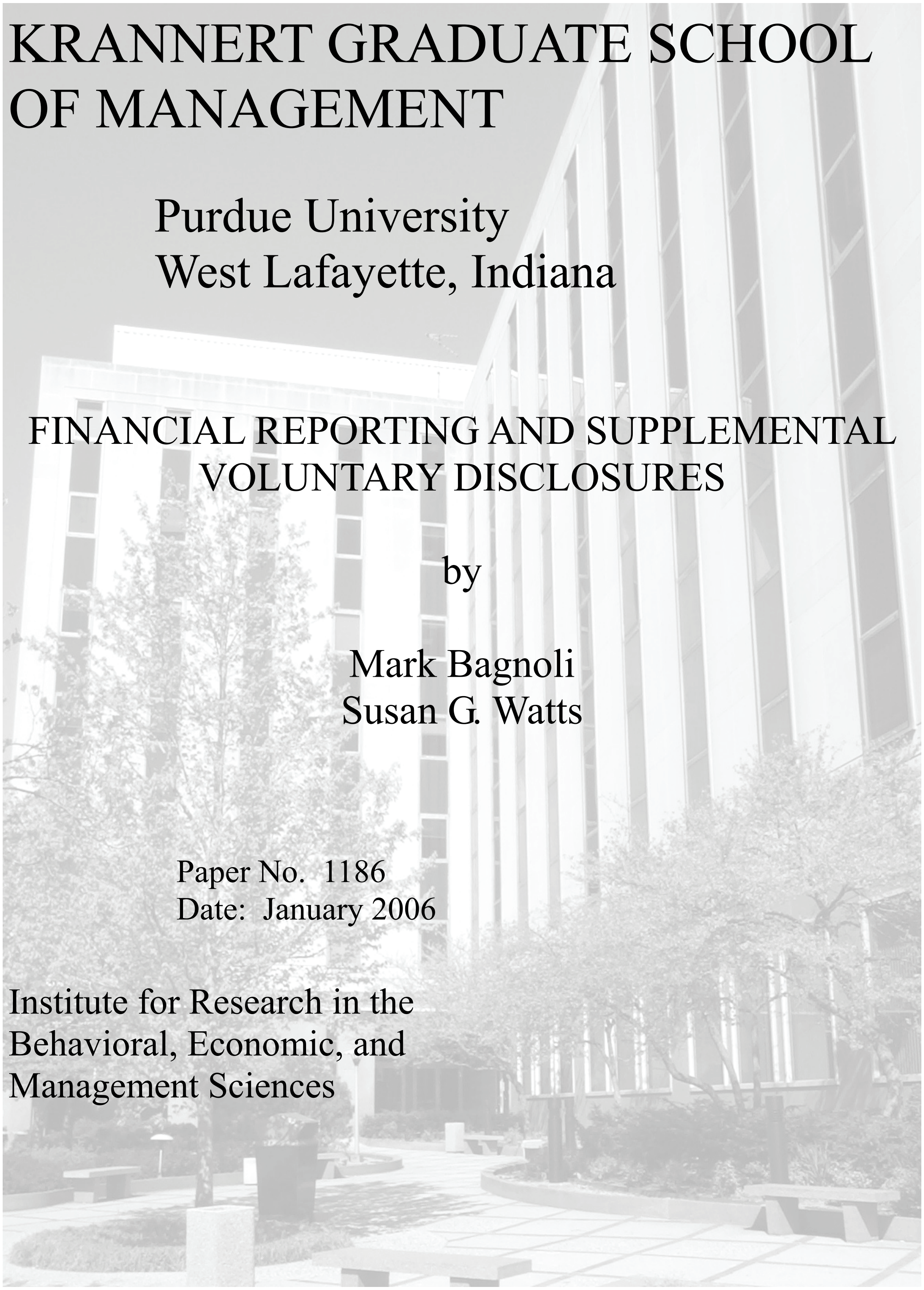

KRANNERT GRADUATE SCHOOL OF MANAGEMENT

Purdue University
West Lafayette, Indiana

FINANCIAL REPORTING AND SUPPLEMENTAL VOLUNTARY DISCLOSURES

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Mark Bagnoli
Susan G. Watts

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Mark Bagnoli
and
Susan G. Watts

Krannert Graduate School of Management
Purdue University
West Lafayette, IN 47907

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Abstract: Using a Verrecchia [1983]–type model, we study the optimal voluntary disclosure strategy of a manager with private information that helps the market interpret financial information the firm is required to report. In equilibrium, the manager's disclosure strategy enhances upward or mitigates downward revisions in the market's estimate of firm value conditional on the firm's financial reports. Hence, what the manager discloses (large or small values of her private information) and the probability of disclosure depend on the information in the firm's financial reports. This leads to testable implications regarding the probability of voluntary disclosure (*e.g.*, firms whose financial reports are more surprising provide more voluntary disclosures), and how earnings and revenue response coefficients depend on the manager's voluntary disclosure strategy. Finally, we show that changes in mandatory disclosure regulations can reduce the probability of voluntary disclosure even though the manager's private information is used to interpret the firm's mandatory disclosures.

Keywords: voluntary disclosure, financial statements, earnings surprise, asymmetric information, price efficiency, good news, bad news

JEL Classification Codes: G1, G14, M41

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1. Introduction.

Much of the analytic work on voluntary disclosures in accounting builds on the work of Verrecchia [1983] or Dye [1985].[1] The objective of this paper is to extend this literature by examining how mandatory financial reporting affects the manager's decision to voluntarily provide supplemental disclosures. In our setting, the manager's private information, if disclosed, reduces the market's uncertainty about how information in the firm's financial reports translates into long–term value and thus is valuable only in the presence of the firm's financial reports. The advantage of this structure is that it allows us to examine how the items in the firm's financial reports, how they translate into long–term firm value, and how they are related affect (1) what the manager chooses to disclose and (2) the probability that the manager makes a voluntary disclosure.

In our model, the firm's financial reports provide multiple pieces of information that the market uses to estimate firm value, and the manager seeks to maximize this value. In the simpler case when the key items in the firm's financial reports (*e.g.*, revenues and expenses) have uncorrelated transitory components,[2] the set of values of the manager's private information that she voluntarily discloses depends on whether the firm's financial report is better or worse than the market expects. As a result, the manager's voluntary supplemental disclosure strategy enhances (mitigates) the market's upward (downward) revision in price conditional on the information in the firm's financial reports. Further, the probability of voluntary disclosure is greater if the firm's mandatory reports contain larger surprises (in either direction). Intuitively, if the firm's reported performance is close to market expectations, the benefits of voluntary disclosure (the associated increase in the firm's stock price) do not exceed the costs, whereas if the firm misses or beats market expectations by a lot, the benefits of voluntary disclosure exceed the costs. When the transitory components in the firm's financial reports covary, the conditions under which the manager's voluntary disclosure decision enhances upward (mitigates downward) price revisions are more complicated; however, similar results on what the manager chooses to disclose and the probability of voluntary disclosure obtain.

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[1] A review and synthesis of the literature is contained in Verrecchia's [2001] survey and Dye's [2001] discussion.

[2] For example, the firm's reported revenues may contain new orders that represent a long–term increase in sales (a permanent component), new orders that do not represent a long–term increase in sales (a transitory component) and/or changes in the firm's pricing power that may be permanent or transitory. Similarly, the firm's reported expenses may contain the effects of long–term strategic decisions designed to lower the firm's production costs (a permanent component), changes in expenses that are temporary (*e.g.*, weather or strike induced effects) and/or permanent or transitory changes in input prices.

Our analysis also allows us to show that there is an important information externality: Changes in mandatory disclosure regulations affect the manager's willingness to voluntarily provide supplemental information and can, in some circumstances, reduce the probability of voluntary disclosure. More specifically, when the transitory components in the firm's financial reports are uncorrelated, regulatory changes that lead to tighter linkages between information in the firm's financial reports and the firm's liquidation value produce positive information externalities by increasing the probability of voluntary disclosure by management. Conversely, regulatory changes that weaken the link produce negative information externalities by lowering the probability of voluntary disclosure.[3] Again, the analysis is more complicated when the transitory components in the firm's financial reports covary but similar information externalities arise. In this case, tightening (loosening) the linkage can lead to either positive or negative information externalities depending, in part, on whether the firm reports good or bad news and on the covariance structure between items in the firm's financial reports. As a result, our model suggests that a cost–benefit analysis of regulatory changes in disclosure policy should include these information externalities and may require careful assessment of the relative importance of voluntary disclosures when the firm reports good versus bad news.

Our model also has several empirical implications. First, it predicts that supplemental disclosures are more likely the more extreme the news in the firm's financial reports. Second, it predicts that relative to managers who do not provide supplemental disclosures, those who do increase the market response to a positive earnings surprise and reduce the market response to a negative earnings surprise. Third, our model predicts that a content analysis of management's supplemental voluntary disclosures is likely to show that if the firm reports good news (bad news), the manager's supplemental disclosure will be designed to enhance (mitigate) the value impact of the news (by indicating that the performance is relatively permanent rather than transitory in the case of good news and vice versa in the case of bad news).

Our work complements a common approach taken in the voluntary disclosure literature which is to view financial reports as altering the information environment for the firm. Papers that take

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[3] This result is similar to Fischer and Stocken's [2004] finding that the introduction of an additional source of information can suppress an expert's ability to communicate his/her private information. Interestingly, the underlying reason for their result is the misalignment of the expert's and decision–maker's preferences in the context of a "cheap talk" game as opposed to the underlying reason in our model—that increases in mandatory disclosures alter the benefits (in the form of increases in the firm's stock price) to providing supplemental disclosures without altering the costs.

this approach (*e.g.*, Verrecchia [1983, 1990], Dye [1985, 1988], Jung and Kwon [1988] or the more recent work by Fischer and Stocken [2001], Dutta and Trueman [2002], Korn and Schiller [2003], Korn [2004]) generally examine comparative static results from changes in the parameters of the probability distribution describing the manager's private information concerning the firm's liquidation value.[4] Einhorn [2005] provides a more detailed formulation of these ideas by explicitly considering a voluntary disclosure model with two signals, each correlated with the firm's liquidation value. The comparative static analyses and Einhorn's more detailed formulation show that the mandatory disclosure of information about the firm's liquidation value reduces the manager's willingness to voluntarily disclose her private signal by reducing the value impact of such a disclosure and does not depend on whether the mandatory disclosure contains good or bad news. Our analysis complements this approach by showing that if the manager's private information reduces uncertainty about how financial statement information translates into firm value, what the manager voluntarily discloses and the probability of a voluntary disclosure both depend on the information contained in the firm's financial reports and whether it is good or bad news.

Our analysis is also closely related to the recent paper by Hughes and Pae [2004]. They focus on acquisition of information about the precision of an estimate and management's decision to voluntarily disclose that information. Our model is a generalization and extension of theirs designed to examine the interaction between mandatory and voluntary disclosure decisions and how the interaction depends on (1) the mandatory disclosure of multiple items in the firm's financial reports, each of which is used to estimate firm value; (2) the relationship between the permanent and transitory components of the information in the firm's financial reports and the firm's liquidation value; (3) the relationship between the transitory components of the items in the firm's financial reports; and (4) whether the firm missed or beat expectations of all of the relevant items reported in its financial statements.

Interestingly, both Hughes and Pae [2004] and Einhorn [2005] describe different mechanisms that lead the manager to endogenously choose between voluntarily disclosing a set of small or a set of large values of her private information. Our analysis indicates that such a result is robust to a wide variety of specifications because it only relies on whether the market price is increasing or decreasing in the manager's private information. In the former case, the manager will voluntarily disclose a set of large values and, in the latter case, the manager will voluntarily disclose a set of

[4] See also the papers cited in the survey by Verrecchia [2001] and Dye's [2001] discussion.

small values.

Finally, our analysis of the interactions between mandatory and voluntary disclosures also complements prior work that focuses on the stewardship role of disclosure (Gigler and Hemmer [1998, 2001], Kwon, Newman and Suh [2001] and Venugopalan [2001]). Each uses a Principal–Agent model to examine how the contract, which motivates the manager's voluntary disclosure of his/her private information to the principal, is affected by alternative mandatory disclosure regimes. Each shows that the costs incurred by the principal to motivate the manager to disclose (the agency costs inherent in the delegation problem) are affected by the different mandatory disclosure regimes.

The remainder of the paper is organized as follows. In Section 2, we present our voluntary disclosure model and solve for the unique equilibrium disclosure strategy. In Section 3, we analyze the interaction between the manager's voluntary disclosure strategy and the information in the firm's financial reports and derive empirical predictions. In Section 4, we analyze the information externalities that arise from changes in financial reporting regulations and examine the impact of mandatory risk disclosures on the manager's voluntary disclosure strategy in Section 5. In Section 6, we extend our analysis to show that our results do not depend on our specific assumption of what the manager's private information is and conclude in Section 7.

2. The Model and Equilibrium.

We model the financial reporting system as providing information that is correlated with the unknown terminal value of the firm, v. We assume that the firm reports a vector $x \in \mathcal{R}^n$ and that, for simplicity, v, x are joint normally distributed ($v, x \sim N(\mu, \Sigma)$), where $\mu = (\mu_v, \mu_1, \mu_2, \ldots, \mu_n)$ is the vector of means and Σ is the variance-covariance matrix with generic element σ_{ij}.[5] Below, we will find it useful to partition Σ so that

$$\Sigma = \begin{pmatrix} \sigma_{vv} & \Sigma_{vx} \\ \Sigma_{xv} & \Sigma_{xx} \end{pmatrix}.$$

Investors are risk neutral and, after observing the firm's financial report, value the firm at $P = \mathrm{E}[v \mid x]$ unless management provides additional, supplementary information that affects their

[5] A natural interpretation of x is that it contains quantitative information provided in the firm's financial statements such as revenues, costs of goods sold, earnings, total assets, working capital, research and development expenditures, operating cash flows, etc.

interpretation of the firm's financial report. Because we have assumed joint normality, a natural way to model this is to assume that the manager has private information about the variance of an element provided in the firm's financial reporting system. Without loss of generality, we will designate this item as the first, making the manager's private information σ_{11}. One interpretation of this set–up is that x_1 is made up of permanent and transitory components and that the market has less information about the mix than does the manager.[6] This structure also captures the idea that the manager is better able (relative to the market) to interpret the firm's historical performance as represented by the information in its financial reports. Alternatively, one can interpret our construct as modeling a manager who has private information about the riskiness of items in the firm's financial reports.[7]

Following Verrecchia [1983, 1990], the manager can choose to voluntarily disclose her private information. If she does so, she must disclose truthfully and incurs a cost $c > 0$.[8] We assume that the manager is risk neutral and seeks to maximize the market value of the firm less any disclosure costs. If the manager discloses her private information, the market uses it in estimating firm value. If the manager does not, the market uses that information in estimating firm value. To represent this formally, assume that the market's priors are represented by a family of log–concave densities of σ_{11}, $g(\sigma_{11}; x)$ with support $[a, b)$, $0 \leq a < b$.[9] Given this, when the manager voluntarily discloses her private information, the market combines it with the information disclosed in the firm's financial report, and assigns the firm a market value of

$$P \equiv \mathrm{E}[v \mid x, \sigma_{11}] = \mu_v + \beta^T(x - \mu_x), \tag{1}$$

where $\mu_x = (\mu_1, \mu_2, \ldots, \mu_n)$ and $\beta = \Sigma_{xx}^{-1}\Sigma_{vx}$. The market's assessment of firm value is linear in the difference between what the firm actually reported and the market's expectation $(x - \mu_x)$

6 For example, if x_1 is revenues, its transitory components might be unusual, temporary fluctuations in orders, temporary changes in the firm's pricing power, or fluctuations in returns. Transitory components in expenses might be unusual, temporary fluctuations in the firm's cost of goods sold, temporary changes in input prices, or fluctuations in shrinkage.

7 This idea is related to, but distinct from, Jorgensen and Kirschenheiter [2003]. In that paper, they assume that the manager has private information about σ_{vv}. Our assumption that the manager knows σ_{11} implies that she has private information about the variance of the liquidation value of the firm conditional on information provided by the financial reporting system, $\mathrm{Var}[v \mid x_1]$. As a result, absent a financial report, the manager's information is not value–relevant.

8 While the details of the analysis would change, we believe that qualitatively similar results would obtain had we followed Dye's [1985] approach of assuming that there was a positive probability that the manager did not have private information rather than assuming a cost to disclosure.

9 See Bagnoli and Bergstrom [2005] for a relatively complete list of log–concave densities and for details on the consequences of assuming log–concavity. Examples of common density functions that are log–concave include the uniform, normal, exponential and power densities.

because (v, x) are joint normal. Also, note that in this case, the manager's disclosure allows the market to compute β^T using the true value of σ_{11}.

Let D represent the set of values of σ_{11} the manager chooses to disclose and N represent the set of values the manager chooses not to disclose. After observing the firm's financial report, if the manager chooses not to disclose her private information, the market estimates firm value as

$$P_N \equiv \mathrm{E}[v \mid x, \sigma_{11} \in N] = \mu_v + \mathrm{E}\big[\beta^T \mid x, \sigma_{11} \in N\big](x - \mu_x). \tag{2}$$

The difference between equations (1) and (2) arises because the market no longer knows the vector of "slope" coefficients in the latter case since the manager chose not to voluntarily provide her private information. Instead, the market estimates the vector of slope coefficients using all of the available information including the fact that the manager chose not to disclose σ_{11}.

Since we are interested in the interaction between the firm's financial reports and the manager's voluntary disclosure decision, we assume that the manager knows the information in the firm's financial reports when making her voluntary disclosure choice.[10] With this information structure, the manager is better off disclosing if the market value of the firm given that the manager discloses σ_{11} less the cost of disclosure exceeds the market value of the firm if the manager does not disclose. Formally, the manager chooses to disclose σ_{11} if

$$\mu_v + \beta^T(x - \mu_x) - c > \mu_v + \mathrm{E}\big[\beta^T \mid x, \sigma_{11} \in N\big](x - \mu_x), \tag{3}$$

chooses not to disclose if the inequality is reversed, and is indifferent if the market values are equal.

Theorem 1: *The equilibrium disclosure strategy for the manager depends on whether the market's expectation of firm value is increasing or decreasing in the manager's private information.*

(i) *If* $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}} > 0$, *then there exists a unique* s_1^*, *such that* $D = \{\sigma_{11} \mid \sigma_{11} \geq s_1^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$.

(ii) *If* $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}} < 0$, *then there exists a unique* s_2^*, *such that* $D = \{\sigma_{11} \mid \sigma_{11} \leq s_2^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$.[11]

Theorem 1 tells us that whether the manager chooses to disclose large or small values of her private information depends on whether the market's estimate of firm value is increasing or

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[10] Since management knows the content of the firm's financial reports prior to making them public, including the manager's voluntary disclosures in the press release describing the firm's financial performance is consistent with the timing in our model.

[11] Part (ii) is a generalization of the main theorem in Hughes and Pae [2004] which contained this result for the case of $n = 1$, $\sigma_{v1} > 0$ and $\sigma_{ij} = 0$, $i \neq j, i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$ (because $n = 1$).

decreasing in her disclosure. The proof basically relies on ideas that can be readily understood by examining Figures 1, 2 and 3. Figure 1 illustrates that when P (the firm's stock price if the manager reveals her private information) is increasing in the manager's private information, once the gain from disclosing σ_{11} is greater than the cost, the gain remains greater for all larger values of σ_{11}, too, for fixed D and N. Thus, in this case, the set of σ_{11} that the manager chooses to disclose is a "greater than" set. Similarly, if P is decreasing in σ_{11}, the set of σ_{11} that the manager chooses to disclose is a "less than" set. The key to understanding the set of disclosed values is to recall the intuition developed in the voluntary disclosure literature: the manager voluntarily discloses the values of her private information that make the firm's stock price high.[12] The same intuition applies to our model. Equation (3) compares the market value of the firm if the manager discloses her private information to the market value of the firm if she does not. In the latter case, the market assigns its expectation of firm value conditional on the manager not providing a supplementary voluntary disclosure (which is necessarily independent of the manager's actual private information) to the firm. This is not true, however, if the manager discloses her private information because the market finds it useful when using the firm's reported financial results to estimate firm value. Hence, the manager, seeking to maximize firm value, discloses when the gain in firm value (the difference between what the market believes the firm is worth based on her disclosure and what the market thinks it is worth conditional on her choosing to not disclose) exceeds the cost of disclosure. As a result, the manager discloses large (small) values of her private information when firm value is increasing (decreasing) in it.

To see that there is a unique critical value that determines whether the manager chooses to voluntarily disclose her private information, consider Figures 2 and 3. They graph the price increase from disclosure for each possible disclosure threshold, s^t (which we will refer to as the Threshold Value Disclosure (TVD) function) and define the equilibrium disclosure threshold, s^* by $TVD(s^*) = c$. The equilibrium threshold value is unique because log–concavity of $g(\sigma_{11}; x)$ ensures that the TVD function is monotone increasing (Figure 2) or decreasing (Figure 3).

3. On the Interaction Between Financial Reports and Voluntary Disclosures.

Having derived the manager's equilibrium voluntary disclosure strategy, we now turn to an analysis of how her strategy is affected by the information provided in the firm's financial reports.

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[12] In many of the voluntary disclosure models, the manager has private information that is positively correlated with the liquidation value of the firm and, generally, voluntarily discloses his/her private information if it is greater than some critical value. See, for example, the discussion in Verrecchia [2001].

To proceed, we need to adopt a sign convention. The issue arises because the relevant information provided in the firm's financial reports include items that are naturally viewed as positively covarying with firm value (*e.g.*, revenues, earnings, assets, etc.) and items that are naturally viewed as negatively covarying with firm value (*e.g.*, expenses and liabilities). To capture this, we assume that good news is equivalent to larger values of $x_i - \mu_i$ for those items that positively covary with firm value but is equivalent to smaller values of $x_j - \mu_j$ for those items that negatively covary with firm value. Obviously, there are exceptions to our general identification that the covariance is positive for revenues, earnings, assets, etc. and negative for expenses and liabilities. For example, greater R & D expense is likely to be positively associated with firm value. Similarly, if the asset is accounts receivable, inventories or an "aggressively" capitalized expenditure, an increase might suggest a slowdown in the firm's business. If so, then each is likely to be negatively associated with firm value. For such exceptions, we retain the identification that when the item positively covaries with the firm's liquidation value, $(x_i - \mu_i) > 0$ represents good news and when the covariance is negative, $(x_i - \mu_i) < 0$ represents good news. The advantage of this convention is that we have a simple mathematical description for when the firm's financial reports contain good (bad) news: $\sigma_{vi}(x_i - \mu_i) > 0 \;\; (\sigma_{vi}(x_i - \mu_i) < 0)$.

Since the equilibrium disclosure policy depends on whether the market's expectation of firm value is increasing or decreasing in the manager's private information, we begin by examining

$$\frac{\partial \mathrm{E}[v \mid x, \sigma_{11}]}{\partial \sigma_{11}} = -\Sigma_{xx}^{-1} \frac{\partial \Sigma_{xx}}{\partial \sigma_{11}} \beta^T (x - \mu_x) - \Sigma_{xx}^{-1} \frac{\partial \Sigma_{vx}}{\partial \sigma_{11}} (x - \mu_x).$$

Since the only entry in Σ_{xx} that is a function of σ_{11} is the first, (which is, in fact, σ_{11}),

$$\frac{\partial \Sigma_{xx}}{\partial \sigma_{11}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

where the matrix on the right hand side contains zeros everywhere but in the first entry. Further, no entry in Σ_{vx} is a function of σ_{11} so,

$$\frac{\partial \mathrm{E}[v \mid x, \sigma_{11}]}{\partial \sigma_{11}} = -\Sigma_{xx}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \beta^T (x - \mu_x). \tag{4}$$

Since each of the slope coefficients (β) depends on all of the components of the variance–covariance structure, equation (4) indicates that whether the firm's market value is increasing or decreasing in the manager's private information is complicated because it depends on the magnitudes of all of these components and on the firm's performance relative to expectations. Thus, we begin with the simpler case, when the transitory components of the items in the financial reports are uncorrelated (Σ_{xx} is a diagonal matrix) and then consider the more general case when they are correlated (Σ_{xx} is not a diagonal matrix).

3.1 Σ_{xx} is a diagonal matrix.

If Σ_{xx} is a diagonal matrix, then $\sigma_{ij} = 0, \quad \forall i, j, \quad i \neq j$. That is, the transitory components of the items in the financial reports are uncorrelated. If each of the relevant items in the firm's financial reports used by the market in estimating firm value are line items on the firm's financial statements, rather than subtotals, and provide incremental value–relevant information, then this assumption is not unreasonable. Further, even if the market uses subtotals (such as earnings or EBITDA), so long as they provide new information (*i.e.*, not every line item that is used to calculate the subtotal is already being used by the market), we can apply this version of our model.

An example may help clarify these points. Consider a simplified income statement in which the firm reports revenue, cost of goods sold and earnings. If both revenue and cost of goods sold are modeled as "true value" plus noise random variables (with, possibly, the "true value" of cost of goods sold a function of the "true value" of revenue), it is reasonable to assume that the noise terms are uncorrelated, that is, Σ_{xx} is a diagonal matrix.[13] In contrast, if the market uses revenue and earnings (equivalently cost of goods sold and earnings), Σ_{xx} is not diagonal because the covariance of revenue and earnings equals the variance of revenue and the covariance of cost of goods sold and earnings equals minus the variance of cost of goods sold. However, the market cannot use all three as one is redundant (earnings equals revenues minus cost of goods sold). Thus, even if the market uses earnings, so long as it is not redundant, we can transform variables back to the original items thereby eliminating subtotals and then apply our analysis in this subsection.

The key advantage of assuming that Σ_{xx} is a diagonal matrix is that $\beta^T = (\frac{\sigma_{v1}}{\sigma_{11}}, \frac{\sigma_{v2}}{\sigma_{22}}, \ldots, \frac{\sigma_{vn}}{\sigma_{nn}})$. Substituting into equation (4),

$$(5) \qquad \frac{\partial \mathrm{E}[v \mid x, \sigma_{11}]}{\partial \sigma_{11}} = \frac{\partial}{\partial \sigma_{11}} \left(\frac{\sigma_{v1}}{\sigma_{11}} (x_1 - \mu_1) \right) = - \left(\frac{1}{\sigma_{11}} \right)^2 \sigma_{v1} (x_1 - \mu_1).$$

Applying Theorem 1, the manager chooses to voluntarily disclose large values of σ_{11} if $\sigma_{v1}(x_1 - \mu_1) < 0$ and small values if $\sigma_{v1}(x_1 - \mu_1) > 0$. Proposition 1 summarizes the relationship between

[13] In particular, one might interpret returns as the noise in revenue and shrinkage as the noise in cost of goods sold. It seems unlikely that stochastic process governing returns is correlated with the stochastic process governing shrinkage. (We thank Dick Dietrich for this example.)

what the manager voluntarily discloses, the type of information of interest from the firm's financial statements and whether the reported information exceeded or missed market expectations.

Proposition 1: *When Σ_{xx} is a diagonal matrix the manager discloses small (large) values of σ_{11} if the firm's financial reports contain good (bad) news.*

Thus, what the manager is willing to voluntarily disclose depends on what is in the financial statements. Intuitively, if the firm reports good news, the manager prefers that the market attribute the outcome to good performance rather than good fortune (to changes in permanent rather than transitory components). As a result, she discloses the variance in the item when it is small since a small variance indicates that the good news is more likely to be the outcome of good performance, thus increasing the news' impact on stock price. Similarly, if the firm reports bad news, the manager prefers that the market attribute the outcome to bad fortune rather than poor performance. Thus, she discloses the variance in the item when it is large since a large variance indicates that the bad news is more likely to simply be the result of bad luck as opposed to poor performance, reducing the news' impact on stock price.

Theorem 1 and equation (5) also provide interesting implications for how the difference between what the firm reports and the market's expectation affects the probability that the manager chooses to voluntarily disclose.

Proposition 2: *When Σ_{xx} is a diagonal matrix, if $s_i^* \in (a, b)$, $i = 1, 2$*

(i) *the probability of voluntary disclosure is declining in c;*

(ii) *the probability of voluntary disclosure is independent of $(x_j - \mu_j)$ for $j = 2, 3, \ldots, n$;*

(iii) *the probability of voluntary disclosure is increasing (decreasing) when $|(x_1 - \mu_1)|$ is increasing (decreasing).*[14]

The comparative static results from changes in the cost of disclosure (part (i)) are standard— the more expensive it is, the less management does.[15] Intuitively, the probability of voluntary disclosure is independent of the other items that the firm reports (part (ii)) because the manager and the market have the same information about these items (there is no asymmetry in their

[14] Part (iii) generalizes Proposition 2 in Hughes and Pae [2004] by extending the analysis to include cases when x_1 negatively covaries with the firm's liquidation value.

[15] One can readily see this by examining either Figures 2 or 3. In the case when P is increasing in the manager's private information, Figure 2, an increase in c shifts the horizontal line up but does not alter the Threshold Value Disclosure (TVD) function. As a result, the critical value increases, reducing the size of the disclosure set D and thus lowering the probability that the manager provides a supplemental voluntary disclosure.

knowledge of the slope coefficients in the linear conditional expectation). As a result, their contribution to firm value is the same whether the manager voluntarily discloses σ_{11} or not and so changes do not affect the benefits or costs of voluntary disclosure.

Part (iii) of Proposition 2 tells us that the TVD function in Figures 2 and 3 shifts up when the surprise in the financial reports (either positive or negative) is larger—the probability of voluntary disclosure is greater for firms that beat or miss market expectations by larger amounts. Intuitively, if the surprise in the financial reports is large and therefore likely to move price by a significant amount, the manager is more likely to provide additional information to help the market "interpret" the surprise. In particular, if the surprise is large and the news is good, the manager is more likely to voluntarily provide supplemental information that enhances the value impact of the information in the financial reports (small σ_{11}). On the other hand, if the surprise is large and the news is bad, the manager is more likely to provide supplemental information that mitigates the value impact of the information in the financial reports (large σ_{11}).

Propositions 1 and 2 can be used to develop testable implications concerning the type of information about the firm's financial reports that managers are likely to voluntarily provide and the frequency with which they do so. An immediate implication is that if the firm beats the consensus estimate (a standard proxy for the market's expectation), our analysis predicts that the manager is more likely to provide supplementary information and that this information will indicate that there was relatively little "luck" associated with the firm's quarterly performance. More importantly, however, our analysis implies that voluntary supplemental disclosures associated with good (bad) news in the firm's financial reports should increase (decrease) the firm's earnings response coefficient (ERC). To see this, rewrite (1) as:

$$\frac{(P-\mu_v)}{\mu_v} = \beta_1 \frac{(x_1-\mu_1)}{\mu_v} + \sum_{j=2}^{n} \beta_j \frac{(x_j-\mu_j)}{\mu_v}.$$

Since μ_v is the market's expectation of the value of the firm prior to learning both the information in the firm's financial reports and any supplemental information management chooses to voluntarily provide, we can rewrite this as CAR $= \alpha +$ ERC $\times$ UE where UE is the scaled earnings surprise (in this case, $(x_1 - \mu_1)/\mu_v$) and everything else is subsumed in the constant term. From Theorem 1, if two firms have the same scaled earnings surprise, the manager that chooses to voluntarily provide supplemental information does so because it enhances the value impact of good news (mitigates the value impact of bad news) in the firm's financial reports. Said differently, for a given earnings surprise, the manager's supplemental disclosures should increase the market response to good news (the ERC rises) and reduce the market response to bad news (the ERC falls).[16] Further, our analysis can form the basis for a content analysis of supplemental voluntary disclosures and, as a result, leads to a natural extension of Francis, Schipper and Vincent [2002] who argue that the increased usefulness of earnings announcements is attributable to increases in concurrent disclosures. Our analysis also suggests an extension to the analysis in Miller [2002]. In particular, it provides the basis for differential predictions for earnings announcements with and without "explanations" and for predictions based on the content of non–financial operating disclosures.[17] Finally, we note that following the methodology employed by Hutton, Miller and Skinner [2003] (to analyze of supplemental information contained in management forecasts) or by Baginski, Hassell and Kimbrough [2004] (to analyze attributions in management forecasts) and applying it to earnings announcements could provide a fairly direct test of the predictions of our model.

Since Proposition 2 is derived under the assumption that there is an interior solution to the manager's voluntary disclosure problem, we need to examine when the solution is not interior to complete our comparative static analysis. Our results are summarized in Proposition 3.

Proposition 3: *If Σ_{xx} is a diagonal matrix and $\sigma_{v1} > 0$, then the manager chooses not to voluntarily disclose her private information if*

$$\frac{-c}{\sigma_{v1}\left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right)} < (x_1 - \mu_1) < 0 \quad \text{or} \quad 0 < (x_1 - \mu_1) < \frac{c}{\sigma_{v1}\left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right)}.$$

If $\sigma_{v1} < 0$, then the manager chooses not to voluntarily disclose her private information if

$$\frac{c}{\sigma_{v1}\left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right)} < (x_1 - \mu_1) < 0 \quad \text{or} \quad 0 < (x_1 - \mu_1) < \frac{-c}{\sigma_{v1}\left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right)}.$$

One immediate consequence of Proposition 3 is that the manager will not voluntarily disclose her private information if the information in the firm's financial reports is close enough to the market's expectation. Combining this with Proposition 2, we observe that it is only when the firm

[16] In addition, this observation produces a testable explanation for the well–documented empirical result that the ERC for bad news tends to be smaller than the ERC for good news.

[17] We should also note that our analysis can be used to fill a gap in the literature identified by Miller [2002, p. 178]: "The analytical literature does not provide predictions for firms with current positive, but impending bad, news." By interpreting some of our x_i's as information disclosed in current financial reports and the rest as information in future financial reports (to the extent that management knows these values), our model predicts that the voluntary disclosures are more likely to focus on good news and that the disclosures will emphasize the value impact of good news and de–emphasize the value impact of bad news.

reports information that is significantly different from the market's expectation that the manager chooses to voluntarily disclose her private information and then she only does so for particular realizations. Thus, firms whose performance differs significantly from market expectations are expected to provide significantly more voluntary disclosure. More formally, if the surprise is small enough ($|(x_1 - \mu_1)|$ is close to zero), the TVD function is always below c—there are no values of the manager's private information for which the threshold value of disclosure exceeds the cost of disclosure.

Interestingly, additional empirical implications can be derived by noticing that the range of values of $(x_1 - \mu_1)$ for which the manager chooses not to disclose her private information depends on the values of a and b, the support of the market's priors over the manager's private information. In particular, if g, the density of σ_{11}, is symmetric about its mean, then the range of values of $(x_1 - \mu_1)$ for which the manager chooses not to disclose her private information is symmetric.[18] Potentially more interesting however is the case is when g is not symmetric.[19] In this case, the set of values of good and bad news that are associated with the manager's decision not to disclose her private information at all are not equal. In particular, if the density of σ_{11} has a long left (right) tail, then the manager is more likely to avoid disclosing information if the firm reports bad (good) news. Thus, our model suggests that firms in financial distress are more likely to provide voluntary disclosures to supplement the firm's financial reports when they contain good news. Similarly, firms that have a record of good performance are relatively more likely to provide supplementary disclosures even when the firm's financial reports contain bad news.

3.2 General Σ_{xx}.

If the transitory components of the items in the financial reports are correlated, then some of the covariances are not zero (Σ_{xx} is not a diagonal matrix), and the analysis changes in an interesting way.[20] In this case, if the manager voluntarily discloses information that helps the

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[18] The uniform density is an example of a log–concave density that is symmetric about its mean.

[19] The power density (with location parameter greater than 1) is an example of such a log–concave density.

[20] Since Σ_{xx} is a symmetric, positive definite matrix, one can find many diagonalized representations. The most common is $\Sigma_{xx} = A\Lambda A^{-1}$ where Λ is a diagonal matrix or the Cholesky decomposition, $\Sigma_{xx} = PIP^T$. Unfortunately, such a transformation does not allow us to generalize the analysis in the previous section. The easiest way to see this is to observe that, in the Cholesky decomposition, P is a lower triangular matrix with diagonal elements $p_{ii} = \sqrt{\sigma_{ii} - \sum_{k=1}^{i-1} p_{ij}^2}$ and non–zero, non–diagonal elements $p_{ij} = (\sigma_{ij} - \sum_{k=1}^{i-1} p_{jk}p_{ik})/p_{ii}$. With this transformation, disclosing σ_{11} becomes equivalent to disclosing P. Unfortunately, this transformation does not alleviate the need to deal with the non–zero covariances. As a result, we work with the more familiar general Σ_{xx} rather than relying on a diagonalization transformation. We thank Anil Arya for his help with these issues.

market better understand the valuation impact of one line item in the firm's financial reports, that same information also improves the market's understanding of the impact of other line items on firm value. In other words, learning σ_{11} not only allows the market to better interpret the realized value x_1 but also the realized value of every other x_j that covaries with x_1.

To see this, consider the case when $n = 2$.[21] In this case,

$$\beta_1 = \frac{(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2})}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})}$$
$$\beta_2 = \frac{(\sigma_{11}\sigma_{v2} - \sigma_{21}\sigma_{v1})}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})}.$$

Note that now that the transitory components covary, the market's uncertainty about how the information in the firm's financial statements maps into firm value is not limited to uncertainty about β_1 but instead extends to all of the slope coefficients.

Substituting the expressions for the slope coefficients into equation (4) yields

$$\frac{\partial \mathrm{E}[v \mid x, \sigma_{11}]}{\partial \sigma_{11}} = \left(\frac{(\sigma_{12}\sigma_{v2} - \sigma_{22}\sigma_{v1})}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^2}\right)\Big[\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)\Big]. \tag{6}$$

Equation (6) shows that how firm value varies in σ_{11} depends on the reported levels of both x_1 and x_2 rather than just the reported level of x_1. Further, because we have assumed that σ_{12} is not zero, how firm value varies with σ_{11} now depends on the whole variance–covariance structure, every element of Σ. As a result, determining whether firm value is increasing or decreasing in σ_{11} becomes more complicated.[22]

More important, equation (6) shows that there is a form of continuity in σ_{12}. In particular, if σ_{12} is a sufficiently small (but not zero), then the manager's optimal disclosure policy is identical to her policy when Σ_{xx} is a diagonal matrix *ceteris paribus*. To see this, note that for σ_{12} sufficiently

[21] The $n = 2$ case is equivalent to the case with arbitrary finite n but with only one covariance, σ_{12}, not equal to zero.

[22] There is, however, an interesting regularity that, in some circumstances, allows us to determine the sign of $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}}$ more easily. Notice that the coefficients on σ_{v1} and $(x_1 - \mu_1)$ (resp. σ_{v2}, and $(x_2 - \mu_2)$) are equal but of opposite sign. Thus, if we can sign one of the terms in (6), we will be able to sign both in many circumstances. Otherwise, comparing relative magnitudes is required to determine whether the manager voluntarily discloses small or large values of σ_{11}. More generally, (4) can be written as

$$\frac{\partial \mathrm{E}[v \mid x, \sigma_{11}]}{\partial \sigma_{11}} = \left(\frac{1}{\Delta_n}\right)^2 \left\{\sum_{i=1}^{n} a_i \sigma_{vi}\right\}\left[-\sum_{i=1}^{n} a_i(x_i - \mu_i)\right].$$

As a result, a version of Proposition 4 holds when the signs of the $a_i\sigma_{v_i}$'s are the same.

small, the first term in equation (6) has the opposite sign as σ_{v1} and the second term has the same sign as $(x_1 - \mu_1)$. Thus, the results in Proposition 1 characterize the manager's optimal disclosure strategy which proves the continuity claimed.[23]

When $n = 2$, the manager's optimal disclosure strategy is described in Proposition 4.

Proposition 4: *For $n = 2$ and $\sigma_{12} \neq 0$, if the firm reports good news about both a revenue–type and an expense–type item and they positively covary or it reports good news about two revenue–type or two expense–type items and the items negatively covary, then the manager voluntarily discloses small values of σ_{11}. If, under the same conditions, the firm reports bad news about both items, then the manager voluntarily discloses large values of σ_{11}. Finally, if the firm reports mixed news then D and N depend on the magnitudes of elements in Σ and the news.*[24]

The key difference between this result and Proposition 1 is that when the transitory components covary, the impact of an increase in σ_{11} on the market's expectation of firm value is significantly more complicated. In particular, Proposition 4 shows that, in some circumstances, whether the derivative is positive or negative depends on the relative magnitudes of the key covariances and the news in the firm's financial reports (*i.e.*, $(x_1 - \mu_1)$ and $(x_2 - \mu_2)$). In other cases, the sign can be determined more easily. Specifically, if the firm reports good news for the two line items in the firm's financial reports, the manager voluntarily discloses small values of her private information (σ_{11}) when (a) the two line items positively covary and one is a revenue–type number and the other an expense–type number, or (b) if the two line items negatively covary and both are revenue–type or both are expense–type numbers. If the firm reports bad news for both line items, the manager voluntarily discloses large values of σ_{11} when either (a) or (b) occurs. The intuition for these results is the same as before.

Proposition 5 shows that, in addition to what is disclosed, the probability of voluntary disclosure also depends on the information contained in the firm's financial reports.

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[23] In fact, if we consider the representation for $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}}$ in the prior footnote, we see that the same form of continuity exists for arbitrary finite n. To see this, note that each a_i for $i = 2, \ldots, n$ is the sum of terms each of which contains a covariance term (σ_{jk}, $j \neq k$) and a_1 is the sum of such terms too with one exception: the first term is the product of the σ_{jj}'s for $j = 2, \ldots, n$. Thus, if one fixes the magnitudes of the $x_i - \mu_i$'s, the σ_{vi}'s for $i = 1, 2, \ldots, n$, and the σ_{jj}'s for $j = 2, \ldots, n$, and if the all the σ_{jk}'s for $j \neq k$ are sufficiently small, then the sign of the first sum is opposite the sign of σ_{v1} and the second sum has the same sign as $(x_1 - \mu_1)$. Thus, the results in Proposition 1 describe the manager's optimal disclosure strategy. And, again, we have a form of continuity—the optimal disclosure strategy is the same when Σ_{xx} is a diagonal matrix as it is when all the off–diagonal elements of Σ_{xx} are sufficiently small.

[24] The formal conditions for when $D = \{\sigma_{11} \mid \sigma_{11} \leq s_2^*\}$ or $D = \{\sigma_{11} \mid \sigma_{11} \geq s_1^*\}$ are described in the Appendix.

Proposition 5: *When $n = 2$ and $\sigma_{12} \neq 0$, if $s_i^* \in (a, b)$, $i = 1, 2$*

(i) *the probability of voluntary disclosure is declining in c;*

(ii) *the probability of voluntary disclosure is declining (increasing) in $(x_2 - \mu_2)$ if $\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0 \quad (\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0)$;*

(iii) *the probability of voluntary disclosure is declining (increasing) in $(x_1 - \mu_1)$ if $-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0 \quad (-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0)$.*

As in the case when the transitory components are uncorrelated, the comparative static result from a change in the cost of disclosure is standard—the more expensive it is, the less management does. Again, this is most readily seen by examining Figures 2 and 3 and observing that an increase in c shifts the horizontal line up thereby shrinking the set of values the manager chooses to voluntarily disclose. The first significant difference that arises when the transitory components of the items in the financial reports covary is that the probability of voluntary disclosure is no longer independent of any of the numbers the firm reports in its financial reports. The key to understanding this is to observe that a change in either $(x_1 - \mu_1)$ or $(x_2 - \mu_2)$ alters the market's expectation of the value of the firm conditional on the information voluntarily disclosed by the manager as well as its expectation of the value of the firm conditional on the manager having chosen not to voluntarily disclose her private information. Thus, what matters is which conditional expectation changes more. If the former increases (decreases) more than the latter, then the probability of voluntary disclosure increases (decreases). This is most easily seen by examining Figures 2 and 3 and observing that the probability of voluntary disclosure increases (decreases) if the TVD function shifts up (down).

So, if x_1 is a revenue–type number (which positively covaries with v) and x_2 is an expense–type number (which negatively covaries with v) and they positively covary, the probability of voluntary disclosure is declining in $(x_2 - \mu_2)$. That is, if the firm's financial reports contain better news about x_2, then $(x_2 - \mu_2)$ becomes more negative and the probability of voluntary disclosure of the manager's private information increases. Similarly, the probability of voluntary disclosure declines if the firm reports worse news. Interestingly, these conclusions may reverse if the items negatively covary (as might be the case if the company's business is counter—cyclical).[25] Further, when x_1 is a revenue–type number and x_2 is an expense–type number and they positively covary, the

[25] That is, if a firm's revenues are inversely related to the economy's business cycle, its expenses are likely to be pro–cyclic, producing a negative covariance in their respective transitory components. For example, if increases in energy prices slow economic growth and thus wage costs, firms that specialize in energy–saving products are likely to have the transitory components in their revenue and expense items negatively covary.

probability of voluntary disclosure is increasing in $(x_1 - \mu_1)$. That is, if the firm reports better news about x_1 in its financial statements, the probability that the manager voluntarily discloses her private information increases. Again, these conclusions may reverse if the items negatively covary.

Perhaps more important, if x_1 is a revenue–type number and x_2 is an expense–type number or vice versa, then bigger surprises work in the same direction—both cause the probability of voluntary disclosure to change in the same direction. However, if one of the items is revenues and the other is net income, then bigger surprises in each work in opposite directions—one causes the probability of voluntary disclosure to increase and the other causes the probability to decrease. This result suggests that event studies that examine the market response to earnings and revenue surprises need to account for the impact of the surprises on the manager's decision to voluntarily provide supplemental disclosures. In particular, if the firm beats both the revenue expectation and the earnings expectation, the impact on the market response will be different than if the firm missed the revenue expectation but beat the earnings expectation, and the difference will, in part, arise from differences in management's decision to provide supplemental disclosures.

4. Information Externalities.

The remaining comparative static involves analyzing the effect of changes in the covariances between the liquidation value of the firm and the information in the firm's financial reports. Intuitively, the covariances measure how tightly linked the firm's liquidation value is to the information in the firm's financial reports and is an alternative proxy for the "mix" of permanent and transitory components in the financial statement items. That is, the covariance is larger (smaller) if the transitory components are a relatively small (large) portion of the values reported. In addition, changes in this covariance can be interpreted in a variety of ways. One interpretation in particular focuses on an important information externality: The effect of changes in financial reporting requirements on the probability that the manager voluntary disclosures her private information which the market then uses for valuation purposes. Our analysis begins with Proposition 6 which describes how the probability of voluntary disclosure depends on the usefulness of the financial reports to the market in the simpler case when the transitory components do not covary.

Proposition 6: *When Σ_{xx} is diagonal and $s_i^* \in (a, b)$ for $i = 1, 2$,*

(i) *the probability of voluntary disclosure is increasing (decreasing) in σ_{v1} when σ_{v1} is positive (negative),*

(ii) *the probability of voluntary disclosure is independent of* σ_{vi} *for* $i = 2, 3, \ldots, n$.

Proposition 6 (part (i)) indicates that if the liquidation value of the firm and x_1 are more tightly related (*i.e.*, $|\sigma_{v1}|$ is large), then the probability of voluntary disclosure of the manager's private information increases. In essence, the probability of voluntary disclosure depends on how useful the information in the financial report is for valuation purposes: the more useful it is (the greater is $|\sigma_{v1}|$), the higher is the probability the manager voluntarily discloses her private information regardless of whether the news in the firm's financial reports is good or bad. The probability of voluntary disclosure is independent of the other covariances (part (ii)) for the usual reason, the market and the manager are symmetrically informed about the valuation implications of all of the information reported in the firm's financial statements except x_1.

Proposition 7: *For general* Σ_{xx}, $n = 2$ *and* $s_i^* \in (a, b)$ *for* $i = 1, 2$,

(i) *the probability of voluntary disclosure is increasing in* σ_{v1} *when* $\sigma_{22}(x_1 - \mu_1) > \sigma_{12}(x_2 - \mu_2)$ *and decreasing when* $\sigma_{22}(x_1 - \mu_1) < \sigma_{12}(x_2 - \mu_2)$,

(ii) *the probability of voluntary disclosure is increasing in* σ_{v2} *when* $-\sigma_{12}(x_1 - \mu_1) > \sigma_{11}(x_2 - \mu_2)$ *and decreasing when* $-\sigma_{12}(x_1 - \mu_1) < \sigma_{11}(x_2 - \mu_2)$.

Intuitively, if disclosing the manager's private information either increases firm value by magnifying the impact of the information in the firm's financial reports or reduces the decline in firm value by minimizing the impact of that information, the manager is more likely to disclose her private information when $|\sigma_{v1}|$ increases.

The empirical implications of changes in the covariance between the liquidation value of the firm and the firm's financial statement information depend on how one interprets a change in the covariance. One interpretation is that the economic activities of the firm have changed making the reported results more correlated with the firm's liquidation value. In this case, the underlying event is likely to be related to what financial analysts refer to as the firm's business strategy.[26] Under this interpretation, Proposition 6 implies that firms for which the change in business strategy creates a tighter (looser) link between reported performance and the firm's liquidation value will be more (less) likely to provide a voluntary disclosure that assists the market in understanding the information in the firm's financial reports.

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[26] We use the terminology found in Palepu, Healy and Bernard [2004] but the idea is present in most financial statement analysis textbooks. Examples include situations in which the firm has announced the imminent introduction of a new product or situations in which a newly introduced product or transitory event is having a significant effect on reported performance.

Alternatively, one can interpret Propositions 6 and 7 as providing empirical predictions about how cross–sectional variations in voluntary disclosure decisions depend on the market's level of uncertainty about how information in the firm's financial reports translates into long–term firm value. (This uncertainty could be proxied by, for example, the variance in analysts' target prices or the variation in analysts' recommendations.) Our analysis suggests that firms for whom the uncertainty is greater are less likely to provide voluntary supplemental disclosures. Further, the difference in the ERCs for firms that do and do not provide voluntary disclosures will be smaller (greater) when the disclosing firms release large (small) values of their private information.[27] Similarly, in many circumstances, there is empirical evidence that can be interpreted as indicating that the liquidation value of the firm correlates differently with alternative reported numbers. For example, Gu et al. [2004], Ertimur et al. [2003], Livnat [2003] and Bagnoli et al. [2001] provide evidence that the market responds differently to revenue and earnings surprises, and Vincent [1999] shows that the market responds more to earnings than funds from operations (FFO) for REITs. Given this interpretation, Propositions 6 and 7 suggest that there are systematic differences in the probability of voluntary disclosure across firms for whom key performance measures differ and therefore systematic differences in ERCs even after controlling for whether the firms report good or bad news in the financial statements.

Perhaps the most interesting implication of Propositions 6 and 7 is that changes in mandatory disclosure requirements produce information externalities even when the voluntarily disclosed information is helpful only in interpreting the effects of the mandatory disclosures on firm value. In some circumstances, the changes increase the likelihood of voluntary disclosure, but in other circumstances, they decrease it. In our model, one mechanism for transmitting the externality is σ_{v1}. Using Proposition 6, if mandatory disclosure requirements increase the covariance (in absolute value) between the liquidation value of the firm and the information mandated to be disclosed, then there is a positive information externality: The change also increases the manager's incentive to voluntarily disclose her private information (the probability of voluntary disclosure increases). Similarly, if the mandatory disclosure requirements decrease the covariance (in absolute value), the probability of voluntary disclosure declines. In both cases, our analysis suggests the need to include the information externality when regulators examine the cost–benefit trade–off from changes in mandatory disclosure requirements. In the more complex case (Proposition 7), a change in

[27] This result follows from noting that a decrease in the probability of voluntary disclosure lowers (raises) the value of TVD in Figure 2 (3) and that the ERC is monotonically related to the magnitude of the TVD.

mandatory disclosure requirements that increases the covariance has an effect on the probability of voluntary disclosure that depends, in part, on whether the firm's financial statements contain good or bad news and how the transitory components in the elements of the firm's financial reports covary. This may be particularly important if regulators place different weights on the amount of information available to investors when firm performance is bad (good), but regardless, our analysis indicates that the information externality is an effect that should be considered when evaluating the impact of a change in mandatory disclosure requirements.

As noted in the introduction, our results complement the results in the prior literature on voluntary disclosure. In this prior work, the manager's private information is generally a signal about the liquidation value of the firm, and the impact of mandatory disclosure is examined by considering the effect of changes in the market's priors on the manager's voluntary disclosure decision. The standard result is that the manager is less likely to voluntarily disclose her signal about the firm's liquidation value if the market has better information about the firm's value.[28] Einhorn [2005] provides a detailed extension of this approach and shows that the probability that the manager discloses her signal about the firm's liquidation value is independent of the realized information in the firm's mandatory disclosure.[29] In our model, the realized value matters because the manager's private information helps the market interpret the information in the firm's financial reports rather than acting as another signal about the firm's liquidation value. In a very different context (a model of cheap talk with misaligned preferences), Fischer and Stocken [2004] derive a result that is similar, in spirit, to ours—an additional source of information can suppress an expert's ability to communicate his/her private information. One interesting difference between Fischer and Stocken's result and ours is that we find that increases in mandatory disclosure can actually increase the likelihood of voluntary disclosure in some circumstances—increases in mandatory disclosures do not always substitute for voluntary disclosures.

We illustrate this idea with an example. Consider a tightening of the regulations on revenue or expense recognition.[30] In both cases, the regulatory change is likely to increase the correlation

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[28] Specifically, the standard result is that the probability of voluntary disclosure is increasing in the market's prior variance of the firm's liquidation value. Thus, if mandatory disclosure is interpreted as reducing the prior variance, then it reduces the probability that the manager voluntarily discloses her signal about the liquidation value of the firm.

[29] Einhorn's result relies on the assumption of joint normality which makes the value of the manager's disclosure depend on the variance–covariance structure but not on the specific realization of the other signal.

[30] For example, in December 1999, the Securities and Exchange Commission released Staff Accounting Bulletin

between current earnings and the firm's liquidation value by reducing the "noise" in revenues or expenses. Proposition 6 suggests that there would be a positive information externality from the change—it would lead to greater voluntary disclosure by the firm. As a result, the benefits of these types of regulatory changes exceed the direct benefits associated with the change in the firm's financial reporting. Similarly, Proposition 7 suggests that there would be a positive information externality from the change when the firm's financial statements contain good news but a negative information externality if they contain bad news. Analogous arguments can be made for the movement away from the use of historical cost and toward the use of market value in the balance sheet (*e.g.*, Statement of Financial Accounting Standards No. 115, 1993) and for the recent regulatory changes in segment reporting (*e.g.*, Statement of Financial Accounting Standards No. 131, 1997).[31]

5. On the Interaction of Voluntary and Mandatory Risk Disclosure.

As currently formulated, our model cannot address the effect of changes in mandated risk disclosures. However, with the increasing emphasis on such disclosures, it is useful and straightforward to extend our model to examine the impact of information about firm risk in financial reports on the manager's voluntary disclosure decision.[32]

To adapt our model to address this issue, we follow the idea in Jorgensen and Kirschenheiter [2003] and assume that there are two sources of variability—industry–wide uncertainty and firm–specific uncertainty. So, let $\sigma_{11} = z + u$, where z represents the industry–wide component of the variance and u represents the firm–specific component which is the manager's private information.

This extension does not alter the results presented but does require a translation. Previously, we modeled the manager's private information as knowing the total variance (σ_{11}) and we described

(SAB) No. 101, "Revenue Recognition in Financial Statements," which was designed to mitigate aggressive revenue recognition. Similarly, in 1997, the AICPA, with the support of the Securities and Exchange Commission and the FASB, adopted SOP 97–2, which is designed to make reported revenues more informative for software companies. Another example is the dispute between the Securities and Exchange Commission and IBM concerning IBM's inclusion of gains from the sale of assets in its selling, general and administrative expenses (see Auerbach [1999]).

[31] While our model focuses on line items in financial reports, it can be adapted to analyze other regulatory changes. For example, consider insider trading regulation. The general rule (10b–5) prohibits trading on material, non–public information but, in 2000, the SEC sought to clarify its enforcement by issuing Rule 10b5–1 which essentially allows insiders to commit to a trading program (usually by scheduling when and how many shares the insider will sell) and roughly exempts such trades from enforcement of insider trading restrictions. Since all such trades must be disclosed, 10b5–1 seemingly reduced the correlation between insider trading activity (much of it is now planned trading) and the firm's liquidation value. If this is, in fact, what happened, then our model suggests that the new rule may have created a negative information externality—the rule reduces the probability of voluntary disclosure by the firm.

[32] See Jorgensen and Kirschenheiter [2003] for a discussion of the regulatory changes that have led to additional required risk disclosure.

the critical value, s^*, which separated the manager's type space into the set that voluntarily disclose her private information (D) and the set that choose not to voluntarily disclose her private information (N). Now, however, the manager's private information is u and so we need to translate the critical value into u^*. As one would expect, with this minor change, Theorem 1 and Propositions 1–7 continue to hold and they allow us to derive Proposition 8.

Proposition 8: *The probability of voluntary risk disclosure is decreasing in the mandated amount of risk disclosure.*

Intuitively, when mandatory risk disclosure reduces the "amount" of variance in the firm's financial performance that is unknown, the benefits from voluntarily disclosing the manager's private information about the unknown part decreases while the cost of disclosing does not. Consequently, the manager optimally decreases the set of values for which she discloses and this change results in a lower probability of voluntary disclosure. Interestingly, in this case, the information externality is consistent—increases in mandatory risk disclosure decrease the manager's incentives to voluntarily disclose her private information. Since increased mandatory risk disclosure crowds out voluntary risk disclosures, it is important for regulators weighing the benefits and costs of proposals to increase mandatory risk disclosure to consider the negative consequences of the induced decrease in voluntary disclosure as part of their overall cost–benefit analysis of the proposed change.

Proposition 8 does not, however, address the issue of whether mandating some risk disclosure increases the total amount of risk information available to the market. The reason is that Proposition 8 tells us that there is a substitution effect: More mandatory risk disclosure results in fewer firms providing voluntary risk disclosure. Consequently, there are three types of firms to consider: (1) those that provide complete risk disclosure both before and after the mandate to provide some risk disclosure; (2) those that provide complete risk disclosure prior to the mandate but only provide the mandated amount after; and (3) those that provide no risk disclosure prior to the mandate and provide the mandated amount after. As a result, mandatory risk disclosure increases the market's access to information about risk for some firms but decreases it for others. Thus, there are situations when mandatory disclosure is, in net, beneficial and others when it is not. We can, however, conclude that if the number of firms providing voluntary risk disclosures is small (the probability of voluntary risk disclosure is low) prior to the regulatory change and if the amount of risk disclosure mandated is large, then the net effect of the regulatory change is likely to be positive.[33] On the other hand, if firms are providing significant amounts of risk disclosure voluntarily and if the regulatory change mandates only a small amount of risk disclosure, then the net effect is likely to be negative. Since the SEC mandate for risk disclosure appears to have been driven by its concern about the lack of voluntary risk disclosure, our analysis suggests that, despite the negative information externality associated with a decline in the amount of voluntary risk disclosure, it is likely that users of financial statements have benefited from the SEC's action.

6. Extensions.

Our previous analysis focused on the manager's private information being the variance (or a component of the variance) of one of the items in the firm's financial reports. In this section, we show that our results do not depend on the particulars of our assumption about what the manager's private information is. For example, if the manager has private information about (i) how one of the items in the firm's financial reports covaries with the firm's liquidation value; or (ii) how two of the items in the firm's financial reports covary, similar results obtain.

To see this, let D represent the set of values of the manager's private information that she chooses to disclose voluntarily and let N represent the set of values that she chooses not to disclose. With the appropriate adjustments (replacing σ_{11} with either σ_{v1} or σ_{12}), equations (1) and (2) continue to represent the market's expectation of firm value given the firm's financial reporting and the manager's voluntary disclosure decision, and equation (3) characterizes the manager's decision to disclose or not. Given this, we have[34]

Theorem 2: *If the manager's private information is that she knows σ_{v1}, the equilibrium disclosure strategy for the manager depends on whether the market's expectation of the value of the firm is increasing or decreasing in the manager's private information.*

(i) *If $\frac{\partial \mathrm{E}[v|x,\sigma_{v1}]}{\partial \sigma_{v1}} > 0$, then there exists a unique t_1^*, such that $D = \{\sigma_{v1} \mid \sigma_{v1} \geq t_1^*\}$ and $N = \{\sigma_{v1} \mid \sigma_{v1} < t_1^*\}$.*

(ii) *If $\frac{\partial \mathrm{E}[v|x,\sigma_{v1}]}{\partial \sigma_{v1}} < 0$, then there exists a unique t_2^*, such that $D = \{\sigma_{v1} \mid \sigma_{v1} \leq t_2^*\}$ and $N = \{\sigma_{v1} \mid \sigma_{v1} > t_2^*\}$.*

Theorem 3: *If the manager's private information is that she knows σ_{12}, the equilibrium disclosure strategy for the manager depends on whether the market's expectation of the value of the firm is*

[33] Similarly, if the market's marginal benefit from additional risk disclosure is declining in the amount of disclosure, then increases in mandatory disclosure are quite likely to provide net benefits because the gains associated with increased risk disclosures by firms that chose not to voluntarily provide such information are more valuable than the losses associated with decreased risk disclosure by those firms that switch from voluntary total disclosure to mandatory partial disclosure.

[34] Since the proofs of Theorems 2 and 3 are virtually identical to the proof of Theorem 1, they are omitted.

increasing or decreasing in the manager's private information.

(i) *If* $\frac{\partial \mathrm{E}[v|x,\sigma_{12}]}{\partial \sigma_{12}} > 0$, *then there exists a unique* w_1^*, *such that* $D = \{\sigma_{12} \mid \sigma_{12} \geq w_1^*\}$ *and* $N = \{\sigma_{12} \mid \sigma_{12} < w_1^*\}$.

(ii) *If* $\frac{\partial \mathrm{E}[v|x,\sigma_{12}]}{\partial \sigma_{12}} < 0$, *then there exists a unique* w_2^*, *such that* $D = \{\sigma_{12} \mid \sigma_{12} \leq w_2^*\}$ *and* $N = \{\sigma_{12} \mid \sigma_{12} > w_2^*\}$.

Interestingly, the intuition for both theorems parallels the intuition for Theorem 1. The manager seeks to enhance the impact of the information contained in the firm's financial reports if that information is leading to an upward revision in the market's estimate of firm value and to minimize the impact if it is leading to a downward revision in the market's estimate of firm value. Thus, if the market's estimate of firm value is increasing in the manager's private information, when she voluntarily discloses her private information, she chooses to disclose only the large values. Similarly, if the market's estimate of firm value is decreasing in the manager's private information, when she voluntarily discloses her private information, she chooses to disclose only the small values.

The following Propositions show that the manager's voluntary disclosure decisions depends on the information in the firm's financial reports.

Proposition 9: *When the manager's private information is that she knows* σ_{v1} *and* $n = 2$,

(i) *if* $\sigma_{22}(x_1 - \mu_1) < \sigma_{12}(x_2 - \mu_2)$, *then* $D = \{\sigma_{11} \mid \sigma_{11} \leq t_2^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} > t_2^*\}$,

(ii) *if* $\sigma_{22}(x_1 - \mu_1) > \sigma_{12}(x_2 - \mu_2)$, *then* $D = \{\sigma_{11} \mid \sigma_{11} \geq t_1^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} < t_1^*\}$.

Proposition 10: *When the manager's private information is that she knows* σ_{12} *and* $n = 2$,

(i) *if* $(\sigma_{12}\sigma_{v1} - \sigma_{11}\sigma_{v2})(\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)) < 0$, *then* $D = \{\sigma_{11} \mid \sigma_{11} \leq w_2^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} > w_2^*\}$,

(ii) *if* $(\sigma_{12}\sigma_{v1} - \sigma_{11}\sigma_{v2})(\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)) > 0$, *then* $D = \{\sigma_{11} \mid \sigma_{11} \geq w_1^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} < w_1^*\}$.

Theorems 2 and 3 and Propositions 9 and 10 illustrate that our general result that the manager's voluntary disclosure decision is affected by the firm's financial reports extends readily to changes in what the manager's private information is. In the context of our model, whether the manager's private information is knowledge of the variance of one of the items reported in the firm's financial statements, how a line item covaries with the firm's liquidation value or just how different line items covary, in every case, the manager's voluntary disclosure decision depends on the information contained in the firm's financial reports. In all of these cases, the intuition is the same because the manager's private information, if disclosed, enhances the market's ability to infer from the information contained in the firm's financial reports.

7. Conclusions.

Employing a Verrecchia–style model of voluntary disclosure (Verrecchia [1983, 1990]), we examine the effects of mandatory financial reporting on the manager's decision to voluntarily provide supplemental information about the firm's financial reports that investors find useful in estimating firm value. We find that the manager's optimal disclosure strategy depends on the information contained in the firm's financial reports in two ways. First, the manager discloses only small or only large values of her private information depending on whether (i) the information contained in the firm's financial reports is value increasing or value decreasing, and (ii) the market's estimate of firm value is increasing or decreasing in the manager's private information. Second, we show that the probability of supplemental voluntary disclosure is also affected by the information contained in the firm's financial reports. In particular, the stochastic properties of the items in the firm's financial reports *as they relate to firm value* affect the type and probability of voluntary supplemental disclosure. For example, if the transitory components in the items in the firm's financial reports are uncorrelated, the probability of supplemental voluntary disclosure is greater (smaller) the better (worse) the news in the firm's financial reports. If the transitory components are correlated, the probability of disclosure increases if it combines with the information in the firm's financial reports to either enhance the market's upward revision in firm value or mitigate the downward revision.

A key empirical prediction from these results is that firms that provide voluntary supplemental disclosures and report good news have larger earnings response coefficients (ERCs) than firms that do not. Similarly, firms that provide voluntary disclosures and report bad news have smaller ERCs than firms that do not. Thus, our analysis provides a new explanation for cross–sectional variation in ERCs and suggests the need to include controls for supplemental voluntary disclosures (which may be found in press releases, conference calls and/or the Management Discussion and Analysis section of the financial reports) in the standard event study regression. A second key empirical prediction is that firms that report good (bad) news about both revenues and earnings are more likely (less likely) to provide voluntary supplemental disclosures. Since the market response to an earnings announcement (and all of the information provided in it) is greater for firms that provide voluntary supplemental disclosures, our analysis suggests that there are cross–sectional differences in ERCs and revenue response coefficients (RRCs) that will depend on both the relationship between reported and expected revenues and earnings numbers and on the manager's decision to provide supplemental voluntary disclosures. Further, we show that these effects are modulated by the amount of uncertainty the market faces regarding how information in the firm's financial reports translates into long term value (uncertainty that can be proxied by the variance in analysts' target prices or by the variation in analysts' recommendations.)

Another important implication of our comparative static results is that there are information externalities associated with changes in financial reporting regulations. In particular, regulatory changes that affect the covariance between items in the firm's financial reports and the firm's liquidation value also affect the manager's voluntary disclosure strategy. The magnitude and direction of the effect (whether there is a positive or negative information externality) depends both on whether the regulatory change increases or decreases the link between items in the firm's financial reports and the firm's liquidation value and on how the market interprets the information in the firm's financial reports. As a result, the benefit–cost considerations of a regulatory change in disclosure should consider both the externality associated with its impact on voluntary disclosure and on the relative importance of increased disclosure when the firm reports good or bad news.

8. Appendix.

Theorem 1: *The equilibrium disclosure strategy for the manager depends on whether the market's expectation of the value of the firm is increasing or decreasing in the manager's private information.*

(i) *If* $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}} > 0$, *then there exists a unique* s_1^*, *such that* $D = \{\sigma_{11} \mid \sigma_{11} \geq s_1^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$.

(ii) *If* $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}} < 0$, *then there exists a unique* s_2^*, *such that* $D = \{\sigma_{11} \mid \sigma_{11} \leq s_2^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$.

Proof: Part (i) To prove that $D = \{\sigma_{11}|\sigma_{11} \geq s_1^*\}$ and $N = \{\sigma_{11}|\sigma_{11} < s_1^*\}$, suppose not. Then there exists ${\sigma_{11}}^A$ and ${\sigma_{11}}^B$ such that ${\sigma_{11}}^A > {\sigma_{11}}^B$ with ${\sigma_{11}}^B \in D$ and ${\sigma_{11}}^A \in N$. The assumption that ${\sigma_{11}}^B \in D$ implies that

$$\mu_v + \beta^T({\sigma_{11}}^B)(x - \mu_x) - c \geq \mu_v + \mathrm{E}\big[\beta^T \mid x, \sigma_{11} \in N\big](x - \mu_x)$$

where $\beta^T({\sigma_{11}}^B)$ means that the vector of "slope" coefficients is computed when $\sigma_{11} = {\sigma_{11}}^B$. Since $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}} > 0$ and we have assumed that ${\sigma_{11}}^A > {\sigma_{11}}^B$,

$$\mu_v + \beta^T({\sigma_{11}}^A)(x - \mu_x) - c > \mu_v + \beta^T({\sigma_{11}}^B)(x - \mu_x) - c \geq \mu_v + \mathrm{E}\big[\beta^T \mid x, \sigma_{11} \in N\big](x - \mu_x).$$

Thus, ${\sigma_{11}}^A \in D$, a contradiction. Hence, D is an interval of relatively large values of σ_{11} and N is an interval of relatively small values of σ_{11}.

To show that there exists a unique s_1^* separating these intervals, note that s_1^* is that value of σ_{11} such that

$$\mu_v + \beta^T(s_1^*)(x - \mu_x) - c = \mu_v + \mathrm{E}\big[\beta^T \mid x, \sigma_{11} < s_1^*\big](x - \mu_x).$$

Rearranging yields

$$\text{(A1)} \qquad \mathrm{E}[v \mid x, \sigma_{11}] - \mathrm{E}[v \mid x, \sigma_{11} < s_1^*] = c.$$

Since we have assumed that the density of σ_{11} is log–concave and since $\mathrm{E}[v \mid x, \sigma_{11}]$ is concave function of σ_{11} as long as firm value is positive, by Theorem 7 in Bagnoli and Bergstrom [2005] (concave transformations of log–concave random variables are also log–concave), the density of $\mathrm{E}[v \mid x, \sigma_{11}]$ is log-concave. Finally, by Theorem 5 in Bagnoli and Bergrstrom, $\mathrm{E}[v \mid x, \sigma_{11}] - \mathrm{E}[v \mid x, \sigma_{11} \in N]$ is monotone increasing because the density of $\mathrm{E}[v \mid x, \sigma_{11}]$ is log-concave. Therefore, the left hand side of (A1) is monotone increasing and, as a result, there is a unique solution to (A1), s_1^*.

Part (ii) The proof that $D = \{\sigma_{11}|\sigma_{11} \leq s_2^*\}$ and $N = \{\sigma_{11}|\sigma_{11} > s_2^*\}$ in this case is essentially identical to the proof in part (i) and is therefore omitted. To show that there exists a unique s_2^* separating these intervals, note that s_2^* is that value of σ_{11} such that

$$\mu_v + \beta^T(s_2^*)(x - \mu_x) - c = \mu_v + \mathrm{E}\big[\beta^T \mid \sigma_{11} > s_2^*\big](x - \mu_x).$$

Rearranging yields

$$\text{(A2)} \qquad \mathrm{E}[v \mid x, \sigma_{11}] - \mathrm{E}[v \mid x, \sigma_{11} > s_2^*] = c.$$

The left hand side of (A2) is $-(\mathrm{E}[v \mid x, \sigma_{11} > s_2^*] - \mathrm{E}[v \mid x, s_2^*])$ which is minus the mean residual lifetime function for $\mathrm{E}[v \mid x, \sigma_{11}]$. Thus, by Theorem 6 in Bagnoli and Bergstrom [2005], the left hand side of (A2) is monotone increasing and, as a result, there is a unique solution to (A2), s_2^*. ∎

Proposition 1: *When Σ_{xx} is a diagonal matrix the manager discloses small (large) values of σ_{11} if the firm's financial reports contain good (bad) news.*

Proof: From Theorem 1, $D = \{\sigma_{11} : \sigma_{11} \leq s_2^*\}$ if P is monotone decreasing in σ_{11}. From (5), P is monotone decreasing if and only if $\sigma_{v1}(x_1 - \mu_1) > 0$, *i.e.*, if σ_{v1} and $(x_1 - \mu_1)$ have the same sign which means that the firm's financial reports contain good news. Similarly, $D = \{\sigma_{11} : \sigma_{11} \geq s_1^*\}$ if P is monotone increasing in σ_{11} and, from (5), P is monotone increasing if and only if $\sigma_{v1}(x_1 - \mu_1) < 0$, *i.e.*, if σ_{v1} and $(x_1 - \mu_1)$ have opposite signs which means that the firm's financial reports contain bad news. ∎

Proposition 2: *When Σ_{xx} is a diagonal matrix, if $s_i^* \in (a, b)$, $i = 1, 2$*

(i) *the probability of voluntary disclosure is declining in c;*

(ii) *the probability of voluntary disclosure is independent of $(x_j - \mu_j)$ for $j = 2, 3, \ldots, n$;*

(iii) *the probability of voluntary disclosure is increasing (decreasing) when $|(x_1 - \mu_1)|$ is increasing (decreasing).*

Proof: s_i^* solves

$$\text{(A3)} \qquad \frac{1}{\sigma_{11}} - \mathrm{E}\left[\frac{1}{\sigma_{11}} \mid x, \sigma_{11} \in N\right] = \frac{c}{\sigma_{v1}(x_1 - \mu_1)}.$$

If P is monotone decreasing in σ_{11} then $N = \{\sigma_{11} : \sigma_{11} > s_2^*\}$ and, by our log–concavity assumption, the LHS of (A3) is monotone increasing in $1/\sigma_{11}$ and thus monotone decreasing in s_2^*. Similarly, if P is monotone increasing in σ_{11}, then $N = \{\sigma_{11} : \sigma_{11} < s_1^*\}$ and, by our log–concavity assumption, the LHS of (A3) is again monotone decreasing in s_1^*. Thus, in either case, if c increases, then D shrinks and the probability of voluntary disclosure declines. To see why equation (A3) is independent of $(x_j - \mu_j)$ for $j = 2, 3, \ldots, n$, note that $\mathrm{E}[v \mid x, \sigma_{11}] = \mu_v + \beta_1(x_1 - \mu_1) + \sum_{j=1}^{n} \beta_j(x_j - \mu_j)$ and $\mathrm{E}[v \mid x, \sigma_{11} \in N] = \mu_v + \mathrm{E}[\beta_1 \mid x_1, \sigma_{11} \in N](x_1 - \mu_1) + \sum_{j=2}^{n} \beta_j(x_j - \mu_j)$. Subtracting, the sums cancel and so the probability of voluntary disclosure is independent of $x_j - \mu_j$ for $j = 2, 3, \ldots, n$. Finally, if $(x_1 - \mu_1) > 0$ and increases, the RHS of (A3) decreases (including the change in the expected value of σ_{11} given x_1) meaning that s_2^* increases and so the probability of voluntary disclosure increases whereas, if $(x_1 - \mu_1) < 0$ and increases, the RHS of (A3) increases (again including the change in the expected value of σ_{11} given x_1) meaning that s_1^* decreases and so does the probability of voluntary disclosure. ∎

Proposition 3: *If Σ_{xx} is a diagonal matrix and $\sigma_{v1} > 0$, then the manager chooses not to voluntarily disclose her private information if*

$$\frac{-c}{\sigma_{v1}\left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right)} < (x_1 - \mu_1) < 0 \quad \textit{or} \quad 0 < (x_1 - \mu_1) < \frac{c}{\sigma_{v1}\left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right)}.$$

If $\sigma_{v1} < 0$, then the manager chooses not to voluntarily disclose her private information if

$$\frac{c}{\sigma_{v1}\left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right)} < (x_1 - \mu_1) < 0 \quad \textit{or} \quad 0 < (x_1 - \mu_1) < \frac{-c}{\sigma_{v1}\left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right)}.$$

Proof: If P is monotone decreasing in σ_{11}, then the solution to (A3) is not interior if the LHS is always smaller than the RHS. Since the LHS is monotone decreasing, this means that

$$\text{(A4)} \qquad \left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right) < \frac{c}{\sigma_{v1}(x_1 - \mu_1)}.$$

Similarly, if P is monotone increasing in σ_{11}, the solution to (A3) is not interior if the LHS is always smaller than the RHS. Since the LHS is monotone increasing, this means that

$$\text{(A5)} \qquad \left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right) < \frac{-c}{\sigma_{v1}(x_1 - \mu_1)}.$$

If $\sigma_{v1} > 0$, then P monotone decreasing requires that $(x_1 - \mu_1) > 0$ and (A4) becomes

$$(x_1 - \mu_1) < \frac{c}{\sigma_{v1}\left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right)}$$

and P monotone increasing requires that $(x_1 - \mu_1) < 0$ and (A4) becomes

$$(x_1 - \mu_1) > \frac{-c}{\sigma_{v1}\left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right)}.$$

Combining these expressions completes the proof when $\sigma_{v1} > 0$. If $\sigma_{v1} < 0$, then P monotone decreasing requires that $(x_1 - \mu_1) < 0$ allowing us to rewrite (A5) as

$$(x_1 - \mu_1) > \frac{c}{\sigma_{v1}\left(\frac{1}{a} - \mathrm{E}\left[\frac{1}{\sigma_{11}}\right]\right)}$$

and P monotone increasing requires that $(x_1 - \mu_1) > 0$ allowing us to rewrite (A5) as

$$(x_1 - \mu_1) < \frac{-c}{\sigma_{v1}\left(\mathrm{E}\left[\frac{1}{\sigma_{11}}\right] - \frac{1}{b}\right)}.$$

Combining these expressions completes the proof when $\sigma_{v1} < 0$. ∎

Proposition 4: *For $n = 2$ and $\sigma_{12} \neq 0$, if the firm reports good news about both a revenue-type and an expense-type item and they positively covary or it reports good news about two revenue-type or two expense-type items and the items negatively covary, then the manager voluntarily discloses small values of σ_{11}. If, under the same conditions, the firm reports bad news about both items, then the manager voluntarily discloses large values of σ_{11}. Finally, if the firm reports mixed news then D and N depend on the magnitudes of elements in Σ and the news.*

Proof: The formal conditions for this Proposition are:

(i) *if the firm reports good news about both x_1 and x_2, $\sigma_{12} > 0$ and $\sigma_{v1}\sigma_{v2} < 0$ or vice versa, then $D = \{\sigma_{11} \mid \sigma_{11} \leq s_2^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$,*

(ii) *if the firm reports bad news about both x_1 and x_2, $\sigma_{12} > 0$ and $\sigma_{v1}\sigma_{v2} < 0$ or vice versa, then $D = \{\sigma_{11} \mid \sigma_{11} \geq s_1^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$,*

(iii) *if the firm reports mixed news then $D = \{\sigma_{11} \mid \sigma_{11} \leq s_2^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$ when* (a) $\sigma_{22}(x_2 - \mu_2) > \sigma_{12}(x_1 - \mu_1)$, $\sigma_{v1} > 0$, and $\sigma_{12}\sigma_{v2} < 0$; (b) *all of the inequalities are reversed;* (c) *x_1 is good news and $\sigma_{22}\sigma_{v1} > \sigma_{12}\sigma_{v2} > 0$;* or (d) *$x_1$ is bad news and $0 > \sigma_{22}\sigma_{v1} > \sigma_{12}\sigma_{v2}$*

(iv) *if the firm reports mixed news then $D = \{\sigma_{11} \mid \sigma_{11} \geq s_1^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$, when* (a) $\sigma_{22}(x_2 - \mu_2) > \sigma_{12}(x_1 - \mu_1)$, $\sigma_{v1} < 0$, and $\sigma_{12}\sigma_{v2} > 0$; (b) *all of the inequalities are reversed,;* (c) *x_1 is bad news and $\sigma_{22}\sigma_{v1} > \sigma_{12}\sigma_{v2} > 0$;* or (d) *$x_1$ is good news and $0 > \sigma_{22}\sigma_{v1} > \sigma_{12}\sigma_{v2}$.*

From Theorem 1, we know that the sign of $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}}$ completely describes the equilibrium D and N sets. Focusing on equation (6), the sign is the sign of

$$\text{(A6)} \qquad \left(\sigma_{12}\sigma_{v2} - \sigma_{22}\sigma_{v1}\right)\left[\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)\right].$$

So, consider (i) of Proposition 4. If the firm reports good news about both x_1 and x_2, $\sigma_{12} > 0$ and $\sigma_{v1}\sigma_{v2} < 0$, then the first term in equation (A6) is negative (positive) and the second term is positive (negative) if σ_{v1} is positive (negative). Thus, the terms have opposite signs and so $\frac{\partial \mathrm{E}[v|x,\sigma_{11}]}{\partial \sigma_{11}} < 0$ implying that $D = \{\sigma_{11} \mid \sigma_{11} \leq s_2^*\}$ *and* $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$. Since the proofs of parts (ii), (iii) and (iv) are analogous, they are omitted. ∎

Proposition 5: *When $n = 2$ and $\sigma_{12} \neq 0$, if $s_i^* \in (a, b)$, $i = 1, 2$*

(i) *the probability of voluntary disclosure is declining in c;*

(ii) *the probability of voluntary disclosure is declining (increasing) in $(x_2 - \mu_2)$ if $\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0 \quad (\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0)$;*

(iii) *the probability of voluntary disclosure is declining (increasing) in $(x_1 - \mu_1)$ if $-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0 \quad (-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0)$.*

Proof: To simplify the notation, let $f(\sigma_{11}) = \mathrm{E}[v \mid x, \sigma_{11}]$. Then s_1^* and s_2^* satisfy

$$\text{(A7)} \qquad f(s_1^*) - \mathrm{E}[f(\sigma_{11}) \mid \sigma_{11} < s_1^*] = c$$

$$\text{(A8)} \qquad f(s_2^*) - \mathrm{E}[f(\sigma_{11}) \mid \sigma_{11} > s_2^*] = c.$$

Part (i): The effect of an increase in c depends on whether the LHS of (A7) (resp. (A8)) is increasing or decreasing in s_1^* (resp. s_2^*) which, in turn, depends on whether f is concave or convex (see Bagnoli and Bergstrom [2005]). Using equation (6),

$$\frac{\partial^2 f(\sigma_{11})}{\partial \sigma_{11}{}^2} = \left(\frac{-2\sigma_{22}(\sigma_{12}\sigma_{v2} - \sigma_{22}\sigma_{v1})}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^3}\right)\left[\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)\right],$$

and so the sign of f'' is minus the sign of f'. So, first consider the case when $f' > 0$. Then, $f'' < 0$ and so f is concave. By Theorem 7 in Bagnoli and Bergstrom, the LHS of (A7) is monotone increasing in s_1^* and therefore, s_1^* is increasing in c. Since $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$, the set of σ_{11} for which the manager does not disclose grows and so the probability of voluntary disclosure declines when c rises. Now consider the other case when $f' < 0$. In this case, $f'' > 0$ and so f is convex. By Theorem 6, the LHS of (A8) is decreasing in s_2^* and therefore, s_2^* is decreasing in c. Since $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$, the set of σ_{11} for which the manager does not disclose grows and so the probability of voluntary disclosure declines when c rises.

Part (ii): The derivative of the LHS of (A7) with respect to $(x_2 - \mu_2)$ is

$$\text{(A9)} \qquad \left(\frac{\sigma_{11}\sigma_{v2} - \sigma_{12}\sigma_{v1}}{s_1^*\sigma_{22} - \sigma_{12}\sigma_{21}}\right) - \mathrm{E}\left[\frac{\sigma_{11}\sigma_{v2} - \sigma_{12}\sigma_{v1}}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \mid x, \sigma_{11} < s_1^*\right].$$

The sign of this derivative depends on the sign of

$$\frac{\partial}{\partial \sigma_{11}}\left(\frac{\sigma_{11}\sigma_{v2} - \sigma_{12}\sigma_{v1}}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}}\right) = \left(\frac{\sigma_{12}}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^2}\right)(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}).$$

Thus, if $\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0$, then (A9) is positive which implies that s_1^* increases in $(x_2 - \mu_2)$. Hence, $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$, the set of σ_{11} for which the manager does not disclose grows and so the probability of voluntary disclosure declines when $(x_2 - \mu_2)$ rises. Further, the conclusion is reversed if $\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0$.

The derivative of the LHS of (A8) with respect to $(x_2 - \mu_2)$ is

$$\text{(A10)} \qquad \left(\frac{\sigma_{11}\sigma_{v2} - \sigma_{12}\sigma_{v1}}{s_1^*\sigma_{22} - \sigma_{12}\sigma_{21}}\right) - \mathrm{E}\left[\frac{\sigma_{11}\sigma_{v2} - \sigma_{12}\sigma_{v1}}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \mid x, \sigma_{11} > s_2^*\right].$$

From the analysis above, if $\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0$, then the sign of (A10) is negative which implies that s_2^* decreases in $(x_2 - \mu_2)$. Hence, $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$, the set of σ_{11} for which the manager does not disclose grows and so the probability of voluntary disclosure declines when $(x_2 - \mu_2)$ rises. Again, the conclusion is reversed if $\sigma_{12}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0$.

Part (iii): The derivative of the LHS of (A7) with respect to $(x_1 - \mu_1)$ is

$$\text{(A11)} \qquad \left(\frac{\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}}{s_2^*\sigma_{22} - \sigma_{12}\sigma_{21}}\right) - \mathrm{E}\left[\frac{\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \mid x, \sigma_{11} < s_1^*\right].$$

The sign of this derivative depends on the sign of

$$\frac{\partial}{\partial\sigma_{11}}\left(\frac{\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}}{s_2^*\sigma_{22} - \sigma_{12}\sigma_{21}}\right) = \left(\frac{-\sigma_{22}}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^2}\right)(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}).$$

Thus, if $-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0$, then (A11) is positive which implies that s_1^* increases in $(x_1 - \mu_1)$. Hence, $N = \{\sigma_{11} \mid \sigma_{11} < s_1^*\}$, the set of σ_{11} for which the manager does not disclose grows and so the probability of voluntary disclosure declines when $(x_1 - \mu_1)$ rises. Further, the conclusion is reversed if $-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0$.

The derivative of the LHS of (A8) with respect to $(x_1 - \mu_1)$ is

$$\text{(A12)} \qquad \left(\frac{\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}}{s_2^*\sigma_{22} - \sigma_{12}\sigma_{21}}\right) - \mathrm{E}\left[\frac{\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \mid x, \sigma_{11} > s_2^*\right].$$

From the analysis above, if $-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) > 0$, then (A12) is positive which implies that s_2^* decreases in $(x_1 - \mu_1)$. Hence, $N = \{\sigma_{11} \mid \sigma_{11} > s_2^*\}$, the set of σ_{11} for which the manager does not disclose grows and so the probability of voluntary disclosure declines when $(x_1 - \mu_1)$ rises. Again, the conclusion is reversed if $-\sigma_{22}(\sigma_{22}\sigma_{v1} - \sigma_{12}\sigma_{v2}) < 0$. ∎

Proposition 6: *When Σ_{xx} is diagonal and $s_i^* \in (a, b)$ for $i = 1, 2$,*

(i) *the probability of voluntary disclosure is increasing (decreasing) in σ_{v1} when σ_{v1} is positive (negative),*

(ii) *the probability of voluntary disclosure is independent of σ_{vi} for $i = 2, 3, \ldots, n$.*

Proof: Consider (A3) and suppose that $\sigma_{v1} > 0$. If P is monotone decreasing in σ_{11} then $N = \{\sigma_{11} : \sigma_{11} > s_2^*\}$ and, by our log–concavity assumption, the LHS of (A3) is monotone decreasing in s_2^* (for details, see the proof of Proposition 2). Similarly, if P is monotone increasing in σ_{11}, then $N = \{\sigma_{11} : \sigma_{11} < s_1^*\}$ and, by our log–concavity assumption, the LHS of (A3) is again monotone decreasing in s_1^*. Since the derivative of the LHS of (A3) with respect to σ_{v1} has the same sign as $-(x_1 - \mu_1)$, if P is monotone decreasing then s_2^* increases and if P is monotone increasing, then s_1^* declines. Therefore, the probability of voluntary disclosure is increasing when $\sigma_{v1} > 0$. Similar reasoning shows that the probability declines if $\sigma_{v1} < 0$. The probability of voluntary disclosure is independent of σ_{v2} because (A3) is independent of σ_{v2}. ∎

Proposition 7: *For general Σ_{xx}, $n = 2$ and $s_i^* \in (a, b)$ for $i = 1, 2$,*

(i) *the probability of voluntary disclosure is increasing in σ_{v1} when $\sigma_{22}(x_1 - \mu_1) > \sigma_{12}(x_2 - \mu_2)$ and decreasing when $\sigma_{22}(x_1 - \mu_1) < \sigma_{12}(x_2 - \mu_2)$,*

(ii) *the probability of voluntary disclosure is increasing in σ_{v2} when $-\sigma_{12}(x_1 - \mu_1) > \sigma_{11}(x_2 - \mu_2)$ and decreasing when $-\sigma_{12}(x_1 - \mu_1) < \sigma_{11}(x_2 - \mu_2)$.*

Proof: Part (i): Let f be defined as in the proof of Proposition 5 so that equations (A7) and (A8) define s_1^* and s_2^* respectively. Given this,

$$\text{(A13)} \qquad \frac{\partial}{\partial \sigma_{v1}}\Big(f(s_1^*) - \mathrm{E}[f(\sigma_{11}) \mid \sigma_{11} < s_1^*]\Big) = \frac{\partial f(s_1^*)}{\partial \sigma_{v1}} - \mathrm{E}\left[\frac{\partial f(\sigma_{11})}{\partial \sigma_{v1}} \mid \sigma_{11} < s_1^*\right]$$

$$\text{(A14)} \qquad \frac{\partial}{\partial \sigma_{v1}}\Big(f(s_2^*) - \mathrm{E}[f(\sigma_{11}) \mid \sigma_{11} > s_2^*]\Big) = \frac{\partial f(s_2^*)}{\partial \sigma_{v1}} - \mathrm{E}\left[\frac{\partial f(\sigma_{11})}{\partial \sigma_{v1}} \mid \sigma_{11} > s_2^*\right],$$

and so the sign of these derivatives depends on whether $\frac{\partial f(\sigma_{11})}{\partial \sigma_{v1}}$ is increasing or decreasing in σ_{11}. Differentiating,

$$\frac{\partial}{\partial \sigma_{11}}\left(\frac{\partial f(\sigma_{11})}{\partial \sigma_{v1}}\right) = \left(\frac{-\sigma_{22}}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^2}\right)\Big[\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)\Big].$$

Thus, if $\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2) > 0$, then (A13) is negative and (A14) is positive. As a result, s_1^* declines and s_2^* increases in σ_{v1} which means that the probability of voluntary disclosure increases in both cases. Similarly, if $\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2) < 0$, then (A13) is positive, (A14) is negative and the probability of voluntary disclosure decreases.

Part (ii): Differentiating (A7) and (A8),

$$\text{(A15)} \qquad \frac{\partial}{\partial \sigma_{v2}}\Big(f(s_1^*) - \mathrm{E}[f(\sigma_{11}) \mid \sigma_{11} < s_1^*]\Big) = \frac{\partial f(s_1^*)}{\partial \sigma_{v2}} - \mathrm{E}\left[\frac{\partial f(\sigma_{11})}{\partial \sigma_{v2}} \mid \sigma_{11} < s_1^*\right]$$

$$\text{(A16)} \qquad \frac{\partial}{\partial \sigma_{v2}}\Big(f(s_2^*) - \mathrm{E}[f(\sigma_{11}) \mid \sigma_{11} > s_2^*]\Big) = \frac{\partial f(s_2^*)}{\partial \sigma_{v2}} - \mathrm{E}\left[\frac{\partial f(\sigma_{11})}{\partial \sigma_{v2}} \mid \sigma_{11} > s_2^*\right],$$

and so the sign of these derivatives depends on whether $\frac{\partial f(\sigma_{11})}{\partial \sigma_{v2}}$ is increasing or decreasing in σ_{11}. Differentiating,

$$\frac{\partial}{\partial \sigma_{11}}\left(\frac{\partial f(\sigma_{11})}{\partial \sigma_{v2}}\right) = \left(\frac{-\sigma_{22}}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^2}\right)\Big[-\sigma_{12}(x_1 - \mu_1) + \sigma_{11}(x_2 - \mu_2)\Big].$$

Thus, if $\sigma_{11}(x_2 - \mu_2) - \sigma_{12}(x_1 - \mu_1) > 0$, then (A15) is negative and (A16) is positive. As a result, s_1^* declines and s_2^* increases in σ_{v2} which means that the probability of voluntary disclosure increases in both cases. Similarly, if $\sigma_{11}(x_2 - \mu_2) - \sigma_{12}(x_1 - \mu_1) < 0$, then (A15) is positive, (A16) is negative and the probability of voluntary disclosure decreases. ∎

Proposition 8: *The probability of voluntary disclosure is declining in the mandated amount of risk disclosure.*

Proof: Let $f(u) = \mathrm{E}[v \mid x, u]$. Then u_1^*, u_2^* are defined by equations (A7) and (A8) after replacing the s_i^*'s with u_i^*'s. Next note that $\frac{\partial \sigma_{11}}{\partial z} = \frac{\partial \sigma_{11}}{\partial u} = \frac{\partial \sigma_{11}}{\partial \sigma_{11}}$ and so $\frac{\partial^2 f}{\partial z \partial u} = \frac{\partial^2 f}{\partial {\sigma_{11}}^2}$. From the proof of Proposition 5, the sign of f'' is minus the sign of f' so that if $f' > 0$, the LHS of (A7) increases, u_1^* increases and the probability of voluntary disclosure declines. Similarly, if $f' < 0$, the LHS of (A8) decreases, u_2^* decreases and the probability of voluntary disclosure declines. ∎

Proposition 9: *When the manager's private information is that she knows σ_{v1} and $n = 2$,*

(i) *if $\sigma_{22}(x_1 - \mu_1) < \sigma_{12}(x_2 - \mu_2)$, then $D = \{\sigma_{11} \mid \sigma_{11} \leq t_2^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} > t_2^*\}$,*

(ii) *if $\sigma_{22}(x_1 - \mu_1) > \sigma_{12}(x_2 - \mu_2)$, then $D = \{\sigma_{11} \mid \sigma_{11} \geq t_1^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} < t_1^*\}$.*

Proof: Differentiating,

$$\frac{\partial \mathrm{E}[v \mid x, \sigma_{v1}]}{\partial \sigma_{v1}} = \frac{\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}}.$$

Thus, $\mathrm{E}[v \mid x, \sigma_{v1}]$ is increasing in σ_{v1} if $\sigma_{22}(x_1 - \mu_1) > \sigma_{12}(x_2 - \mu_2)$ and vice versa. ∎

Proposition 10: *When the manager's private information is that she knows σ_{12} and $n = 2$,*

(i) *if $(\sigma_{12}\sigma_{v1} - \sigma_{11}\sigma_{v2})(\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)) < 0$, then $D = \{\sigma_{11} \mid \sigma_{11} \leq w_2^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} > w_2^*\}$,*

(ii) *if $(\sigma_{12}\sigma_{v1} - \sigma_{11}\sigma_{v2})(\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)) > 0$, then $D = \{\sigma_{11} \mid \sigma_{11} \geq w_1^*\}$ and $N = \{\sigma_{11} \mid \sigma_{11} < w_1^*\}$.*

Proof: Differentiating,

$$\frac{\partial \mathrm{E}[v \mid x, \sigma_{12}]}{\partial \sigma_{12}} = \left(\frac{\sigma_{12}\sigma_{v1} - \sigma_{11}\sigma_{v2}}{(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^2}\right)(\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2) bigr).$$

Thus, $\mathrm{E}[v \mid x, \sigma_{12}]$ is increasing in σ_{12} if $(\sigma_{12}\sigma_{v1} - \sigma_{11}\sigma_{v2})(\sigma_{22}(x_1 - \mu_1) - \sigma_{12}(x_2 - \mu_2)) > 0$ and vice versa. ∎

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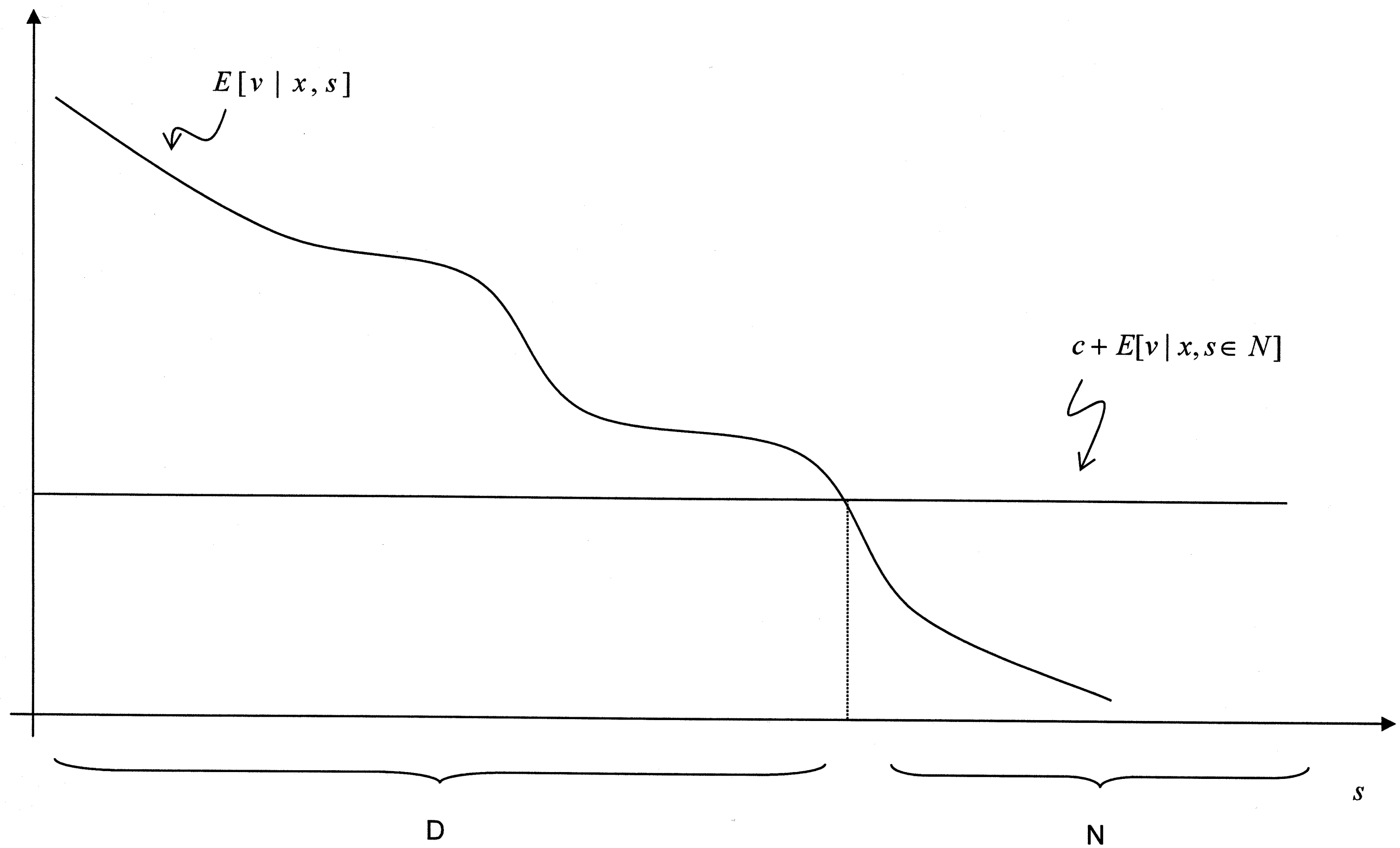
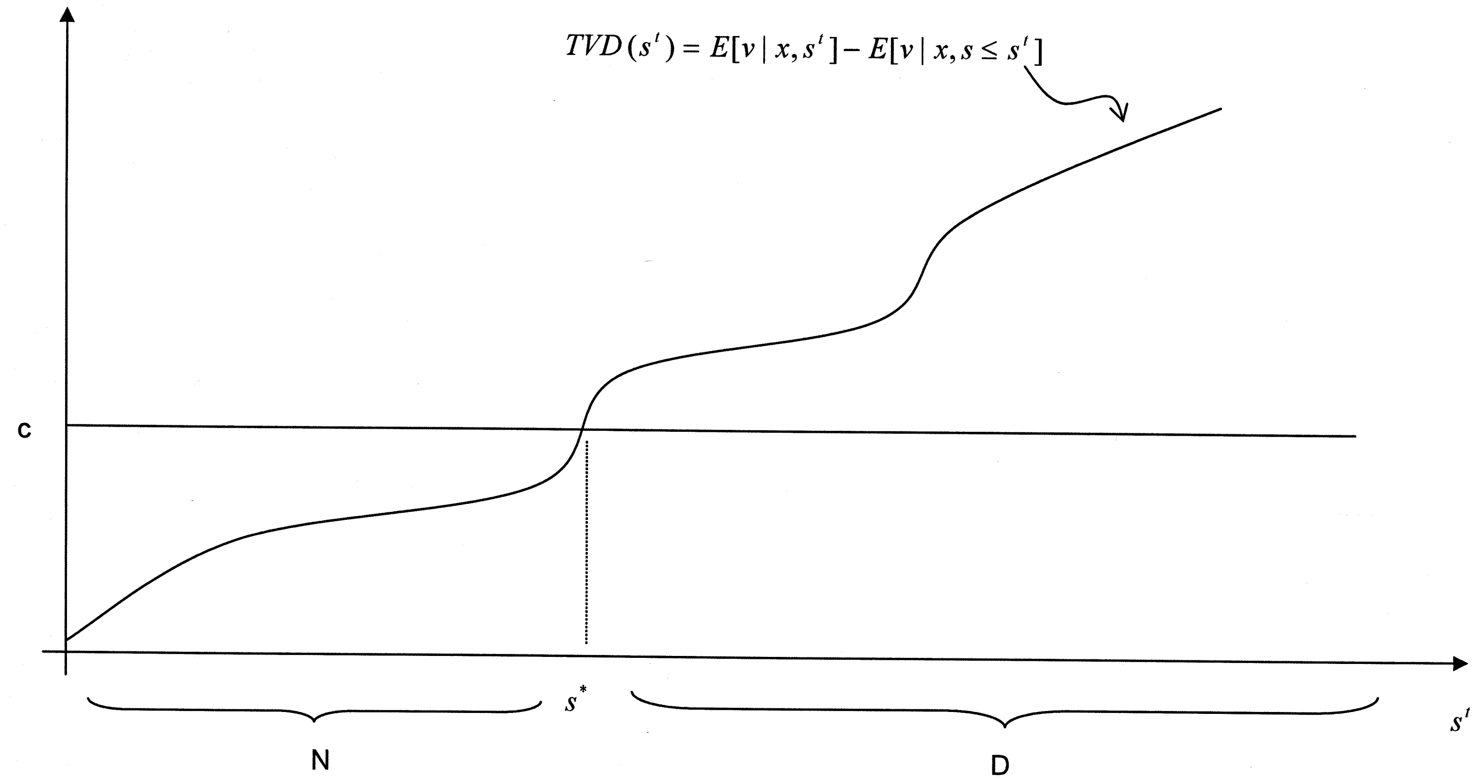
Figure 1
Structure of the Disclosure and Non-Disclosure sets of *s*



The market's expectation of the value of the firm if the manager discloses that her private information is $E[v \mid x, s]$ and the market's expectation of the value of the firm if the manager does not disclose her private information is $E[v \mid x, s \in N]$. The manager prefers to disclose if the former exceeds the latter plus the cost of disclosure, *c*. Thus, when the firm's stock price is declining in *s*, the manager chooses to disclose small values: *D* is a set of small values of *s*.

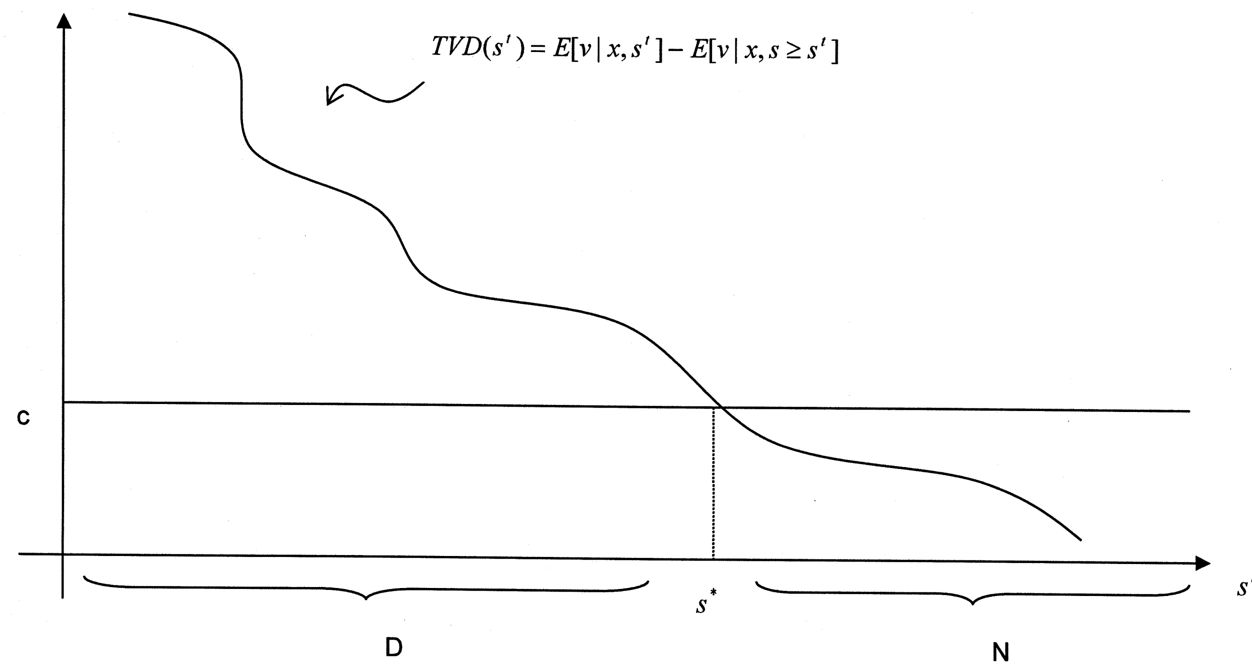
Figure 2
Existence and Uniqueness of s^* when P is Increasing in the Manager's Private Information



The Threshold Value Disclosure function ($TVD(s') = E[v \mid x, s'] - E[v \mid x, s \leq s']$) describes the difference in the market's expectation of the firm's liquidation value when the value of the manager's private information equals the market's conjecture about the largest value of s that is not disclosed. In equilibrium, s^* is the equilibrium cut-off value (separating the set of values that the manager will voluntarily disclose and the set she will not) and solves $TVD(s^*) = E[v \mid x, s^*] - E[v \mid x, s \leq s^*] = c$.

Figure 3

Existence and Uniqueness of s^* when P is Decreasing in the Manager's Private Information



The Threshold Value Disclosure function ($TVD(s') = E[v \mid x, s'] - E[v \mid x, s \geq s']$) describes the difference in the market's expectation of the firm's liquidation value when the value of the manager's private information equals the market's conjecture about the smallest value of s that is not disclosed. In equilibrium, s^* is the equilibrium cut-off value (separating the set of values that the manager will voluntarily disclose and the set she will not) and solves $TVD(s^*) = E[v \mid x, s^*] - E[v \mid x, s \geq s^*] = c$.