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RAMSEY FISCAL AND MONETARY POLICY UNDER STICKY PRICES AND LIQUID BONDS

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1. INTRODUCTION

THERE HAS BEEN recent renewed interest in the issue of optimal fiscal and monetary policy. The benchmark framework approaches this issue from the point of view of a Ramsey planner. However recent focus has been on model economies where inflation matters and is costly to society in real terms. In this paper we provide an alternative setup of a sticky-price dynamic stochastic general equilibrium model with Ramsey optimal fiscal and monetary policy whereby there can exist an interest rate spread between two classes of nominally risk free bonds (government and private bonds) *à la* Canzoneri and Diba (2005). The crucial difference between the bonds here is that government bonds provide some liquidity service. Thus private agents may want to hold assets in the form of government debt in exchange for their liquidity service although they pay a lower return than the private bond. Canzoneri and Diba (2005) provide the factual example that, " ... [U].S. Treasury bills clearly facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, and individuals hold them in money-market accounts that offer checking services." We investigate how this new feature alters the equilibrium characterization of the Ramsey allocation, and modifies the trade-off between price stability and income-tax stability found in recent papers, such as Schmitt-Grohé and Uribe (2004a) and Siu (2004), on Ramsey optimal policy in sticky price environments.

We show that government debt dynamics affect the intertemporal allocations of resources via the Ramsey planner's sequence of implementability constraints. This does not happen in standard models where government debt provides no liquidity service. In our model, government debt modifies the

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marginal utility value of future streams of budget surpluses and also directly via the money-bond transactions technology constraint, in the sequence of implementability constraints. This is because government bonds are valued by the private sector in terms of their transactions service, and this is taken into account by the planner in designing an optimal plan for its fiscal and monetary policy. The existence of liquid, interest-bearing government bonds creates a spread between the returns on illiquid private bonds and liquid government bonds that acts as an additional tax instrument.[1] This suggests an avenue for fiscal policy, in terms of government debt with liquidity services (via the interest-rate spread), to alter the trade-offs between a limited number of distorting tax instruments faced by a Ramsey planner who wishes to approximate market completion in a world without real state-contingent assets.

We find that the more sticky prices become, the more the optimal Ramsey plan favors price stability *but* the planner can also afford a less distortionary and *less volatile* income tax scheme. The latter result is opposite to that of existing literature, for example Schmitt-Grohé and Uribe (2004a) and Siu (2004). This is because in our model the dynamics of liquid government bonds affect the government's sequence of implementability (intertemporal solvency) constraints. Thus the planner uses the interest-spread channel which alters the dynamics of liquid government bonds, as a means of satisfying the constraints, in designing its optimal tax and monetary policy plan. In doing so, the planner does not have to rely so much on using distortionary income tax or costly inflation to meet its expected intertemporal solvency constraints.

In earlier literature on optimal fiscal and monetary policy, the analyses were often carried out using competitive flexible-price monetary models without capital, for example, Lucas and Stokey (1983), Calvo and Guidotti (1993), and Chari, Christiano, and Kehoe (1991). The general conclusion was that optimal fiscal-monetary policy entails a volatile and serially uncorrelated inflation rate while labor income tax is smooth. This is because the planner uses surprise inflation as a lump-sum tax on household financial wealth, while minimizing the distortionary effect of labor income tax. Thus real government bonds act as a shock absorber to maintain a constant path for the labor income tax rate.

In the seminal works of Schmitt-Grohé and Uribe (2004a) and Siu (2004), the authors provide a variation on the results found in the optimal fiscal-monetary policy literature. In such economies, inflation is costly in terms of real resources such that the planner has to trade-off between minimizing tax distortions and minimizing costly inflation volatility. On one hand, in order to minimize tax distortions on private work incentives, the planner would like to use unexpected variations in the price level as a means for taxing household wealth, which leads to greater inflation volatility. This is the same effect found in the earlier class of flexible price competitive economies. On the other, the existence of price-adjustment cost affects household welfare via their feasibility constraint. This discourages the planner from trading off unexpected inflation with labor income tax variations, resulting in lower inflation volatility. Schmitt-Grohé and Uribe (2004a) find that the second effect

[1] One can envision that the private sector can also issue liquid assets or bonds (e.g. credit cards, commercial paper and etc.). However, for the sake of clarity and exposition, we assume that there only exist a nominally risk-free private bond that is illiquid and the liquid government bond.

dominates. In other words, for modest degrees of price stickiness, the tension is resolved in the direction in favor of price stability or low inflation volatility. Furthermore, the tax rate on labor is still reasonably smooth or "near random walk", but this tends to be less so, when there is imperfect competition; or even less when there exist sticky prices. Siu (2004) also has very similar conclusions. Siu (2004) specifically reports that under an optimal Ramsey policy, the volatility of inflation decreases while that of the labor tax rate increases as the degree of price stickiness in the economy rises. He also finds that the tax distortion can be smoothed over time.[2]

The new addition in our model has a close counterpart in Canzoneri and Diba (2005). However, they were concerned with the issue of price level determinacy in a deterministic, partial-equilibrium and flexible-price model with simple monetary- and fiscal-policy rules. In their economy, fiscal policy can provide a nominal anchor, even when monetary policy does not. Their result arises because government bonds can provide liquidity services and this allows bonds to affect the equilibrium process for inflation. They allow for bonds to enter a cash-in-advanced (CIA) constraint and to act as imperfect substitutes for money. We generalize their assumption to a general equilibrium production economy with costly price adjustment. Furthermore, we consider optimal policy from the point of view of the benchmark Ramsey planner.

The remainder of the paper is organized as follows. We outline the model primitives in Section 2. We show how a decentralized equilibrium, defined in Section 3, can be supported as a Ramsey planning problem in Section 4. We then deduce the implications of the introduced liquid-bond feature in the model for Ramsey optimal taxation and monetary policy, in Section 5. We calibrate the model and perform some numerical experiments to study the behavior of the Ramsey equilibria in Section 6. We conclude in Section 7.

2. The Model

Consider an economy populated by a continuum of infinitely lived identical households on $[0, 1]$. Each period $t \in \mathbb{N}$, household derives utility from consumption, c_t, and leisure, $1 - h_t$ where time endowment is unity and h_t is the fraction of time spent working. Households are also monopolistic firms producing differentiated intermediate goods. Fiscal and monetary policy will be determined jointly by a Ramsey planner. We begin by specifying the exogenous stochastic processes in the model.

2.1 Exogenous stochastic processes

There are two exogenous forcing processes in the model. These can be interpreted as demand and supply shocks. On the demand side, government spending is a Markov process, where

$$\ln g_t = (1 - \rho_g) \ln \overline{g} + \rho_g \ln g_{t-1} + u_{g,t};\ \ \rho_g \in [0, 1), u_{g,t} \sim \text{i.i.d.} \left(0, \sigma_g^2\right). \tag{1}$$

[2]The result in Siu (2004) and Schmitt-Grohé and Uribe (2004a), in terms of a near-unit-root feature of optimal income tax, echoes the outcome in Aiyagari, Marcet, Sargent, and Seppälä (2002). In Aiyagari, Marcet, Sargent, and Seppälä (2002), the model is perfectly competitive but features incomplete markets where there is only real non-state-contingent government debt.

where $\bar{g}$ is steady state government consumption. On the supply side, economy-wide shocks to production technology is given by the Markov process

$$\ln z_t = \rho_z \ln z_{t-1} + u_{z,t};\ \ \rho_z \in [0,1), u_{z,t} \sim \text{i.i.d.}\left(0, \sigma_z^2\right). \tag{2}$$

It is assumed that $(g_t, z_t)' \in \mathcal{S}$ where $\mathcal{S} \subset \mathbb{R}_+^2$ is compact.

2.2 *Household-firm problem*

Households are monopolistic firms producing a differentiated intermediate good. Define Y_t as the total final demand for aggregate output, $\widetilde{P}_t$ as the firm-specific price charged by each firm, and P_t the aggregate price level. Thus the demand for this monopolist's good is $d\left(\widetilde{P}_t/P_t\right) Y_t$, where $d'\left(\widetilde{P}_t/P_t\right) < 0$, $d(1) = 1$, and $d'(1) < -1$. The household-firm employs labor, $\widetilde{h}_t$, with a competitive nominal wage $w_t P_t$, and produces using a technology

$$d\left(\frac{\widetilde{P}_t}{P_t}\right) Y_t = z_t \widetilde{h}_t \tag{3}$$

Because each household-firm is monopolistic, it can set $\widetilde{P}_t$, and following Rotemberg (1982), we assume it faces a real convex cost of price adjustment

$$C\left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}}\right) = \frac{\theta}{2}\left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}} - \overline{\Pi}\right)^2 \tag{4}$$

where θ will be a parameter governing the degree of price-stickiness and $\overline{\Pi} \geq 1$ is steady-state inflation.

Let $m_t = M_t/P_t$ and $b_t = B_t/P_t \in \mathcal{B} \subset \mathbb{R}_+$ respectively denote real money balances and real government bond holdings determined at the end of period t. Define $\Pi_t = P_t/P_{t-1}$ and $p_t = \widetilde{P}_t/P_t$ respectively, as gross inflation and a firm-specific price relative to the average price level. Let R_t be the one-period nominally risk-free gross return on government bond holdings, $b_t^* \in \mathcal{B}^* \subset \mathbb{R}$ be a private bond that pays a nominally risk-free return of R_t^* in period $t+1$, and $\tau_t \in [0,1]$ be the flat tax rate on labor income. The sequence of household budget constraints is given by

$$\begin{aligned} c_t + m_t + b_t + b_t^* \leq{}& \frac{m_{t-1}}{\Pi_t} + R_{t-1}\frac{b_{t-1}}{\Pi_t} + R_{t-1}^*\frac{b_{t-1}^*}{\Pi_t} \\ &+ \left[p_t Y_t d(p_t) - w_t \widetilde{h}_t - \frac{\theta}{2}\left(\frac{p_t}{p_{t-1}}\Pi_t - \overline{\Pi}\right)^2\right] + (1-\tau_t) w_t h_t. \end{aligned} \tag{5}$$

for $t = 0, 1, 2, \ldots$. The household's time-0 payoff is measured as the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{6}$$

where $\mathbb{E}_0$ is the mathematical expectations operator, taken over the sequence of functions $U(c_t, h_t)$ measurable with respect to the information set generated by $\{z_t, g_t, b_t^*, b_t\}$ at time 0.[3] $U(\cdot)$ satisfies

[3]Specifically at time zero, the information set or sigma algebra is $\mathcal{F}_0 = \mathcal{B}_0 \times \mathcal{B}_0^* \times \mathcal{S}$, where $\mathcal{F}_0 \subset \mathcal{F}_1 \cdots \subset \mathcal{F}_t$.

the Inada conditions: $\lim_{c\searrow 0} U_c(c,h) = +\infty$ and $\lim_{l\searrow 0} U_l(c,h) = +\infty$ where $l := 1-h$. The household maximizes (6) subject to (5) and a cash-in-advance (CIA) constraint:

$$m_t + k(b_t) \geq c_t. \tag{7}$$

The transactions service of bonds is reflected in the function $k(b_t)$ which satisfies the following properties, which are similar to Canzoneri and Diba (2005) except for a minor modification to allow for endogenous output determination in our model.

ASSUMPTION 1: The function $k(b_t)$ satisfies:

A1 $k(b_t) = 0$ for $b_t \leq 0$;

A2 $k'(b_t) > 0$ and $k''(b_t) < 0$ for $b_t > 0$;

A3 $\lim_{b\searrow 0} k'(b_t) < 1$, $\lim_{b\nearrow +\infty} k'(b_t) = 0$ and $\lim_{b\nearrow +\infty} k(b_t) < c_t$.

Assumption A1 ensures that negative bond holdings do not provide any transactions value so that $b_t \in \mathcal{B} \subset \mathbb{R}_+$, and A2 ensures that positive government bond holdings provide increasing transactions service, but the marginal transactions service is decreasing. Lastly, A3 ensures that these bonds are never sufficient to fund all consumption purchases.[4] That is, there will still be positive holdings of money.[5]

Let the Lagrange multiplier on the constraints (7) and (5) be μ_t and λ_t, respectively, and the multiplier on the technology constraint (3), when inserted into (5) be $mc_t\lambda_t$, where mc_t is the real marginal cost of production for a firm. The first-order conditions are

$$c_t: \qquad U_c(c_t, h_t) = \lambda_t + \mu_t \tag{9}$$

$$b_t^*: \qquad \lambda_t = \beta R_t^* \left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right) \tag{10}$$

$$b_t: \qquad \lambda_t = R_t \beta \left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right) + \mu_t k'(b_t) \tag{11}$$

$$m_t: \qquad \lambda_t = \beta \left(\frac{\lambda_{t+1}}{\Pi_{t+1}}\right) + \mu_t \tag{12}$$

[4] In terms of practical implementation, to ensure the CIA binds at all times and still satisfies positive money holdings, we will assume shocks with small bounded supports, and admit only the parameter $\lim_{b\nearrow +\infty} k(b_t) = \phi$ such that for sufficiently large steady-state consumption, $\bar{c} > \phi$, consumption c_t will almost surely be bounded above $k(b_t)$ for all t and all histories leading up to and including date t.

[5] Alternatively we could have modeled the CIA constraint as

$$m_t + k(b_t)\, c_t \geq c_t. \tag{8}$$

where k still satisfies Assumption 1. This would be closer to the CIA constraint in the endowment economy of Canzoneri and Diba (2005), where $c_t = y = 1$. In this case, m_t will be strictly positive since c_t is nonnegative under the Inada conditions, and $k(b_t) \in (0,1)$. However, this assumption creates additional nonlinearities in the optimality conditions with respect to liquid bonds for households and the planner, without affording much difference in the qualitative implications of the model.

$$h_t: \qquad U_h(c_t, h_t) = -\lambda_t (1-\tau_t) w_t \tag{13}$$

$$\widetilde{h}_t: \qquad \frac{w_t}{z_t} = mc_t \tag{14}$$

$$\widetilde{P}_t: \qquad \lambda_t \left[Y_t d(p_t) + p_t Y_t d'(p_t) - \theta \left(\frac{\Pi_t p_t}{p_{t-1}} - \overline{\Pi} \right) \frac{\Pi_t}{p_{t-1}} - mc_t Y_t d'(p_t) \right] + \beta \left[\lambda_{t+1} \theta \left(\frac{\Pi_{t+1} p_{t+1}}{p_t} - \overline{\Pi} \right) \frac{\Pi_{t+1} p_{t+1}}{p_t^2} \right] = 0 \tag{15}$$

for all states and dates $t \in \mathbb{N}$. The last two conditions (14) and (15), respectively, characterize the optimal labor demand by the household-firm and the optimal price-setting condition which depends on expected future prices. These first-order conditions are quite standard, apart from (11).

2.3 Symmetric pricing equilibrium

In equilibrium, there is no trade of the private bond. However it can be shown that the interest rate on the private bond must still be positive in equilibrium.

LEMMA 1: In equilibrium $b_t^* = 0$ but $R_t^* > 1$.

Proof: Identical households have no desire to borrow or lend to each other on the private asset market so that $b_t^* = 0$ in equilibrium. From the optimality conditions (9) and (12), we have

$$\mu_t = \lambda_t \left(\frac{R_t^* - 1}{R_t^*} \right) \tag{16}$$

for all states and dates $t \in \mathbb{N}$. By the Inada condition on consumption, it must be that $c_t > 0$, and along with Assumption 1, the CIA constraint must bind so that $\mu_t > 0$, and with optimality such that $\lambda_t > 0$, for all states and dates $t \in \mathbb{N}$, then (16) implies that $R_t^* > 1$ for all states and dates $t \in \mathbb{N}$. □

Also, in a symmetric equilibrium, all household-firms charge the same price, so that $p_t = 1$. That is, all households will charge the same price as the average price, or $\widetilde{P}_t = P_t$, for all t. Given the same production technology and competitive wage rate, it must be that the amount of labor supplied by each household equals its demand in its production such that $h_t = \widetilde{h}_t$. The demand for each monopolist's good is $d(p_t) Y_t$ so that the elasticity of demand for each good is $\epsilon(p_t) = d'(p_t) p_t Y_t / d(p_t) Y_t$.

In a symmetric equilibrium, $p_t = 1$ so that under our assumption that $d(1) = 1$, we get the elasticity of demand faced by each household-firm is constant, $\eta \equiv d'(1) < -1$. Since the marginal revenue for each monopolist is $[1 + \epsilon(p_t)] d(p_t) Y_t$, in the symmetric equilibrium, marginal revenue for all monopolists becomes $(1+\eta) Y_t$. The optimal pricing condition (15), together with the fact that in a symmetric equilibrium, $Y_t = z_t h_t$ and also using (14), can be expressed as

$$\left(\Pi_t - \overline{\Pi}\right) \Pi_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right] + \frac{\eta z_t h_t}{\theta} \left[\frac{1+\eta}{\eta} - \frac{w_t}{z_t} \right]. \tag{17}$$

after taking conditional expectations. This is an expectations-augmented Phillips curve, which says that time-t inflation depends on the contemporaneous gap between real marginal cost and steady-state real marginal cost, $\eta^{-1}(1+\eta)$, and expected discounted next-period inflation. Also, the greater is the cost of prices adjustment, $\theta \to \infty$, the closer is expected discounted next-period inflation to current inflation. That is, prices are expected not to change very much the more costly is price adjustment. The greater is the elasticity of demand, $\eta \to -\infty$, the more positive and sensitive is the response of current inflation to real marginal cost (limiting case of perfect competition).

2.4 *Resource constraint*

The resource constraint is given by

$$z_t h_t = c_t + g_t + \frac{\theta}{2}\left(\Pi_t - \overline{\Pi}\right)^2 \tag{18}$$

which is the market clearing condition for consumption goods, private and government, where some of that produced resources is dissipated in terms of a price-adjustment cost.

2.5 *Government budget constraint*

The sequence of government budget constraints is

$$M_t + B_t + \tau_t P_t w_t h_t = M_{t-1} + R_{t-1}B_{t-1} + P_t g_t. \tag{19}$$

This says that government spending and the payment of public debt with interest, is financed with either the issue of new money, new debt or income tax receipts. We can re-write this in real terms as

$$m_t + b_t + \tau_t w_t h_t = \frac{m_{t-1}}{\Pi_t} + \frac{R_{t-1}b_{t-1}}{\Pi_t} + g_t \tag{20}$$

for $t \in \mathbb{N}$. Notice that with higher inflation, the government can relax the one-period government budget constraint by lowering the real liability of money holding m_{t-1}/Π_t. This also makes the real gross return on government bonds, R_{t-1}/Π_t, depend on the state of inflation.

3. DECENTRALIZED EQUILIBRIUM

The following defines the competitive or decentralized equilibrium for a given feasible policy rule.

DEFINITION 1: Given policy rule $\{\tau_t, R_t, R_t^*\}_{t=0}^{\infty}$, a decentralized equilibrium is the sequence of bounded allocations $\{c_t, h_t, m_t, w_t, \Pi_t, mc_t, b_t\}_{t=0}^{\infty}$ respecting the optimality conditions (9)-(14) and (17), satisfying the feasibility constraints (18) and (19) and the transversality condition

$$\lim_{s\to\infty} \mathbb{E}_t \left(\prod_{i=0}^{s} R_{t+i}^{-1}\right)(R_{t+s}B_{t+s} + M_{t+s}) = 0, \tag{21}$$

for given stochastic processes (1)-(2).

4. RAMSEY PROBLEM

We cast the fiscal and monetary policy problem in terms of a Ramsey planning problem which implements a decentralized equilibrium. First we characterize the equilibrium using the primal approach as in Atkinson and Stiglitz (1980), Lucas and Stokey (1983), and Chari, Christiano, and Kehoe (1995), which characterizes the equilibrium in terms of allocations (and the inflation rate) as far as possible. This is done so that we can show, in a condensed way, how the introduction of liquid government bonds, b_t, can alter the Ramsey equilibrium allocations. In order to analyze the qualitative behavior of these dynamics, we will characterize and solve the Ramsey problem using the dual approach, as set out in Appendix B.

The following proposition shows that the equilibrium plan under such a Ramsey planner also satisfies the condition of a decentralized equilibrium in Definition 1.

PROPOSITION 1: The plans $\{c_t, h_t, \Pi_t, mc_t, b_t, R_t^*\}_{t=0}^{\infty}$ respecting the resource constraint (18), the sequence of government budget constraints:

$$\begin{aligned} c_t - k(b_t) + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)} \right) h_t \\ = \frac{c_{t-1} - k(b_{t-1})}{\Pi_t} + \frac{\left[R_{t-1}^* - \left(R_{t-1}^* - 1\right) k'(b_{t-1})\right] b_{t-1}}{\Pi_t} + g_t \end{aligned} \tag{22a}$$

for $t \geq 1$ and

$$c_0 - k(b_0) + b_0 + \left(mc_0 z_0 + \frac{U_h(c_0, h_0)}{U_c(c_0, h_0) / \left(2 - R_0^{*-1}\right)} \right) h_0 = \frac{M_{t-1} + R_{-1} B_{-1}}{P_{-1} \Pi_0} + g_0 \tag{22b}$$

the expectational Phillips curve

$$\left(\Pi_t - \overline{\Pi}\right) \Pi_t = \beta \mathbb{E}_t \left[\frac{U_c(c_{t+1}, h_{t+1}) / \left(2 - R_{t+1}^{*-1}\right)}{U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right] + \frac{\eta z_t h_t}{\theta} \left[\frac{1+\eta}{\eta} - mc_t \right] \tag{23}$$

and the sequence of present-value implementability constraints,

$$\begin{aligned} \mathbb{E}_t \sum_{s=0}^{\infty} \frac{\beta^s}{\Delta_{t,t+s}} \frac{U_c(c_{t+s}, h_{t+s})}{\left(2 - R_{t+1}^{*-1}\right)} \Bigg\{ \left[1 + \frac{\left(R_{t+s}^* - 1\right)\left(1 - k'(b_{t+s})\right)}{R_{t+s}^* - \left(R_{t+s}^* - 1\right) k'(b_{t+s})} \right] c_{t+s} + \left(mc_{t+s} - 1\right) z_{t+s} h_{t+s} \\ - \frac{\left(R_{t+s}^* - 1\right)\left(1 - k'(b_{t+s})\right)}{R_{t+s}^* - \left(R_{t+s}^* - 1\right) k'(b_{t+s})} k(b_{t+s}) + \frac{U_h(c_{t+s}, h_{t+s}) h_{t+s}}{U_c(c_{t+s}, h_{t+s}) / \left(2 - R_{t+1}^{*-1}\right)} \Bigg\} \\ = \frac{U_c(c_t, h_t)}{\left(2 - R_t^{*-1}\right)} \left[\frac{R_{t-1}^* - \left(R_{t-1}^* - 1\right) k'(b_{t-1}) b_{t-1} + c_{t-1} - k(b_{t-1})}{\Pi_t} \right] \end{aligned} \tag{24}$$

where $\Delta_{t,t+s} = \prod_{i=1}^{s} \left[1 - \left(1 - R_{t+i-1}^{*-1}\right) k'(b_{t+i-1})\right]$, for all states and $t = 0, 1, 2, ...$, and given initial conditions $\left(R_{-1} B_{-1} + M_{-1}\right) / P_{-1}$ also satisfy the decentralized equilibrium in Definition 1.

Proof: See Appendix A. □

REMARK 1: The LHS of (24) is the expected present value of the stochastic stream of utility value of the government's real budget surpluses, which takes into account private agents' optimal plans given the government's strategy. However, this is augmented by: (a) monopolistic competitive distortions; and (b) private demand for liquidity, which would reduce to similar implementability contraints for flexible-price economies if $mc_{t+s} = 0, \forall t, s \geq 0$. The RHS is the utility value of existing government budget deficit at the beginning of time $t \geq 0$.

REMARK 2: Note that $\{b_{t+s}\}_{s=0}^{\infty}$ appears in the implementability constraint (24). This is not the case in the one-bond and one-interest-rate models typically found in the literature. For instance, compare this with equation (29) of Schmitt-Grohé and Uribe (2004a). Thus we have an additional channel, in the dynamics of liquid government bonds, via which fiscal policy can alter the constraints faced by the planner. We defer further discussion of this to Section 5.

REMARK 3: The existence of costly price adjustment implies that the Ramsey plan underlying the primal form of the decentralized equilibrium can no longer be described by a single sequence of present-value implementability constraint as is usually done in flexible-price economies. There is also a sequence of sticky-price constraints facing the planner, which is summarized by (23).

The intuition from Schmitt-Grohé and Uribe (2004a) is that the sequence of prices is uniquely determined when real allocations are obtained in the primal form of a flexible price equilibrium. These prices then imply a sequence of real discount factors that ensure the transversality condition in the competitive equilibrium is respected in all dates and states. However, when a Phillips curve exists under sticky prices, it imposes an additional constraint on the across-state and across-date feasibility of allocations. So in order for the resulting Ramsey plan to deliver a sequence of prices that is consistent with that in a decentralize equilibrium, the plan has to satisfy both the decentralized equilibrium's transversality condition and the Phillips curve constraints, and the sequence of implementability constraints.[6]

In our model, the planner can use the additional liquid-bond interest-rate instrument to alleviate some of the constraints imposed by sticky-prices on the implementability constraints. This alters some of the trade-off between price stability and across-state and across-time labor tax smoothing result found by Siu (2004) and Schmitt-Grohé and Uribe (2004a). While it is not possible to show analytically how liquid government bonds alleviate these constraints in the model, we can show these results numerically in Section 6. Nevertheless, we can deduce and discuss the implications of the liquid government bond for Ramsey equilibrium allocations in the next section.

5. LIQUID-BOND IMPLICATIONS FOR OPTIMAL POLICY

Government policy can be pinned down as a sequence $\{R_t^*, R_t, \tau_t\}_{t=0}^{\infty}$, where the first interest rate can be thought as monetary policy pinning down the pricing kernel with respect to the private bond, b_t^*, which then pins down optimal private consumption in the Euler equation (10).

Given $\{R_t^*\}_{t=0}^{\infty}$, combining the marginal utility of consumption (9) and (10), real money demand optimality (12), and optimal labor supply decision (13), we have:

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = (1 - \tau_t) w_t \left(\frac{R_t^*}{2R_t^* - 1} \right), \tag{25}$$

[6] Siu (2004) showed that sticky prices effectively impose a cost on the planner in using inflation as means of making ex post real debt state-contingent and that this outcome would approximately be equivalent to a real economy that rules out real state-contingent bond markets as in Aiyagari, Marcet, Sargent, and Seppälä (2002).

so that the planner determines the consumption-leisure margin by setting income tax policy $\{\tau_t\}_{t=0}^{\infty}$ for given real wage rate w_t. Equation (25) is the usual consumption-leisure intratemporal condition with the qualification that with money demand through the CIA constraint, the opportunity cost of money, R_t^* affects the marginal utility of consumption via the marginal cost of liquidity holding, μ_t.

Consider now the policy instrument R_t. Combining (10)-(12), we can express the optimal demand for government bonds as

$$k'(b_t) = \frac{R_t^* - R_t}{R_t^* - 1}. \tag{26}$$

At the optimum, the household will demand government bonds up to the point where the marginal transactions value of such bonds are equal to the marginal opportunity cost of holding government bonds, relative to the private bond which pays a return of R_t^*. Notice that as long as $b_t > 0$ it must be that, $R_t^* - R_t > 0$ since $k'(b_t) > 0$. Thus, as long as the government issues bonds with transactions value for private agents, there will exist an interest-rate spread in the model.[7]

A consequence of liquid bond demand is that real money demand is now affected by the process of government bonds, b_t, directly. This can be seen by combining the CIA constraint (7), when it binds, with (9) to yield real money demand as $m_t = U_c^{-1}(\lambda_t + \mu_t) - k(b_t)$ and λ_t and μ_t are pinned down by (10)-(12) which explicitly involve the demand for government bonds $k'(b_t)$. Hence, there is an intratemporal effect of the policy spread $R_t^* - R_t$ that determines the distribution of household liquidity holdings between money and government bonds.

Finally, government bonds affect optimal inflation dynamics (15) through the real marginal cost of production, mc_t, and this comes directly from its immediate effect on the marginal value of wealth λ_t in (11) and hence optimal labor supply and demand, (13) and (14). This has an indirect effect on the Phillips curve constraint for the planner (23).

Thus, the existence of liquid b_t allows the planner to exploit the spread, $R_t^* - R_t$, and therefore the path of b_t in order to satisfy its sequence of implementability constraints in (24) and sticky-price constraints (23) in return for smoothing labor income tax across states and dates and also maintaining stability in costly inflation, under the Ramsey optimal plan.

To gain further insight into the role of liquid government bonds and the effect of the interest-rate spread in affecting the optimal fiscal and monetary policy, we use numerical solutions and simulations in the next section.

6. PROPERTIES OF RAMSEY EQUILIBRIUM

In this section, we present numerical solutions and examples of the Ramsey equilibrium. First, we consider how the optimal Ramsey program behaves in environments with and without sticky prices

[7] There are many empirical studies, notably Weil (1989), Giovannini and Labadie (1991) , Bansal and Coleman (1996), and Canzoneri, Cumby, and Diba (2002), that find a sizeable equity premium, or a large spread between the average return on equity and the return on treasury bills. In our model, care has to be taken to interpret the interest-rate spread literally as an "equity premium". As Canzoneri and Diba (2005) suggest, one might attempt to measure our return on the illiquid private bond, R^* , using consumption and price data on our household's Euler equation. Further, one can take the return on liquid government bonds, R, as that for a three-month T-bill. In that instance, our notion of an interest-rate spread, $R^* - R$, should have a magnitude that is close to what is observed as the equity premium.

and/or liquid bonds, using a baseline calibration. Second, we examine the behavior of the Ramsey plan within successively more sticky price environments, when we allow for liquid government bonds. Third, we investigate further the role that bond liquidity plays, by repeating the previous experiment across different degrees of bond liquidity. Last, we consider the role played by bond liquidity in the face of technology or government spending shocks individually, using the baseline sticky-price calibration.

In order to implement the model numerically, we impose functional forms on the model's primitives. We assume the period utility of the representative household to be $U(c,h) = \ln c + \delta \ln (1-h)$. A bonds transactions-service function, which satisfies Assumption 1, is $k(b) = \phi \left(1 - e^{-\frac{b}{c}}\right)$ where $\phi \leq \overline{c}$ and $\overline{c}$ is steady-state consumption. This functional form is similar to that used by Canzoneri and Diba (2005) in their numerical example.

TABLE 1
BASELINE FULL MODEL (SP) CALIBRATION

Parameter	Value	Description
β	0.956	Subjective discount factor
s_g	0.2	Share of government consumption in GDP
$\overline{\Pi}$	1.042	Gross inflation rate
$\overline{z}$	1	Steady-state level of technology
δ	3.017	Labor supply parameter
ϕ	0.149	Bond substitutability parameter
b/zh	0.44	Share of government debt in GDP
θ	$17.5/4$	Degree of price stickiness
η	-6	Elasticity of demand
ρ_z	0.82	Autocorrelation of technology
σ_z	0.0229	Std. deviation of technology shock
ρ_g	0.9	Autocorrelation of government spending
σ_g	0.0302	Std. deviation of government spending shock

The baseline sticky-price-liquid-bond economy (denoted later as SP) is calibrated using post-war US data. The calibration is summarized in Table 1. The calibration of β, given steady-state inflation $\overline{\Pi} = 1.042$, ensures that steady-state nominal return on the private bond is $R^* = 1.09$. Given the share of government debt in GDP of about 44 per cent per annum, we can calibrate ϕ to ensure that the interest rate spread, $R^* - R$ in steady state is about 5 percent, following the findings of Bansal and Coleman (1996). The parameter δ is solved endogenously using the government budget constraint at steady state, and is consistent with a fraction of hours worked, $h = 0.2$. The details of calibrating ϕ and δ can be found in Appendix C. The rest of the parameters follow the calibration of Schmitt-Grohé and Uribe (2004a). We employ a second-order accurate perturbation method by Schmitt-Grohé and Uribe (2004b) to solve for the optimal state transition and policy functions around the non-stochastic steady-state.

TABLE 2
TAX RATE PROPERTIES OF VARIOUS ECONOMIES

Economies	R^*	R	Π	τ
		Unconditional mean		
FP* ($\phi = \theta = 0$)	1.014	–	0.969	0.427
SP* ($\phi = 0$)	1.090	–	1.040	0.341
FP ($\theta = 0$)	1.063	1.000	1.017	0.390
SP	1.090	1.018	1.042	0.370
		Percentage standard deviation		
FP* ($\phi = \theta = 0$)	0.532	–	0.532	0.893
SP* ($\phi = 0$)	0.009	–	0.007	0.160
FP ($\theta = 0$)	0.113	0.069	0.119	0.229
SP	0.005	0.037	0.001	0.091
		Autocorrelation		
FP* ($\phi = \theta = 0$)	0.960	–	0.963	0.891
SP* ($\phi = 0$)	0.783	–	0.997	0.929
FP ($\theta = 0$)	0.908	0.916	0.759	0.153
SP	0.684	0.958	0.181	0.777

Notes:
1. Where unstated, $\theta = 17.5/4$ and $\phi = 0.149$.
2. Statistics of $H = 500$ simulations of length $T = 100$.
3. The first 500 periods of time series were discarded.

6.1 Equilibrium Comparisons

We will focus on comparing alternative model settings and assessing the models' qualitative tax properties using the baseline calibration in Table 1, unless otherwise stated. This exercise is reported in Table 2. The four settings we consider here are:

- Model FP*: Flexible prices, $\theta = 0$, where government bonds are not liquid, $\phi = 0$.
- Model SP*: Sticky prices, $\theta = 17.5/4$, where government bonds are not liquid, $\phi = 0$.
- Model FP: Flexible prices, $\theta = 0$, and government bonds provide liquidity, $\phi = 0.149$.
- Model SP: Sticky prices, $\theta = 17.5/4$, and government bonds provide liquidity, $\phi = 0.149$.

We do not report the case where there are flexible prices and perfectly competitive markets since the results are well known in the literature, as summarized by Chari, Christiano, and Kehoe (1991). Furthermore, we wish to focus on the role of liquid bonds and its implications for optimal policy in more recent model economies with non-competitive markets.

Steady state tax properties: Consider the steady-state or unconditional mean properties reported in Table 2. When government bonds do not provide liquidity (FP* and SP*), $\phi = 0$, the instrument R_t becomes redundant. This is obvious from the identical stochastic discount factor in (10) and (11), so that $R_t = R_t^*$ for almost all sample paths. More interestingly, when $\phi \neq 0$, the steady-state or mean spread $R^* - R$ is higher under sticky prices (SP) than under flexible prices (FP). Also, gross inflation is higher under SP than FP. Labor income tax rate is almost identical across FP and SP.

An intuition for these steady state results is that under SP, the planner can use inflation as a tax on monopolistically competitive profits. As shown by Schmitt-Grohé and Uribe (2004a) the most costly it is to change prices, the more, in their case, is the inflation rate closer zero, so that the nominal interest rate deviates more from the Friedman rule of zero nominal interest.[8] In our case, we have normalized the steady-state inflation rate to 4.2 per cent per annum to be consistent with post-war US data. So the analogous result we have is that in the SP economy the planner pushes the inflation rate to its "zero" at 4.2 per cent, compared to the more deflated level of 1.7 per cent in the FP economy. There is also a larger tax on liquidity as measured by both R and R^* when moving from FP to SP. This is our equivalent of the increased deviations from the Friedman rule when moving from FP to SP.

We show this further in Figure 1(a) where we plot the results of the asymptotic unconditional mean of the key variables over different values for the degree of price stickiness, θ. (We leave a similar analysis on business-cycle properties to Section 6.2 later.) The difference here is we focus only on sticky price economies. It can be seen that as θ increases, the interest-rate spread rises. Inflation tax increases toward its steady state, while government bond holdings and income tax rate τ falls with θ. As prices become more sticky, the planner is more concerned about the resource cost of inflation deviation. The monetary policy aspect of the planner's policy involves further deviation from the Friedman rule by increasing R^*. However, the fiscal policy aspect of the optimal Ramsey plan involves lowering R relatively to R^* so as to create a larger spread in $R^* - R$ and thereby altering the level of liquid bond holdings. As suggested earlier in Section 5, this only distorts the distribution of liquidity holdings between money and government bonds, but by lowering the level of government bonds the planner can adjust its sequence of implementability constraints (24) by lowering average income tax and instead increasing the tax on liquidity services.

The intuition for this is that increasing deviations from the Friedman rule is called for in order to indirectly tax monopoly profits, but this results in a larger tax on money. To offset this effect on money holdings, the planner engineers a higher tax on liquidity holdings in terms of the government bond, by increasing the spread in $R^* - R$, which on the demand side of the cash-only goods market, is also used to purchase cash consumption goods.[9] Also the planner must deliver a lower tax on labor so that there is more production of the consumption good. Thus, the optimal plan causes the quantity of liquid government bonds to fall and money holdings to rise (Figure 1(b)). With increasing spread in $R^* - R$, households shift from holding government bonds to holding more money for purchasing within-period cash consumption goods.

[8] A peculiarity of the Schmitt-Grohé and Uribe (2004a) model is that their cost of inflation which enters their Phillips curve assumes a zero inflation rate steady state, whereas in their quantitative exercise, steady state gross inflation is calibrated to 1.042 or a steady-state inflation rate of 4.2 per cent. This is what we also use, but to be consistent in the model we have normalized our inflation cost function (4) such that $\overline{\Pi} = 1.042$.

[9] Recall we do not further distinguish between cash and credit goods, following Canzoneri and Diba (2005).

Business-cycle tax properties: Table 2 also reports the volatility (second panel) and persistence (bottom panel) properties of the four comparative economies. It can be seen that the Ramsey optimal policy involves relatively less volatile inflation when there is a sticky-price cost to inflation (SP and SP*) compared to when inflation volatility is costless (FP and FP*). In the flexible price economy (FP) without the interest-rate spread, the planner uses ex post inflation volatility to induce greater variability of ex post real return on government debt so that government debt act as a shock absorber in order to maintain smoother taxes across-states (volatility) and dates (persistence). This becomes less vital when there exists bond liquidity (FP*) so that labor tax rate and inflation volatilities are lower relative to FP, since now an optimal Ramsey plan can induce state-contingent real debt ex post via the tax on bond liquidity, as measure by R^* relative to R. This channel becomes more important where there is a sticky-price constraint exerting cost on the planner to use inflation to make ex post real debt state contingent. Thus one can observe in both SP* and SP, there is even less volatility induced in labor income tax or inflation as the planner uses the interest-rate spread instead to satisfy the implementability constraints in the optimal policies.

A similar intertemporal smoothing idea can be seen in the autocorrelation coefficients across the four economies. Tax on labor generally stays highly persistent, whereas inflation becomes less persistent, but the interest spread becomes more persistent.

6.2 *Business-cycle properties under price stickiness*

In this second part of the numerical exercise, we consider in more detail how the existence of liquid government bonds alter the Ramsey plan in sticky-price environments, in terms of across-state and across-date allocations. These two features are summarized by volatility (in percentage standard deviations) and persistence (first-order autocorrelations) statistics of the key tax instrument variables.[10] We fix the parameter that determines bond liquidity at the baseline value of $\phi = 0.149$, and then consider a subset of increasingly sticky-price economies, as measured by θ.

Figure 2(a) plots the Monte Carlo simulated probability densities of standard deviation of the tax instruments and Figure 2(b) plots the averages of the same statistics with their respective 90% confidence intervals, as a function of the degree of price stickiness, θ.[11] In the face of shocks to government spending and technology, optimal policy is geared towards greater price stability. It can be seen that as θ rises from a near flexible-price economy ($\theta \approx 0$) to a very sticky-price one ($\theta = 8$), the volatility of inflation, Π decreases. However, we also see a rise in the volatility of R relative to R^* (and therefore in the volatility of liquid bond, b). It can also be seen that labor income tax, τ, becomes less volatile as θ increases.

In order to achieve lower inflation volatility since inflation is more costly as price stickiness rises, the planner creates more volatility in the ex post real return on government debt and the government debt itself. The greater volatility in the return on government debt and the debt itself means that the

[10] Additional results on other variables in the system are available on request.

[11] Each density function or each point on the graphs in the lower panel represents an averaged statistic, for an economy indexed by θ, for Monte Carlo simulations of length $T = 100$ repeated for $H = 1000$ history paths. The sample histories are kept the same across all θ's.

planner can use debt as a shock absorber whilst minimizing the shock absorbing role of inflation or labor income tax when financing government spending. This result affirms the intuition discussed in Section 6.1.2. – labor income tax becomes less volatile as θ increases – in contrast with that of Siu (2004) and Schmitt-Grohé and Uribe (2004a) in the relevant parameter domain of θ. Specifically, Siu (2004) showed that as price-stickiness increases the volatility of labor income tax rate rises because the planner in that case forgoes minimizing labor tax volatility in favor of a lower inflation volatility. Our result is different because government bonds are held by households partly to provide liquidity. Thus, instead of distorting labor supply and hence output by increasing the volatility of labor tax rate, the planner in our model chooses to distort the distribution of liquidity between government bonds and money. Thus we see a greater volatility on R and b, while a lower volatility on τ as θ rises.

Part of the optimal tax program involves intertemporal smoothing of taxes and therefore allocations. Figure 3(a) plots the Monte Carlo simulated probability densities of the first-order autocorrelation of the tax instruments and Figure 3(b) plots the averages of the same statistics with their respective 90% confidence intervals, as a function of the degree of price stickiness, θ. We can see that as θ moves from a flexible price economy to one which has a lot stickiness ($\theta = 8$), labor income tax rate, τ, becomes more persistent and the monetary policy aspect of the Ramsey policy, R^* and the return on liquid government debt, R are both quite persistent. The converse is true for inflation. In order to minimize the costly effect of inflation when price stickiness increases, the optimal program makes inflation less and less autocorrelated so that, in combination with less volatile inflation, the cost of inflation is smaller.

Finally, Figure 4(a) plots the Monte Carlo simulated probability densities of the contemporaneous correlation of the tax instruments with output, and Figure 4(b) plots the averages of the same statistics with their respective 90% confidence intervals, as a function of the degree of price stickiness, θ. A negative correlation between R and y suggests that in good times the planner would like to partially reduce its debt burden by lowering the return on government debt. This is equivalent to increasing the tax rate on bond liquidity. Similarly, in good times, when y is high, the planner would like to tax labor, τ, at a higher rate. Both these outcomes are consistent with a planner that aims to smooth out tax distortions over time and across states.

In summary, we find that the more sticky prices become, the optimal Ramsey plan favors more price stability but the planner can also afford a less distortionary income tax. That is as price stickiness increases, the less volatile and persistent is inflation and the less volatile is labor income tax, but the more volatile and persistent is the interest rate on liquid bonds and the quantity of government bonds. Also, the relative interest-rate spread volatility is increasing with the degree of price stickiness, reflecting the increasing use of the tax on bond liquidity across states.

6.3 Robustness and the effect of bond liquidity

In this third exercise, we investigate the effect of the government bond liquidity on the optimal policy plan for feasible values of ϕ. This exercise allows us to see how ϕ affects the optimal policy plan when both technology and government-spending shocks are present, and also serves as a check on the sensitivity of our previous result in Section 6.2. We repeat the exercise of analyzing the optimal policy under different price-stickiness environments, across different values of ϕ. Here we will focus on the unconditional means and standard deviations of the tax instruments.

Figure 5 plots the volatility of the key variables as functions of a set of economies indexed by (ϕ, θ), where each economy (as a point on the surfaces) is made to share the same set of histories of stochastic technology and government spending shocks. Thus we can consider the effect on the optimal volatility of our variables of interest as we vary the degree of price stickiness θ, for different cases of ϕ.

We obtained the following results. First, with more price stickiness and given a particular degree of bond-money substitutability, ϕ, there is a rise in the volatility of government bond return, R, relative to the market-bond return, R^*, but a fall in the volatility of inflation and labor tax. In other words, the government can use debt as a shock absorber in order to lower two kinds of social costs – inflation cost which increases with price stickiness and labor distortion cost which increases with the volatility of income tax. This again affirms the result from Section 6.2 for various computationally feasible values of ϕ.

Second, for each given price stickiness level, θ, the greater is ϕ the more the planner can afford to reduce the uncertainty of inflation and labor tax rates while increasing the volatility of the interest spread between market and bond returns. Intuitively, in an economy with greater liquidity effect of government bonds (higher ϕ), the "cost" of using bond tax is lower relative to the cost of using inflation tax and labor tax. This is because for equal opportunity cost of holding liquid bonds $R^* - R$, a higher money-bond substitutability results in a larger demand for government bonds which means a larger tax base in terms of bond tax, since $k'(b; \phi) > k'\left(b; \widetilde{\phi}\right)$ for all $\widetilde{\phi} > \phi$. This argument is shown graphically in Figure 6. This effect is further enhanced by the planner allowing for a lower spread on average, $R^* - R$, as shown in Figure 7, as ϕ increases. Thus, with relatively greater holdings of liquid government bonds as ϕ rises, the planner allows for more volatility on the bond rate – a surprise interest-rate tax, given inflation tax is too costly – for a given degree of price stickiness.

6.4 Liquid bonds and individual shocks

In this last exercise, we break the analysis of the effect of ϕ down to individual shocks to technology and government spending. This is shown by impulse response analysis. This allows one to study the optimal Ramsey plan under sticky prices when liquid government bonds matter, and how it matters in the faces of a supply-side or a demand-side shock.

In Figure 8 we consider a one-standard-deviation $(\sigma_g = 0.023)$ positive shock to government spending. In Figure 9 we consider a one-standard-deviation $(\sigma_z = 0.03)$ shock to technology. We keep the parameterization of the model as in the baseline case in Table 1 but vary ϕ. For example, under the positive government spending shock, the optimal policy plan generates a persistent decline in the interest spread, $R^* - R$, in order to encourage more government bond holdings. Labor taxes are also raised but kept on a persistently positive deviation path, while consumption and real money holdings fall. With higher bond liquidity effect, ϕ, the path of inflation and labor tax are kept remarkably similar to the case with near zero bond liquidity, while the optimal plan allows the interest spread to adjust by larger amplitudes and thus using government bond holdings more as the shock absorber. A similar effect can be seen in Figure 9 for the case of the technology shock.

As these impulse responses show, the effect of government bond liquidity, ϕ, serves to provide an optimal surprise interest-rate tax avenue, while the optimal responses of inflation and labor income tax are remarkably stable or unchanged across degrees of money-bond substitutability, ϕ.

7. Conclusion

We constructed a model where government bonds provide liquidity service, an idea that goes back to the work of Tobin (1965) and Patinkin (1965) and supported by the observation that US Treasury bills have a role in facilitating transactions.

We showed in the paper that when a government bond plays a dual role of providing liquidity as well as a traditional function as a financial asset, it alters the Ramsey optimal fiscal and monetary policy equilibrium allocations. We found that in environments of increasing price stickiness inflation becomes less volatile and less persistent and labor income tax is less volatile. However, both the quantity of government debt and its return to the debt holder become more volatile and more persistent. Further, the labor income tax rate remains very persistent, reflecting a tax-smoothing outcome. Also, the interest-rate spread is increasing with the degree of price stickiness, reflecting the increasing tax on bond liquidity. Thus, with increasing price-stickiness the Ramsey optimal monetary policy is to stabilize inflation, foregoing the shock-absorbing role of inflation in creating an ex post state-contingent government debt. The corresponding optimal fiscal policy is to minimize labor income tax distortions, over time (tax smoothing) and across states (lower volatility). In return for the gain in low inflation volatility and low intertemporal income tax distortions, the optimal policy uses liquid government bonds as a means of shock absorption. We show that this result is robust across feasible parameterizations of bond liquidity and also in the face of government spending shocks and technology shocks.

APPENDIX A
PROOF OF PROPOSITION 1

First show that the plans $\{c_t, h_t, \Pi_t, mc_t, b_t, R_t^*\}_{t=0}^{\infty}$ satisfying Definition 1 also satisfy (18), (22a)-(24). Use (7) to eliminate m_t, (26) to eliminate R_t, and (13)-(14) to eliminate τ_t, from the real government budget constraint (20). This yields (22a)-(22b) for $t \geq 0$. Using (11), (10) and (12) we can construct $\lambda_t = U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)$ for all t and all states, and use this to eliminate λ_t and λ_{t+1} from (17) to yield (23). To show that the decentralized equilibrium satisfies the time-t implementability constraint, for $t, s \geq 0$, (19) can be written as

$$M_{t+s} + B_{t+s} + P_{t+s}\tau_{t+s} mc_{t+s} z_{t+s} h_{t+s} = R_{t+s-1}B_{t+s-1} + M_{t+s-1} + P_{t+s}g_{t+s}. \tag{27}$$

Let $D_{t+s} := \prod_{i=0}^{s} R_{t+i-1}^{-1}$ and $W_{t+s} := R_{t+s-1}B_{t+s-1} + M_{t+s-1}$.

Thus we can write $B_{t+s} = (W_{t+s+1} - M_{t+s}) R_{t+s}^{-1}$. Substituting these definitions into (27), and multiplying (27) with D_{t+s} we obtain

$$D_{t+s}M_{t+s}\left(1 - R_{t+s}^{-1}\right) + D_{t+s}R_{t+s}^{-1}W_{t+s+1} - D_{t+s}W_{t+s} = D_{t+s}\left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right).$$

Summing this from $s = 0$ to $S > 0$, and taking expectations conditional on information at time t:

$$\mathbb{E}_t \sum_{t=0}^{S} \left[D_{t+s}M_{t+s}\left(1 - R_{t+s}^{-1}\right) - D_{t+s}\left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right)\right]$$
$$= \mathbb{E}_t D_{t+S+1}W_{t+S+1} + D_t W_t.$$

Let $S \to \infty$ and invoking (21), we have $\lim_{S\to\infty} \mathbb{E}_t D_{t+S+1}W_{t+S+1} = 0$ and thus,

$$\mathbb{E}_t \sum_{t=0}^{\infty} \left(\prod_{i=1}^{s} R_{t+i-1}^{-1}\right) \left[M_{t+s}\left(1 - R_{t+s}^{-1}\right) - \left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right)\right] = W_t. \tag{28}$$

Making use of (10) to find $R_t^* R_{t+1}^* \cdots R_{t+s-1}^*$, we can derive

$$\mathbb{E}_t \left[\beta^s \left(\frac{\lambda_{t+s}P_t}{\lambda_t P_{t+s}}\right) \prod_{i=1}^{s} R_{t+i-1}^*\right] = 1.$$

Multiply both sides of (28) with this to obtain

$$\mathbb{E}_t \sum_{t=0}^{\infty} \left(\prod_{i=1}^{s} R_{t+i-1}^{-1} R_{t+i-1}^*\right) \frac{\beta^s \lambda_{t+s}}{P_{t+s}} \left[M_{t+s}\left(1 - R_{t+s}^{-1}\right)\right.$$
$$\left. - \left(P_{t+s}g_{t+s} - P_{t+s}\tau_{t+s}mc_{t+s}z_{t+s}h_{t+s}\right)\right] = \frac{\lambda_t W_t}{P_t}.$$

and using (26), (7), (13)-(14) and $\lambda_t = U_c(c_t, h_t) / \left(2 - R_t^{*-1}\right)$, to eliminate R_{t+s}, λ_t, λ_{t+s}, M_{t+s}/P_{t+s}, and using (18) to eliminate g_{t+s} we can obtain (24).

Going backwards. Now show that $\{c_t, h_t, \Pi_t, mc_t, b_t, R_t^*\}_{t=0}^{\infty}$ satisfying (18), (22a)-(24) can implement the decentralized equilibrium in Definition 1. Suppose that the economy is determined by the Ramsey plan satisfying (18), (22a)-(24). The planner can construct λ_t that satisfies (11), (10), (12), and (13)-(14) and (7). From these and (22a) we can recover $\{\tau_t, m_t, g_t\}$ that satisfy (19). Given λ_t

and λ_{t+1} we can recover (17) from (23). Further $\{R_t\}$ can be recovered from (26) for given $\{b_t, R_t^*\}$. It remains to show that the decentralized equilibrium's transversality condition will not be violated. Since (19) can be recovered, re-write this at $t+s$ in time-t value as

$$\mathbb{E}_t \sum_{t=0}^{S} \left[\frac{D_{t+s} M_{t+s}}{P_t D_t} \left(1 - R_{t+s}^{-1}\right) - \frac{D_{t+s}}{P_t D_t} \left(P_{t+s} g_{t+s} - P_{t+s} \tau_{t+s} mc_{t+s} z_{t+s} h_{t+s}\right) \right] = \mathbb{E}_t \frac{D_{t+S+1}}{P_t D_t} W_{t+S+1} + \frac{W_t}{P_t}. \quad (29)$$

Since the time-t implementability constraint is satisfied in the Ramsey plan, the limit of the LHS of (29) necessarily exists when $S \to \infty$, and this limit is W_t/P_t such that the present value of the government budget surpluses equals exactly the initial condition on government liabilities. This implies $\lim_{S\to\infty} \mathbb{E}_t D_{t+S+1} W_{t+S+1} = 0$. And re-writing for the definition of D_{t+S+1} and W_{t+S+1}, we have

$$\lim_{s\to\infty} \mathbb{E}_t \left(\prod_{i=0}^{s} R_{t+i}^{-1} \right) \left(R_{t+s} B_{t+s} + M_{t+s}\right) = 0$$

which is (21). $\square$

APPENDIX B
THE RAMSEY DUAL PROBLEM FOR NUMERICAL COMPUTATIONS

The Lagrangian for the Ramsey problem is

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(c_t, h_t) + \lambda_t^c \left[U_c(c_t, h_t) - \lambda_t \left(2 - \frac{1}{R_t^*}\right) \right] + \lambda_t^b \left[\lambda_t - \beta R_t^* \mathbb{E}_t \frac{\lambda_{t+1}}{\Pi_{t+1}} \right] \\
& + \lambda_t^s \Bigg[c_t - k(b_t) + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)}{\lambda_t} \right) h_t - \frac{c_{t-1} - k'(b_{t-1})}{\Pi_t} \\
& - \frac{\left(R_{t-1}^* - \left(R_{t-1}^* - 1\right) k(b_{t-1})\right) b_{t-1}}{\Pi_t} - g_t \Bigg] \\
& + \lambda_t^r \left[z_t h_t - c_t - g_t - \frac{\theta}{2} \left(\Pi_t - \overline{\Pi}\right)^2 \right] + \lambda_t^p \Bigg[\beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right) \\
& + \frac{\eta}{\theta} z_t h_t \left(\frac{1+\eta}{\eta} - mc_t \right) - \left(\Pi_t - \overline{\Pi}\right) \Pi_t \Bigg] \Bigg\}
\end{aligned}$$

with the first-order conditions for $t \geq 1$,

$$U_c(c_t, h_t) + \lambda_t^c U_{cc}(c_t, h_t) + \lambda_t^s - \beta \mathbb{E}_t \frac{\lambda_{t+1}^s}{\Pi_{t+1}} - \lambda_t^r = 0$$

$$U_h(c_t, h_t) + \lambda_t^s \left(mc_t z_t + \frac{U_{hh}(c_t, h_t) h_t + U_h(c_t, h_t)}{\lambda_t} \right) + \lambda_t^r z_t + \lambda_t^p \frac{\eta}{\theta} z_t \left(\frac{1+\eta}{\eta} - mc_t \right) = 0$$

$$\begin{aligned}
- \lambda_t^c \left(2 - \frac{1}{R_t^*}\right) + \lambda_t^b - \lambda_{t-1}^b \frac{R_{t-1}^*}{\Pi_t} - \frac{\lambda_t^s}{\lambda_t^2} U_h(c_t, h_t) h_t & \\
- \lambda_t^p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t^2} \left(\Pi_{t+1} - \overline{\Pi}\right) \Pi_{t+1} \right) + \frac{\lambda_{t-1}^p}{\lambda_{t-1}} \left(\Pi_t - \overline{\Pi}\right) \Pi_t & = 0
\end{aligned}$$

$$-\frac{\lambda_t^c\lambda_t}{(R_t^*)^2}-\lambda_t^b\beta\mathbb{E}_t\frac{\lambda_{t+1}}{\Pi_{t+1}}+\beta\mathbb{E}_t\frac{\lambda_{t+1}^s}{\Pi_{t+1}}\left(1-k'(b_t)\right)b_t=0$$

$$\lambda_t^s\left[1-k'(b_t)\right]-\beta\mathbb{E}_t\mathbb{E}_t\frac{\lambda_{t+1}^s}{\Pi_{t+1}}\left\{\left[R_t^*-(R_t^*-1)\left(k'(b_t)+b_tk''(b_t)\right)\right]+k'(b_t)\right\}=0$$

$$\lambda_{t-1}^bR_{t-1}^*\frac{\lambda_t}{\Pi_t^2}+\frac{\lambda_t^s}{\Pi_t^2}\left[R_{t-1}^*-\left(R_{t-1}^*-1\right)k'(b_{t-1})\right]b_{t-1}$$
$$+\frac{\lambda_t^s}{\Pi_t^2}\left[c_{t-1}-k(b_{t-1})\right]-\theta\lambda_t^r\left(\Pi_t-\overline{\Pi}\right)+\left(\frac{\lambda_{t-1}^p\lambda_t}{\lambda_{t-1}}-\lambda_t^p\right)\left(2\Pi_t-\overline{\Pi}\right)=0$$

$$\lambda_t^s=\frac{\eta}{\theta}\lambda_t^p$$

$$U_c(c_t,h_t)=\lambda_t\left(2-\frac{1}{R_t^*}\right)$$

$$\lambda_t=\beta R_t^*\mathbb{E}_t\frac{\lambda_{t+1}}{\Pi_{t+1}}$$

$$c_t-k(b_t)+b_t+\left(mc_tz_t+\frac{U_h(c_t,h_t)}{\lambda_t}\right)h_t=\frac{c_{t-1}-k(b_{t-1})}{\Pi_t}$$
$$+\frac{\left(R_{t-1}^*-\left(R_{t-1}^*-1\right)k'(b_{t-1})\right)b_{t-1}}{\Pi_t}+g_t$$

$$z_th_t=c_t+g_t+\frac{\theta}{2}\left(\Pi_t-\overline{\Pi}\right)^2$$

$$\left(\Pi_t-\overline{\Pi}\right)\Pi_t=\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\left(\Pi_{t+1}-\overline{\Pi}\right)\Pi_{t+1}\right)+\frac{\eta}{\theta}z_th_t\left(\frac{1+\eta}{\eta}-mc_t\right).$$

and the first-order conditions for $t=0$,

$$U_c(c_0,h_0)+\lambda_0^cU_{cc}(c_0,h_0)+\lambda_0^s-\beta\mathbb{E}_0\frac{\lambda_1^s}{\Pi_1}-\lambda_0^r=0$$

$$U_h(c_0,h_0)+\lambda_0^s\left(mc_0z_0+\frac{U_{hh}(c_0,h_0)h_0+U_h(c_0,h_0)}{\lambda_0}\right)$$
$$+\lambda_0^rz_0+\lambda_0^p\frac{\eta}{\theta}z_0\left(\frac{1+\eta}{\eta}-mc_0\right)=0$$

$$-\lambda_0^c\left(2-\frac{1}{R_0^*}\right)+\lambda_0^b-\frac{\lambda_0^s}{\lambda_0^2}U_h(c_0,h_0)h_0-\lambda_0^p\beta\mathbb{E}_0\left(\frac{\lambda_1}{\lambda_0^2}\left(\Pi_1-\overline{\Pi}\right)\Pi_1\right)=0$$

$$-\frac{\lambda_0^c\lambda_0}{(R_0^*)^2}-\lambda_0^b\beta\mathbb{E}_0\frac{\lambda_1}{\Pi_1}+\beta\mathbb{E}_0\frac{\lambda_1^s}{\Pi_1}\left(1-k'(b_0)\right)b_0=0$$

$$\lambda_0^s\left[1-k'(b_0)\right]-\beta\mathbb{E}_0\mathbb{E}_0\frac{\lambda_1^s}{\Pi_1}\left\{\left[R_0^*-(R_0^*-1)\left(k'(b_0)+b_0k''(b_0)\right)\right]+k'(b_0)\right\}=0$$

$$\frac{\lambda_0^s}{\Pi_0^2}\left[R_{-1}^*-\left(R_{-1}^*-1\right)k'(b_{-1})\right]b_{-1}$$

$$+\frac{\lambda_0^s}{\Pi_0^2}\left[c_{-1}-k\left(b_{-1}\right)\right]-\theta\lambda_0^r\left(\Pi_0-\overline{\Pi}\right)-\lambda_0^p\left(2\Pi_0-\overline{\Pi}\right)=0$$

$$\lambda_0^s=\frac{\eta}{\theta}\lambda_0^p$$

$$U_c\left(c_0,h_0\right)=\lambda_0\left(2-\frac{1}{R_0^*}\right)$$

$$\lambda_0=\beta R_0^*\mathbb{E}_0\frac{\lambda_1}{\Pi_1}$$

$$c_0-k\left(b_0\right)+b_0+\left(mc_0z_0+\frac{U_h\left(c_0,h_0\right)}{\lambda_0}\right)h_0=\frac{c_{-1}-k\left(b_{-1}\right)}{\Pi_0}+\frac{\left(R_{-1}^*-\left(R_{-1}^*-1\right)k'\left(b_{-1}\right)\right)b_{-1}}{\Pi_0}+g_0$$

$$z_0h_0=c_0+g_0+\frac{\theta}{2}\left(\Pi_0-\overline{\Pi}\right)^2$$

$$\left(\Pi_0-\overline{\Pi}\right)\Pi_0=\beta\mathbb{E}_0\left(\frac{\lambda_1}{\lambda_0}\left(\Pi_1-\overline{\Pi}\right)\Pi_1\right)+\frac{\eta}{\theta}z_0h_0\left(\frac{1+\eta}{\eta}-mc_0\right).$$

where $\lambda_{-1}^c=\lambda_{-1}^b=\lambda_{-1}^s=\lambda_{-1}^r=\lambda_{-1}^p=0$.

Appendix C
Calibrating ϕ and δ

From the Ramsey planner's version of the government budget constraint, we have at steady state

$$\left[\overline{c}-k\left(\overline{b}\right)\right]\left(1-\overline{\Pi}^{-1}\right)+\overline{b}\left(1-\frac{\left(\overline{R}^*-\left(\overline{R}^*-1\right)k'\left(\overline{b}\right)\right)}{\overline{\Pi}}\right)+\left(\overline{mcz}+\frac{U_h\left(\overline{c},\overline{h}\right)}{\overline{\lambda}}\right)\overline{h}-s_g\overline{z}\overline{h}=0 \tag{30}$$

and given our assumption on functional forms, we have

$$U_h\left(\overline{c},\overline{h}\right)=-\delta/\left(1-\overline{h}\right),\ k\left(\overline{b}\right)=\phi\left(1-e^{-\frac{\overline{b}}{\overline{c}}}\right),\ k'\left(\overline{b}\right)=\frac{\phi}{\overline{c}}e^{-\frac{\overline{b}}{\overline{c}}}.$$

Given $\overline{h}$ and s_g, we can solve for $\overline{c}$ from the resource constraint (18) at steady state. And $\overline{\Pi},\overline{b},\overline{R}^*$ are known values, while $\overline{\lambda}$ can be solved from the first-order condition $U_c\left(\overline{c},\overline{h}\right)=1/\overline{c}=\overline{\lambda}\left(2-1/\overline{R}^*\right)$. Using the optimality condition (26) at steady state, we can calibrate ϕ from

$$k'\left(\overline{b}\right)=\frac{\phi}{\overline{c}}e^{-\frac{\overline{b}}{\overline{c}}}=\frac{\overline{R}^*-\overline{R}}{\overline{R}^*-1}$$

given an estimate of $\overline{R}$. Once all the required values are known, one can solve for δ from (30).

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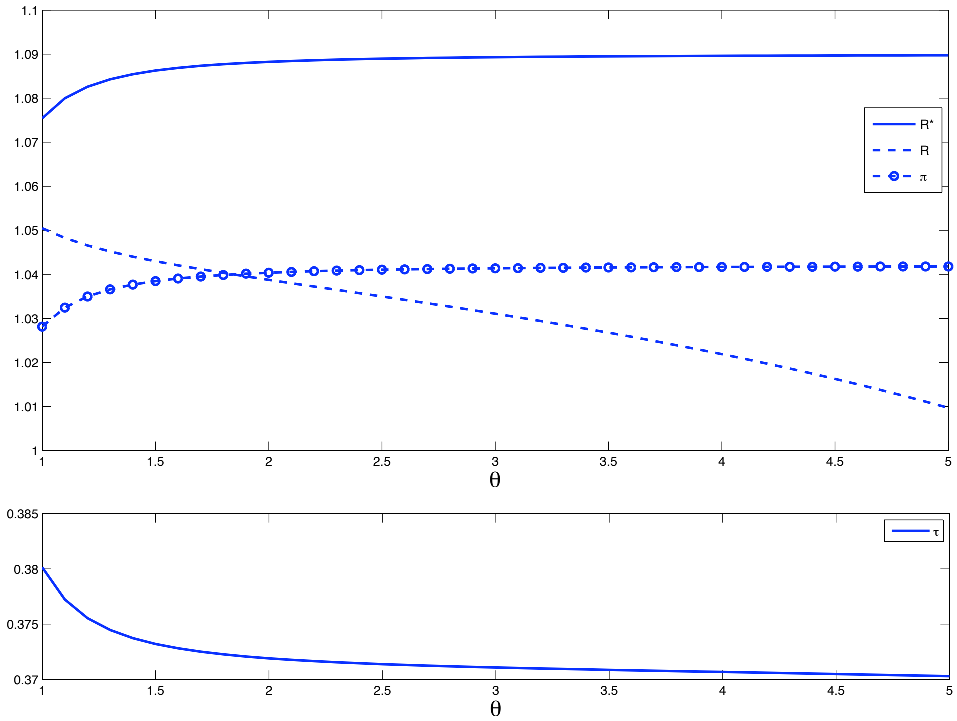
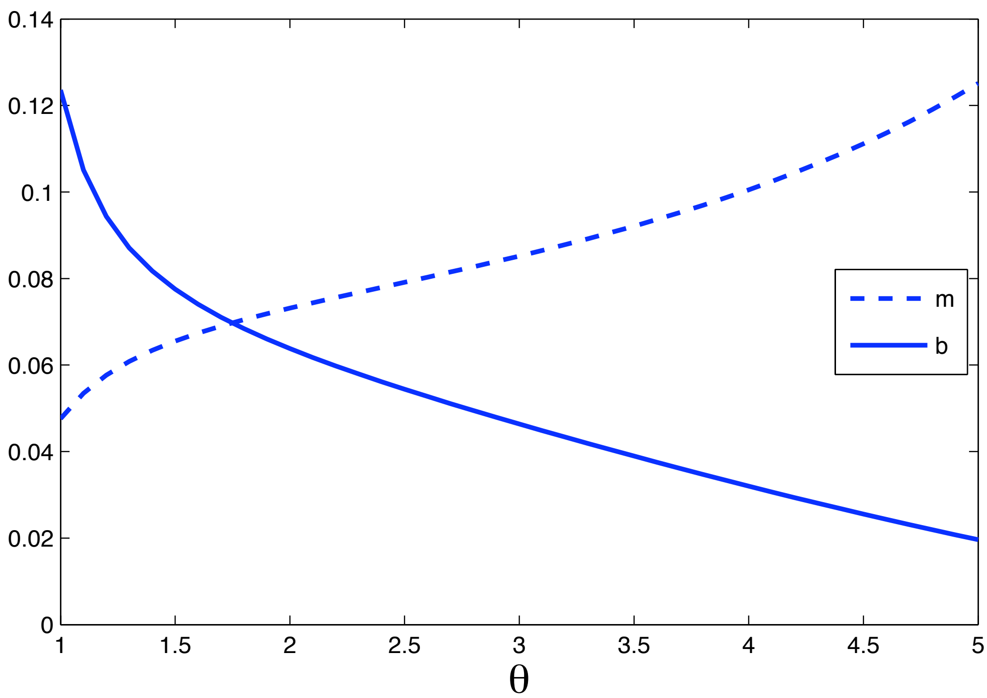
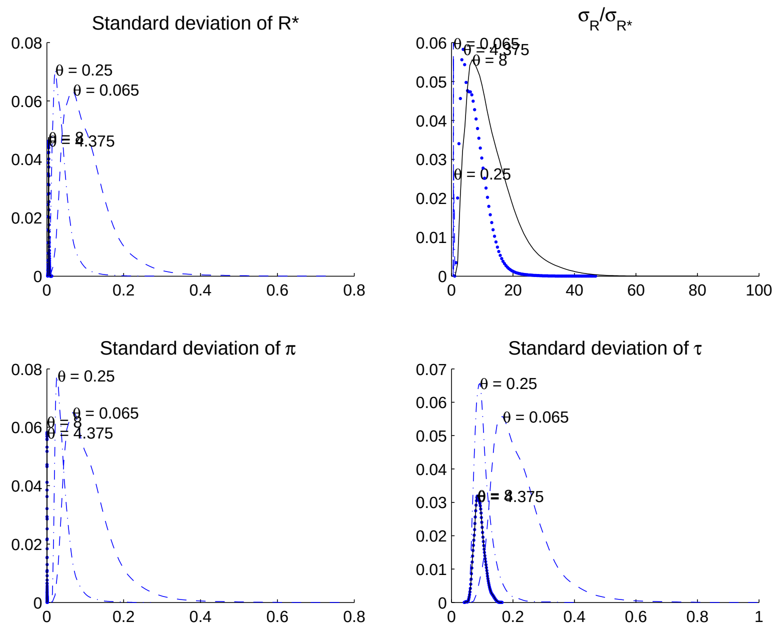


(a) Tax instruments.

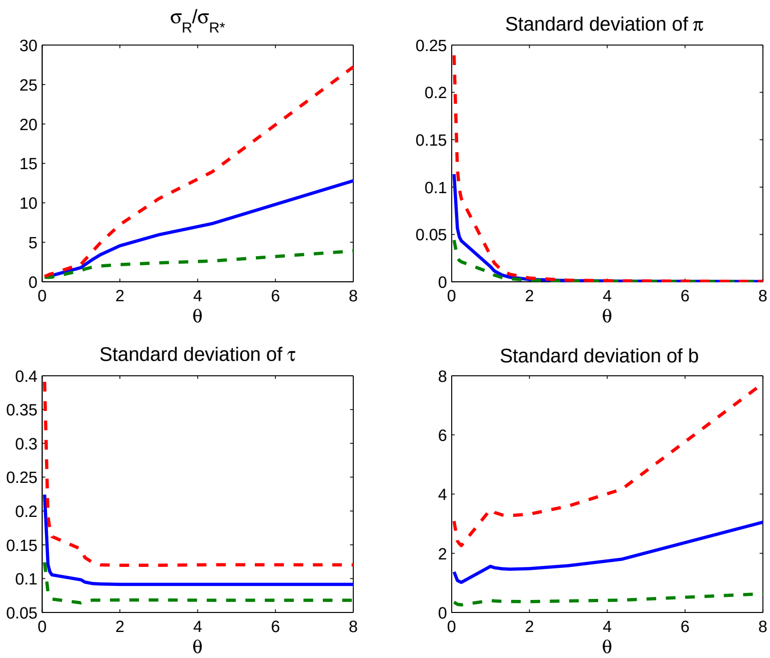


(b) Liquid bond and real money balance.

Fig. 1. Unconditional means under increasing price stickiness environments.

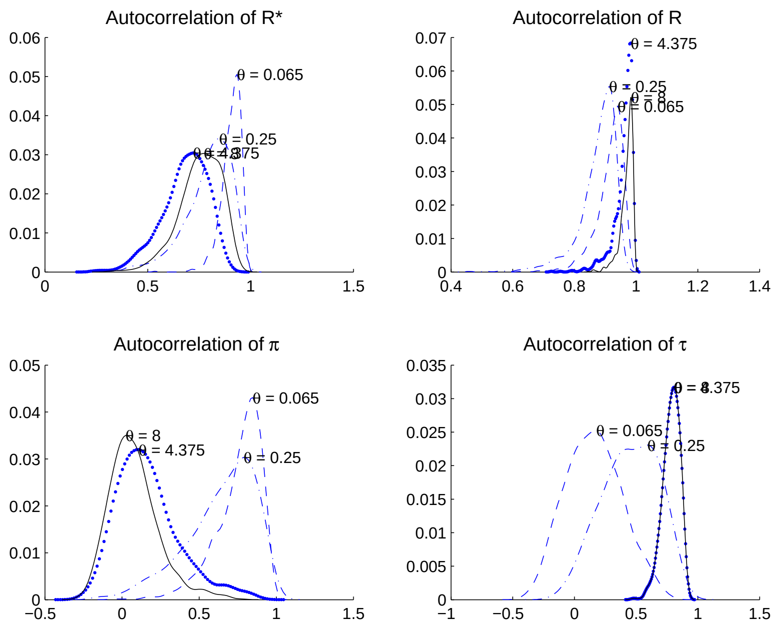


(a) Simulated standard deviation probability densities as functions of θ.

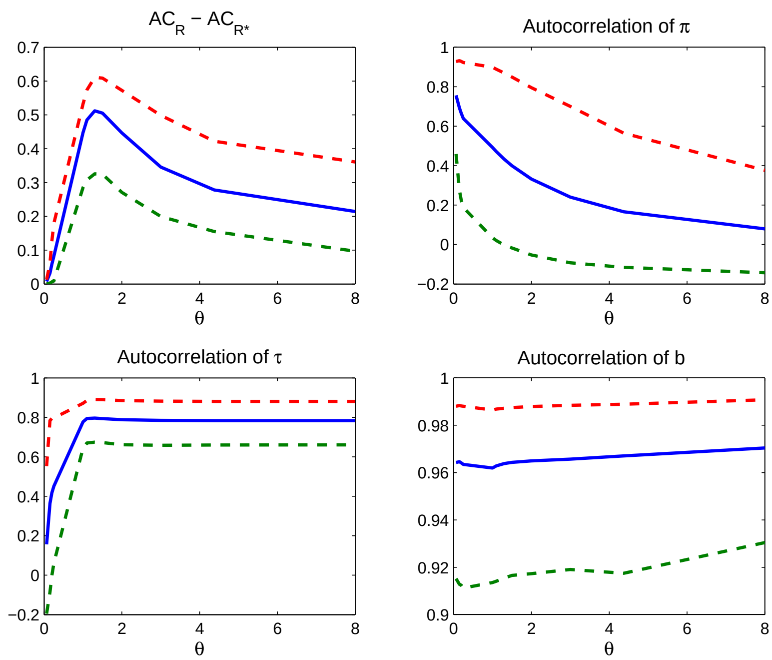


(b) Average for Monte Carlo simulation with 90% confidence intervals.

Fig. 2. Tax instrument volatilities in percentage standard deviation under increasing price stickiness environments.

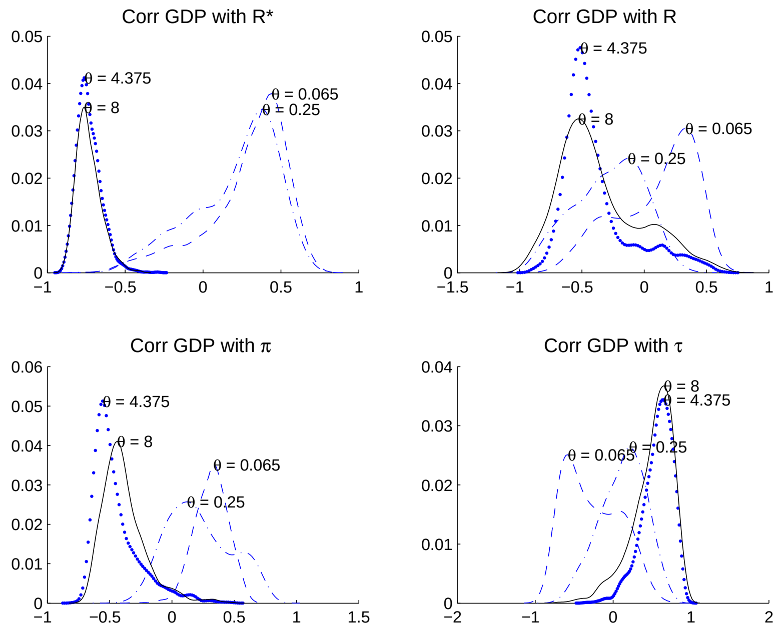


(a) Simulated autocorrelation probability densities as functions of θ.

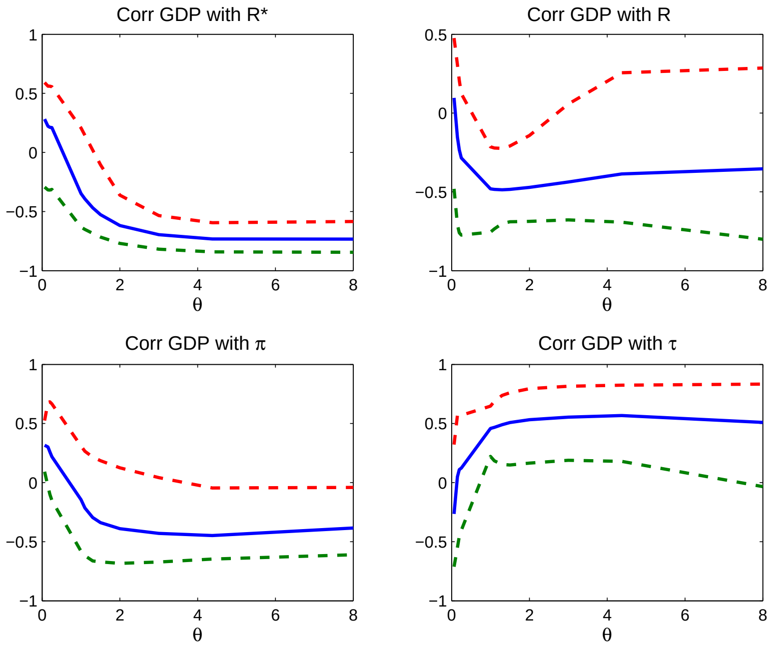


(b) Average for Monte Carlo simulation with 90% confidence intervals.

Fig. 3. Tax instrument autocorrelations under increasing price stickiness environments.



(a) Simulated cross-correlation probability densities as functions of θ.



(b) Average for Monte Carlo simulation with 90% confidence intervals.

Fig. 4. Tax instrument contemporaneous correlation with GDP.

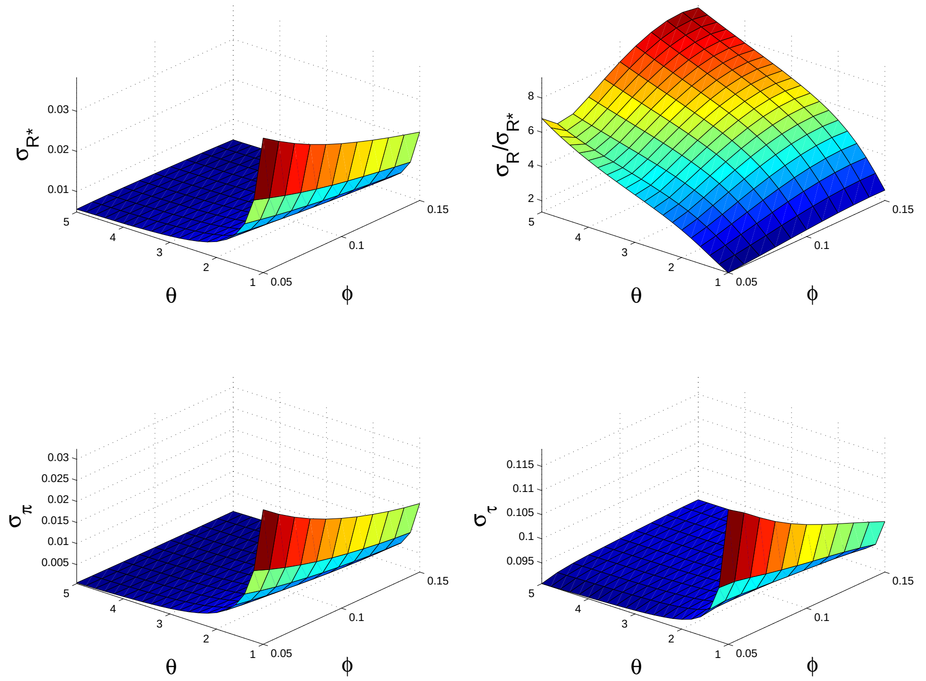


Fig. 5. Tax-instrument volatilities as functions of economies indexed by (ϕ, θ). Each point on the surfaces are generated by the same set of histories of exogenous shocks. Averages of statistics for Monte Carlo simulation $T = 100, H = 1000$.

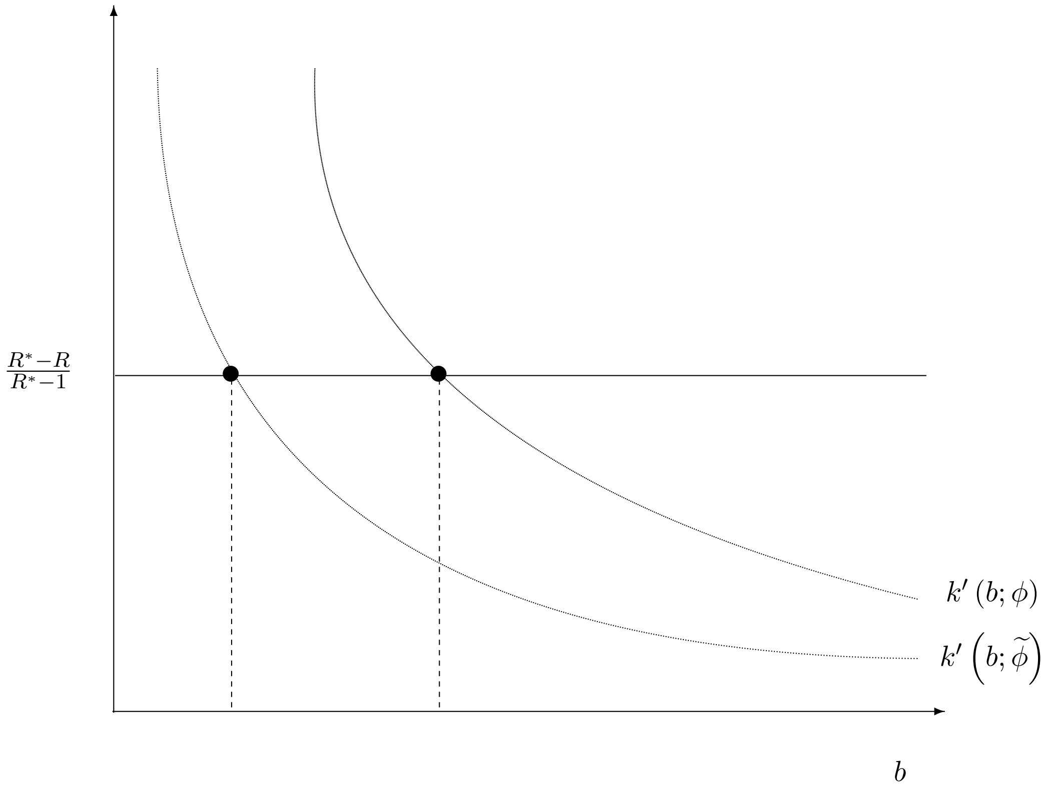


Fig. 6. Example with $\phi > \widetilde{\phi}$. For equal opportunity cost of holding government bonds, a higher ϕ, results in higher bond holdings.

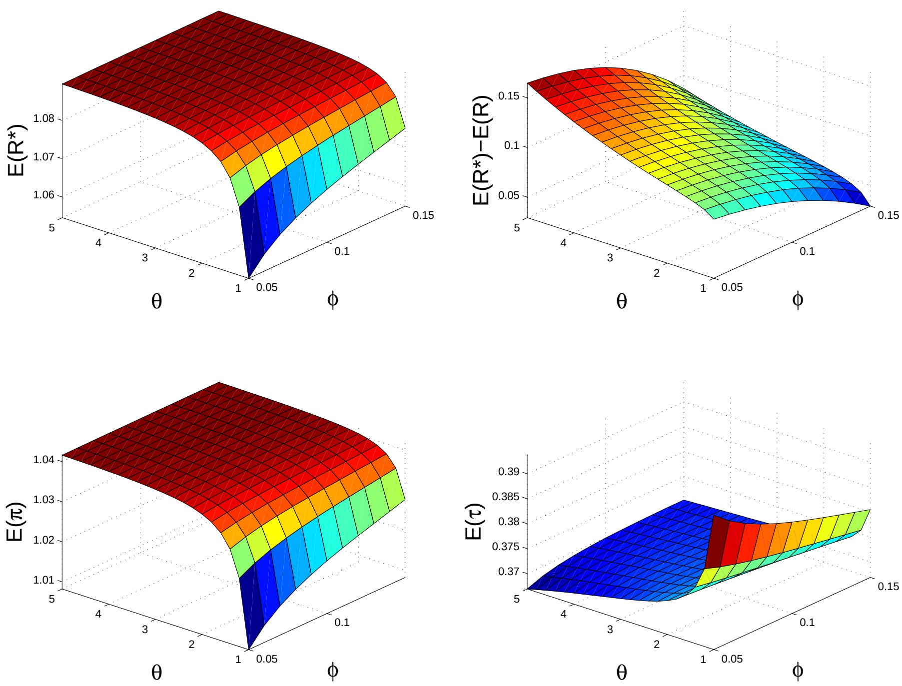


Fig. 7. Tax-instrument unconditional means as functions of economies indexed by (ϕ, θ). Each point on the surfaces are generated by the same set of histories of exogenous shocks. Averages of statistics for Monte Carlo simulation $T = 100, H = 1000$.

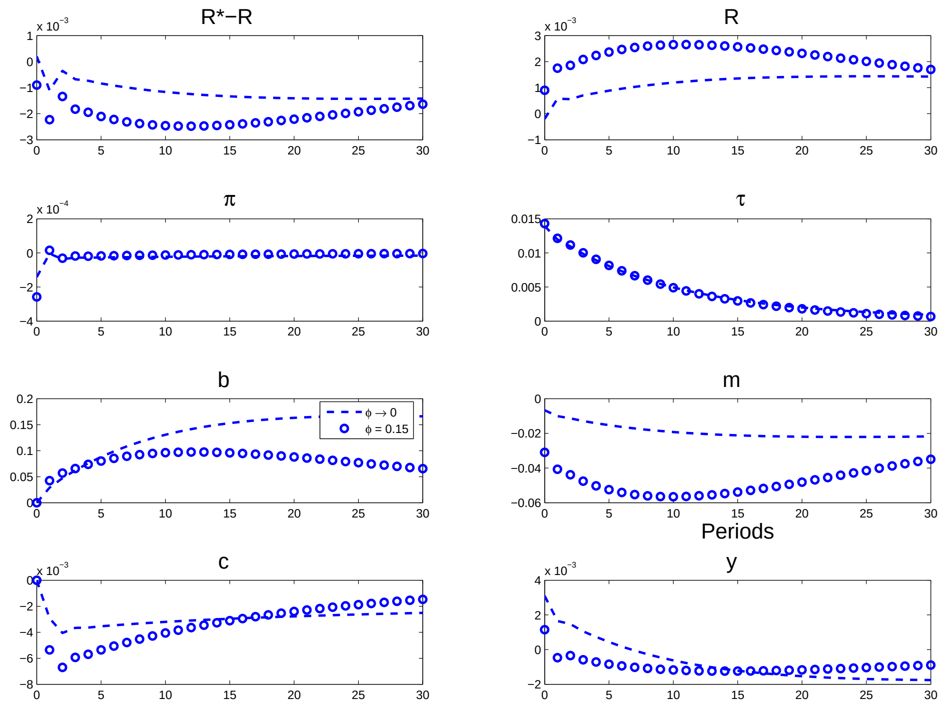


Fig. 8. Impulse response functions to one standard deviation ($\sigma_g = 0.03$) i.i.d. government spending shock in two economies with different government bond liquidity, ϕ.

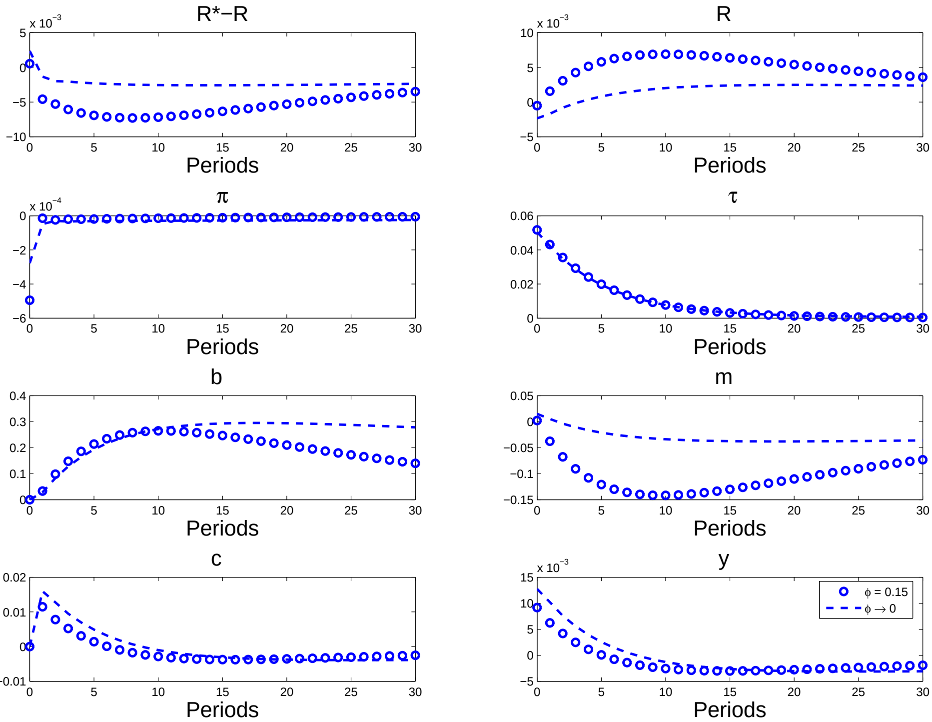


Fig. 9. Impulse response functions to one standard deviation ($\sigma_z = 0.023$) i.i.d. technology shock in two economies with different government bond liquidity, ϕ.