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Implicit Policy Preferences and Trade Reform by Tariff Aggregates*

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Implicit Policy Preferences and Trade Reform by Tariff Aggregates

Abstract:

Pressure from negotiators on agricultural tariff reform in the Doha Round is favouring commitments to reduce "average" tariffs over a range of commodities. This stems from the perceived need for "flexibility" in protection levels, particularly for some highly protected product groups like sugar, dairy products and rice. Yet reforms that reduce the average tariff across agricultural products but raise tariff dispersion may well reduce welfare and therefore defy the spirit of the negotiations. This paper develops a practical approach to identifying the policy preferences implicit in existing tariff patterns and employs these preferences in formulating mathematical programs that represent the primary policy formation process. These are solved and the effects explored of reform by reductions in either the arithmetic or the trade value weighted average of tariffs. In applications to the EU and Japan, tariff dispersion is found to increase with either averaging formula but by more in the trade value weighted case.

1. Introduction:

The approach employs the concept of implicit welfare weights, as developed in the early work of Tyers (1990). Policy formation is beset by inequalities of influence that stem either from the preferences of fully informed voters¹ or from information asymmetries and hence divergences in lobbying power by producer and other groups². Policy weights offer a reduced form approach to representing such unequal influences in models of the policy formation process.³ The weights are derived by solving the "inverse optimum" problem. Conventional policy problems begin with weighted welfare functions and pose the question, what are the optimal interventions? The first order conditions associated with such problems, however, make it possible to invert this question and to ask, what are the welfare weights that make a set of observed interventions optimal? This paper generalises the derivation of such weights, as they apply to the choice of import tariffs, through the use of a full global general equilibrium analysis. It also extends the scope of their derivation, at least potentially, to the full spectrum of commodities in the GTAP Database. The weights, once derived, are then

¹ Seek, for example, Downs (1957) and Grossman and Helpman (2001: Part I; and 2002: Ch.2) and Mayer (1984).

² See Olsen 1965, Anderson (1980), Anderson and Hayami (1986) and Grossman and Helpman (2001: Part II; and 2002: Part II).

³ Zusman (1976) modelled the policy process as a bargaining game and showed that the resulting political equilibrium might be simulated as the outcome of the maximisation of a policy preference function over the objectives of interest groups in which the weights attached to each are constants reflecting differential influence.

used in the primary policy problem which is, this time, constrained by commitments to reductions in tariff aggregates such as might emerge from the Doha Round of trade negotiations.

The welfare function employed is of the Bergsonian type, which aggregates measures of preparedness to pay across producer and other interests in the economy. The weights then indicate the marginal value to the policy process of an extra dollar of benefit to each production and non-production interest group.⁴ The Cournot assumption is made about international policy interactions, so that the policy process in any one region takes the tariffs in others as given. Producer interests are identified in each domestic commodity market based on real incomes from factors that are sector specific in the short run, along with a single aggregate of consumer and tax-payer interests. The current pattern of distortions would be used to derive implicit weights for this function and the thus-weighted welfare function applied to comparisons of alternative "average" tariff reforms.

Once the weights are derived, a key element of this research is the reconstruction of the primary policy problem, this time with the known weights, to examine the effects on tariff dispersion of negotiated reductions in various measures of the "average" tariff across broad groups of products. Popular tariff aggregation measures include the trade restrictiveness index (TRI: Anderson and Neary 1994), the augmented TRI (Bach and Martin 2001), an arithmetic average (as used in the Uruguay Round) or a trade-weighted average (Bureau and Slavatici 2004). If the conclusion of the Doha Round yields a commitment to a quantified reduction in the level of any one of these indices, the objective of the analysis is then to identify what underlying tariff structure would be chosen by negotiating governments, whether this structure would yield an increase or a decrease in tariff dispersion and whether it would result in an improvement in the country's economic welfare when preparedness to pay measures are equally weighted.

⁴ The Bergsonian social welfare function is a reduced form representation of a game between domestic interests, for whom a welfare transformation surface can be defined as in Tyers (1990: Figure 1) or as a domestic counterpart of the international game represented by Bagwell and Staiger (2002: Figure 2.4). the weights represent the slope of the transformation surface at the policy equilibrium.

2. On what the weights depend:

This is best illustrated with a simple example. Consider a single commodity market in a small open economy that is undistorted except for a tariff on imports in that market, as shown in Figure 1. The home expenditure share on this commodity is small, domestic demand and supply are linear in the home price, p, which is distorted relative to the import price, p^* , by a specific tariff-come-subsidy, t, so that $p = p^* + t$. In this case there is only one producer interest and its preparedness to pay for protection depends on its quasi-rent, V_P . The non-producer interest can be thought of as an amalgam of consumers and taxpayers, on whom the welfare effects of protection can be measured in terms of the sum of consumer surplus and tariff revenue, V_C . As the tariff is changed, then, so also the welfare measures for producer and non-producer interests change. Indeed, they map out a transformation surface or welfare possibility frontier like that shown in Figure 2.

The policy formation problem, then, seeks to identify the point on this surface that maximises a social welfare function in V_P and V_C , the simplest conceivable of which is $W = w_P V_P + w_C V_C$. This has linear social indifference curves of slope $-w_P / w_C$. The social optimum is at a point of tangency, so that this slope is the same as the marginal rate of transformation between producer and non-producer welfare $(dV_C / dV_P = w_P / w_C)$. In this otherwise undistorted, small economy, when the weights are equal and the slope is unity, a point on the transformation surface is selected at which t=0. When producer interests outweigh consumer interests, however, a point above this is chosen to the right of this one, where t>0 and $w_P / w_C > 1$.

The inverse optimum problem, then, begins with a knowledge of the chosen tariff, *t*, and seeks to identify the corresponding marginal rate of transformation, dV_P/dV_C . The weights therefore depend on the behaviour of the underlying commodity market and, particularly, on the magnitudes of its demand and supply elasticities. They embody the economic cost of protection in that, where demand and supply are comparatively elastic and dead weight losses are high, the transformation surface has greater curvature, so that a comparatively costly tariff increase requires a larger weight on producer welfare. This is evident in Figure 2, which shows the transformations surfaces for cases in which demand is elastic and inelastic. The elastic

case has visibly greater curvature. This is further evident from the plots of the slopes of the two frontiers in Figure 3. When demand is elastic, delivering benefits to producers is shown to require movement to more steeply sloped parts of the surface, implying much larger relative weights on producer welfare.

3. Deriving the weights in a more general model:

Consider an economy with *n* producer interests, *i*, each of whose welfare measured in equivalent income, V_i , depends on a tariff of power ρ_i . If inequalities and information asymmetries in the policy process cause it to behave as if a central planner were maximising a weighted welfare function, that function might take the linear form:

(1)
$$W = \sum_{i=1}^{n} w_i V_i + w_C V_C ,$$

where

(2)
$$V_i = V_i(\rho_1, ..., \rho_n), \quad V_C = V_C(\rho_1, ..., \rho_n)$$

are structural (state) equations that embody the behaviour of the economy and its response to the tariffs, and V_c is an aggregation of the income equivalents of the welfare of non-producer interests with w_c its associated weight.

The implicit policy planner faces a constrained multivariate optimisation problem, to choose $(\rho_1, ..., \rho_n)$ to maximise (1), subject to the functional constraints (2). This problem can be simplified, at least in principle, by substituting (2) into (1) so that the maximisation problem becomes unconstrained in $(\rho_1, ..., \rho_n)$ and the first order conditions are:

(3)
$$\sum_{i=1}^{n} \frac{\partial V_i}{\partial \rho_j} w_i + \frac{\partial V_C}{\partial \rho_j} w_C = 0 \quad \forall \ j = 1, n .$$

These are *n* conditions on *n* unknown tariff powers. Provided the state equations can be constructed and are differentiable, this problem is readily solved.

Now imagine that the policy process has already performed its magic and yielded powers of the tariff that are observed as $(\rho_1^*, ..., \rho_n^*)$, but the n+1 weights are unknown. The implicit weights can then be solved from the same first order conditions, with the derivatives evaluated at $(\rho_1^*, ..., \rho_n^*)$:

(4)
$$\sum_{i=1}^{n} \frac{\partial V_i(\underline{\rho})}{\partial \rho_j} (\underline{\rho} = \underline{\rho}^*) \cdot w_i + \frac{\partial V_C(\underline{\rho})}{\partial \rho_j} (\underline{\rho} = \underline{\rho}^*) \cdot w_C = 0 \quad \forall \ j = 1, n$$

The minor problem arises that there are n+1 weights and only *n* first order conditions, so that the weights are available only in relative magnitudes. The inverse optimisation problem is readily solved, however, with the addition of a condition that pins down their average value, such as that their arithmetic average is unity:

(5)
$$\sum_{i=1}^{n} w_i + w_c = n+1 .$$

Relations (4) and (5) then offer n+1 conditions in n+1 unknowns, enabling a solution for the vector of weights: $(w_1, ..., w_n, w_C)$ via the inversion of an augmented welfare response matrix, *H*:

(6)
$$\begin{bmatrix} w_1 \\ .. \\ w_n \\ w_C \end{bmatrix} = \begin{bmatrix} h_{1,1} & .. & h_{1,n} & h_{1,n+1} \\ .. & .. & .. & .. \\ h_{n,1} & .. & h_{n,n} & h_{n,n+1} \\ 1 & .. & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ .. \\ 0 \\ n+1 \end{bmatrix},$$

where the elements of the *H* matrix are:

(7)
$$H = \begin{bmatrix} \frac{\partial V_1}{\partial \rho_1}(\underline{\rho}^*) & \dots & \frac{\partial V_n}{\partial \rho_1}(\underline{\rho}^*) & \frac{\partial V_C}{\partial \rho_1}(\underline{\rho}^*) \\ \dots & \dots & \dots \\ \frac{\partial V_1}{\partial \rho_n}(\underline{\rho}^*) & \dots & \frac{\partial V_n}{\partial \rho_n}(\underline{\rho}^*) & \frac{\partial V_C}{\partial \rho_n}(\underline{\rho}^*) \\ 1 & \dots & 1 & 1 \end{bmatrix}.$$

Critical to the analysis are the income equivalent welfare measures,

 $(V_1,...,V_n,V_C)$ and the economic behavioural (state) equations, (2). For these, our analysis draws on the standard GTAP global comparative static model (Hertel et al. 1997). This model offers a widely familiar behavioural link between a variety of government interventions and factor incomes, regional incomes and regional utility levels. It also offers compatibility with the GTAP Database that allows disaggregation of products and services into at least 50 groups. For the purpose of estimating implicit weights, however, it has two drawbacks. First, it includes a single consolidated household in each region and so it does not directly yield measures of interest group welfare. And second, even if it did include appropriate measures of group welfare, while the complete model does imply a set of state conditions (2), the equations from which are large in number and formulated mostly in proportional changes, making it extremely cumbersome to construct the derivatives in (4) analytically.

Welfare measures:

Producer interests are assumed to be motivated by a concern over income earned by primary factors that are specific to their particular production process in the short run. GTAP has five primary factors: land, production labour, professional labour, physical capital and natural resources. A short run closure⁵ is constructed in which land, physical capital and natural resources are completely immobile between sectors. Income to these three primary factors is then the primary interest of producer groups. As indicated in the Appendix, this income, appropriately deflated by consumption price indices for each group, is then suitable as a money-metric for group welfare. Unfortunately, since GTAP offers only the one private household in each region, it is not possible to construct group specific indices of consumption prices with which to deflate group incomes. It is therefore assumed that the consumption preferences of all groups are identical and group incomes are deflated by regional consumer price indices.⁶ The group incomes therefore take the form:

(7)
$$V_i = \frac{R_i K_i + R_i^N N_i + R_i^A A_i}{P_C}, \quad \forall \ i=1,n,$$

where R_i , R_i^N and R_i^A are the rental rates on physical capital, natural resources and land, respectively, and P_C is the regional consumer price index.

To calculate the income to non-producer interests (consumers and tax-payers), group incomes are subtracted from regional GNP and deflated by the regional consumer price index:

(8)
$$V_C = \frac{GNP_i}{P_C} - \sum_{i=1}^n V_i$$
.

⁵ In models like GTAP the number of variables exceeds the number of equations, requiring that some variables be set as exogenous. The choice as to which variables are made exogenous and which remain endogenous is referred to as the closure.

⁶ The GTAP deflator used is the private consumption price, ppriv(r).

Changes in welfare measures due to tariffs:

Rather than construct the welfare response matrix, H, from state equations (2) that are condensed from the full algebraic representation of GTAP, the elements of the matrix are instead derived numerically by running GTAP repeatedly for small changes in the powers of the tariffs, ρ . A single simulation, say one for a marginal change to the power of tariff j, $\Delta \rho_j$, enables the observation of corresponding changes in the welfare measures, $\Delta V_1, ..., \Delta V_n, \Delta V_c$ and the approximation of the

derivatives $\frac{\partial V_1}{\partial \rho_j}, ..., \frac{\partial V_n}{\partial \rho_j}, \frac{\partial V_C}{\partial \rho_j}$, which are the elements of the *j*th row of the *H* matrix.

The GTAP model is used to make n such simulations, one for each of the tariff items, to complete the matrix. Once the augmented H matrix is constructed the implicit weights are solved for using equation (6).

4. The weights implicit in tariff formation by the European Union and Japan

The approach described in Section 2, above, is applied to the two-region, four product group aggregation of the GTAP Database presented in Table 1. A focus on either the EU or Japan is served by aggregating to two regions (the EU and the Rest of the World or Japan and the Rest of the World), since tariffs in GTAP differ depending on the region of origin.⁷ Aggregation to two regions is therefore a means of constructing the region-generic average tariff for each imported product group.⁸ Products are aggregated into four groups, two of which are agricultural. The second of these, "other agriculture", includes both raw agricultural products and processed foods. The observed average tariffs and the implied weights, calculated as in Section 2, above, are given in Table 2.⁹

The results show that the tariff formation process in both countries behaves as if considerably more weight is attached to producer interests than to non-producer

⁷ Tariff differences by region of origin arise in GTAP even where non-discriminatory tariffs are applied. They are due to differences in the commodity composition of imports from different regions. Aggregations turn up higher average tariffs against regions from which the product group comprises a

comparatively large proportion of sub-products on which higher tariffs are charged. ⁸ The representation of the EU in GTAP incorporates intra-EU trade. Here the tariffs associated with

imports to the EU from the Rest of the World are considered, via the GTAP variable *tms*.

⁹ The *H* matrix is constructed from repeated simulations of the GTAP model using the Gempack and RunGTAP softwares.

interests, with "other agriculture" and manufacturing attracting the highest implicit weights in the EU and both agricultural sectors the highest in Japan. It is notable that the average tariff on industrial products is lower in the EU than that on agricultural products yet the implicit weight is also large. A high weight on industrial protection is required, however, to justify even low tariffs because protection of this sector in the EU imposes substantial external costs on other sectors and hence on other fixed factor real incomes. As discussed in Section 2, where protection causes comparatively large dead weight losses, the transformation surface between producer and other interests has greater curvature and so large weights are required to justify high tariffs. In Japan, all the weights depart from unity by less than in the EU, suggesting its existing distortions incur smaller dead weight losses, so that there is comparatively little curvature of the transformation surface.

Also notable is the comparatively low weight on non-producer interests, particularly in the EU. This is because these weights reflect the tariff formation process alone and it has traditionally been a game amongst producer interests. It may well be, however, that the imbalance in this process in favour of producer interests is redressed in other policy arenas, such as those driving direct taxation and transfers. The purpose of this exercise is not to infer the biases in the overall policy formation process but to focus on those implicit in that which determines the mix of tariffs levied on imports.

5. Predicting the policy response to negotiated reductions in average tariffs

Once the biases implicit in the policy formation process have been estimated in the form of the vector of weights, the primary policy problem can be reconstructed with the now-known weights, to examine the effects on tariff dispersion of negotiated reductions in various measures of the "average" tariff across broad groups of products.

Tariff aggregation formulae:

Popular tariff aggregation measures include the arithmetic average (as used in the Uruguay Round), the trade-weighted average (Bureau and Slavatici 2004), the trade restrictiveness index (TRI: Anderson and Neary 1994) and the augmented TRI (Bach and Martin 2001). Consider the simplest of these first: the arithmetic average of powers of tariffs,

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(9)
$$\overline{\rho}_0 = \frac{1}{n} \sum_{i=1}^n \rho_i$$

and the trade value weighted average of powers of tariffs,

(10)
$$\overline{\rho}_{1} = \sum_{i=1}^{n} \left[S_{i}^{M} \cdot \rho_{i} \right],$$

where S_i^M is the share of product *i* in the cif value of imports.¹⁰ Unfortunately, unless the policy process employs a set of exogenous prior import shares, the import shares in the trade value weighted formulation are endogenous:

(11)
$$S_i^M = \frac{M_i(\underline{\rho}) \cdot \rho_i}{\sum_{j=1}^n M_j(\underline{\rho}) \cdot \rho_j}$$

At the outset, these shares will be assumed exogenous and to be assigned based on historical trade data. The possibility that shares would be updated will be considered subsequently.

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Reformulating the primary policy problem:

The most sophisticated reformulation of the primary policy problem would embody all the analytics of the GTAP model. Imagine that the set of endogenous variables defining the performance of the economy in GTAP makes up the vector \underline{x} . The primary policy problem, then, is as follows:

Problem 1: Choose the vector ρ to

Maximise

subject to

$$W = \sum_{i=1}^{n} w_i V_i + w_C V_C ,$$

$$\underline{V} = \underline{V}(\underline{x})$$

$$\underline{x} = \underline{x}(\underline{\rho})$$

$$\frac{1}{n} \sum_{i=1}^{n} \rho_i \le \overline{\rho}_0$$

¹⁰ It is also possible to average the elements of the vector of corresponding ad-valorem tariff rates, $\underline{\tau}$, in which case the formulae are different and might yield a different pattern of protection if such averages were adopted.

$$\sum_{i=1}^{n} \left[S_{i}^{M} \cdot \rho_{i} \right] \leq \overline{\rho}_{1}$$
$$\rho \geq \underline{0}.$$

In this reformulation the set of equations $\underline{x} = \underline{x}(\underline{\rho})$ are from the structure of the GTAP model, while the set $\underline{V} = \underline{V}(\underline{x})$ formulate the welfare measures in terms of GTAP endogenous variables. In effect, this reformulation does no more than add inequality constraints on the average tariff which can be made to bind at a variety of levels of the average. The complexity of the approach, however, stems from the incorporation of all the model's structural equations as equality constraints.

Reformulation as a linear program:

Imagine the substitution of $\underline{x} = \underline{x}(\underline{\rho})$ into $\underline{V} = \underline{V}(\underline{x})$, yielding the functions (2), $\underline{V} = \underline{V}(\underline{\rho})$. These can then be linearized around the zero tariff level, $\underline{\rho} = \underline{1}$, so that (12) $V_i = a_i + \sum_{i=1}^n \frac{\partial V_i}{\partial \rho_i} (\underline{\rho} = \underline{1}) \cdot \rho_j \quad \forall i = l, n$.

This is convenient because the gradient terms can be calculated by solving the GTAP model for small deviations in $\underline{\rho}$, this time from an initial equilibrium that is free of tariff distortions ($\underline{\rho} = \underline{1}$). It can be done in the same way as the elements of the *H* matrix in (7) are derived. The key difference is that the linearization is this time around the zero tariff point. The reason for this will be clear once the simpler policy problem has been formulated.

The simpler reformulation is then

Problem 2: choose the vector, ρ to:

Maximise $W = \sum_{j=1}^{n} v_j \rho_j = \underline{v} \cdot \underline{\rho}$ Subject to $\frac{1}{n} \sum_{i=1}^{n} \rho_i \leq \overline{\rho}_0$ $\sum_{i=1}^{n} \left[S_i^M \cdot \rho_i \right] \leq \overline{\rho}_1$ $\underline{\rho} \geq \underline{1}$.

Here the coefficients in the objective function are most important. Since total weighted welfare is the weighted sum of group welfare:

(13)
$$W = \sum_{i=1}^{n} w_i \left[\sum_{j=1}^{n} \frac{\partial V_i}{\partial \rho_j} \left(\underline{\rho} = \underline{1} \right) \cdot \rho_j \right] + w_C \sum_{j=1}^{n} \frac{\partial V_C}{\partial \rho_j} \left(\underline{\rho} = \underline{1} \right) \cdot \rho_j ,$$

it is the equivalent of the following sum over product lines:

(14)
$$W = \sum_{j=1}^{n} \left[\sum_{i=1}^{n} w_i \frac{\partial V_i}{\partial \rho_j} \left(\underline{\rho} = \underline{1} \right) + w_C \frac{\partial V_C}{\partial \rho_j} \left(\underline{\rho} = \underline{1} \right) \right] \cdot \rho_j \; .$$

And, so,

(15)
$$v_j = \sum_{i=1}^n w_i \frac{\partial V_i}{\partial \rho_j} \left(\underline{\rho} = \underline{1}\right) + w_C \frac{\partial V_C}{\partial \rho_j} \left(\underline{\rho} = \underline{1}\right), \quad \forall \ j = 1, n$$

Each v_j is a product of a row of the matrix *H* and the weights vector, where the elements of *H* are this time evaluated at the zero tariff point. Thus, the full vector is:

(13)
$$\underline{v} = \begin{bmatrix} \frac{\partial V_1}{\partial \rho_1} (\underline{\rho} - \underline{1}) & \frac{\partial V_n}{\partial \rho_1} (\underline{\rho} = \underline{1}) & \frac{\partial V_C}{\partial \rho_1} (\underline{\rho} = \underline{1}) \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial V_1}{\partial \rho_n} (\underline{\rho} = \underline{1}) & \cdots & \frac{\partial V_n}{\partial \rho_n} (\underline{\rho} = \underline{1}) & \frac{\partial V_C}{\partial \rho_n} (\underline{\rho} = \underline{1}) \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ w_C \end{bmatrix}$$

It is now clear why the elements of the matrix of gradients cannot be evaluated at $\underline{\rho} = \underline{\rho}^*$. Were this to be the case, by the first order conditions from which the weights are derived, (4), the vector of coefficients would be precisely zero ($\underline{v} = \underline{0}$). This is an envelope result that is true by definition since $\underline{\rho} = \underline{\rho}^*$ is "optimal" with the calculated weights.

The bounds imposed on the average powers of the tariffs, $\overline{\rho}_0$ and $\overline{\rho}_1$, can be set to values larger than, and including, unity. If unity is chosen, there is a unique solution to the problem, $\underline{\rho} = \underline{1}$ (free trade). If the average of observed tariff powers, $\overline{\rho} = \overline{\rho}^*$, is chosen, the problem yields a set of "optimal" tariff powers that only approximates the observed tariff powers. It is not precise because this simpler formulation is a linearized approximation to the full primary policy problem, detailed earlier, and it can be expected to be least accurate when departures from free trade are largest. Indeed, since the objective function is linear in the tariff powers, *Problem 2* cannot be realistically unconstrained. The larger the average is permitted to be, the larger will be the "optimal" tariffs. In reality, even weighted gains from tariffs peter out when the tariffs are too high.

Illustration of Problem 2 for the EU and Japan:

The coefficients from the objective function of *Problem 2* are listed in Table 3. These are in weighted 1997 US \$ billions per tariff point. As the largest sector protected by tariffs, it is not surprising that the EU's industrial sector has the highest policy payoff per tariff point. The payoff is also large for the highly weighted "other agriculture" sector, however, and it is negative on services since the implicit welfare weight on services real income is very low (Table 2) and protection of the services sector would reduce fixed factor real income in the other more highly weighted sectors. In the case of Japan, the very high tariffs on agriculture and the miniscule tariffs on industrial products shift the payoffs in favour of agricultural protection and away from tariffs on industrial products and services.

These coefficients are employed in *Problem 2* and the problem solved using the GAMS software. Results are presented in Table 4. They exhibit the unfortunate tendency of linear programming to yield "corner solutions". When tariffs are constrained by the arithmetic average, the *Problem 2* raises the tariff on the product with the largest objective function coefficient – the one that yields the maximum weighted US\$ billions per tariff point – to the maximum extent, leaving the less rewarding tariffs at zero (powers at unity). This is a consequence of the absence from *Problem 2* of diminishing returns to the policy process of raising any single tariff indefinitely.

When the binding constraint is the import value weighted average of tariff powers, *Problem 2* turns up a slightly different result. In the case of the EU, because industrial imports constitute almost two thirds of imports from non-EU regions, the tariff on industrial goods cannot be raised very far before the constraint binds. Tariffs on "other agriculture", however, are almost as rewarding but their import value share is only six per cent. It is therefore a better option for the policy process to raise the tariff on other agricultural goods to the maximum extent available (until the weighted average

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tariff ceiling is reached). This leads to very high average tariffs on other agricultural goods and zero tariffs on other products. In the case of Japan the same patter emerges except that the switch is between a tariff on "other agriculture" and one on crops.

Reformulation as a non-linear program

To avoid the unfortunate tendency of the linear formulation to yield unrealistic corner solutions, a first step is to recognise that there are diminishing returns in weighted welfare to indefinite increases in individual tariffs. This is because, as tariffs rise, they tend to choke off imports of the products to which they are applied and they raise costs in other sectors that carry high implicit welfare weights. The extent of these diminishing returns can be gauged by the results discussed earlier, that when the objective function coefficients are evaluated at the free trade equilibrium, as in the formulation of *Problem 2* $\underline{v}(\underline{\rho} = \underline{1}) = \underline{v}^0 \neq \underline{0}$. When they are evaluated at the equilibrium distorted by the observed tariffs (the "optimal" policy equilibrium), however, they are all zero, $\underline{v}(\rho = \rho^*) = \underline{v}^0 = \underline{0}$.

Again, the brute force approach to incorporating this behaviour would be to return to the full-blown *Problem 1*. Yet it is also possible to represent this non-linearity in a problem that is only slightly more complex than *Problem 2*. This is achieved by recognising that the objective function coefficients themselves depend on the levels of their associated tariff powers. Thus, $\underline{v} = \underline{v}(\underline{\rho})$ and $\underline{v}'(\underline{\rho})$ is a negative definite matrix. The simplest way to construct these functions is to ignore cross effects between tariffs and recognise that the own-tariff relationship must take the form of the concave function illustrated in Figure 2. Concavity is required of this function because, were it to be either linear or strictly convex, the objective function terms $v_i(\rho_i) \cdot \rho_i$ would not be increasing in ρ_i , even at $\rho_i = 1$. This approach is considered first.

It is simplest to fit a quadratic function on the three conditions that: when $v = v^0$, $\rho = 1$, when v = 0, $\rho = \rho^*$ and when $\rho = 1$, $\frac{dv}{d\rho} = 0$. The result is the following: (14) $v_i = \frac{v_i^0}{1 + 2 e^{i \pi t} e^{-\pi t}} \left[\rho_i^{*2} - \rho_i^2 - 2(\rho_i^* - \rho_i) \right], \quad \forall i = 1, n$,

(14)
$$V_{i} = \frac{1}{1 - 2\rho_{i}^{*} + \rho_{i}^{*2}} \left[\rho_{i}^{*} - \rho_{i}^{*} - 2(\rho_{i} - \rho_{i}) \right], \quad \forall i = 1, n ,$$

where the v_i^0 terms are the coefficients of the objective function calculated for *Problem* 2 and the ρ_i^* terms are the observed tariffs, presumed to be "optimal" at the outset. These augmented coefficients then allow the construction of a third, non-linear, primary policy problem:

Problem 3: choose the vector, ρ to:

Maximise $W = \sum_{j=1}^{n} \left\{ \frac{v_j^0 \rho_j}{1 - 2\rho_j^* + \rho_j^{*2}} \left[\rho_j^{*2} - \rho_j^2 - 2(\rho_j^* - \rho_j) \right] \right\}$ Subject to $\frac{1}{n} \sum_{i=1}^{n} \rho_i \le \overline{\rho}_0$ $\sum_{i=1}^{n} \left[S_i^M \cdot \rho_i \right] \le \overline{\rho}_1$ $\underline{\rho} \ge \underline{1}.$

This problem has the nice property that, like *Problem 1*, it yields an unconstrained optimum. Unfortunately, however, the quadratic function for v does not allow this unconstrained maximum to occur precisely at $\underline{\rho} = \underline{\rho}^*$. To achieve this with precision, a higher order polynomial might be calibrated to the shape shown in Figure 4, using the additional conditions that

(15)
$$\frac{\partial}{\partial \rho_i} \left[v_i(\underline{\rho}) \cdot \rho_i \right] = 0 \text{ and } \frac{\partial^2}{\partial \rho_i^2} \left[v_i(\underline{\rho}) \cdot \rho_i \right] < 0 \text{ when } \underline{\rho} = \underline{\rho}^*.$$

Even this step would lack the interactive behaviour linking tariffs in one sector to gains in others. For that, a fully interactive polynomial form would be required.

For the present purpose, however, it is sufficient to exploit the nonlinearities in *Problem 3* to illustrate their effects on tariff formation behaviour. To do this, the objective function coefficients in this problem are calibrated so that the unconstrained optimum tariff power vector is $\underline{\rho} = \underline{\rho}^*$.¹¹ When this is done, the tendency to yield

¹¹ This entails using calibrated values for the elements of $\underline{\rho}^*$ as they appear in the objective function. The behaviour of the vector of coefficients, $\underline{\mathcal{V}}$, remains consistent with Figure 1 except that the horizontal intercept is shifted, usually to the right, so that the unconstrained optimum is correct. With this

corner solutions disappears and the effects of constraining the averages are more intuitive.

Illustration of the nonlinear Problem 3 for the EU and Japan:

At first, *Problem 3* is solved unconstrained, yielding the optimal (observed) tariff power vector, $\underline{\rho} = \underline{\rho}^*$. Then the problem is subject to a binding constraint on the arithmetic average tariff power. In successive solutions the average tariff bound is reduced any changes in the mix and dispersion of tariffs are noted. Then the problem is subjected to a binding constraint on the trade value weighted average tariff. Again, successive solutions are subjected to reductions in this bound. The results are listed in Table 5.

In the case of the EU, as the arithmetic average is restricted, the policy process as represented in *Problem 3* tends to sacrifice the tariff on crops (a sector with a smaller weight than "other agriculture" or industry and a smaller import value share) while it retains comparatively high tariffs on "other agriculture" and industrial products. If the trade value weighted average is constrained, however, the cuts are heaviest in industry tariffs and least in "other agriculture". Although Problem 3 yields more balanced, and hence more realistic, cuts than the corner solutions of Problem 2, the pattern is similar. When the arithmetic average is constrained the sector with the largest import value share tends to retain its protection. When it is the import value weighted average that is constrained, the retention of protection to the large-share sector (industry) restricts more tightly the protection that can be allocated to the other sectors. The policy process, as simulated by *Problem 3*, then tends to sacrifice the industry tariff in favour of retaining high tariffs on crops and "other agricultural" imports. The pattern for Japan is similar, except that the redistribution is from the tariff on "other agriculture" to that on crops. The very small tariff on industrial imports is sacrificed early whichever tariff average is used.

Indicators of the welfare implications of these different responses are offered in Table 6. First, a trade value weighted measure of the coefficient of variation of tariff rates is listed. Whichever average measure is constrained, *Problem 3* yields increased

approximation, the condition that $\mathcal{V}(\underline{\rho}=\underline{\rho}^*)=\underline{0}$ is therefore sacrificed for the purpose of applying *Problem* 3 in search of more realistic policy formation behaviour than that shown in *Problem 2*.

tariff dispersion by this measure, while proportional cuts in tariffs would not. Comparing the two average measures, there is a tendency for tariff dispersion to increase less when cuts are made to the arithmetic mean than when they are made to the trade value weighted average. This is due to the large cuts made to the high-weight industrial tariff when the trade value weighted average is constrained.

Next, the effects on equally weighted welfare are given. Tariff reform actually reduces equally weighted welfare in the EU, for two possible reasons. First, the removal of tariffs alone is a second best reform given the remaining distortions represented in GTAP (mainly taxes on production exports and factor use). And second, the tariffs shift the terms of trade in favour of the EU. Indeed, the observed ones are less than "optimal" even with unweighted (meaning equally weighted) welfare, if the trade elasticities implied by GTAP can be believed. Japan's smaller economy very likely has smaller optimum tariffs in equally weighted welfare terms and so equally weighted welfare increases with tariff reform. Consistent with the effects on tariff dispersion, binding the trade weighted average tariff impairs equally weighted welfare more than when the policy process is bound by the arithmetic average. Finally, weighted welfare is shown to be reduced more by the trade weighted average tariff constraint. This implies that, at least in the EU, such a constraint would place more strain on the policy formation process than a bound on the arithmetic average tariff.

6. Conclusion

When countries' trade policy formation processes are unbalanced, favouring some producer groups relative to others and all producer groups relative to nonproduction interests, an expressed preference for "flexibility" in the Doha Round can be read as an indication that some tariffs will be politically easier to cut than others. Average cut formulae are therefore under consideration in order to offer this flexibility. In order to evaluate alternative measures, however, it is important to have some means by which the policy process, constrained by a committed reduction in an average tariff measure, might be predicted. How would it alter the mix, and hence the dispersion, of tariffs across commodities? This paper offers a practical approach to predicting this behaviour.

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It is assumed that the tariff formation process is differently influenced by producers of different products and that its resulting behaviour can be reflected in a set of implicit weights in a Bergsonian social welfare function. When tariffs are observed as an output of their formation process, the implicit weights can be calculated from the "inverse optimum" problem. It is then assumed that these weights reflect a political equilibrium that precedes a shock to the process, in this case the advent of the Doha Round. While the Round might be successful in bringing about commitments to reduced average tariffs, the old implicit weights are assumed to reflect continued biases in tariff formation which determine the adjustment of the tariff mix across products.

Once implicit welfare weights have been derived, the tariff formation process is represented as a mathematical programming problem. Two alternative approaches are trialled, with a non-linear programming problem offering the most credible results. Two different measures of the average tariff are constructed as constraints: the arithmetic average and the trade value weighted average. When these are bound, the tariff mixes that result are quite different. In illustrative analyses of the tariff formation processes in the EU and Japan, binding either measure of the average tariff leads to increased tariff dispersion and reduced overall (equally weighted as well as unequally weighted) welfare. Binding the arithmetic average, however, appears to yield smaller increases in dispersion as the policy process is free to maintain tariff levels on products with high trade value shares.

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Figure 1: Single market example

Figure 2: Single market: producer vs consumer/taxpayer welfare frontier





Figure 4



Table 1: Illustrative GTAP aggregation

| Regions: |
|---|
| European Union, EU, or Japan |
| Rest of World <i>RoW</i> |
| Primary factors: |
| Land, A |
| Production labour, L |
| Professional labour, S |
| Physical capital, K |
| Natural resources, N |
| Products: |
| Crops |
| Other agriculture |
| Industry (manufacturing, mining, minerals and energy) |
| Services |
| Source: GTAP Database, Version 5. |

| | Ad valorem tariff rate, % | Implied welfare weight |
|--------------------------|---------------------------|------------------------|
| European Union | | |
| Producer interests: | | |
| Crops | 14.6 | 1.162 |
| Other agriculture | 34.7 | 1.303 |
| Industry | 4.2 | 1.257 |
| Services | 0.0 | 0.952 |
| Non-production interests | | 0.325 |
| Japan | | |
| Producer interests: | | |
| Crops | 65.4 | 1.021 |
| Other agriculture | 42.1 | 1.076 |
| Industry | 1.9 | 0.951 |
| Services | 0.0 | 1.018 |
| Non-production interests | | 0.933 |

Table 2: Average tariffs and implied welfare weights

Source: Analysis using GTAP and the estimation method described in the text. The observed tariff rates are averages from the GTAP Database Version 5.

| | European Union | | Japan | |
|-------------------|----------------|--------------|----------------|--------------|
| | Coefficient, v | Import share | Coefficient, v | Import share |
| Crops | 6.74 | 0.049 | 22.9 | 0.057 |
| Other agriculture | 20.24 | 0.057 | 23.9 | 0.106 |
| Industry | 25.70 | 0.632 | 8.05 | 0.614 |
| Services | -7.13 | 0.262 | -4.32 | 0.223 |

Table 3: Objective function coefficient vectors, \underline{y}^{a} , and import value shares ^b

a These are the weighted return to the policy process of increments to tariffs, measured in 1997 US\$ billions per tariff point.

b Import value shares are for imports from non-EU sources only, evaluated at agents (domestic) prices. Source: Analysis using GTAP and the estimation method described in the text, equation (13).

Table 4: "Optimal" tariff powers from *Problem 2*, for different constraints on the arithmetic and trade value weighted averages

| | Crops | Other agric | Industry | Services |
|---------------------------|-------|-------------|----------|----------|
| European Union | | | | |
| Arithmetic average | | | | |
| $\overline{\rho}_1 = 1$ | 1.0 | 1.0 | 1.0 | 1.0 |
| $\overline{\rho}_1 = 1.1$ | 1.0 | 1.0 | 1.4 | 1.0 |
| $\overline{\rho}_1 = 1.3$ | 1.0 | 1.0 | 2.2 | 1.0 |
| Trade value weighted | | | | |
| $\overline{\rho}_2 = 1$ | 1.0 | 1.0 | 1.0 | 1.0 |
| $\bar{\rho}_2 = 1.1$ | 1.0 | 2.8 | 1.0 | 1.0 |
| $\overline{\rho}_2 = 1.3$ | 1.0 | 6.3 | 1.0 | 1.0 |
| Japan | | | | |
| Arithmetic average | | | | |
| $\overline{\rho}_1 = 1$ | 1.0 | 1.0 | 1.0 | 1.0 |
| $\overline{\rho}_1 = 1.1$ | 1.0 | 1.4 | 1.0 | 1.0 |
| $\overline{\rho}_1 = 1.3$ | 1.0 | 2.2 | 1.0 | 1.0 |
| Trade value weighted | | | | |
| $\bar{\rho}_2 = 1$ | 1.0 | 1.0 | 1.0 | 1.0 |
| $\overline{\rho}_2 = 1.1$ | 2.75 | 1.0 | 1.0 | 1.0 |
| $\overline{\rho}_2 = 1.3$ | 6.26 | 1.0 | 1.0 | 1.0 |

Source: Solutions of Problem 2 in the text.

| | Crops | Other agric | Industry | Services |
|--|-------|-------------|----------|----------|
| European Union | • | | | |
| Arithmetic average | | | | |
| Unconstrained, $\overline{\rho}_1 = 1.135$ | 1.146 | 1.347 | 1.042 | 1.000 |
| $\overline{\rho}_1 = 1.10$ | 1.068 | 1.296 | 1.036 | 1.000 |
| $\overline{\rho}_1 = 1.05$ | 1.000 | 1.176 | 1.024 | 1.000 |
| Trade value weighted | | | | |
| Unconstrained, $\overline{\rho}_2 = 1.054$ | 1.146 | 1.347 | 1.042 | 1.000 |
| $\overline{\rho}_2 = 1.04$ | 1.130 | 1.334 | 1.023 | 1.000 |
| $\overline{\rho}_2 = 1.03$ | 1.117 | 1.324 | 1.009 | 1.000 |
| Proportional cut ^a | | | | |
| Unconstrained, $\bar{\rho}_2 = 1.054$ | 1.146 | 1.347 | 1.042 | 1.000 |
| $\overline{\rho}_2 = 1.04$ | 1.108 | 1.257 | 1.031 | 1.000 |
| $\overline{\rho}_2 = 1.03$ | 1.081 | 1.193 | 1.023 | 1.000 |
| Japan | | | | |
| Arithmetic average | | | | |
| Unconstrained, $\overline{\rho}_1 = 1.273$ | 1.654 | 1.421 | 1.019 | 1.000 |
| $\overline{\rho}_1 = 1.2$ | 1.488 | 1.311 | 1.000 | 1.000 |
| $\overline{\rho}_1 = 1.1$ | 1.242 | 1.158 | 1.000 | 1.000 |
| Trade value weighted | | | | |
| Unconstrained, $\bar{\rho}_2 = 1.094$ | 1.654 | 1.421 | 1.019 | 1.000 |
| $\overline{\rho}_2 = 1.06$ | 1.541 | 1.275 | 1.000 | 1.000 |
| $\overline{\rho}_2 = 1.03$ | 1.403 | 1.066 | 1.000 | 1.000 |
| Proportional cut ^a | | | | |
| Unconstrained, $\bar{\rho}_2 = 1.094$ | 1.654 | 1.421 | 1.019 | 1.000 |
| $\overline{\rho}_2 = 1.06$ | 1.417 | 1.269 | 1.012 | 1.000 |
| $\overline{\rho}_2 = 1.03$ | 1.209 | 1.134 | 1.006 | 1.000 |

Table 5: "Optimal" tariff powers from *Problem 3*, for different constraints on the arithmetic and trade value weighted averages

a Proportional cuts are in tariff rates rather than tariff powers.

Source: Solutions of *Problem 3* in the text.

| | Coefficient of variation of tariff rates, trade value weighted, % ^a | Unweighted welfare: % departure from unconstrained ^b | Weighted welfare: % departure from unconstrained |
|--|--|---|--|
| European Union Arithmetic average | | | |
| Unconstrained, $\overline{\rho}_1 = 1.135$ | 147 | 0.0 | 0.0 |
| $\overline{\rho}_1 = 1.10$ | 151 | -0.6 | -0.6 |
| $\overline{\rho}_1 = 1.05$ | 153 | -1.3 | -2.8 |
| Trade value weighted | | | |
| Unconstrained, $\overline{\rho}_2 = 1.054$ | 100 | 0.0 | 0.0 |
| $\overline{\rho}_2$ =1.04 | 193 | -1.3 | -0.2 |
| $\overline{\rho}_2 = 1.03$ | 255 | -2.2 | -0.8 |
| Proportional cut | | | |
| Unconstrained, $\overline{\rho}_2 = 1.054$ | 147 | 0.0 | 0.0 |
| $\overline{\rho}_2 = 1.04$ | 147 | -1.0 | -0.8 |
| $\overline{\rho}_2 = 1.03$ | 147 | -1.7 | -2.2 |
| $\overline{\rho}_2 = 1.00$ | 0.0 | -3.9 | -9.9 |
| Japan | | | |
| Arithmetic average | 240 | 0.0 | 0.0 |
| Unconstrained, $\rho_1 = 1.273$ | 240 | 0.0 | 0.0 |
| $\overline{\rho}_1 = 1.2$ | 298 | 1.8 | -1.8 |
| $\overline{\rho}_1 = 1.1$ | 298 | 2.7 | -3.7 |
| Trade value weighted | • 10 | | |
| Unconstrained, $\overline{\rho}_2 = 1.094$ | 240 | 0.0 | 0.0 |
| $\overline{\rho}_2 = 1.06$ | 308 | 1.6 | -1.6 |
| $\overline{\rho}_2$ =1.03 | 372 | 2.3 | -2.8 |
| Proportional cut | • • • | | |
| Unconstrained, $\overline{\rho}_2 = 1.094$ | 240 | 0.0 | 0.0 |
| $\overline{\rho}_2 = 1.06$ | 240 | 2.0 | -3.2 |
| $\overline{\rho}_2 = 1.03$ | 240 | 2.6 | -6.7 |
| $\overline{\rho}_2 = 1.00$ | 0.0 | 3.6 | -10.1 |

 Table 6: Welfare implications of *Problem 3* responses to different constraints on the arithmetic and trade value weighted averages

a Trade value weighted standard deviation of tariff rates divided by correspondingly weighted average of tariff rates.

b Here a constant coefficient calculation is made (without diminishing returns) using the values of $\underline{v}(\underline{\rho}=1,\underline{w}=1)$. Note that tariff reform reduces equally-weighted welfare in the EU, possibly because the removal of tariffs alone is a second best reform given remaining distortions or, more likely, because the tariffs shift the terms of trade in favour of the EU and hence are less than "optimal" even with equally weighted welfare given the trade elasticities implied by GTAP can be believed. Source: Solutions of *Problem 3* in the text.

Appendix

The assessment of welfare changes in terms of income equivalents should account for changes in both prices and income. What is needed is the income equivalent of a change from price-income vector (P_0, Y_0) to (P_1, Y_1) and this is:

(A1)
$$\Delta W = \left(Y_1 + EV\left(\boldsymbol{P}_{\boldsymbol{\theta}}, \boldsymbol{P}_1, U_1\right)\right) - Y_0 ,$$

where the EV is the dollar amount that the household would be indifferent between accepting in lieu of the change in prices alone and it depends on the change in the expenditure function evaluated at the post price-change level of utility:

(A2)
$$EV(\boldsymbol{P}_{\boldsymbol{\theta}},\boldsymbol{P}_{\boldsymbol{I}},\boldsymbol{U}_{1}) = e(\boldsymbol{P}_{\boldsymbol{\theta}},\boldsymbol{U}_{1}) - e(\boldsymbol{P}_{\boldsymbol{I}},\boldsymbol{U}_{1}).$$

The GTAP household is assumed to have Cobb-Douglas preferences over three expenditure types each of which are CES subaggregates home and foreign goods. The consumer price index in GTAP, used in the analysis in the text, is a composite Cobb-Douglas-CES index consistent with the household's expenditure function. For simplicity of exposition here, consider the single stage Cobb-Douglas case. The expenditure function is defined by the following optimisation problem:

(A3)
$$e(\boldsymbol{P}, U) = \min \sum_{i=1}^{n} P_i X_i \quad subject \ to \ U = \prod_{i=1}^{N} X_i^{\alpha_i} \ge \overline{U} \ .$$

Solving this yields the Hicksian (compensated) demand curves:

(A4)
$$X_i^h = \frac{\alpha_i}{P_i} \frac{\prod_{j=1}^N P_j^{\alpha_j}}{\prod_{j=1}^N \alpha_j^{\alpha_j}} \overline{U}$$

and minimum expenditure becomes:

(A5)
$$e(\boldsymbol{P},U) = \frac{\prod_{j=1}^{N} P_{j}^{\alpha_{j}}}{\prod_{j=1}^{N} \alpha_{j}^{\alpha_{j}}} \overline{U} = \frac{\overline{U}}{\prod_{j=1}^{N} \alpha_{j}^{\alpha_{j}}} \cdot \prod_{j=1}^{N} P_{j}^{\alpha_{j}},$$

which is just a scaled Cobb-Douglas index of prices. Define the consumer price index as:

$$(A6) P_C = \prod_{j=1}^N P_j^{\alpha_j}$$

If $V(\mathbf{P}, Y)$ is the indirect utility function, minimum expenditure is also $e[\mathbf{P}, V(\mathbf{P}, Y)] = Y$. Then:

(A7)
$$\frac{\prod_{j=1}^{N} P_{j}^{\alpha_{j}}}{\prod_{j=1}^{N} \alpha_{j}^{\alpha_{j}}} V(\boldsymbol{P}, \boldsymbol{Y}) = \boldsymbol{Y} \quad \text{and} \quad V(\boldsymbol{P}, \boldsymbol{Y}) = \frac{\prod_{j=1}^{N} \alpha_{j}^{\alpha_{j}}}{\prod_{j=1}^{N} P_{j}^{\alpha_{j}}} \boldsymbol{Y} .$$

This provides all the ingredients for ΔW . First the EV is

$$EV(\boldsymbol{P}_{\boldsymbol{\theta}},\boldsymbol{P}_{\boldsymbol{I}},\boldsymbol{U}_{1}) = e(\boldsymbol{P}_{\boldsymbol{\theta}},\boldsymbol{U}_{1}) - e(\boldsymbol{P}_{\boldsymbol{I}},\boldsymbol{U}_{1}) = \frac{P_{0}^{C}}{\prod_{j=1}^{N} \alpha_{j}^{\alpha_{j}}} \frac{\prod_{j=1}^{N} \alpha_{j}^{\alpha_{j}}}{P_{1}^{C}} Y_{1} - Y_{1}$$

which is:

(A8)
$$EV = Y_1 \left(\frac{P_0^C}{P_1^C} - 1\right) = -Y_1 \left(\frac{\Delta P^C}{P_1^C}\right).$$

The income equivalent of the combined income and price changes is then:

$$\Delta W = Y_1 - Y_0 + EV(P_0^C, P_1^C, Y_1) = Y_1 - Y_0 - Y_1 \frac{\Delta P^C}{P_1^C} ,$$

which can be expressed in proportional change form as:

(A9)
$$\frac{\Delta W}{W} = \frac{Y_1 \left(1 - \frac{\Delta P^C}{P_1^C}\right) - Y_0}{Y_0} \cong \frac{\Delta Y}{Y_0} - \frac{\Delta P^C}{P_1^C} .$$

This is, approximately, the proportional change in real GNP or group income.