\beta), 1] \\ [q_2, 1] \text{ with } q_1 \leq \frac{1}{2} < q_2 & \text{otherwise.} \end{cases}$$

Here sellers benefit from advertising low price limits when they have a strong bargaining position, i.e., $r^* = r_C < q_2$ when $q_1 > \frac{1}{2}$. Otherwise, they will advertise a high price.

### 7.2 Equilibrium Average Sale Prices

In the benchmark example $p_1$ and $p_2$ are equally likely outcomes among all the transactions observed. This allows us to easily calculate average market sale price ($\bar{p}$) and its coefficient of variation ($c.v.$) for the baseline parameters where $q_1 = .37, q_2 = .86$ and

$q_3 = .92$. The result is in the mid-column of Table 1.

| | $(S=3, B=2)$ (strong competition) | $(S=2, B=2)$ (moderate competition) | $(S=2, B=3)$ (weak competition) |
|---|---|---|---|
| | | $\boldsymbol{\theta} = \boldsymbol{\theta}_F$ | |
| $r^* =$ | $[0, q_1]$ | 0.5 | $0.7 < q_2$ |
| $\overline{p} =$ | 0.53 | 0.68 | 0.85 |
| $c.v. =$ | 0.44 | 0.27 | 0.1 |
| | | $\boldsymbol{\theta} = \boldsymbol{\theta}_C$ | |
| $r^* =$ | $0.27 < q_1$ | $[q_2, 1]$ | $[q_3, 1]$ |
| $\overline{p} =$ | 0.27 | 0.62 | 0.76 |
| $c.v. =$ | 0 | 0.4 | 0.3 |
| | | $\boldsymbol{\theta} = \boldsymbol{\theta}_N$ | |
| $r^* =$ | $[0, 1]$ | $[0, 1]$ | $[0, 1]$ |
| $\overline{p} =$ | 0.53 | 0.62 | 0.76 |
| $c.v. =$ | 0.44 | 0.4 | 0.3 |
| | | $\boldsymbol{\theta} = \boldsymbol{\theta}_X$ | |
| $r^* =$ | 0.27 | 0.5 | 0.72 |
| $\overline{p} =$ | 0.27 | 0.5 | 0.72 |
| $c.v. =$ | 0 | 0 | 0 |

Table 1

Perhaps the most remarkable finding is that sellers are not necessarily better off in markets with full commitment rather than weaker commitment to the posted price. Indeed, when $S = B = 2$ mean sale prices are the lowest under a policy of fixed prices. The reason is that the expected demand matches exactly the store's capacity, $\lambda = 1$, and the market is so small that there is high risk of having no buyers (25%). To insure against this risk, stores compete aggressively posting low prices. This reduces $\overline{p}$ since, under fixed prices, demand pressure cannot be exploited to 'bargain prices up.'

Interestingly, average sale prices are higher *even if* buyers can bargain prices down, i.e., under price ceilings. Why? Promising possible price reductions is a very effective way to compete for buyers in market scenarios where stores expect low demand with high probability (here, one customer arrives with 50% chance). Thus, sellers can entice buyers to their store by posting a price that is high but it is likely to be reduced.

More generally, two elements provide incentives to aggressively compete for customers by advertising low prices: great risk of having unsold inventory ($\Pr[n = 0]$ high) and small expected demand ($\lambda$ small). These factors have a particularly strong effect on $r$ when sellers cannot increase their attractiveness by committing to possible price reductions and when they cannot exploit large demand realizations to bid prices up. Thus, we expect that sellers should fare better under fixed prices, rather than price ceilings, in markets that are either large or have large expected demand. We validate this intuition, next, by varying $\lambda$ in a small market and then by simulating several large markets.

**7.3 Prices and Market Composition**

To build intuition on how market composition affects sellers' strategies and distribution of sale prices, we modify the basic example to consider a buyer's market ($S = 3 > B = 2$) and a seller's market ($S = 2 < B = 3$). The distribution of buyers in the market changes and so does the distribution of sale prices as illustrated below:

| $(S, B)$ | $\Pr[p_1]$ | $\Pr[p_2]$ | $\Pr[p_3]$ |
|---|---|---|---|
| $(3, 2)$ | $2/3$ | $1/3$ | $0$ |
| $(2, 2)$ | $1/2$ | $1/2$ | $0$ |
| $(2, 3)$ | $2/8$ | $3/8$ | $3/8$ |

Table 2 reports values of key endogenous variables as we change market structure adding a seller or adding a buyer (moving left or right of the third column). A central observation is that sale prices fall and their dispersion grows as a seller is added to the market (second column). This reflects the increased competition for customers, as expected demand at a store falls to $\lambda = 0.66$ from $\lambda = 1$. The opposite naturally occurs in a seller's market, i.e., by adding a buyer (last column).

Remarkably, adding a store to the market raises the competition so much that advertised prices end up *below* the minimum negotiated price, i.e., $r^* = 0.27 < q_1$. This is particularly striking under price ceilings, when sellers simply give up on promising discounts to attract customers; instead, they choose to charge a low fixed price. The opposite occurs under price floors: stores advertise their readiness to always negotiate by posting $r^* \leq q_1$. Hence, fixed prices and price ceilings are revenue-equivalent pricing conventions, in a small buyer's market; the same is true for negotiations and price floors.

The analysis is almost symmetric in the opposite scenario of a small seller's market. Here price ceilings and negotiations are revenue equivalent but the symmetry is not exact since $r^*$ is not identical under price floors or fixed prices (although close).

**7.4 Customer's Base, Prices and the Incidence of Negotiations**

To study how expansions of the customer's base affect sale prices we simulated a larger market. Figure 4 considers the same parameters of Figure 3, and plots average equilibrium sale prices at a store (panel $a$) and their coefficient of variation (panel $b$) as, keeping fixed $S$, the customer base $\lambda$ and the market size $B + S$ expand.[9]

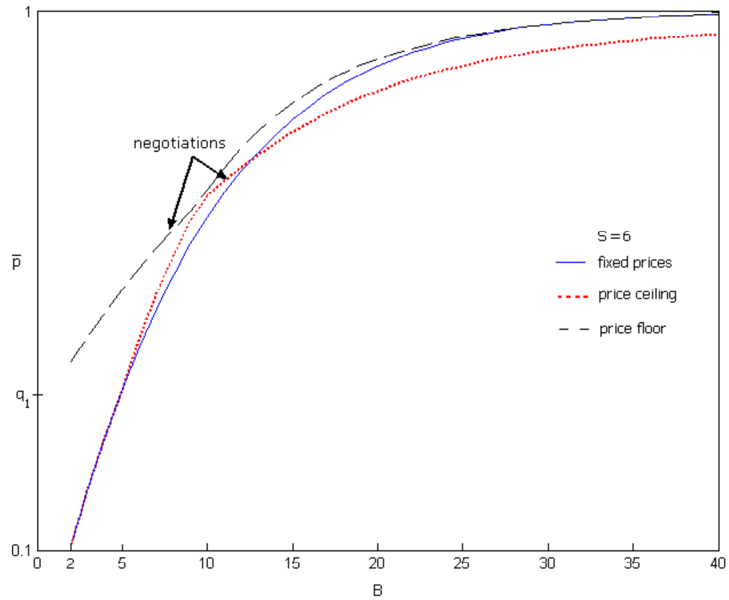


Figure 4a

Panel ($a$) confirms the intuition that average sale prices are positively correlated to the customer base. Notably, the curve $\bar{p}_F$ envelopes all other curves, which means that stores' expected profits are generally the highest under price floors. The intuition is simple. Although sellers may advertise the lowest $r$ under price floors (see Figure 3) they are also free to raise prices when demand is high. This explains two additional observations. First, when $\lambda$ is very small, then expected profits are higher under negotiations than fixed prices (the opposite is true when $\lambda$ is large). With few customers per seller, high-demand realizations are rare so there is less to gain from posting a low fixed price $r$ relative to always bargaining. Second, committing to a policy of price discounts is superior to charging fixed prices in small markets with demand expected to be close to capacity, i.e., $\bar{p}_C > \bar{p}_X$ when $\lambda \approx 1$. Price ceilings allow sellers to post a high $r$ while still competing effectively for the few customers.

Panel ($b$) indicates that the dispersion of sale prices is hump-shaped (except under fixed prices, when it is zero) because as $B$ moves above 2 stores can get more customers more frequently. Eventually the coefficient of variation drops since excess demand is so likely that most trades tend to occur at high prices. Of course, dispersion is the highest

[9] Larger markets where $B$ varies or $B + S$ is constant and $\lambda$ varies produce qualitatively similar results.

under bargaining (it is the envelope of the two curves) as each $n$ necessarily implies a different sale price, and it converges to dispersion under price ceiling since in the latter case as $B$ becomes large sellers eventually post $r^* \geq q_B$ (i.e., under $\boldsymbol{\theta}_C$ stores always negotiate). Conversely, as $B$ contracts to 2, price dispersion under negotiations and price floors coincides as $r^* \leq q_1$ (i.e., under $\boldsymbol{\theta}_F$ stores always negotiate).

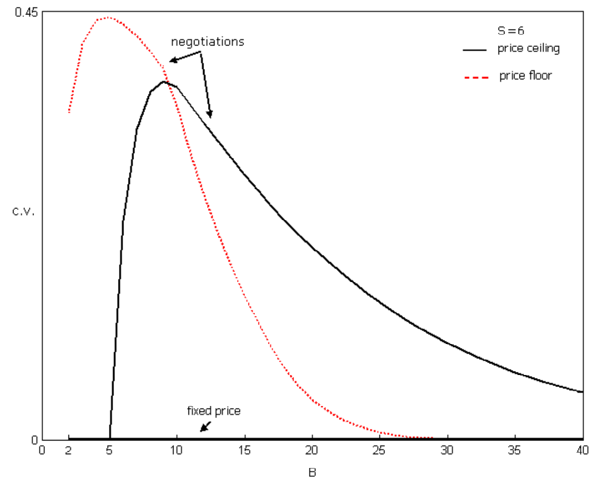


Figure 4b

We expand on these considerations in Figure 5, reporting the endogenous probabilities of haggling, under price ceilings and floors.

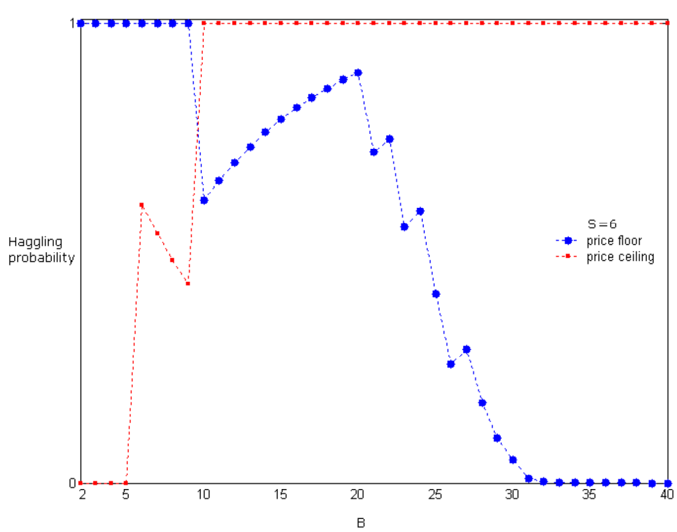


Figure 5 - Haggling probabilities

It is immediate that the extent of negotiations hinges on both available commitment and customer base. We see that there is more haggling under price floors than ceilings, if $\lambda$ is small. As the customer base expands the general trend is less frequent haggling under price floors and more frequent under price ceilings. Thus, for $\lambda$ large haggling is certain under price ceilings but absent under price floors (when sale prices jump only in the unlikely event of exceptionally large demand). The incidence of haggling is non-monotonic since both $r$ and the expected demand grow with $B$; these intensive and extensive margin effects have opposing effects. For instance, if under price floors an increase in expected demand raises $r$ just a little, then negotiations may become more likely.

This discussion has an interesting implication. Suppose it is the sellers that jointly select the 'pricing convention' to be adopted on the market. Suppose also that full commitment to the advertised price is unfeasible but haggling involves a variable resource cost for the seller (e.g., time or personnel cost). Then the model predicts that in markets with a wide customer base prices would be advertised as subject to increase (e.g. real estate prices in L.A. county) but we would see prices advertised as subject to reductions in markets with a smaller customer base (e.g. real estate prices in Tippecanoe county).

## 8 Final Remarks

We have studied a directed search market with capacity-constrained sellers and homogenous buyers. Sellers compete for customers by means of price advertisements. These may differ from the sale price depending on available commitment and the demand shock realized by the seller. We have fully characterized the equilibrium distribution of these prices as functions of the parameters describing the market. Hence, the model can provide empirically testable predictions of market behavior.

For instance, consider markets where sellers have a large customer base. The analysis predicts high advertised prices when sellers can be prevented from trading above the 'sticker price.' Here, each single deal is negotiated and sellers offer large discounts only if business is unusually slow (as car dealerships seem to do). On the other hand, the model predicts low advertised prices in markets where consumers are not protected against sudden sale-price hikes. Here, sales occur at a fixed (advertised) price with occasional high-price sales when demand is unusually large (as motels seem to do).

## References


Abreu, D. (1986). Extremal Equilibria of Oligopolistic Supergames. Journal of Economic Theory 39, 191-225

Aliprantis C. D., and Border K. C. M.R. (1994). *Infinite Dimensional Analysis: A Hitchhiker's Guide*. 1st Edition. Springer-Verlag, Heidelberg and New York

Arbatskaya M. (2004). Ordered Search. Manuscript, Emory University

Baye M. R., Kovenock, D. and De Vries, C (1992). It Takes Two to Tango: Equilibria in a Model of Sales. Games and Economic Behavior 4, 493-510

Baye M.R., Morgan J. and Scholten P. (2004). Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site. Journal of Industrial Economics (forthcoming)

Benoit, J., Kennes J. and King I. (2000). Bidding for Labor. Review of Economic Dynamics 3, 619-649

Burdett, K. and Judd, K. L. (1983). Equilibrium Price Dispersion. Econometrica, 51 (4), 955-969

Burdett K., Shi, S. and Wright, R. (2001). Pricing and Matching with Frictions. Journal of Political Economy 109 (5), 1060-1085

Camera G. and A. Delacroix (2004). Trade Mechanism Selection in Markets with Frictions. Review of Economic Dynamics 7(4), 851-868

Camera G. and Corbae D. (1999). Money and Price Dispersion. International Economic Review, 40, 985-1008

Camera G. and Selcuk C. (2004). A Note on Bargaining Under Capacity Constraints and Excess Demand. Manuscript, Purdue University

Carlson J. and P. McAfee (1983). Discrete Equilibrium Price Dispersion. Journal of Political Economy, 91 (4), 480-493

Coles M.G. and Eeckhout J. (2003). Indeterminacy and directed search. Journal of Economic Theory 111, 265-276

Hoel P. G., S.C. Port and C. J. Stone (1971). Introduction to Probability Theory. Houghton Mifflin, Boston

Pratt, J., Wise, D., and Zeckhauser R. (1979). Price Differences in Almost Competitive Markets. Quarterly Journal of Economics, 93, 189-211

Varian H. (1980). A Model of Sales. American Economic Review 70, 651-659

## Appendix

**Proof that $\mathbf{v} = \mathbf{v}^*$ implies $v_i = v_i^* = \frac{1}{S}$**

Suppose $r = r^*$ and $v_h^* > 0$ but $v_i^* \neq v_h^*$ for $i \neq h$. Suppose a buyer is at store $i$. Then, $\sum_{n=1}^{B-1} f_n(v_i^*; B-1)$ is the probability that store $i$ is visited by at least one more customer. Recall that $p_n$ is non-decreasing in $n$ and that $f_n(v_i^*; B-1)$ falls in $v_i^*$ for $n$ small and grows for $n$ large. Thus, as $v_i^*$ rises above $v_h^*$ we have that $U_i$ falls as the probability of trading at low prices (the probability that $n$ is small) falls, while the probability of trading at high prices (the probability that $n$ is large) rises. Therefore if $v_i^* > v_h^*$ then $U_i < U_h$. But then setting $v_i^* = 0$ is optimal. It follows that $W(r^*) = 0$, which is not an equilibrium. Seller $i$ could improve his payoff by setting $r < r^*$. If $v_i^* < v_h^*$ then $U_i > U_h$ and this cannot be an equilibrium either since it implies $v_i^* = 1$ and $v_h^* = 0$. This this contradicts our conjecture $v_h^* > 0$, made above. Thus, in a symmetric equilibrium $v_i = v_i^* = \frac{1}{S}$ for all $i$.■

**Proof of Lemma 3**

Suppose that store $i$ posts $r \in [0,1]$ and every other store posts $r^* \in [0,1]$ (possibly different than $r$). Given demand $n$, denote by $p_n$ and $p_n^*$ the sale prices at store $i$ and in any other store; sale prices satisfy (4).

Suppose every buyer selects the vector $\mathbf{v}^*$, visiting store $i$ with probability $v_i^*$ and any other store with probability $v^*$. Now consider the representative buyer, whose strategy vector is $\mathbf{v}$. According to (10) this buyer is indifferent between stores $i$ and any other store if $U_i = U$ that is if

$$\sum_{n=0}^{B-1} \frac{f_n(B-1, v_i^*)(1-p_{n+1})}{n+1} = \sum_{n=0}^{B-1} \frac{f_n(B-1, v^*)(1-p_{n+1}^*)}{n+1}. \tag{27}$$

where we notice that

$$\sum_{n=0}^{B-1} \frac{f_n(B-1, v)}{n+1} = \sum_{n=1}^{B} \frac{f_n(B, v)}{Bv} = \frac{1-(1-v)^B}{Bv}.$$

Using (8) we have $v^*$ as a function of $v_i^*$ so we define the function $g : [0,1] \to \mathbb{R}_+$ where

$$g(v_i^*) = \frac{U}{U_i} = \frac{\sum_{n=0}^{B-1} \frac{f_n(B-1, v^*)(1-p_{n+1}^*)}{n+1}}{\sum_{n=0}^{B-1} \frac{f_n(B-1, v_i^*)(1-p_{n+1})}{n+1}}. \tag{28}$$

It follows from (10) that $v_i = 0$ if $g(v_i^*) > 1$, $v_i \in [0,1]$ if $g(v_i^*) = 1$ and $v_i = 1$ if $g(v_i^*) < 1$.

Now we examine some of the properties of $g$. To start, $g'(v_i^*) > 0$. To see why notice that $\frac{\partial f_n(B-1,v_i^*)}{\partial v_i^*} < 0$ for $n$ small and $\frac{\partial f_n(B-1,v_i^*)}{\partial v_i^*} > 0$ for $n$ large. Also, $\left\{\frac{1-p_{n+1}}{n+1}\right\}$ is a strictly decreasing sequence, since $p_n$ is non-decreasing in $n$ (from (4)). It follows that $\frac{\partial U_i}{\partial v_i^*} < 0$ and $\frac{\partial U}{\partial v^*} < 0$, since less weight is given to low prices (low $n$) and more to high prices (high $n$) as the probability that any other buyer visits that store rises. We also have $\frac{\partial U}{\partial v_i^*} = \frac{\partial U}{\partial v^*}\frac{\partial v^*}{\partial v_i^*} > 0$ since $\frac{\partial v^*}{\partial v_i^*} < 0$ from (8). Therefore $g'(v_i^*) > 0$.

Now consider the end points of $g$. When $v_i^* = 0$ we have $U_i = 1 - p_1$ since $(v_i^*)^0 (1 - v_i^*)^{B-1} = 1$ for $v_i^* = 0$. Since $v^* = \frac{1}{S-1}$ (from (8)) then we can write

$$g(0) \quad = \frac{1}{1-p_1} \sum_{n=0}^{B-1} \frac{f_n(B-1, \frac{1}{S-1})(1-p_{n+1}^*)}{n+1}.$$

When $v_i^* = 1$ we have $U_i = \frac{1-p_B}{B}$ (because $(v_i^*)^{B-1} (1 - v_i^*)^0 = 1$) and $U = 1 - p_1^*$. Thus,

$$g(1) = \quad \frac{1-p_1^*}{1-p_B} B.$$

Note that

$$\begin{array}{lll} g(0) \leq 1 & \text{if} & \sum_{n=0}^{B-1} \frac{f_n(B-1,\frac{1}{S-1})}{n+1}(1-p_{n+1}^*) \leq 1 - p_1 \\ g(1) \geq 1 & \text{if} & B(1-p_1^*) \geq 1 - p_B. \end{array} \tag{29}$$

It should be obvious that since $g'(v_i^*) > 0$, if $g(0) \leq 1 \leq g(1)$ then by the intermediate value theorem there is a unique $\hat{v}_i^*$ such that

$$\hat{v}_i^* = \{v_i^* \in [0,1] : g(v_i^*) = 1\}.$$

Since $g$ is a continuous function of $r$ and $r^*$ then we let $\hat{v}_i^* : [0,1]^2 \to [0,1]$, a continuous function. Since $\frac{\partial p_n}{\partial r} \geq 0$ and $\frac{\partial p_n^*}{\partial r^*} \geq 0$ then we have $\frac{\partial U}{\partial r^*} \leq 0 \leq \frac{\partial U_i}{\partial r}$ so that $\frac{\partial g(\hat{v}_i^*)}{\partial r^*} \leq 0 \leq \frac{\partial g(\hat{v}_i^*)}{\partial r}$. Using $g(\hat{v}_i^*) = 1$ and the implicit function theorem we get $\frac{d\hat{v}_i^*}{\partial r^*} = -\frac{\frac{\partial g(\hat{v}_i^*)}{\partial r^*}}{\frac{\partial g(\hat{v}_i^*)}{\partial \hat{v}_i^*}} \geq 0$. That is, $\hat{v}_i^*$ is non-decreasing in $r^*$. Similarly, $\frac{d\hat{v}_i^*}{\partial r} \leq 0$. In particular it should be clear that $\hat{v}_i^* = 0$ when $g(0) = 1$ (since $g(v_i^*) > 1$ for all $v_i^* > 0$), $\hat{v}_i^* = 1$ when $g(1) = 1$ (since $g(v_i^*) < 1$ for all $v_i^* < 1$) and $\hat{v}_i^* \in (0,1)$ when $g(0) < 1 < g(1)$.

We now determine the fixed points in the buyer's strategy.

**Case 1. Sale prices are independent of posted prices**

Here we have $p_n^* = p_n$ for all $n$ and all $(r, r^*)$. Clearly from (28) we have $g(v_i^*) = 1$ if and only if $v_i^* = v^* = \frac{1}{S}$. Moreover, $g(0) < 1 < g(1)$ since $p_1 \leq p_n \leq p_B$ for any $1 < n < B$. Since $v_i \in [0,1]$ when $g(\frac{1}{S}) = 1$, then it follows that $v_i = v_i^* = v^* = \frac{1}{S}$ is a fixed point to the strategy of buyers.

**Case 2. Sale prices are a function of posted prices**

Now suppose $p_n^* \neq p_n$ for some $n$ and some $(r, r^*)$.

In this case, (4) tells us that the sale price is a non-decreasing function of the posted price, in general, and it is strictly decreasing for some $n \geq 1$ and some posted prices. Of course this depends on the pricing convention adopted and $n$, as discussed in section 3.2 (see Table 1). In particular, we have to realize that $\frac{\partial p_n}{\partial r} = 1$ for some $n \geq 1$ and some $r \in [0,1]$ and $\frac{\partial p_n^*}{\partial r^*} = 1$ for some $n \geq 1$ and some $r^* \in [0,1]$. It follows that we have $\frac{\partial U}{\partial r} < 0$ or $\frac{\partial U}{\partial r^*} < 0$ for some $(r, r^*) \in [0,1]^2$. The immediate implication is that (29) is not satisfied for all $(r, r^*)$; that is, $g(0) > 1$ or $g(1) < 1$ for some $(r, r^*) \in [0,1]^2$. Thus, we must examine the behavior of $g(0)$ and $g(1)$.

Consider $g(0)$. We note that $g(0) < 1$ for all $r = r^*$ since in this case $1 - p_1 = 1 - p_1^*$. Since $\frac{\partial p_1}{\partial r} \geq 0$ it easily follows that $g(0) < 1$ also for $r < r^*$. Notice that, for some pricing convention used and for some $r^*$, there can may or may not be an $r > r^*$ such that $g(0) > 1$ (for example under fixed prices, we have $p_n = r$ and $p_n^* = r^*$ so $r \approx 1$ implies $g(0) > 1$). Thus consider two cases, given $r^*$ and a pricing convention: (i) $g(r) = 1$ for some $r \in (r^*, 1]$ and (ii) $g(r) < 1$ for all $r \in [0,1]$.

- Case (i): In this case we let $\alpha_0(r^*) \in (r^*, 1]$ denote the value of $r$ such that $g(0) = 1$; since $\frac{\partial(1-p_1)}{\partial r} \leq 0$ and $\frac{\partial p_n^*}{\partial r^*} \geq 0$ it follows that $\alpha_0(r^*)$ is unique, $\alpha_0'(r^*) \geq 0$, and $g(0) \geq 1$ for all $r \geq \alpha_0(r^*)$. Also, since $g'(v_i^*) > 0$ then if $r \geq \alpha_0(r^*)$ we have $g(v_i^*) \geq 1$ for all $v_i^* \in [0,1]$.

- Case (ii): Of course, $g(0) < 1$ may hold for all $r$ given some $r^*$ and some pricing convention; for example, if $p_n = \min(q_n, r)$ and $r^* = q_B$ then $g(0) < 1$ since $p_n^* = q_n$ so that $1 - p_1 = 1 - \min(q_1, r) \geq 1 - q_1 > \sum_{n=0}^{B-1} \frac{f_n(B-1, \frac{1}{S-1})}{n+1}(1 - q_{n+1})$. In this case, $\alpha_0(r^*)$ does not exist. Therefore, for notational convenience we define a variable $K > 1$ and say $g(0) \geq 1$ for all $r \geq K$; this is equivalent to stating that $g(0) < 1$ for all $r$ since $r \in [0,1]$. Thus, let $\bar{\alpha}(r^*) = \alpha_0(r^*)$ if there exists some $r \in [0,1]$ such that $g(0) = 1$, and we let $\bar{\alpha}(r^*) = K$ otherwise.

Consider $g(1)$. Notice that $g(1) > 1$ when $r = r^*$ (since $p_1^* \leq p_B$ and $B \geq 2$); since $\frac{\partial p_B}{\partial r} \geq 0$ this implies $g(1) > 1$ for $r \geq r^*$. Of course, we may have $g(1) < 1$ for $r < r^*$, given some pricing convention and some $r^*$.

- If $g(1) = 1$ for some $r \in [0,1]$ then, by an argument similar to the above, it follows that there exists an $\alpha_1(r^*) \in [0, r^*)$ with $\alpha_1'(r^*) \geq 0$, such that if $r \leq \alpha_1(r^*)$ then $g(1) \leq 1$. Since $g'(v_i^*) > 0$ then if such an $\alpha_1(r^*)$ exists and $r \leq \alpha_1(r^*)$, then we have $g(1) \leq 1$ for all $v_i^* \in [0,1]$. Notice that $\alpha_1(r^*) < \alpha_0(r^*)$.

- If $g(1) > 1$ for every $r \in [0,1]$ then $\alpha_1(r^*)$ does not exist; for example, let $p_n = \min(q_n, r)$ and $r^* = q_B$ in which case $B(1-\min(q_1,r)) \geq B(1-q_1) > 1-q_B$. In these instances, for convenience we say that say $g(1) \leq 1$ for all $r \leq -K$; this is equivalent to stating that $g(1) > 1$ for all $r$ since $r \geq 0$. Thus, let $\underline{\alpha}(r^*) = \alpha_1(r^*)$ if there exists some $r \in [0,1]$ such that $g(1) = 1$, and we let $\underline{\alpha}(r^*) = -K$ otherwise.

Notice that $\underline{\alpha}(r^*) < \bar{\alpha}(r^*)$ so $(\underline{\alpha}(r^*), \bar{\alpha}(r^*))$ is nonempty. We also emphasize that the set $A = (\underline{\alpha}(r^*), \bar{\alpha}(r^*)) \cap [0,1] \subseteq [0,1]$. That is the bounds $\underline{\alpha}(r^*)$ and $\bar{\alpha}(r^*)$ may not be binding. Thus let $\underline{r}^* = \max(0, \underline{\alpha}(r^*))$ be the smallest element in $A$ and $\bar{r}^* = \min(\bar{\alpha}(r^*), 1)$ be the highest element in $A$. We then can then partition the unit interval as follows

$$[0,1] = [0, \underline{r}^*) \bigsqcup [\underline{r}^*, \bar{r}^*] \bigsqcup (\bar{r}^*, 1],$$

and note that $[0, \underline{r}^*) = \emptyset$ if $\underline{\alpha}(r^*) = -K$ (since $\underline{r}^* = 0$) and $(\bar{r}^*, 1] = \emptyset$ if $\bar{\alpha}(r^*) = K$ (when $\bar{r}^* = 1$) while $[0,1] \supseteq [\underline{r}^*, \bar{r}^*] \neq \emptyset$ always. Since $\alpha_0'(r^*), \alpha_1'(r^*) \geq 0$ then $\frac{\partial \bar{r}^*}{\partial r^*}, \frac{\partial \underline{r}^*}{\partial r^*} \geq 0$.

Observe that $r^* \in [\underline{r}^*, \bar{r}^*]$. This is because if $\underline{r}^* > 0$ then $\underline{r}^* = \max(0, \underline{\alpha}(r^*)) = \alpha_1(r^*) < r^*$. If $\bar{r}^* < 1$ then we have $\bar{r}^* = \min(\bar{\alpha}(r^*), 1) = \alpha_0(r^*) > r^*$. If $\underline{r}^* = 0$ then $\underline{r}^* \leq r^* \in [0,1]$ and if $\bar{r}^* = 1$ then $\bar{r}^* \geq r^* \in [0,1]$. Thus $r^* \in [\underline{r}^*, \bar{r}^*]$.

Now, recall that if $r \in [\underline{r}^*, \bar{r}^*]$ then $g(0) \leq 1 \leq g(1)$. In that case we have $U_i = U$ for $v_i^* = \hat{v}_i^* \in [0,1]$. Thus, we summarize the discussion above as follows:

$$\text{if } r \in \begin{cases} [0, \underline{r}^*) \\ [\underline{r}^*, \bar{r}^*] \\ (\bar{r}^*, 1] \end{cases} \text{then } U_i \begin{cases} > U & \text{for } v_i^* \in [0,1] \\ = U & \text{for } v_i^* = \hat{v}_i^*,\ U_i > U \text{ for } v_i^* \in [0, \hat{v}_i^*), \text{ else } U_i < U \\ < U & \text{for } v_i^* \in [0,1]. \end{cases}$$

Recall that $[0, \underline{r}^*)$ and $(\bar{r}^*, 1]$ can be empty sets, as well as $[0, \hat{v}_i^*)$ if $\hat{v}_i^* = 0$ and $(\hat{v}_i^*, 1]$ if $\hat{v}_i^* = 1$.

Now consider the representative buyer's best response correspondence $v_i$ from (10):

$$\text{if } r \in \begin{cases} [0, \underline{r}^*) \\ [\underline{r}^*, \bar{r}^*] \\ (\bar{r}^*, 1] \end{cases} \text{then } v_i \begin{cases} = 1 & \text{for } v_i^* \in [0,1] \\ = [0,1] & \text{if } v_i^* = \hat{v}_i^*,\ v_i = 1 \text{ if } v_i^* \in [0, \hat{v}_i^*), \text{ else } v_i = 0 \\ = 0 & \text{for } v_i^* \in [0,1]. \end{cases}$$

Hence, the symmetric equilibrium strategy $v_i = v_i^*$ is such that for any $i$ we have

$$v_i^* = v(r, r^*) = \begin{cases} 1 & \text{if} \quad r \in [0, \underline{r}^*) \\ \hat{v}_i^* & \text{if} \quad r \in [\underline{r}^*, \bar{r}^*] \\ 0 & \text{if} \quad r \in (\bar{r}^*, 1]. \end{cases}$$

Observe that $v : [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous function. It is easy to determine from (27) and (8) that $v_i = v_i^* = \frac{1}{S}$ only when $r = r^*$. Since $\frac{d\hat{v}_i^*}{\partial r} \leq 0 \leq \frac{d\hat{v}_i^*}{\partial r^*}$ then we have

$$v_i = v_i^* \begin{cases} \geq \frac{1}{S} \geq v^* & \text{if} \quad r < r^* \\ = v^* = \frac{1}{S} & \text{if} \quad r = r^* \\ \leq \frac{1}{S} \leq v^* & \text{if} \quad r > r^*. \end{cases}$$

**Proof of Lemma 4**

Consider first the case $\boldsymbol{\theta} = \boldsymbol{\theta}_N$. It is obvious that any $r = r^* \in [0,1]$ is a symmetric equilibrium since $p_n$ and $p_n^*$ are independent of posted prices.

Now consider any case where $\boldsymbol{\theta} \neq \boldsymbol{\theta}_N$. This implies $\frac{\partial p_n}{\partial r} > 0$ for some $n$ and some $r \in [0,1]$, from (4).

Consider (11) for seller $i$ so that we have

$$W(r, r^*) = \sum_{n=0}^{B} f_n(B, v_i^*) p_n$$

where $v_i^*$ satisfies (15) and $p_n$ satisfies (4). We have earlier established that $f_n(B, v_i^*)$ is continuous in $r$ and $r^*$ (since $v_i^*$ is continuous) and $p_n$ is continuous in $r$. Thus, $W : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous in both arguments (being a linear combination of continuous functions). It lays in the compact set $[0,1]$ since $\sum_{n=0}^{B} f_n(B, v_i^*) = 1$ and $p_n \in [0,1]$.

Recall the definition of the value function $\hat{W}(r)$ and of the correspondence of maximizers $\mu(r^*)$. By Berge's Maximum Theorem it follows that $\hat{W}(r)$ is continuous and the "argmax" correspondence $\mu$ is upper hemicontinuous with compact values. By Theorems 14.11 and 14.12 in Aliprantis et al. we also have that $\mu$ has a closed graph. We can then apply Kakutani's fixed point theorem to determine that the set of fixed points of $\mu$ is compact and non-empty.

Now observe from Lemmas 3 and 4 that $\hat{v}_i^* = 0$ is possible only if $r = \bar{r}^*$ (when $g(0) = 1$ is possible) while $\hat{v}_i^* = 1$ is possible only if $r = \underline{r}^*$ (when $g(1) = 1$ is possible). In all other instances $\hat{v}_i^* \in (0,1)$. In particular, in a symmetric equilibrium $r = r^*$ so $v_i^* = \frac{1}{S}$ according to (15) and (8).■

**Proof of Theorem 5**

Lemmas 7 and 8 jointly establish existence of a symmetric equilibrium. Here we characterize the solution $r^*$ and discuss its uniqueness.

We start by using (7) and (9) to rearrange $U_i$ and $U$ as

$$U_i = \tfrac{1}{Bv_i^*}\sum_{n=1}^{B} f_n\left(v_i^*, B\right)\left(1-p_n\right) \qquad \text{and} \qquad U = \tfrac{1}{Bv^*}\sum_{n=1}^{B} f_n\left(v^*, B\right)\left(1-p_n^*\right). \tag{30}$$

since

$$U_i = \sum_{n=0}^{B-1} \frac{f_n(B-1, v_i^*)(1-p_{n+1})}{n+1} = \frac{1}{Bv_i^*}\sum_{n=1}^{B} f_n\left(v_i^*, B\right)\left(1-p_n\right).$$

Since $p_0 = 0$ we can use $U_i$ to rearrange (11) as

$$W = \sum_{n=1}^{B} f_n\left(v_i^*, B\right) p_n = \sum_{n=1}^{B} f_n\left(v_i^*, B\right) - Bv_i^* U_i = 1 - \left(1-v_i^*\right)^B - Bv_i^* U_i.$$

Since $v_i^*$ must satisfy $U_i = U$ then we can substitute for $U_i$ by using (30) so we get

$$W = \sum_{n=1}^{B} f_n\left(v_i^*, B\right) - \tfrac{v_i^*}{v^*}\sum_{n=1}^{B} f_n\left(v^*, B\right)\left(1-p_n^*\right). \tag{31}$$

For notational simplicity, we let $f_n(v) = f_n(v, B) \equiv \binom{B}{n} v^n (1-v)^{B-n}$ so that

$$\tfrac{\partial f_n(v)}{\partial v} = f_n(v)\tfrac{n-Bv}{v(1-v)}.$$

Recall that seller $i$ chooses $r$ taking as given $r^*$. This influences $v_i^*$ and $v^*$ via the equality $U_i = U$, as indicated in (15). That is we have $\frac{\partial W}{\partial r} = \frac{\partial W}{\partial v_i^*}\frac{\partial v_i^*}{\partial r}$. We know from Lemma 3 that $\frac{\partial v_i^*}{\partial r} \leq 0$ and we know that $v_i^*|_{r=r^*} = \frac{1}{S}$. Thus we can characterize the equilibrium $r^*$ by studying $\frac{\partial W}{\partial v_i^*}$ for $v_i^* \in [0,1]$. In particular, we note that if a unique equilibrium exists, then it must be that $\left.\frac{\partial W}{\partial v_i^*}\right|_{r=r^*} = 0$.

From (31) above, recalling that $v^* = \frac{1-v_i^*}{S-1}$, then we have

$$\begin{aligned}\tfrac{\partial W}{\partial v_i^*} &= \sum_{n=1}^{B} f_n\left(v_i^*\right)\tfrac{n-Bv_i^*}{v_i^*(1-v_i^*)} - \tfrac{1}{(v^*)^2}\sum_{n=1}^{B} f_n\left(v^*\right)\tfrac{1-p_n^*}{S-1} \\ &+ \tfrac{v_i^*}{v^*}\sum_{n=1}^{B} f_n\left(v^*\right)\tfrac{(n-Bv^*)}{v^*(1-v^*)}\tfrac{1-p_n^*}{S-1}.\end{aligned} \tag{32}$$

In equilibrium $r = r^*$ and $v_i^* = v^* = \frac{1}{S}$. We can then evaluate (32) at $r = r^*$, which means also imposing $v_i^* = v^* = \frac{1}{S}$. Specifically, define

$$\Delta\left(r^*\right) = \left.\tfrac{\partial W}{\partial v_i^*}\right|_{v_i^*=\frac{1}{S}, r=r^*} = \tfrac{\sum_{n=1}^{B} M_n p_n^* - A}{S-1}$$

where $\Delta : [0,1] \to \mathbb{R}$ exploits the following definitions: $f_n \equiv f_n\left(\frac{1}{S}\right)$

$$M_n = S^2 f_n\left(1 - \tfrac{n-\lambda}{S-1}\right)$$

and

$$\textstyle\sum_{n=1}^{B} M_n = S^2\left(1 - f_0 - \frac{f_1}{S}\right) \quad > A = S^2\left(1 - f_0 - f_1\right) > 0. \tag{33}$$

We obtain the above, when $v = \frac{1}{S}$, using $\sum_{n=1}^{B} f_n = 1 - \left(\frac{S-1}{S}\right)^B$ and $\sum_{n=1}^{B} f_n \frac{n-Bv}{v(1-v)} = B\left(\frac{S-1}{S}\right)^{B-1}$. We also define

$$\omega = \frac{\sum_{n=1}^{B} M_n q_n - A}{S-1}.$$

We now study the behavior of $\Delta(r^*)$ on $[0,1]$ under every pricing convention, denoting $\Delta'(r^*) = \frac{\partial \Delta(r^*)}{\partial r^*}$. Notice that in equilibrium we must have $\Delta(r^*) = 0$ so that using the expression above in equilibrium we must have

$$\textstyle\sum_{n=1}^{B} f_n p_n = 1 - f_0 - f_1 + \frac{\sum_{n=1}^{B} f_n (n-\lambda) p_n}{S-1} \tag{34}$$

Since $p_0 = 0$ we can write $E[n] = \sum_{n=1}^{B} f_n n = \lambda = Bv$ and

$$\textstyle\sum_{n=1}^{B} f_n (n-\lambda) p_n = \sum_{n=1}^{B} f_n n p_n - \lambda \sum_{n=1}^{B} f_n p_n = E[np] - E[n]E[p] = \mathrm{cov}[n,p]$$

where $p \in P \cup \{0\}$ is a random variable represented profits. Thus, in equilibrium we need

$$E[p] = 1 - f_0 - f_1 + \frac{\mathrm{cov}[n,p]}{S-1}. \tag{35}$$

**1) Case $\theta = \theta_X$**

Here we have $p_n^* = r^*$ for all $n = 1, 2, ..B$. Notice from (15) and (30) that $\left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} < 0$ always. Thus the set of maxima must satisfy $\Delta(r^*) = 0$, i.e. $\left.\frac{\partial W}{\partial v_i^*}\right|_{r=r^*} = 0$. We have

$$\Delta(r^*) = \quad \frac{r^* \sum_{n=1}^{B} M_n - A}{S-1}. \tag{36}$$

Notice that $\Delta'(r^*) = \frac{\sum_{n=1}^{B} M_n}{S-1} > 0$ from (33). Also, $\Delta(0) = \frac{-A}{S-1} < 0$ and $\Delta(1) = \frac{\sum_{n=1}^{B} M_n - A}{S-1} > 0$ from (33). Therefore, the Intermediate Value Theorem establishes there exists a unique $r_X \in (0,1)$ such that if $r^* = r_X$ then $\Delta(r^*) = 0$. Solving (36) we obtain

$$r_X \quad = \frac{A}{\sum_{n=1}^{B} M_n}$$

It follows that $r = r^* = r_X$ is the unique maximum of $W$, hence the unique equilibrium.

**2) Case $\theta = \theta_F$**

Here $p_n^* = \max(q_n, r^*)$. Thus, from (15) and (30) we have $\left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} < 0$ only if $r^* \geq q_1$ and $\left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} = \Delta'(r^*) = 0$ when $r^* \in [0, q_1)$. Thus, we have

$$\left.\frac{\partial W}{\partial r}\right|_{r=r^*} = \begin{cases} 0 & \text{if} \quad r^* \in [0, q_1) \\ \Delta(r^*) \left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} & \text{if} \quad r^* \in [q_1, 1] \end{cases}$$

It follows that we must concentrate on studying $\Delta(r^*)$ on $[q_1, 1]$. Obviously, if $\Delta(r^*) > 0$ on that set, then we have $\frac{\partial W}{\partial r}\big|_{r=r^*} < 0$ for all $r^* \in [q_1, 1]$ so we have $r^* \in [0, q_1)$ is the equilibrium set. To have a unique interior equilibrium we need $\Delta(r^*) = 0$ for some $r^* \in (q_1, 1)$. Thus study $\Delta$ on the set $[q_1, 1]$, where $\Delta$ is continuous but not continuously differentiable.

Recall that we have defined $q_{B+1} = 1$. Thus, suppose $r^* = r_j \in [q_j, q_{j+1}) \subset [q_1, 1]$ for some $j = 1, 2, ...B$. Then we have

$$\Delta(r_j) = \frac{r_j \sum_{n=1}^{j} M_n + \sum_{n=j}^{B} M_n q_n - q_j M_j - A}{S-1} \tag{37}$$

$$\Rightarrow \Delta'(r_j) = \frac{\sum_{n=1}^{j} M_n}{S-1} \quad \text{for } r_j \in [q_j, q_{j+1}).$$

Call the expression in square brackets $Q_n$ (which might be negative or positive). Observe that $\{Q_n\}$ is a decreasing sequence with $Q_1 > 0$ always and $Q_B < 0$ if $B > S$. Recall also that $r_j < q_n$ for $n \geq j+1$. Thus when $B \leq S$, we have $M_n \geq 0$ for all $n$ and therefore $\Delta'(r_j) > 0$ for $r_j \in [q_j, q_{j+1})$ and $\Delta(r_j) < \Delta(r_{j+1})$ if $B \leq S$. Therefore, $\Delta(r^*)$ is strictly increasing in $r^* \in [q_1, 1]$ if $B \leq S$.

If $B > S$ then there exists some $1 < \bar{n} < B$ such that $M_n \geq 0$ for $n \leq \bar{n}$ and $M_n < 0$ for $n > \bar{n}$. Since from (33) we have $\sum_{n=1}^{B} M_n > 0$, then $\sum_{n=1}^{j} M_n > \sum_{n=1}^{B} M_n > 0$ for all $j < B$ because $M_n \geq 0$ for $n$ small and $M_n < 0$ for $n$ large. Thus $\Delta'(r_j) > 0$ for $r_j \in [q_j, q_{j+1})$ for all $j$, if $B > S$ . Now, observe that $B > S$ then we have $\Delta(r_j) < \Delta(r_{j+1})$ since $M_n \geq 0$ for $n \leq \bar{n}$ and $M_n < 0$ for $n > \bar{n}$. Thus we have

$$\Delta'(r^*) = \begin{cases} 0 & \text{if} \quad r^* \in [0, q_1) \\ \frac{\sum_{n=1}^{j} M_n}{S-1} > 0 & \text{if} \quad r^* \in [q_j, q_{j+1}) \text{ for all } j = 1, 2..., B-1 \end{cases}$$

We see that $\Delta$ is continuous and from (33)

$$\Delta(1) = \frac{\sum_{n=1}^{B} M_n - A}{S-1} > 0$$

Since $\Delta'(r^*) = 0$ for $0 \leq r^* < q_1$ then $\Delta(r^*) = \Delta(q_1)$ for $0 \leq r^* < q_1$ where

$$\Delta(q_1) = \frac{\sum_{n=1}^{B} M_n q_n - A}{S-1}.$$

Since $\Delta(r^*)$ is a continuous increasing function on $[q_1, 1]$, then if $\Delta(q_1) < 0$ we have that there exists a unique $r_F \in (q_1, 1)$ associated to a unique $j = 1, 2, ..., B$ such that if $r^* = r_F \in [q_j, q_{j+1})$ then $\Delta(r^*) = 0$. Using (37) we obtain

$$r_F = \frac{A - \sum_{n=j}^{B} M_n q_n + M_j q_j}{\sum_{n=1}^{j} M_n} .$$

We note that

$$\Delta(q_1) < 0 \Leftrightarrow \omega < 0.$$

Thus, if $\omega < 0$ then $\Delta(q_1) < 0$ and there exists a unique equilibrium $r^* = r_F$. If $\omega \geq 0$ then we have $\Delta(r^*) > 0$ for all $r^* \in (q_1, 1]$ and $\Delta(q_1) \geq 0$ hence we have a continuum of equilibria $r^* \in [0, q_1]$.

**3) Case** $\theta = \theta_C$

Here we have $p_n^* = \min(q_n, r^*)$. Thus, from (15) and (30) we have $\left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} < 0$ only if $r^* \in [0, q_B)$ and $\left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} = \Delta'(r^*) = 0$ when $r^* \in [q_B, 1]$. Thus, we have

$$\left.\frac{\partial W}{\partial r}\right|_{r=r^*} = \begin{cases} \Delta(r^*) \left.\frac{\partial v_i^*}{\partial r}\right|_{r=r^*} & \text{if} \quad r^* \in [0, q_B) \\ 0 & \text{if} \quad r^* \in [q_B, 1]. \end{cases}$$

It follows that we must concentrate on studying $\Delta(r^*)$ on $[0, q_B)$. Obviously, if $\Delta(r^*) < 0$ on that set then $\left.\frac{\partial W}{\partial r}\right|_{r=r^*} > 0$ for all $r^* \in [0, q_B)$ so we have $r^* \in [q_B, 1]$ is the equilibrium set. To have a unique interior equilibrium we need $\Delta(r^*) = 0$ for some $r^* \in (0, q_B)$. Thus study $\Delta$ on the set $[0, q_B)$ where $\Delta$ is continuous but not continuously differentiable.

Recall that we have defined $q_0 = 0$. Thus, suppose $r^* = r_{j-1} \in [q_{j-1}, q_j) \subset [0, q_B)$ for some $j = 1, ...B$. Then

$$\begin{aligned} \Delta(r_{j-1}) &= \tfrac{\sum_{n=1}^{j} M_n q_n - M_j q_j + r_{j-1} \sum_{n=j}^{B} M_n - A}{S-1} \\ \Rightarrow \Delta'(r_{j-1}) &= \tfrac{\sum_{n=j}^{B} M_n}{S-1} \quad \text{for } r_{j-1} \in [q_{j-1}, q_j). \end{aligned} \tag{38}$$

We always have $\Delta(0) = -\frac{A}{S-1} < 0$ and $\Delta(r^*) = \Delta(q_B)$ for $q_B \leq r^* \leq 1$ (since $\Delta'(r^*) = 0$ on $[q_B, 1]$) where

$$\Delta(q_B) = \tfrac{\sum_{n=1}^{B} M_n q_n - A}{S-1}.$$

we notice that $\Delta(q_B) > 0 \Leftrightarrow \omega > 0$. Thus, define $r_C \in (0, q_B)$ associated to a unique $j = 1, 2, ..., B$ such that if $r^* = r_C \in [q_{j-1}, q_j)$ then $\Delta(r^*) = 0$. Using (38) we obtain

$$r_C = \tfrac{A - \sum_{n=1}^{j} M_n q_n + M_j q_j}{\sum_{n=j}^{B} M_n}.$$

Now consider the slope of $\Delta$. If $B \leq S$ we have $M_n \geq 0$ for all $n$ and therefore $\Delta'(r^*) \geq 0$ for all $r^* \in [0, q_B)$. Since $\Delta(q_B) > 0$ since $\omega > 0$ in this case, then we have that $r^* = r_C$ is the unique equilibrium, by the intermediate value theorem.

If $B > S$ then $M_n \geq 0$ for $n$ small and $M_n < 0$ for $n$ large. In particular we have $\Delta'(r_{B-1}) = \frac{M_B}{S-1} < 0$ and $\Delta'(r_0) = \frac{\sum_{n=1}^{B} M_n}{S-1} > 0$ (due to (33)). Hence, $\Delta(r^*)$ is a hump-shaped continuous function on $[0, q_B)$. Thus, there exists some value $1 < \overline{\overline{n}} \leq B - 1$ such that $\Delta\left(r^{*}\right)$ decreases for $r^{*} \geq q_{\overline{\overline{n}}}$ and increases otherwise. Here we may have three cases. If $\omega > 0$ then $\Delta\left(q_{B}\right) > 0$ hence we again have $r^{*} = r_{C}$ as the unique equilibrium. In this case $\frac{\partial W}{\partial r}\Big|_{r=r^{*}} = 0$ at $r^{*} = r_{C}$ and $W$ decreases when moving away from $r_{C}$. If $\omega \leq 0$ then $\Delta\left(q_{B}\right) \leq 0$. In this case we may have two sub-cases:

- A first sub-case is $\Delta(r^{*}) < 0$ for all $r^{*} \in [0, q_{B})$. Here $\frac{\partial W}{\partial r}\Big|_{r=r^{*}} > 0$ for all $r^{*} \in [0, q_{B})$ so we have a continuum of equilibria on $[q_{B}, 1]$

- The second sub-case is $\Delta(r^{*}) < 0$ for all $r^{*} \in [0, q_{j}) \cup [q_{j+k}, q_{B})$ for some $1 \leq j < k \leq B-1$ and $\Delta(r^{*}) > 0$ for all $r^{*} \in [q_{j+1}, q_{j+k-1})$. Here, $\Delta(r^{*}) = 0$ for two elements $r_{j}^{*} \in [q_{j}, q_{j+1})$ and $r_{j+k-1}^{*} \in [q_{j+k-1}, q_{j+k})$ respectively. Here $\frac{\partial W}{\partial r}\Big|_{r=r^{*}} > 0$ for $r^{*} \in [r_{j+k-1}^{*}, q_{B})$ so $r_{j+k-1}^{*}$ must be a minimum, and $\frac{\partial W}{\partial r}\Big|_{r=r^{*}} < 0$ for $r^{*} = r_{j}^{*}$ so $r_{j}^{*}$ and $[q_{B}, 1]$ must be both local maxima. Obviously $W|_{r^{*}=r_{j}^{*}} < W|_{r^{*}=q_{B}}$ since in both case $v_{i}^{*} = \frac{1}{S}$. It follows that we have a continuum of equilibria on $[q_{B}, 1]$.

Finally, we prove that (i) $r_{F} = r_{X}$ if and only if $r_{X} \geq q_{B}$ and (ii) $r_{C} = r_{X}$ if $r_{X} \leq q_{1}$ and $r_{C} > r_{X}$ if $r_{X} > q_{1}$. To do so, define $Y \equiv \sum_{n=1}^{B} M_{n}$.Then:

(i) If $r_{F} = r_{X} = \frac{A}{Y}$, then clearly $j = B$. Hence, $r_{F} \geq q_{B}$ and so $r_{X} \geq q_{B}$. Now prove $r_{X} \geq q_{B} \Rightarrow r_{F} = r_{X}$. Let $r_{X} \geq q_{B}$. For any $j \leq B-1$ we have $r_{F} > q_{j+1}$; since this contradicts the definition $r_{F} < q_{j+1}$ then it must be that $r_{F} = r_{X}$. To prove it, notice that

$$r_{F} < q_{j+1} \Leftrightarrow \frac{A-\sum_{n=j+1}^{B} M_{n} q_{n}}{Y-\sum_{n=1}^{j} M_{n}} < q_{j+1} \Leftrightarrow r_{X} < q_{j+1} + \sum_{n=j+1}^{B} \frac{M_{n}(q_{n}-q_{j+1})}{Y} \equiv G_{j+1}$$

We have $G_{j+1} > G_{j}$ since

$$q_{j+1} + \sum_{n=j+1}^{B} M_{n}\left(\frac{q_{n}-q_{j+1}}{Y}\right) > q_{j} + \sum_{n=j}^{B} \frac{M_{n}(q_{n}-q_{j})}{Y} \quad \Leftrightarrow 1 > \frac{\sum_{n=j+1}^{B} M_{n}}{Y}$$

for $\sum_{n=j+1}^{B} M_{n} < 0$ or $\sum_{n=j+1}^{B} M_{n} > 0$, since $Y > \left|\sum_{n=j+1}^{B} M_{n}\right| > 0$. Now, for $j = B-1$ we have $r_{X} < q_{B}$, which is not true.. So $r_{F} < q_{j+1}$ does not hold for any $j \leq B-1$, when $r_{X} \geq q_{B}$. Hence, $r_{F} = r_{X}$. Notice that $r_{F} < r_{X}$ is not generally true for $r_{X} < q_{B}$. To prove it let $r_{X} \in [q_{B-1}, q_{B})$ and let $\lambda$ be large. Then when $j = B-1$ we have $q_{B} > r_{F} > r_{X} \geq q_{B-1}$ since $-M_{B} q_{B} > -r_{X} M_{B}$.

(ii) If $r_{C} = r_{X}$, then clearly $h = 1$, which implies $r_{C} \leq q_{1}$, hence $r_{X} \leq q_{1}$. The converse is easily seen true. To prove that if $r_{X} > q_{1}$ then $r_{C} > r_{X}$, we just have to show that

$r_C > r_X$ for $h = 2$ (since $r_C$ can only increase as $h$ grows above 2). Let $r_X > q_1$. Notice that

$$r_C = \frac{A - \sum_{n=1}^{h-1} M_n q_n}{Y - \sum_{n=1}^{h-1} M_n} > r_X = \frac{A}{Y} \Leftrightarrow \sum_{n=1}^{h-1} M_n q_n < \frac{A}{Y} \sum_{n=1}^{h-1} M_n$$

For $h = 2$ we have $r_C > r_X$ as $M_1 q_1 < \frac{A}{Y} M_1 \Rightarrow q_1 < r_X$ (recall that $M_1 > 0$).■

**Proof of Lemma 6**

Let $n = 1, 2, ..., B$. Recall that in a symmetric equilibrium the probability of having $n$ customers, $\Pr[n, n \neq 0]$ is independent of the commitment technology as is given by (17). Thus the distribution of buyers at any store is independent of $\boldsymbol{\theta}$. When $\boldsymbol{\theta} = \boldsymbol{\theta}_F$ then we have $p_n \geq q_n$ for all $n$, while $p_n = q_n$ for all $n$ when $\boldsymbol{\theta} = \boldsymbol{\theta}_N$. It follows that $\bar{p}_N \leq \bar{p}_F$. When $\boldsymbol{\theta} = \boldsymbol{\theta}_C$ then $p_n \leq q_n$ for all $n$ and so $\bar{p}_C \leq \bar{p}_N \leq \bar{p}_F$. Clearly $\bar{p}_X = r_X$. Since $r_X - r_C$ and $r_X - r_F$ may be positive or negative for $r_X \in (q_1, q_B)$ we do not have a clear relationship between $\bar{p}_X$ and average sale prices under weaker commitment technologies. However, if $r_X$ is close to $r_C$ and $r_F$, then it is clear that $\bar{p}_C \leq \bar{p}_X \leq \bar{p}_F$ since (i) $p_n \geq r_F \approx r_X$ with positive probability, when $\boldsymbol{\theta} = \boldsymbol{\theta}_F$ and (ii) $p_n \leq r_C \approx r_X$ with positive probability, when $\boldsymbol{\theta} = \boldsymbol{\theta}_C$.■

**Proof of Lemma 7**

Fix $\lambda \in \mathbb{R}_+$ and let $B = \lambda S$ and let $S \to \infty$ (alternatively let $S = B/\lambda$ and let $B \to \infty$). We see that $\lim_{S\to\infty} \text{cov}(x_i, x_j) = 0$, i.e. $x_i$ and $x_j$ are independent random variables. This implies that as the size of the market grows unbounded, we can focus only on the marginal probabilities, that is the probability that any given seller is visited by $n$ buyers. In this case, this marginal probability distribution is $\text{bin}(B, 1/S)$. As $S \to \infty$ the binomial distribution converges to a Poisson with parameter $\lambda$ (see Hoel et al., chapter 3). Thus, (17) implies (25) as the market grows large while keeping $\lambda$ constant.■

**Proof of Lemma 8**

If we set $B = S\lambda$ and let $S \to \infty$ we have

$$\begin{aligned}
\lim_{S\to\infty} \tfrac{M_n}{S^2} &= \lim_{S\to\infty} f_n(B, \tfrac{1}{S}) \left(1 - \tfrac{n-\lambda}{S-1}\right) = \tfrac{e^{-\lambda}\lambda^n}{n!} \\
\lim_{S\to\infty} \tfrac{A}{S^2} &= \lim_{S\to\infty} \left[1 - \left(1 - \tfrac{1}{S}\right)^{\lambda S} - \lambda \left(1 - \tfrac{1}{S}\right)^{\lambda S - 1}\right] = 1 - e^{-\lambda} - e^{-\lambda}\lambda
\end{aligned}$$

Then, from (16) we obtain (26). It is immediate that $\frac{dr_X}{d\lambda} > 0$. To demonstrate the other claims let $f_n\left(\frac{1}{S}\right) = f_n$. Then, use (35) and notice that $\text{cov}(n, p) > 0$ (for $n = 0$ we have $p_0 = 0$ and $p_n > 0$ otherwise) and it is minimized when $p_n = r$ for all $n$. Also, $\text{cov}[n, p] < \infty$ since $E[np] < \infty$. When $B = S\lambda$ then $\lim_{S\to\infty} \frac{\text{cov}(n,p)}{S-1} = 0$. Thus, an approximate solution for $r$ in a large economy must solve $\sum_{n=1}^{B} f_n p_n = 1 - f_0 - f_1$. This leads to the expressions in (26). For instance, if $p_n = r_X$ for all $n$, then

$$r_X = 1 - \frac{f_1}{1 - f_0} = 1 - \frac{\lambda}{e^{\lambda} - 1}.$$

If $r_F \neq r_X$ (i.e. if $r_X < q_B$) then we have

$$r_F \textstyle\sum_{n=1}^{j} f_n + \sum_{n=j+1}^{B} f_n q_n = 1 - f_0 - f_1$$

for some $1 \leq j \leq B-1$. Since $\sum_{n=j+1}^{B} f_n q_n > \sum_{n=j+1}^{B} f_n r_F$ and $1-f_0-f_1 = r_X \sum_{n=1}^{j} f_n + \sum_{n=j+1}^{B} f_n r_X$, then $r_F < r_X$. Similarly, $r_C > r_X$ if $r_X > q_1$.

Finally, to demonstrate that $r_F$ and $r_C$ are increasing in $\lambda$ notice that in equilibrium we must have $E[p] = 1 - f_0 - f_1$. It is obvious that $1 - f_0 - f_1$ grows in $\lambda$ more than $E[p]$ if $r$ is constant (as $p \in (0, 1)$). Thus $r$ must increase in $\lambda$.$\blacksquare$