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WORKING PAPERS IN ECONOMICS AND ECONOMETRICS

Voting Power in the Australian Senate: 1901-2004

Alex Robson
School of Economics
College of Business and Economics
The Australian National University

Working Paper No. 480

ISBN 086831 480 3

June, 2007

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June 1, 2007

Abstract

Indices of voting power are intended to measure the *a priori* degree of influence that a voter or party can expect to have in framing legislation or passing motions. Commonly used measures include those proposed by Shapley and Shubik (1954), Banzhaf (1965) and Deegan and Packel (1978). This paper computes these power indices for the Australian Senate for the period 1901-2004. The introduction of the Single Transferable Vote in the Senate in 1949 appears to have had a profound effect on the voting power of both major parties, as well as on the degree of concentration of voting power.

1 Introduction

The Australian Department of the Senate claims that "proportional representation is a somewhat complicated electoral system, which ensures that political parties gain representation in proportion to their share of the vote. The result has been that the membership of the Senate is now a truer reflection of the voters' support of the different political parties."[1]

Whether this is indeed the case is open to question, for the notion of what exactly is meant by a "true reflection of voters' support" has been under sustained attack ever since the work of Arrow (1963) and Gibbard (1973) and Sattherthwaite (1974). Even if we accept that proportional representation reflects voters' true preferences in the number of seats, raw numbers of seats do not always translate into political power.

The concept of *voting power* and indices of voting power can be used to address these issues. This note computes power indices for parties in the Australian Senate for the period

*School of Economics, College of Business and Economics, Australian National University, Canberra, ACT, 2600, Australia. Email: alex.robson@anu.edu.au Ph +61 2 6125 4909. This paper was prepared for the 2007 Australian Law and Economics Conference, 1-2 June, Rydges Lakeside Hotel, Canberra.

[1] See Senate Brief No 1 (September 2006) "Electing Australia's Senators" at http://www.aph.gov.au/Senate/pubs/briefs/brief01.htm

1901-2004, and compares indices of voting power to actual fractions of seats held. We also develop and compute a summary measure of concentration of voting power, which is closely related to existing measures of concentration found in the industrial organisation and public choice literature.

2 Measures of Voting Power

2.1 Background: Cooperative Games with Transferable Utility

Consider a set of N agents. A *cooperative game* in such a group consists of a feasible utility set for the grand coalition N, as well as a utility set for every non-empty subset of N, including the coalitions with only one agent. Therefore, there are $2^N - 1$ possible non-empty subsets, for which we need to assign a utility possibility set, which is just a list of utilities that each agent gets if various coalitions happen to form.

In a game with *transferable utility*, each such utility set is a feasible set of cooperative opportunities: if the agents in a given coalition can all agree on the utility set, they can enforce *any* utility distribution in this set. The course of action which must be taken to achieve this distribution is ignored. This means that there is only one number associated with each coalition (namely the sum of the utilities of individuals in that coalition), instead of a vector of utilities.

More formally, a *coalitional or cooperative game with transferable payoff or transferable utility* (a TU game) consists of a finite set N of *players* and a function $v : \mathcal{P}(N) \to \mathbb{R}$ (where $\mathcal{P}(N)$ is the set of all subsets of agents) that represents the *worth* $v(S)$ of the the *coalition* $S \subseteq N$. The function $v(S)$ is the primitive data of the model and is called the *characteristic function* of the game. An *allocation* or *imputation* is a collection of utilities $\{x_i : i \in N\}$. An allocation is *feasible* if $\sum_{i \in N} x_i = v(N)$. The set of all imputations for a game is denoted by $E(v)$.

2.1.1 Simple and Weighted Majority Games

The general model described above can be applied to legislatures with only a few minor modifications. If a particular motion or piece of legislation is before the legislature, the motion will either pass or not pass (fail). If motions must pass by a strict majority of legislators voting in favour, then (in the absence of abstentions) there are only two relevant kinds of coalitions that can form: those which can force a motion to pass (*winning coalitions*), and those which cannot (*losing coalitions*).

Without loss of generality, with transferable utility we can set $v(S) = 1$ if S is winning, and $v(S) = 0$ if S is not winning, so that:

$$v(S) = \begin{cases} 1 & \text{if } |S| > \frac{N}{2} \\ 0 & \text{if } |S| \leq \frac{N}{2} \end{cases} \tag{1}$$

The game in (1) is called a *simple majority game*. We can use this model of simple games for legislatures because every coalition that forms is either winning or losing, with nothing in between.

The Australian Senate is comprised of members of various political parties, and to the extent that there is tight party discipline, we can also model the Senate as a *weighted majority game*. This consists of a set N of agents, a collection of *weights* $\{w_i > 0 : i = 1, \ldots, N\}$ and a *quota* q. We represent a weighted majority game by:

$$[q; w_1, \ldots, w_N]$$

In this environment, a coalition S is winning if:

$$\sum_{i \in S} w_i \geq q$$

Setting $q = \frac{N}{2} + 1$ and $w_i = 1$ for all i gets us back to the situation in which each legislator is regarded as an individual voter. If party i wins w_i seats and there is tight party discipline (so that each party always votes on issues as a bloc), then we have a weighted majority game. The characteristic function of the weighted majority game is:

$$v(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} w_i \geq q \\ 0 & \text{if } \sum_{i \in S} w_i < q \end{cases}$$

2.2 Voting Power and Indices of Voting Power

The motivation behind the concept of voting power can be illustrated using the following simple example. In the very first Australian Senate following the election on 29 March 1901, the distribution of Senate seats among the parties was as follows:

Party	**Protectionist**	**Free Trade**	**ALP**
Seats	11	17	8
Percentage of Seats	0.30556	0.47222	0.22222
Winning Coalitions Containing Party i	$\{P, FT\}$ $\{P, ALP\}$ $\{P, FT, ALP\}$	$\{FT, P\}$ $\{FT, ALP\}$ $\{P, FT, ALP\}$	$\{ALP, FT\}$ $\{P, ALP\}$ $\{P, FT, ALP\}$

With 36 Senators in total, to pass a motion in the Senate a party would require a clear majority or 19 votes. So in the 1901 Senate, no party could pass legislation on its own –

the fact that the Free Trade Party had a plurality of seats was irrelevant as far the practical consideration of actually passing laws was concerned. Indeed, the Protectionists and the ALP could form a coalition and pass legislation without needing any Free Trade votes, which is what actually occurred in both the House of Representatives and the Senate.[2]

This situation illustrates the proposition that using raw numbers of seats to determine political power or influence is clearly inappropriate. Because all of the winning coalitions in the above game require at least two of the parties, none of them could be said to have more political influence that the other. Any reasonable measure of voting power should have each party having equal power or influence.

The concept of *voting power* simply formalises this reality that raw numbers of seats are not always a good indicator of how likely certain political parties or individuals will be pivotal in the sense that they can turn a losing coalition into a winning one. We now turn to three such measures that have been developed and used in the literature.

2.2.1 The Shapley-Shubik Power Index

Shapley and Shubik (1954) introduced an index for measuring an individual's voting power in a committee. They consider all $n!$ possible arrangements of voters. They view a voter's power as the *a priori* probability that he will be *pivotal* in some arrangement of voters. Pivotalness requires that:

1. If every voter before i in the arrangement votes in favour of the bill, and if every voter after i in the arrangement votes against the bill, then the bill would fail; and
2. If voter i and every voter before i in the arrangement votes in favour of the bill, and if every voter after i in the arrangement votes against the bill, then the bill would pass.

The *Shapley-Shubik power index* for voter i is simply the number of arrangements of voters in which voter i satisfies the two conditions above, divided by the total number of arrangements of voters. It therefore assigns a player the probability that the voter will cast the deciding vote if all arrangements of voters are equally likely. The expected frequency with which a party or voter is the pivot, over all possible alignments of the voters, is an indication of the voter's power. [3]

[2] The Protectionists held 31 seats in the House of Representatives, with the Free Traders holding 28 and the ALP 14. There were two independents. The Protectionists (led by Prime Minister Edmund Barton), passed the White Australia Policy, tariffs, and industrial arbitration laws all with the support of the ALP. Bolton (2000, at page 31) concludes that the promise of the White Australia Policy guaranteed ALP support for the Protectionists.

[3] Gul (1989) provides a non-cooperative foundation to the Shapley value.

Consider all possible orderings of the N voters, legislators or parties, and consider all the ways in which a winning coalition can be built up. There are $N!$ possible orderings of the players. For each one of these orderings, some unique player will join the coalition and turn it from a losing coalition into a winning coalition. In other words, there will be a unique pivotal voter for each possible permutation of voters. The number of times that player i is pivotal, divided by the total number of possible alignments, is player i's power. That is:

$$\varphi_i^{SS} = \frac{\text{Number of arrangements in which } i \text{ is pivotal}}{N!}$$

where it is assumed that each of the $N!$ alignments is equally probable. If S is a winning coalition and $S - \{i\}$ is losing, then i is pivotal. Let $s = |S|$. Given the size of S, the number of ways of arranging the previous $s-1$ voters is $(s-1)!$. Also, the number of ways in which the remaining $(N-s)$ voters can be arranged is $(N-s)!$. Therefore, given S, the total number of ways that voter i can be pivotal is simply:

$$(s-1)!\,(N-s)!$$

We therefore get:

$$\varphi_i^{SS} = \frac{\displaystyle\sum_{S:\, i \text{ is pivotal}} (s-1)!\,(N-s)!}{N!} \tag{2}$$

[see, for example, Owen (1995), page 265 or Felsenthal and Machover (1998) page 197].[4] To illustrate how to compute this index, let us go back to the 1901 election results and consider the weighted majority game:

$$\left[19;\ \underset{\text{(Prot)}}{11},\ \underset{\text{(Free Trade)}}{17},\ \underset{\text{(ALP)}}{8}\right]$$

The $3! = 6$ possible ways of arranging the parties are:

$$\begin{array}{ll} \{P, \underline{FT}, ALP\} & \{FT, \underline{ALP}, P\} \\ \{P, \underline{ALP}, FT\} & \{ALP, \underline{P}, FT\} \\ \{FT, \underline{P}, ALP\} & \{ALP, \underline{FT}, P\} \end{array}$$

where the pivotal player in each arrangement is underlined. Therefore it is easy to see that:

$$\varphi_i^{SS} = \frac{1}{3} \text{ for } i = P, FT, ALP$$

On the other hand, consider the 1910 election. In that election the results give the following weighted majority game:

$$\left[19;\ \underset{\text{(ALP)}}{22},\ \underset{\text{(Liberal)}}{14}\right]$$

Calculations are even more straightforward here. The Liberal Party is never pivotal here, and the ALP is always pivotal. Therefore:

$$\varphi_{ALP}^{SS} = 1 \text{ and } \varphi_{Liberal}^{SS} = 0$$

[4] φ^{SS} (and indeed all power indices that we compute) have the property that $\sum_i \varphi_i = 1$

2.2.2 The Banzhaf Power Index

Banzhaf's (1965) index is also concerned with the fraction of possibilities in which a voter is pivotal, but only considers the *combinations* of such possibilities, rather than *permutations*. In other words, Banzaf's approach does not worry about the order in which voters are arranged. To distinguish between the Banzhaf approach and the Shapley-Shubik approach, consider any winning coalition $S \cup \{i\}$. If i leaves this group and S turns into a losing coalition, then i is said to be *critical* (the ways in which the coalition S can be arranged are ignored). The *(normalised) Banzhaf power index* of voter i is the number of coalitions in which i is critical, divided by the total number of all such coalitions in which some voter is critical. Mathematically, we have:

$$\varphi_i^{Bz} = \frac{\theta_i}{\sum_{j=1}^{N} \theta_j} \tag{3}$$

where θ_i is the number of coalitions in which i is critical (also known as the number of "swings" for i). Returning to the 1901 election results, the coalitions in which some player i is critical are $\{P, FT\}$, $\{FT, ALP\}$ and $\{ALP, P\}$. Therefore, $\sum_{j=1}^{N} \theta_j = 6$ and:

$$\varphi_i^{Bz} = \frac{1}{3} \text{ for } i = P, FT, ALP$$

which is the same as φ_i^{SS} (this need not always be the case). On the other hand, consider again the 1910 election. We get:

$$\varphi_{ALP}^{Bz} = 1 \text{ and } \varphi_{Liberal}^{Bz} = 0$$

which, again, is the same as φ_i^{SS}.

2.2.3 The Deegan-Packel Index

Another index of voting power that has received some attention in the literature is that proposed by Deegan and Packel (1978). Their measure is based on the notion of *minimal winning coalitions* (MWCs), which are coalitions that become losing if *any* single voter is removed.[5] Following Riker's (1962) *size principle*, they argue that coalitions exceeding the minimal winning ones will not form (why bother recruiting more supporters if your coalition is already winning?) They also assume that each MWC is equally likely, and that members of MWCs split any gains equally.

[5] Note that this is not the same thing as i being critical. Player i can be critical and turn a winning coalition into a losing one, but that does not mean that other players in the coalition are also critical. In other words, a minimal winning coalition is one in which *all* members are critical.

These assumptions uniquely determine the following power index. Suppose that player i is a member of the minimal winning coalitions $\{S_1, S_2, \ldots, S_{K_i}\}$, which have $s_1, s_2, \ldots, s_{K_i}$ members respectively. Then the *total Deegan-Packel (DP) power* of i is:

$$TDPP_i = \sum_{k=1}^{K_i} \frac{1}{s_{K_i}}$$

and the *Deegan-Packel power index* of i is:

$$\varphi_i^{DP} \equiv \frac{TDPP_i}{\sum_{j=1}^{N} TDPP_j}$$

Returning to the 1901 election results, note that the minimal winning coalitions are $\{P, FT\}$, $\{FT, ALP\}$ and $\{ALP, P\}$. Therefore,

$$TDPP_i = \frac{1}{2} + \frac{1}{2} = 1 \text{ for } i = P, FT, ALP$$

and so:

$$\varphi_i^{DP} = \frac{1}{3} \text{ for } i = P, FT, ALP$$

which, again, is the same as φ_i^{SS} and φ_i^{Bz} (again, this need not always be the case). On the other hand, consider again the 1910 election. The minimal winning coalition is simply $\{ALP\}$, and so we get:

$$\varphi_{ALP}^{DP} = 1 \text{ and } \varphi_{Liberal}^{DP} = 0$$

which, again, is the same as φ_i^{SS} and φ_i^{Bz}.

3 Results

3.1 Background: Australian Senate Elections, 1901-2004

I computed the Shapley Shubik Index, Normalised Banzhaf Index, and Deegan-Packel Index for the every political party (including independents) in every Australian Senate for the years 1901 to 2004 under two different assumptions. The first set of estimates assumes that all parties are separate entities and do not collude or form long run agreements with one another. In particular, this assumption means that the Liberal Party and the National Party might potentially vote differently on any given piece of legislation.

The second set of estimates begins in 1949, and uses the fact that since 1949 the Liberal and National parties have formed the Coalition when in government and in opposition. Under this second assumption, the Coalition is treated as a single party. Computing the power indices using these two different assumptions allows us to compute the gains (or losses) from this power-sharing arrangement and to investigate how they have changed over time and how they have impacted on the voting power of other parties.

In interpreting and discussing the estimates below, there are several other key institutional developments that should be kept in mind.[6]

1. *Plurality Voting:* From 1901 to 1917, Australia had a plurality voting system in both houses.[7] In the Senate, all states used the "block vote" (a generalisation of the plurality voting system for multi-member constituencies).

2. *Preferential Voting:* The 1918 Commonwealth Electoral Act introduced the alternative vote in the House of Representatives and preferential block voting in the Senate.

3. *Proportional Representation:* In 1949 the Single Transferable Vote (STV) was introduced in the Senate.[8]

4. *Size:* The number of senators increased in each state in 1949 (36 to 60), 1975 (60 to 64), and 1984 (64 to 76). In Senate elections usually only half the Senators face re-election. Double dissolutions were held in 1914, 1951, 1974, 1975, 1983 and 1987.

5. *Compulsory Voting:* Compulsory voting at Federal Elections was introduced in 1924.

Items 1 and 2 on this list are of particular interest here because according to *Duverger's Law*, whether a voting system is preferential or not determines the effective number of parties that compete in elections, which will also influence the indices of voting power.[9] A companion paper [Robson (2007)] to this paper provides an empirical analysis of the consequences of the 1918 Electoral Act for the effective number of parties in both houses of parliament.

[6] Farrell and McAllister (2006) provide an excellent analysis and summary of voting methods used in Australian elections since Federation.

[7] The only exception was the 1901 election, in which the various states were allowed to used their own systems. In the House of Representatives, NSW, VIC and WA used the plurality voting system. In the Senate, all states used the "block vote", or multi-member plurality. A companion paper [Robson (2007)] to this paper provides an empirical analysis of the consequences of the 1918 Electoral Act for the effective number of parties in both houses of parliament.

[8] The actual counting of Senate votes works as follows. Suppose the quota is 100 votes, and that candidate A receives 200 first preference votes. A wins a Senate seat. A "transfer value" is calculated for A's surplus vote, which is equal to his surplus votes divided by his total, which here is $100/200 = 1/2$. Then, each of these 200 ballots is scrutinsed to determined the second preferences of these voters. Suppose, for example, that 50 voters ranked candidate B second. Then these 50 votes are multiplied by A's transfer value (1/2) to give a total of 25, which is then added to the number of first placed votes which B receives.

If after this process no other candidate reaches the quota, the candidate with the lowest number of votes is excluded. Their second preferences are distributed among the other candidates, at full value, just as they are in House of Representatives elections.

[9] See Duverger (1963)

Data on election results were obtained from http://elections.uwa.edu.au.[10] This data is in turn partly based on Hughes and Graham (1968). Under the first assumption, for the current Senate the power indices for each party are:[11]

Table 1: Voting Power in the 2004 Senate (first assumption)

Party (Number of Seats)	Fraction of Seats Held	φ_i^{SS}	φ_i^{Bz}	φ_i^{DP}
ALP (28)	0.36842	0.17619	0.15888	0.10455
Liberals (33)	0.43421	0.44286	0.43925	0.24242
Nationals (5)	0.06579	0.14286	0.14019	0.1803
Democrats (4)	0.05263	0.07619	0.08411	0.12424
Greens (4)	0.05263	0.07619	0.08411	0.12424
Family First (1)	0.01316	0.04286	0.04673	0.11212
Country Liberals (1)	0.01316	0.04286	0.04673	0.11212

On the other hand, under the second assumption we get:

Table 2: Voting Power in the 2004 Senate (second assumption)

Party (Number of Seats)	Fraction of Seats Held	φ_i^{SS}	φ_i^{Bz}	φ_i^{DP}
ALP (28)	0.36842	0.03333	0.02778	0.1
Coalition (38)	0.5	0.83333	0.86111	0.5
Democrats (4)	0.05263	0.03333	0.02778	0.1
Greens (4)	0.05263	0.03333	0.02778	0.1
Family First (1)	0.01316	0.03333	0.02778	0.1
Country Liberals (1)	0.01316	0.03333	0.02778	0.1

Under the second assumption, the ALP's voting power drops dramatically and for all indices is the same as the minor parties and independents. If we assume that the Country Liberal party Senator (Senator Nigel Scullion of the Northern Territory) joins the Coalition on every vote, then the Coalition has an absolute majority and all power indices become 1 for the Coalition. Scullion is a Minister and is also Deputy Leader of the Nationals in the Senate, so table 2 may not provide a true picture of *a priori* voting power in the current Senate. On the other hand, the National Party's Senator Barnaby Joyce has not voted with

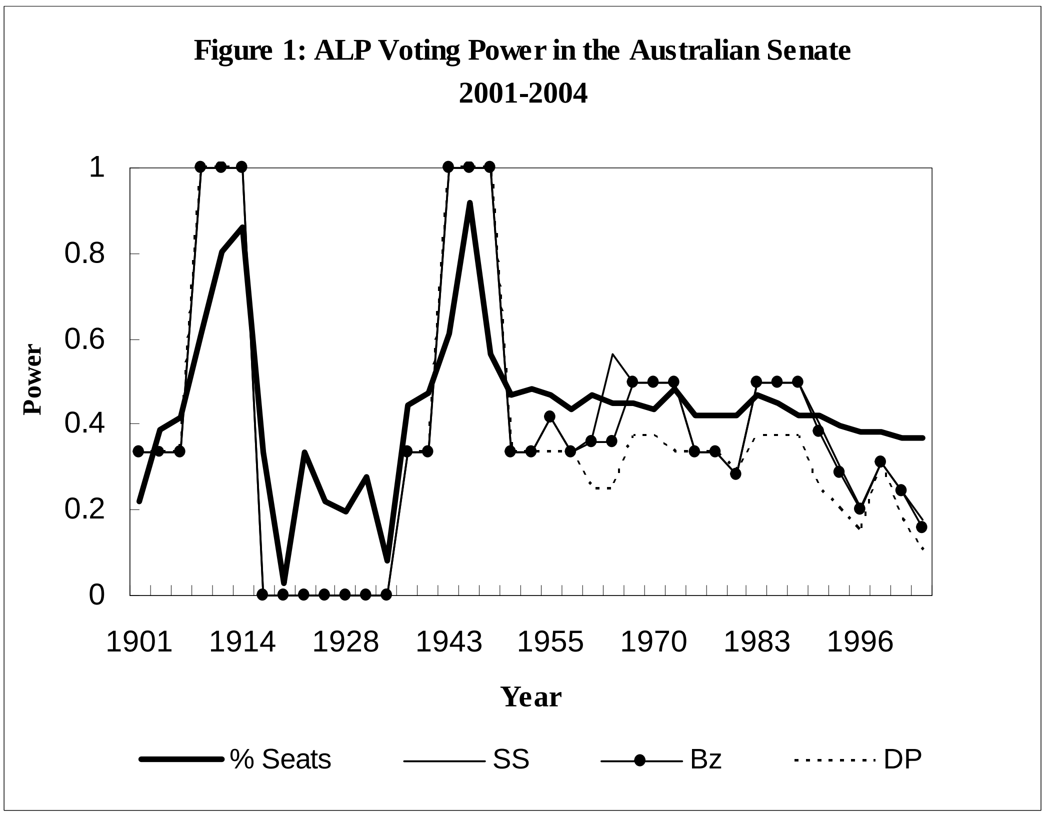
[10]See Sharman (2002) for comments on this website as a resource for political scientists and other researchers.

[11]To maintain consistency with data over the 1901-2001 period, the numbers used are for the party affiliations of Senators immediately after their election. Following the 2004 election, Victorian Senator Julian McGauran defected from the Nationals to the Liberal Party. This would affect the results in table 1, but not table 2 below which treats the Liberals and Nationals as a single voting bloc.

the Coalition on every motion or bill before the current Senate. If we regard Joyce as an independent voter, table 2 again becomes relevant for assessing voting power in the current Senate, with Joyce's voting power replacing Scullion's in table 2.

3.2 Voting Power of the ALP: 1901-2004

Figure 1 presents the three measures of voting power discussed above for the Australian Labor Party between 1901 and 2004, together with the fraction of Senate seats held by the ALP.[12]



For the ALP the Shapley-Shubik and Banzhaf indices track each other closely over the entire history of the Senate – and indeed all indices give the same results up until 1953. Each index indicates that before the 1910 election, the ALP, Protectionists and Free Traders (called the "Anti-Socialists" in 1907) shared voting power equally. Even though the ALP held a plurality of seats in 1904 and 1907, this did not transfer into greater voting power. In 1910 the ALP won both houses and had a clear majority in the Senate until 1917, at which point the ALP split over conscription. 11 ALP Senators formed a coalition with the

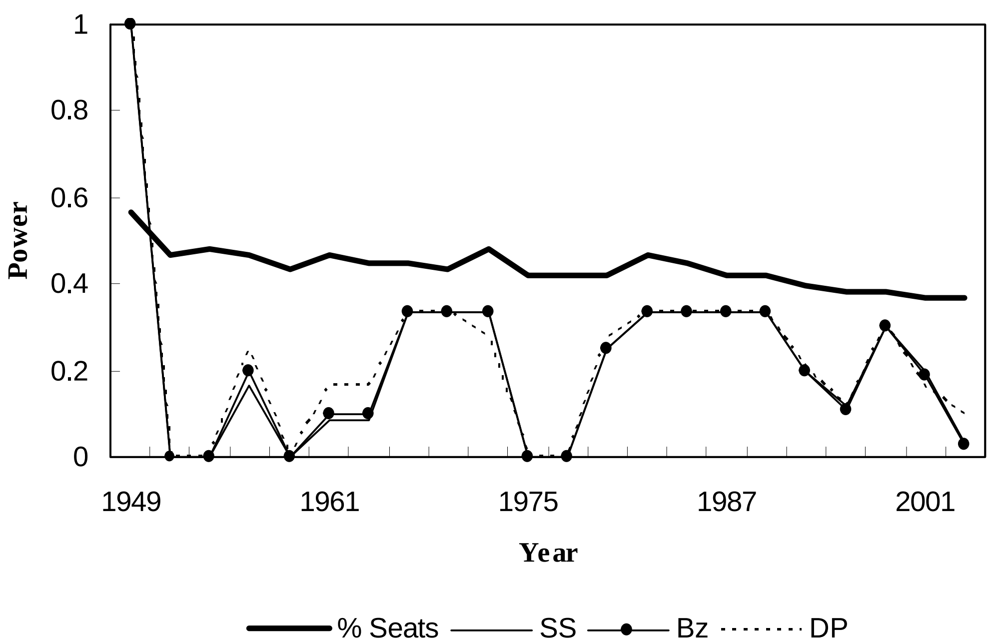
[12]Full results for all parties between 1901 and 2004 are available in spreadsheet form upon request.

Liberals to form the Nationalists, who then held a clear majority (24 of 36) of Senate seats. Thus ALP power (using all indices) falls to zero in 1917 and remains at that level until 1937. Following the 1943 election the ALP again held a majority (22 of 36) seats, and their voting power was one until the 1949 election.

The post 1949 period is interesting for two reasons. First, the number of Senators nearly doubled in 1949, to 60. Second, Senate elections moved to the Single Transferable Vote method, a form of proportional representation. Figure 1 suggests that these reforms coincided with the beginning of a what appears to be a long-term downward trend in the fraction of Senate seats held by the ALP. This trend has lasted 50 years and looks like continuing. But the fraction of seats only tells part of this story – the voting power indices also suggest that there has been a steady fall in the voting power of the ALP in the post-war period.[13]

These indices were computed under the assumption that the Liberals and Nationals did not act as a single Coalition party. If this assumption is changed, what happens to the ALP's post-war voting power? The results are reported in Figure 2 below, and show that the ALP's voting power is significantly reduced.

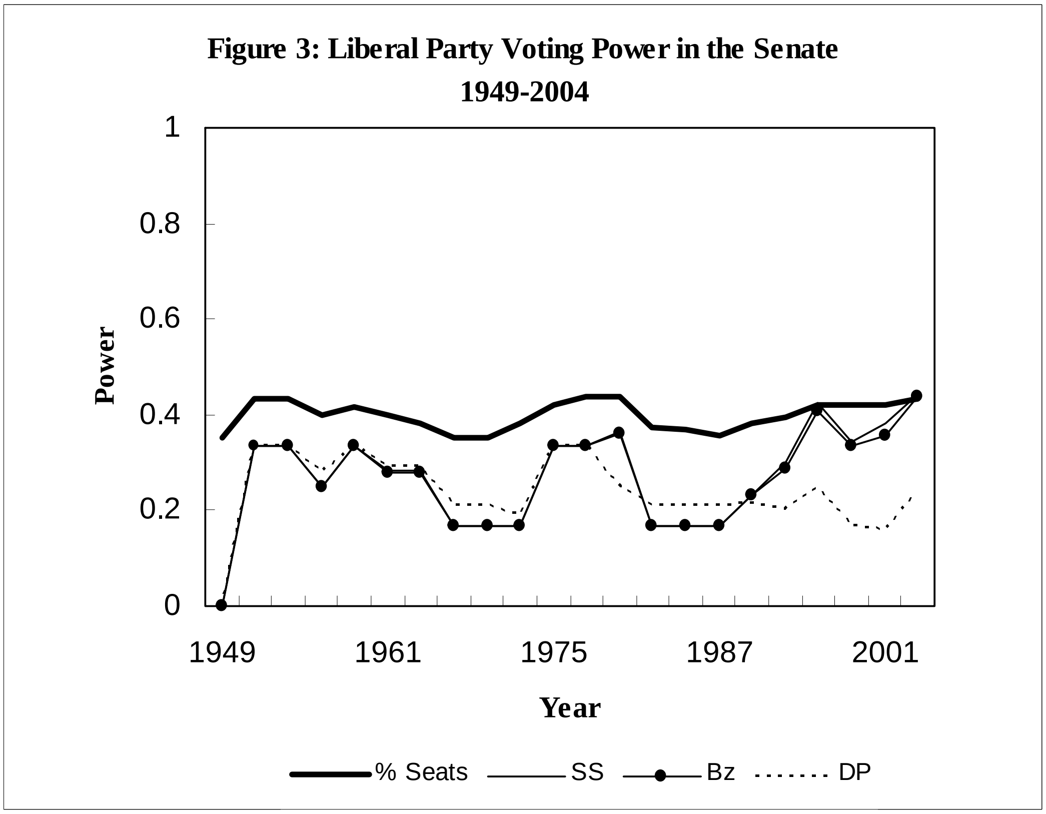
Figure 2: ALP Voting Power in the Senate
(Coalition votes as a bloc)
1949-2004



[13]Following the 2004 election, the current Senate the indices for the ALP are: $\varphi_{ALP}^{SS} = 0.17619$, $\varphi_{ALP}^{Bz} = 0.15888$ and $\varphi_{ALP}^{DP} = 0.10455$, a 70-year low for the ALP.

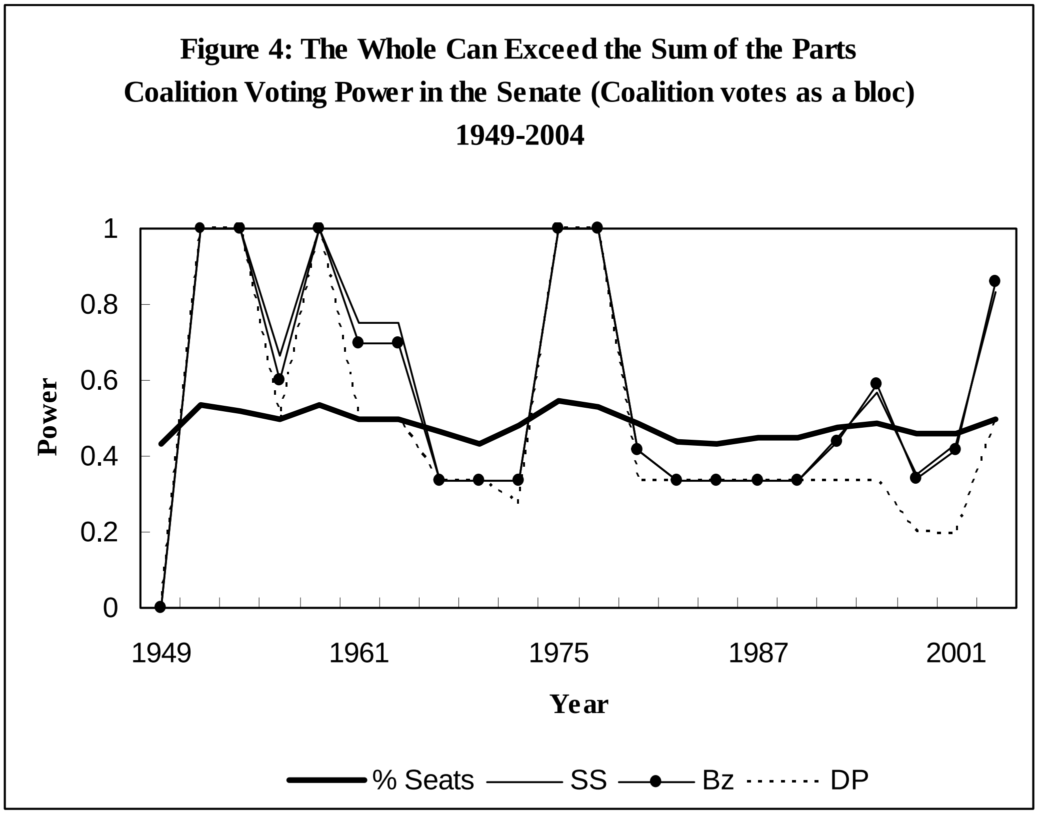
3.3 Voting Power of the Liberal Party and Coalition

Figure 3 presents the three measures of voting power discussed above for the Liberal Party between 1949 and 2004, together with the fraction of Senate seats held by the Liberals.



Again, the ALP the Shapley-Shubik and Banzhaf indices track each other closely, with minor differences between the two. The DP index is based on minimal winning coalitions instead of the notion of pivotalness and so as expected gives different results. Figure 3 suggests that the Liberal Party's relatively modest gains in the Senate since the late 1980s in terms of seats has translated into a significant increase in voting power.

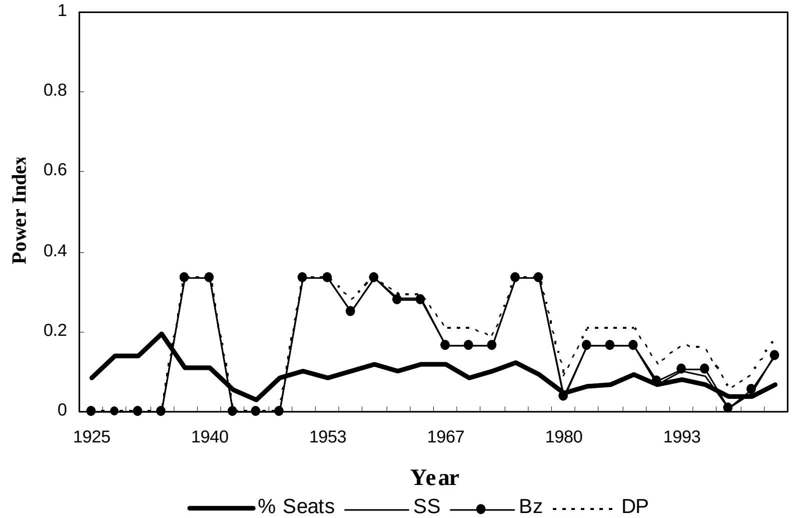
The indices in Figure 3 were computed under the assumption that the Liberals and Nationals did not act as a single Coalition party. If this assumption is changed, we can estimate the voting power of the Coalition. The results are reported in Figure 4 below. In nearly all cases, the formation of the Coalition has increased the total voting power of the Nationals and Liberals above what it would have been if they had acted as separate entities. In terms of indices of voting power, the Coalition agreement has paid off.



3.4 Other Parties: The National Party, Australian Democrats and Australian Greens

Figures 5, 6 and 7 below presents the three measures of voting power discussed above for the National Party (1925-2004), Australian Democrats (1977-2004) and Australian Greens (1990-2004) under the assumption that the Nationals and Liberals act as separate entities. These estimates therefore provide upper bounds for the voting power of these minor parties over the relevant periods. In contrast with the major parties, for the minor parties the indices of voting power almost always exceed the fraction of seats obtained by each party (which in turn exceeds the fraction of primary votes they obtain).

Figure 5: National Party Voting Power in the Australian Senate 1925-2004*



*1925-1974 "Country Party"; 1975-1980 "National Country Party"; 1983-2004 "National Party"

Figure 6: Australian Democrats Voting Power in the Senate 1977-2004

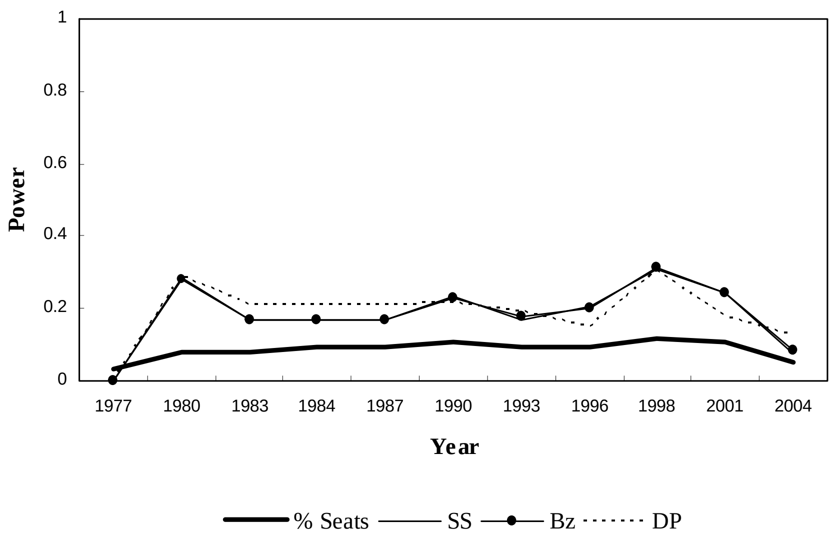
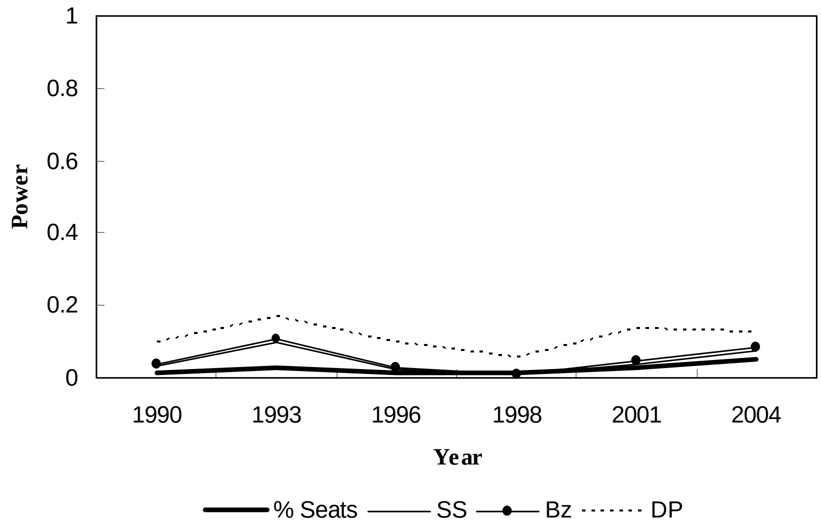


Figure 7: Voting Power of the Australian Greens in the Senate 1990-2004



4 Concentration of Voting Power

In the industrial organisation (IO) literature, *concentration measures* provide a summary of how competitive an industry is. Consider, for example, the sidely used *Herfindahl–Hirschman index* of market concentration:

$$H = \sum_{i=1}^{N} s_i^2 \tag{4}$$

where, $s_i \equiv q_i/Q$ is firm i's market share. Note that H does indeed measure market concentration. For example, if a single firm in the industry is "dominant" and has a very high market share, then the H is large (close to one). On the other hand, if no firm "dominates", then each of the s_i are small, and so $\sum_{i=1}^{N} s_i^2$ would be close to zero. In general, $0 \leq H \leq 1$, with a lower value indicating a "more competitive" market. More generally, following the axiomatization of concentration indices by Encaoua and Jacquemin (1980), we could use any index of the form:

$$R(\mathbf{s}) = \sum_{i=0}^{N} s_i g(s_i) \tag{5}$$

where g is an arbitrary, non-decreasing function with the property that $sg(s)$ is convex in s.

Similar measures are used in political science to analyse the "effective number" of political parties. Let v_i be the number of primary or first-place votes obtained by party i in an election, and let s_i be the number of seats obtained by party i in an election. Natural measures of the "effective" number of votes or seats received by each party in an election are:

$$ENV \equiv \left[\sum_{i=1}^{N} \left(\frac{v_i}{v} \right)^2 \right]^{-1} \tag{6}$$

and

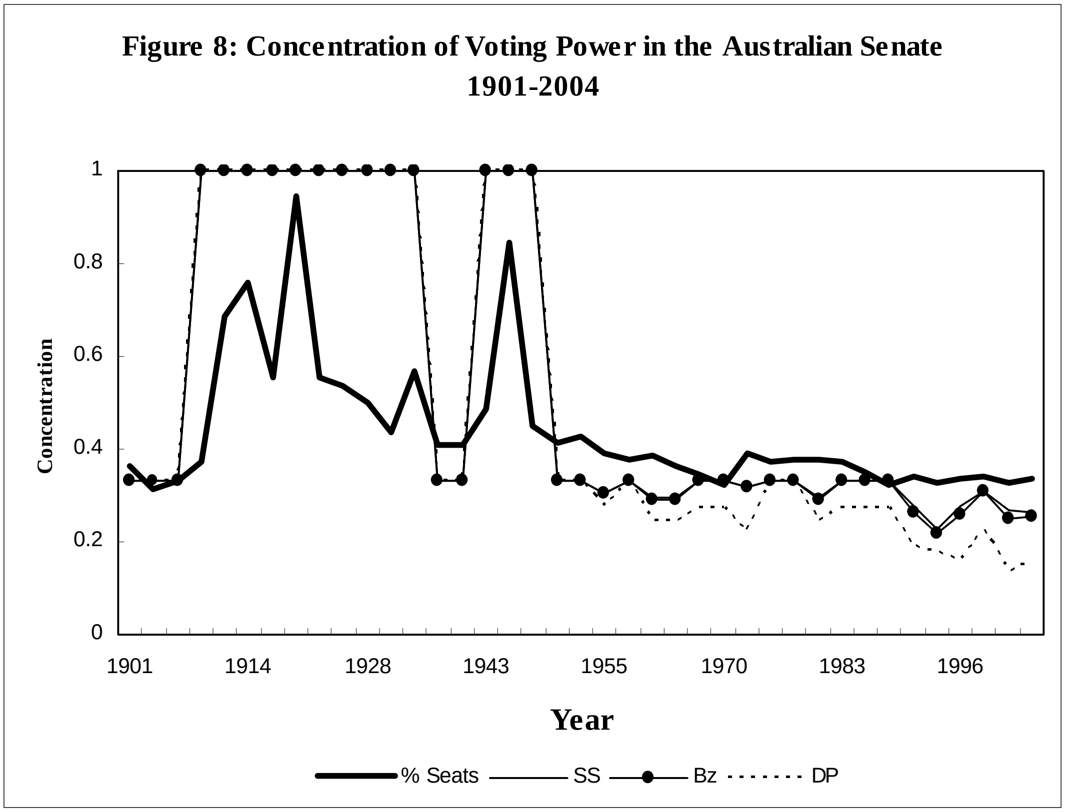
$$ENS \equiv \left[\sum_{i=1}^{N} \left(\frac{s_i}{s} \right)^2 \right]^{-1} \tag{7}$$

where $v = \sum_i v_i$ and $s = \sum_i s_i$. See for example, Muller (2003, page 273). These measures are just inverses of the HH index. We can compute similar measures with φ^{SS}, φ^{Bz} and φ^{DP} to give an overall picture off the concentration of voting power in the Australian Senate. That is, we could compute:

$$\varphi_{Con} = \sum_{i=1}^{N} \varphi_i^2 \tag{8}$$

for each of our indices. Since $0 \leq \varphi_i \leq 1$ for all i, we also have $0 \leq \varphi_{Con} \leq 1$. A low number would indicate that voting power is relatively dispersed, whereas a higher number indicates relatively concentrated voting power.

Figure 8 below computes the concentration measure in (8) using fraction of seats, φ^{SS}, φ^{Bz} and φ^{DP}, under the assumption that the Liberals and Nationals vote as separate parties in the post-1949 period.



The results show that voting power was relatively heavily concentrated before the introduction of proportional representation in 1949, but concentration has been low and has declined steadily since that reform. The introduction of the single transferable vote not only reduced concentration of seats and primary votes, but also reduced the concentration of voting power.

5 Conclusion

This paper computed indices of voting power for the Australian Senate for its entire existence using three well-known indices used in the literature. The indices are intended to provide an *a priori* estimate of the degree of influence that a political party or voter can expect to have over which legislation is passed. Our results show that the introduction in 1949 of the single transferable vote in the Senate has had profound, long term effects on voting power. First, power has become more dispersed. Second, the voting power of the ALP has been in long term decline following this reform. Third, the Coalition parties have maintained their voting power and have recently increased it significantly above their seat shares – a rarity for larger parties in the postwar era.

The primary disadvantage of the measures presented and computed here is that they assume that all coalitions are equally likely to form. But political parties are well entrenched in Australia and ideological, product branding or reputational considerations mean that some coalitions are more likely to form than others. Incorporating these features into power indices would require different measures to be developed and estimated.

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