NCCC-134
APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

Hedging in Presence of Market Access Risk

by

Glynn T. Tonsor

Suggested citation format:

Tonsor, G. T. 2008. “Hedging in Presence of Market Access Risk.” Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St. Louis, MO. [http://www.farmdoc.uiuc.edu/nccc134].

Hedging in Presence of Market Access Risk

Glynn T. Tonsor*
June 2008

*Working paper presented at the 2008 NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management
St. Louis, Missouri, April 21-22, 2008*

*Assistant Professor (gtonsor@msu.edu) in the Department of Agricultural, Food, and Resource Economics, Michigan State University. We thank Bob Myers and attending participants at the 2008 NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management for helpful comments on an earlier draft of this manuscript. All errors and omissions are the responsibility of the authors.

Hedging in Presence of Market Access Risk

Abstract

Existing literature predominantly assumes perfect knowledge of production methods when deriving optimal futures position hedging rules. This paper relaxes this assumption and recognizes situations where producers interested in hedging may not know the exact input mix that will subsequently be used in their physical operations. This uncertainty is built into a conceptual model subsequently used to demonstrate the impacts of this risk on optimal hedging behavior.

Key words: distiller grains, hedging, market access risk, risk management

Introduction

In many risk management studies, hedging strategies are based on the implicit assumption of perfect knowledge of underlying production methods. Most existing literature has focused solely on price risk, but even those considering yield/production risk assume the production method to be employed is known. In some industries, managers have flexibility in selecting from alternative methods to produce a given output. For instance, an emerging example is how livestock feeding practices are changing with the increasing quantity of distiller's grains with solubles (DGS) being produced by ethanol plants. In particular, DGS are rapidly entering livestock ration formulations (primarily as a partial substitute for corn) leading to a range of alternative input mixes that may be applicable to a given producer.

Livestock producers utilizing DGS not only face price uncertainty, but also uncertainty regarding availability of DGS. In this paper we use the term *access risk* to describe situations of uncertainty regarding market access (e.g., access to production inputs). Livestock producers may have limited access to DGS for an array of reasons. First, DGS exports have exceeded expectations of many forecasters reducing the amount of DGS on the domestic market. Secondly, an increasingly common practice of ethanol plants is to utilize contracts to remove DGS price risk. Utilization of DGS contracts can further reduce the local availability of DGS to livestock producers. Finally, transportation innovations (e.g., development of specialized railcars to improve DGS handling) have provided ethanol plants with additional marketing flexibility further reducing availability at times for local livestock producers.

One could argue that each of these three reasons for limited access simply reflects relative values of DGS and those livestock producers could simply pay more for DGS and resolve these access hurdles. On the other hand, it is important to note that certain producers (for instance those of smaller operational size) may be quantity rationed by ethanol plants as larger (and maybe more regular) volume transactions are preferable for ethanol plant management. A recent survey conducted by the United States Department of Agriculture's National Agricultural Statistics Service (USDA NASS) and the Nebraska Corn Development, Utilization, & Marketing Board provides supporting evidence of this quantity rationing. In particular, the survey found the average size of feedlots utilizing ethanol co-products to be three times as large (1,276 head) as the average size of those not feeding ethanol co-products (416 head). Cattle feedlot operators that are not currently using ethanol co-products indicated that availability is the primary impediment. Collectively, these points suggest that quantity rationing may be experienced by some cattle feedlot operations seeking DGS access.

This research analyzes how uncertainty in the availability of inputs to be utilized at a future point in time impacts producer hedging decisions and effectiveness. We focus on the specific case of a cattle feedlot producer who faces uncertainty regarding availability of future DGS. If the producer is not quantity rationed with respect to DGS he will (assuming favorable prices for current discussion simplicity) purchase both DGS and corn. However, if the availability of DGS is limited, the producer will purchase a higher amount of corn. Stated differently, the producer is uncertain at the time of initiating hedges of the actual feed mix that will be employed.

While this analysis focuses on an application specific to cattle feedlot operators facing access risk regarding DGS, other related examples exist including uncertainty in the biodiesel industry on availability of animal fats (as opposed to traditional soybean oil) as a primary feedstock, or uncertainty in the cattle industry regarding availability of winter wheat for grazing (as opposed to increasing the quantity of forage purchases). The theoretical model used to characterize optimal hedging decisions in the presence of access risk is derived in the same spirit as currently prevailing literature (e.g., Vukina, Li, Holthausen; Haigh and Holt). This analysis also contrasts the optimal hedging rules incorporating market access risk with those in prevailing literature implicitly assuming constant market access. Finally, evaluations based upon historical prices are conducted to further highlight the implications of this market access issue and corresponding hedging behavior in an application to cattle feedlot producers.

Conceptual Model:

Our model is developed assuming the cattle feeding process takes five months to complete (Kastens and Schroeder, 1994). The model specifies feeder cattle price realization to occur two months into the future ($t+2$) from the date initial hedging decisions are made (t). All feed purchases are assumed to be made in the spot market two periods after cattle placement ($t+4$). The final time period ($t+7$) is when the producer sells fattened cattle (the operation's primary output). The assumption of one feed purchasing time and price is imposed to facilitate focus of the analysis on input market access risk rather than risk associated with inputs being purchased over multiple periods, each of which would have access risk. To further focus on the effect of market access risk on optimal hedging decisions, we consider feeder cattle cash purchases and associated hedging to be exogenous and not impacted by the input market access risk of focus in this work. This assumption allows us to keep the problem at hand a two-choice variable problem and to concentrate on the market access issues of interest (e.g., Vukina, Lie, and Holthausen).

The problem facing the livestock producer is further compounded by the fact that he not only faces price risks but faces uncertainty regarding the availability of key inputs. In particular, the producer faces market access risk in the form of by-products (e.g., distiller's grains with solubles from ethanol plants). If the producer has access to such by-products he will purchase both corn and by-products. However, if market access is limited and by-products are unavailable, the producer will purchase a higher amount of corn, in lieu of the unavailable by-product. Stated differently, the producer is uncertain at the time of initiating hedges of the actual feed mix that will be employed.

To access the implications of this market access risk on hedging behavior, our analysis adopts a utility maximization approach where the producer is assumed to have a mean-variance utility function.[1] Consistent with multiple applications in the literature, the mean-variance approach allows us to easily incorporate and focus on the uncertainty regarding market access into our analysis (Kroner and Sultan; Gagnon, Lypney, and McCurdy; Haigh and Holt; Vukina, Li, and Holthausen). Furthermore, work by Lence (1995, 1996) suggests that alternative approaches such as the commonly applied minimum variance hedge (MVH) may be sub-optimal and Garcia, Adam, and Hauser have provided evidence of the usefulness of mean-variance approaches. The problem faced by the producer can be formulated as:

$$(1) \qquad MAX \quad \{ E_t[\pi_{t+7}] - \frac{\theta}{2} * \sigma^2(\pi_{t+7}) \}$$

where E_t denotes the expectation operator based on information available at time *t*, π_{t+7} denotes profit realized at time *t+7*, θ is the risk parameter (positive under risk aversion), and σ^2 denotes the variance operator.[2]

Profit is defined on a per head basis as:

$$(2) \qquad \begin{aligned} \pi_{t+7} &= LC_{t+7}^{CP} * Q^{LC} - C_{t+4}^{CP} * Q_{NO\_DGS}^{C} + \delta_{t+4} * C_{t+4}^{CP} * Q_{DGS}^{C} - \delta_{t+4} * DGS_{t+4}^{CP} * Q^{DGS} \\ &+ f^{LC} * (LC_{t+7}^{FP} - LC_{t}^{FP} - tc^{LC}) + f^{C} * (C_{t+4}^{FP} - C_{t}^{FP} - tc^{C}) - K \end{aligned}$$

where $Q_{NO\_DGS}^{C}$, Q_{DGS}^{C}, and Q^{DGS} are quantities specifying the conversion of inputs (corn fed when DGS are unavailable, corn offset by DGS when DGS are fed, and DGS, respectively) into output in the form of live cattle (Q^{LC}); LC_j^{CP}, C_j^{CP}, and DGS_j^{CP}, represent cash prices of live cattle, corn, and DGS at time *j*, respectively; LC_j^{FP} and C_j^{FP} represent futures market prices for contracts maturing at time *j* of live cattle and corn, respectively; f^{LC} and f^{C} are futures market position choice variables for live cattle and corn, respectively; transaction costs of hedging on the futures market are denoted for live cattle and corn hedges by tc^{LC} and tc^{C}, respectively; δ_{t+4} is a binary variable equal to one if the producer has access to DGS at time *t+4* and zero otherwise; and K denotes other costs assumed fixed in our model (e.g., labor, feeder cattle purchase, etc.). Variables with time subscripts of *t+4* or *t+7* denote unknown, stochastic variables. Formulation of the problem in equations (1) and (2) successively captures the presence of both price and input market access risk.

The variance of profit (as specified in equation (2)) conditional on information available at time *t* can be expressed as:

---

[1] As noted by Chen, Roberts, and Thraen, this framework is equivalent to expected utility if net profits are normally distributed. Meyer has also demonstrated that the mean-variance approach is consistent with expected utility modeling under a weaker set of restrictions.

[2] All expectations and variance terms are conditional on the information set available at time *t*; accompanying notation is omitted for presentation simplicity.

$$
\begin{aligned}
(3)\qquad & \sigma^2(\pi_{t+7}) = Q^{LC^2} * \sigma^2_{LC^{CP}_{t+7}} + Q^{C}_{NO\_DGS}{}^2 * \sigma^2_{C^{CP}_{t+4}} + Q^{C}_{DGS}{}^2 * \sigma^2_{\delta_{t+4}C^{CP}_{t+4}} \\
& + Q^{DGS^2} * \sigma^2_{\delta_{t+4}DGS^{CP}_{t+4}} + (f^{LC})^2 * \sigma^2_{LC^{FP}_{t+7}} + (f^{C})^2 * \sigma^2_{C^{CP}_{t+4}} \\
& + 2Q^{LC}\begin{pmatrix} -Q^{C}_{NO\_DGS}\sigma_{LC^{CP}_{t+7},C^{CP}_{t+4}} + Q^{C}_{DGS}\sigma_{LC^{CP}_{t+7},\delta_{t+4}C^{CP}_{t+4}} - Q^{DGS}\sigma_{LC^{CP}_{t+7},\delta_{t+4}DGS^{CP}_{t+4}} \\ + f^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}} + 2Q^{LC} f^{C}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}} \end{pmatrix} \\
& + 2Q^{C}_{NO\_DGS}\left(-Q^{C}_{DGS}\sigma_{C^{CP}_{t+4},\delta_{t+4}C^{CP}_{t+4}} + Q^{DGS}\sigma_{C^{CP}_{t+4},\delta_{t+4}DGS^{CP}_{t+4}} - f^{LC}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}} - f^{C}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\right) \\
& + 2Q^{C}_{DGS}\left(-Q^{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},\delta_{t+4}DGS^{CP}_{t+4}} + f^{LC}\sigma_{\delta_{t+4}C^{CP}_{t+4},LC^{FP}_{t+7}} + f^{C}\sigma_{\delta_{t+4}C^{CP}_{t+4},C^{FP}_{t+4}}\right) \\
& - 2Q^{DGS}\left(f^{LC}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},LC^{FP}_{t+7}} + f^{C}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},C^{FP}_{t+4}}\right) + 2f^{LC}f^{C}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}
\end{aligned}
$$

where $\sigma^2_a = \text{var}(a)$ and $\sigma_{ab,cd} = \text{cov}(ab,cd)$.

Substituting equations (2) and (3) into equation (1) leaves an expression containing the two choice variables of optimal futures positions (f^{LC} and f^{C}). The first-order conditions for an extremum are:

$$
\begin{aligned}
(4)\qquad & \frac{\partial E_t[U_{t+7}]}{\partial f^{LC}} = E(\Delta LC^{FP}) - \theta\{ f^{LC}\sigma^2_{LC^{FP}_{t+7}} + Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}} - Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}} \\
& + Q^{C}_{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},,LC^{FP}_{t+7}} - Q^{DGS}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},,LC^{FP}_{t+7}} + f^{C}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}\} = 0
\end{aligned}
$$

$$
\begin{aligned}
(5)\qquad & \frac{\partial E_t[U_{t+7}]}{\partial f^{C}} = E(\Delta C^{FP}) - \theta\{ f^{C}\sigma^2_{C^{FP}_{t+4}} + Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}} \\
& - Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}} + Q^{C}_{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},,C^{FP}_{t+4}} - Q^{DGS}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},,C^{FP}_{t+4}} + f^{LC}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}\} = 0
\end{aligned}
$$

where $E(\Delta LC^{FP}) = LC^{FP}_{t+7} - LC^{FP}_{t} - tc^{LC}$ and $E(\Delta C^{FP}) = C^{FP}_{t+4} - C^{FP}_{t} - tc^{C}$. The first-order conditions (4) and (5) are comprised of two linear equations in two unknowns (f^{LC} and f^{C}). Utilizing Cramer's rule the optimal futures market positions (f^{LC^*} and f^{C^*}) are identified as:

(6)

$$
f^{LC^*} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \begin{pmatrix} \frac{E(\Delta LC^{FP})}{\theta\sigma^2_{LC^{FP}_{t+7}}} - \frac{E(\Delta C^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} \\ + \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{C}_{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} \\ + \frac{Q^{DGS}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{Q^{C}_{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{DGS}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \end{pmatrix}
$$

(7)

$$f^{C^*} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \left(\begin{aligned} & \frac{E(\Delta C^{FP})}{\theta\sigma^2_{C^{FP}_{t+4}}} - \frac{E(\Delta LC^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} \\ & + \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{C}_{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} \\ & + \frac{Q^{DGS}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{C}_{DGS}\sigma_{\delta_{t+4}C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{DGS}\sigma_{\delta_{t+4}DGS^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \end{aligned} \right)$$

where $\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}} = \frac{(\sigma_{LC^{FP}_{t+7}C^{FP}_{t+4}})^2}{\sigma^2_{C^{FP}_{t+4}}\sigma^2_{LC^{FP}_{t+7}}}$ is the square of the correlation coefficient between corn and live cattle futures market prices. As noted by Vukina, Li, and Holthausen, existence of optimal hedging positions requires that futures prices not be perfectly correlated (e.g., $1 \neq \rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}}$). To further evaluate the implications of market access risk on optimal hedging positions, it is useful to decompose the covariance terms associated with market access risk. The decomposition utilized here (based on a sufficient but not necessary assumption of multivariate normality) results in: [3]

(8.1) $\sigma_{\delta_{t+4}C^{CP}_{t+4},LC^{FP}_{t+7}} = E[\delta_{t+4}]\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}} + E[C^{CP}_{t+4}]*\sigma_{\delta_{t+4},LC^{FP}_{t+7}}$,

(8.2) $\sigma_{\delta_{t+4}DGS^{CP}_{t+4},LC^{FP}_{t+7}} = E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}} + E[DGS^{CP}_{t+4}]*\sigma_{\delta_{t+4},LC^{FP}_{t+7}}$,

(8.3) $\sigma_{\delta_{t+4}C^{CP}_{t+4},C^{FP}_{t+4}} = E[\delta_{t+4}]\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}} + E[C^{CP}_{t+4}]*\sigma_{\delta_{t+4},C^{FP}_{t+4}}$, and

(8.4) $\sigma_{\delta_{t+4}DGS^{CP}_{t+4},C^{FP}_{t+4}} = E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}} + E[DGS^{CP}_{t+4}]*\sigma_{\delta_{t+4},C^{FP}_{t+4}}$,

where $E[\delta_{t+4}]$ denotes the producers expectation of the random probability of DGS being available at time *t+4*.[4] Incorporation of equations (8.1)-(8.4) into equations (6) and (7) results in:

[3] Readers are referred to Appendix A for additional details on derivation of the covariance terms presented in equations (8.1)-(8.4).

[4] That is, $E[\delta_{t+4}] = X$ implies the producer places a probability of X that he will have access.

$$
\begin{aligned}
f^{LC^*} &= \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} \\
&* \left\{
\begin{array}{l}
\dfrac{E(\Delta LC^{FP})}{\theta\sigma^2_{LC^{FP}_{t+7}}} - \dfrac{E(\Delta C^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} \\
- \dfrac{Q^{C}_{DGS}\left(E[\delta_{t+4}]\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}} + E[C^{CP}_{t+4}]*\sigma_{\delta_{t+4},LC^{FP}_{t+7}}\right)}{\sigma^2_{LC^{FP}_{t+7}}} \\
+ \dfrac{Q^{DGS}\left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},,LC^{FP}_{t+7}} + E[DGS^{CP}_{t+4}]*\sigma_{\delta_{t+4},LC^{FP}_{t+7}}\right)}{\sigma^2_{LC^{FP}_{t+7}}} \\
+ \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\
+ \dfrac{Q^{C}_{DGS}\left(E[\delta_{t+4}]\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}} + E[C^{CP}_{t+4}]*\sigma_{\delta_{t+4},C^{FP}_{t+4}}\right)\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\
- \dfrac{Q^{DGS}\left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}} + E[DGS^{CP}_{t+4}]*\sigma_{\delta_{t+4},C^{FP}_{t+4}}\right)\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}}
\end{array}
\right\}
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
f^{C^*} &= \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} \\
&* \left\{
\begin{array}{l}
\dfrac{E(\Delta C^{FP})}{\theta\sigma^2_{C^{FP}_{t+4}}} - \dfrac{E(\Delta LC^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} \\
- \dfrac{Q^{C}_{DGS}\left(E[\delta_{t+4}]\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}} + E[C^{CP}_{t+4}]*\sigma_{\delta_{t+4},C^{FP}_{t+4}}\right)}{\sigma^2_{C^{FP}_{t+4}}} \\
+ \dfrac{Q^{DGS}\left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}} + E[DGS^{CP}_{t+4}]*\sigma_{\delta_{t+4},C^{FP}_{t+4}}\right)}{\sigma^2_{C^{FP}_{t+4}}} \\
+ \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\
+ \dfrac{Q^{C}_{DGS}\left(E[\delta_{t+4}]\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}} + E[C^{CP}_{t+4}]*\sigma_{\delta_{t+4},LC^{FP}_{t+7}}\right)\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\
- \dfrac{Q^{DGS}\left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}} + E[DGS^{CP}_{t+4}]*\sigma_{\delta_{t+4},LC^{FP}_{t+7}}\right)\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}}
\end{array}
\right\}
\end{aligned}
\tag{10}
$$

. The optimal hedging positions presented in equations (9) and (10) consist of speculative (first two terms in parentheses), risk-minimizing (both own- and cross-price terms), and market access risk components.

Omitting Market Access Risk

To further analyze the impact of market access risk, it is instructive to introduce a series of alternative restricting assumptions and examine how the optimal hedging positions (equations 9 and 10) adjust. First, to compare our model to existing literature omitting market access risk consideration, we evaluate optimal hedging when DGS are known with certainty to not be available (e.g., $Q_{DGS}^{C} = Q^{DGS} = 0; Q_{NO\_DGS}^{C} > 0; E[\delta_{t+4}] = 0; \sigma_{\delta_{t+4}A,B} = 0\ \forall A,B$). In this situation, (9) and (10) reduce to:

$$(11)\ f_{NO-DGS}^{LC^*} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \left\{ \begin{array}{l} \dfrac{E(\Delta LC^{FP})}{\theta\sigma^2_{LC^{FP}_{t+7}}} - \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} - \dfrac{E(\Delta C^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\ + \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \end{array} \right\}$$

$$(12)\ f_{NO-DGS}^{C^*} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \left\{ \begin{array}{l} \dfrac{E(\Delta C^{FP})}{\theta\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} - \dfrac{E(\Delta LC^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\ + \dfrac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \dfrac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \end{array} \right\}$$

If a) the producer is infinitely risk averse ($\theta \sim \infty$) or expected returns from holding futures positions are zero, b) corn and live cattle futures prices are uncorrelated, and c) corn (live cattle) cash prices are uncorrelated with live cattle (corn) futures prices, then (11) and (12) further reduce to:

$$(11.1)\ f_{NO-DGS}^{LC^*} = -\frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} \text{ and}$$

$$(12.1)\ f_{NO-DGS}^{C^*} = \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}}.$$

Note that equations (11.1) and (12.1) are traditional OLS hedge ratios, similar to those presented by Haigh and Holt. Therefore, naïve hedging is optimal only if 1) the producer is infinitely risk averse or expects returns from holding futures positions to be zero; 2) cash and futures prices across commodities are uncorrelated; 3) futures prices across commodities are uncorrelated; *and* 4) distiller's grains are known not to exist.

It is also instructive to consider the other extreme case where DGS are known with certainty to be available ($E[\delta_{t+4}] = 1$) and will be fed (e.g., $Q_{DGS}^{C} > 0; Q^{DGS} > 0; Q_{NO\_DGS}^{C} > 0; \sigma_{\delta_{t+4}a,b} = \sigma_{a,b} \forall a,b; \sigma_{\delta_{t+4},c} = 0 \forall c$) . If we evaluate optimal hedging under these conditions, then (9) and (10) reduce to:

$$(13)\ f_{DGS}^{LC^*} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \left\{ \begin{array}{l} \frac{E(\Delta LC^{FP})}{\theta\sigma^2_{LC^{FP}_{t+7}}} - \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} - \frac{Q^{C}_{DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} \\ + \frac{Q^{DGS}\sigma_{DGS^{CP}_{t+4},,LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} - \frac{E(\Delta C^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\ + \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\ + \frac{Q^{C}_{DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{DGS}\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \end{array} \right\}$$

$$(14)\ f_{DGS}^{C^*} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \left\{ \begin{array}{l} \frac{E(\Delta C^{FP})}{\theta\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} - \frac{Q^{C}_{DGS}\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} \\ + \frac{Q^{DGS}\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} - \frac{E(\Delta LC^{FP})\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\theta\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\ - \frac{Q^{C}_{NO\_DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{C}_{DGS}\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \\ - \frac{Q^{DGS}\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \end{array} \right\}$$

If a) the producer is infinitely risk averse or expected returns from holding futures positions are zero, b) corn and live cattle futures prices are uncorrelated c) corn (live cattle) cash prices are uncorrelated with live cattle (corn) futures prices, *and* d) distiller's grains are known to exist then (13) and (14) further reduce to:

$$(13.1)\ f_{DGS}^{LC^*} = -\frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{Q^{DGS}\sigma_{DGS^{CP}_{t+4},,LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}}$$

$$(14.1)\ f_{DGS}^{C^*} = \frac{\left(Q^{C}_{NO\_DGS} - Q^{C}_{DGS}\right)\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{DGS}\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}}$$

Equation 13.1 reveals the optimal live cattle futures position decision rule under these assumptions to contain both a live cattle own-price risk component (first term) and a DGS cross-price risk component (second term). Similarly equation 14.1 shows the optimal corn futures position decision rule to contain both a corn own-price risk component (first term) and a DGS cross-price risk component (second term). Hence, naïve hedging consistent with analyzes omitting consideration of DGS market access and use (e.g., equations 11.1 and 12.1) is optimal only if a) the producer is infinitely risk averse or expected returns from holding futures positions are zero, b) corn and live cattle futures prices are uncorrelated c) corn (live cattle) cash prices are uncorrelated with live cattle (corn) futures prices, d) DGS cash prices are uncorrelated with live cattle and corn futures prices, *and* e) there is no reduction in corn purchases when DGS products are fed (e.g., $Q^{C}_{DGS} = 0$).

Market Access Risk: Isolated from Speculation and Price Correlation

An alternative comparison of our model to existing research can be made by returning to the general optimal hedging solutions of our model (equation 9 and 10) and focusing more narrowly on market access risk by imposing the restrictions that a) the producer is infinitely risk averse or expected returns from holding futures positions are zero, b) corn and live cattle futures prices are uncorrelated, and c) corn (live cattle) cash prices are uncorrelated with live cattle (corn) futures prices. Imposing these assumptions results in equations (9) and (10) reducing to:

$$(15)\quad f^{LC^*} = -\frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{\left(Q^{DGS}E[DGS^{CP}_{t+4}] - Q^{C}_{DGS}E[C^{CP}_{t+4}]\right)\sigma_{\delta_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{Q^{DGS}E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}}$$

(16)

$$f^{C^*} = \frac{\left(Q^{C}_{NO\_DGS} - Q^{C}_{DGS}E[\delta_{t+4}]\right)\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{\left(Q^{DGS}E[DGS^{CP}_{t+4}] - Q^{C}_{DGS}E[C^{CP}_{t+4}]\right)\sigma_{\delta_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{DGS}E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}}$$

As previously noted, to obtain traditional OLS hedging ratios, we must further assume that DGS cash prices as well as DGS access are totally uncorrelated with live cattle and corn futures prices. Consider the weaker assumption that DGS prices are uncorrelated with both futures contracts but DGS access is correlated with each futures contract[5]:

$$(15.1)\quad f^{LC^*} = -\frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{\left(Q^{DGS}E[DGS^{CP}_{t+4}] - Q^{C}_{DGS}E[C^{CP}_{t+4}]\right)\sigma_{\delta_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}}$$

$$(16.1)\quad f^{C^*} = \frac{\left(Q^{C}_{NO\_DGS} - Q^{C}_{DGS}E[\delta_{t+4}]\right)\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{\left(Q^{DGS}E[DGS^{CP}_{t+4}] - Q^{C}_{DGS}E[C^{CP}_{t+4}]\right)\sigma_{\delta_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}}$$

Equations (15.1) and (16.1) reveal that as long as DGS access is correlated with prices of the two available futures contracts and expenditures on DGS don't precisely equate savings on corn purchases, traditional OLS hedging ratios are not optimal.

For completeness in our evaluation, also consider the alternative assumption that DGS access is uncorrelated with both futures contracts but DGS prices are correlated with each futures contract:

$$(15.2)\quad f^{LC^*} = -\frac{Q^{LC}\sigma_{LC^{CP}_{t+7},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}} + \frac{Q^{DGS}E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}}}{\sigma^2_{LC^{FP}_{t+7}}}$$

$$(16.2)\quad f^{C^*} = \frac{\left(Q^{C}_{NO\_DGS} - Q^{C}_{DGS}E[\delta_{t+4}]\right)\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}} + \frac{Q^{DGS}E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}}{\sigma^2_{C^{FP}_{t+4}}}$$

Equations (15.2) and (16.2) reaffirm that as long as DGS prices are correlated with prices of the two available futures contracts, and some non-zero probability of market access exist, traditional OLS hedging ratios are not optimal as they omit cross-hedging components.

Comparative Statics: Isolated Impacts of Market Access Factors on Hedging Rules

Additional insights can be provided by our model through a series of comparative static evaluations. This paper concentrates on shifts in price correlations and market access components unique to the market access evaluation underlying our objectives. In particular, returning to the general optimal hedging solutions of our model (equations 9 and 10) we identify a set of

[5] This is admittedly an unreasonable assumption in the context of DGS prices. However, this scenario is included here because in the more general "market access risk" sense of other applications, this situation may be applicable.

comparative statics results related to optimal live cattle and corn hedging positions that are presented in Table 1. Note that denominators of the ten expressions in Table 1 are positive as $(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})>0$. Due to the multivariate and market access complexity of our underlying model, none of the presented comparative static results can be unambiguously signed.

A. Correlation of DGS Prices and Futures Market Prices

It is useful to assume at this point that a producer anticipates some non-zero probability of market access and would prefer to feed DGS (e.g., $Q^{DGS}E[\delta_{t+4}]>0$). Based upon this assumption (without which our model collapses to a traditional two-commodity, price-risk hedging model as previously shown), we identify the optimal hedge in live cattle futures to be increasing (less negative) in the covariance of DGS cash prices and live cattle futures prices. This finding is consistent with typical cross-hedging intuition. The effect of correlation between DGS cash prices and each futures prices on the optimal hedge in live cattle and corn futures depends on the covariance between live cattle and corn future prices.

The impact of covariability of DGS cash and live cattle futures prices on optimal corn futures hedge is inversely related to the covariance of live cattle and corn futures prices. Furthermore, the optimal corn futures hedge is increasing in the covariance of DGS cash and corn futures prices.

B. Correlation of Market Access Probability and Futures Market Prices

It is useful again to make a simplifying assumption. In particular, assume at this point that a producer's current set of expectations are such that if DGS are available and fed, the producer anticipates a cost savings. In the absence of changes in the time period of feeding or in quality (and hence price) of animals fed DGS (both of which are assumed nonexistent in this work and currently debated in the animal science literature), it seems unlikely that a rational producer would feed DGS such that total DGS expenditures exceed savings in offset corn purchases. As such, we now assume that expected prices and input quantity selection result in expenditures on DGS being less than the savings in offset corn purchases (e.g., $Q^{DGS}E[DGS^{CP}_{t+4}]<Q^{C}_{DGS}E[C^{CP}_{t+4}]$).

Based upon this assumption, the next set of evaluations in table 1 reveal the optimal live cattle futures hedge to be decreasing with rises in the covariance between probability of DGS market access and live cattle futures prices. The impact of market access and corn futures price correlation on the optimal live cattle hedge is found to depend on the correlation of live cattle and corn futures market prices.

Table 1 also shows the impact of covariability of market access and live cattle futures prices on optimal corn futures hedges to be dependent on the correlation of live cattle and corn futures market prices. Further, we find the optimal corn futures hedge to be decreasing in the covariation of market access and corn futures price, again provided that savings in offset corn purchases exceed associated expenditures on DGS purchases.

C. Expected Probability of Market Access

Since DGS are not known with certainty to be available or unavailable, we also evaluate how optimal hedging positions adjust to changes in expectations regarding availability. Table 1 reveals the effect on both live cattle and corn hedging positions to be ambiguous. This impact depends on the sign and relative valuations of multiple covariance and variance terms as well as the input mix coefficients (e.g., Q_{DGS}^{C} and Q^{DGS}).

D. DGS Use Intensity

Given variation in individual producer preference and perceptions regarding feeding DGS (as well as current disagreement in the animal science literature), we also examine the impact of changes in the selected quantity of DGS that a producer may hold (Q^{DGS}). As in our evaluation of market access probability effects, table 1 shows the effect of a producer altering the DGS quantity to be fed if available to be ambiguous and to depend on the sign and relative valuations of covariance of DGS prices and market access with both futures contract prices, as well as producer expected probability of market access and anticipated DGS prices.

Historical Evaluation:

The discussion above identifies the importance of multiple covariance and variance terms as well as producer expectations on prices and market access likelihood. To gain additional insights on the impact of market access risk on optimal hedging (particularly since many key comparative statics of interest in table 1 can not be unambiguously signed.), we have identified estimates of many of these key parameters by analyzing historical price data. In particular, we have gathered monthly cash and futures market price data from May 1995 to December 2007. These price series are utilized to calculate historical variance and covariance values; starting in January of 1996 we identified variances and correlations using all historical data available at the time.[6] Descriptive statistics of these variables are presented in table 2. To obtain expected corn and live cattle cash price estimates, we followed Taylor, Dhuyvetter, and Kastens and Tonsor, Dhuyvetter, and Mintert, respectively, and used 1 year and 4 year historical average basis values to adjust current futures market prices.

While historical variance/covariance and price estimates are necessary conditions in further evaluating the comparative statics of table 1, they are not sufficient. Additional valuations of input mix quantities (e.g., DGS use intensity and offset corn quantities), producer expectations regarding market access, and covariability of market access probability and each futures market price are also needed, but are not readily available from historical data sources. To proceed we present a set of assumed values for these terms in table 3. In particular, we assume that each key element can take on one of five values. The selection of input mix quantities spans a range of DGS inclusion from 0% to 40% (in 10% increments) of the total ration, Corresponding corn quantities are identified to make all rations equivalent to a diet of 55 corn bushels when DGS are not being utilized.[7] Given that producer expectations regarding future DGS access likely vary, we allow corresponding probability estimates to range from 0% to 100% (in 25% increments). Finally, assumptions must be made about the covariability of market access probability and futures market prices. We considered five alternatives values for these two covariance series. Utilizing mean values from the historical

---

[6] While a multitude of methods are available for identifying variance/covariance terms (e.g., implied volatility, GARCH modeling, etc.) we initially utilized simpler, backward-looking historical methods. This selection stems from our paper's primary interest being in evaluating the impact of market access and not alternative variance forecasting methods.

[7] Here we assume it takes 55 corn bushels to finish the steer in 5 months (*t+2* to *t+7* in our model). For instance, if a producer prefers a DGS inclusion rate of 20%, table 3 implies that 0.36 tons (720 lbs) and 40 bushels of corn (2,240 lbs) would be fed instead of 0 tons of DGS and 55 bushels of corn.

data of the most recent twelve years (1996-2007) presented in table 2 and the assumed values shown in table 3, we now can return to table 1 and attempt to sign our comparative static results of interest.

We first note that a key simplifying assumption we made in our presentation of the comparative statics (and underlying our entire analysis) appears to be reasonable. In particular, use of average prices over the past twelve years and all considered input quantity mixes result in expenditures on DGS being less than the savings in offset corn purchases (e.g., $Q^{DGS}E[DGS_{t+4}^{CP}] < Q_{DGS}^{C}E[C_{t+4}^{CP}]$).

I. Correlation of DGS Prices and Futures Market Prices

Maintaining the assumption that the producer of interest anticipates some non-zero probability of market access and would prefer to feed DGS (e.g., $Q^{DGS}E[\delta_{t+4}] > 0$), both optimal live cattle and corn hedges are increasing in the covariance of their own futures price and DGS cash prices. The optimal live cattle (corn) hedge is increasing in the covariability of DGS cash prices and corn (live cattle) futures prices as the covariance between live cattle and corn future prices is negative (table 2).

II. Correlation of Market Access Probability and Futures Market Prices

The finding of our historical evaluation that use of DGS results in expenditures on DGS being less than savings in offset corn purchases (e.g., $Q^{DGS}E[DGS_{t+4}^{CP}] < Q_{DGS}^{C}E[C_{t+4}^{CP}]$), implies that optimal live cattle and corn hedges are reduced with increases in the covariance between probability of DGS market access and live cattle and corn futures prices, respectively. Similarly, optimal live cattle (corn) hedges are reduced with increases in the covariance between probability of DGS market access and corn (live cattle) futures prices as live cattle and corn futures prices are negatively correlated (table 2).

III. Expected Probability of Market Access

Changes in a producer's expected probability of market access have different directional impacts on optimal live cattle and corn hedging positions. More specifically, based upon our historical evaluation (and for all consider input mix quantities) optimal live cattle and corn hedges increase and decrease, respectively, with enhancement in the probability a producer assigns to market access. As previously noted, the live cattle and corn hedging positions in our model take on negative and positive values, respectively, reflecting the short and long positions that correspond to our profit equation (2). As such, the impact of producers assigning a higher probability to future market access implies that smaller (less negative and less positive, in live cattle and corn respectively) positions become optimal.

IV. DGS Use Intensity

Unlike the above examples, our historical evaluation is unable to unambiguously identify the effect of a producer's selection of DGS intensity use. The optimal live cattle futures hedge increases with DGS use quantities in situations characterized by the covariance of market access probability with both live cattle and corn futures prices being non-negative and at least one covariance being positive (e.g., $\sigma_{\delta_{t+4},LC_{t+7}^{FP}} \geq 0; \sigma_{\delta_{t+4},C_{t+4}^{FP}} \geq 0;$ and $\sigma_{\delta_{t+4},LC_{t+7}^{FP}} > 0$ or $\sigma_{\delta_{t+4},C_{t+4}^{FP}} > 0$). Conversely, if either of the two futures prices is negatively correlated with the probability of market access, then optimal live cattle futures hedges decrease with increases in DGS use.

The previous two statements regarding impacts on live cattle hedging apply regardless of the probability of market access assumed in each evaluation. This differs from corn hedges as the impact of DGS intensity of use on optimal corn hedging is found to be relatively more sensitive to assumed market access probabilities and covariability of access with futures prices. In situations where the covariance of market access probability with both live cattle and corn futures prices is non-negative, the optimal corn futures hedge increases with rising DGS use intensity. However, if either of the two futures prices is negatively correlated with the probability of market access, then the impact of changes in DGS use on optimal corn futures hedges is not unambiguous and depends both on assumed market access probability and specific covariance values.

Conclusions and Implications:

Most existing risk management studies identify hedging strategies based on the implicit assumption of perfect knowledge of underlying production methods. However, in some instances, managers have flexibility in selecting from alternative methods to produce a given output. An example focused on in this analysis is how livestock feeding practices are changing with the increasing quantity of distiller's grains with solubles (DGS) being produced by ethanol plants. In particular, DGS are rapidly entering livestock ration formulations (primarily as a partial substitute for corn) leading to a range of alternative input mixes that may be applicable to a given producer. This paper notes that these livestock producers utilizing DGS not only face price uncertainty, but also uncertainty regarding availability of DGS. In this paper we use the term *access risk* to describe situations of uncertainty regarding market access (e.g., access to production inputs).

This research analyzes how uncertainty in the availability of inputs to be utilized at a future point in time (access risk) impacts producer hedging decisions and effectiveness. The developed model and subsequent analyses demonstrate the impact of incorporating market access risk into optimal hedging rules. While this analysis focused on an application specific to cattle feedlot operators facing access risk regarding DGS, other related examples exist including uncertainty in the biodiesel industry on availability of animal fats (as opposed to traditional soybean oil) as a primary feedstock, or uncertainty in the cattle industry regarding availability of winter wheat for grazing (as opposed to increasing the quantity of forage purchases). Future work could further parameterize the DGS feeding issues analyzed here as additional information becomes available or examine other examples (e.g. biodiesel feedstocks) that may also be characterized by uncertainty on future input mixes.

References

Bohrnstedt, G.W. and A.S. Goldberger. “On the Exact Covariance of Products of Random Variables.” *Journal of the American Statistical Association*. 64:328(Dec. 1969):1439-1442.

Chen, G., M.C. Roberts, and C.S.Thraen. “Managing Dairy Profit Risk Using Weather Derivatives.” *Journal of Agricultural and Resource Economics*. 31(2006):653-666.

Gagnon, L., G.J. Lypney, and T.H. McCurdy. “Hedging Foreign Currency Portfolios.” *Journal of Empirical Finance*. 5(Sept. 1998): 197-220.

Garcia, P., B.D.Adam, and R.J. Hauser. “The Use of Mean-Variance for Commodity Futures and Options Hedging Decisions.” *Journal of Agricultural and Resource Economics*. 19(1994):32-45.

Haigh, M.S. and M.T. Holt. “Hedging Multiple Price Uncertainty in International Grain Trade.” *American Journal of Agricultural Economics*. 82(Nov. 2000): 881-896.

Kastens, T.L. and T.C. Schroeder. (1994). Cattle Feeder Behavior and Feeder Cattle Placements. *Journal of Agricultural and Resource Economics*. 19, 337-348.

Kroner, K.F. and J. Sultan. “Time Varying Distributions and Dynamic Hedging with Foreign Currency Futures.” *Journal of Financial and Quantitative Analysis*. 28(1993): 535-551.

Lence, S.H. “The Economic Value of Minimum-Variance Hedges.” *American Journal of Agricultural Economics*. 77(May 1995): 353-364.

Lence, S.H. “Relaxing the Assumptions of Minimum-Variance Hedging.” *Journal of Agricultural and Resource Economics*. 21(July 1996): 39-55.

Meyer, J. “Two-Moment Decision Models and Expected Utility” *American Economic Review*. 77(1987): 421-430,

Taylor, M.R., K.C. Dhuyvetter, and T.L. Kastens. (2006). Forecasting Crop Basis Using Historical Averages Supplemented with Current Market Information. *Journal of Agricultural and Resource Economics*. 31, 549-567.

Tonsor, G.T., K.C. Dhuyvetter, and J.R. Mintert. (2004). Improving Cattle Basis Forecasting. *Journal of Agricultural and Resource Economics*. 29, 228-241.

United States Department of Agriculture (USDA) National Agricultural Statistics Service (NASS). *Ethanol Co-Products Used for Livestock Feed*. NASS Publication Sy 6-1, June 2007. Accessed on April 7, 2008 at: http://www.nass.usda.gov/Newsroom/2007/06_29_2007_a.asp

Vukina, T. D. Lie, and D.M. Holthausen. “Hedging with Crop Yield Futures: A Mean-Variance Analysis.” *American Journal of Agricultural Economics*. 78(Nov. 1996): 1015-1025.

Table 1. Comparative Static Results

Correlation of DGS Prices and Futures Market Prices

$$\frac{\partial f^{LC^*}}{\partial \sigma_{DGS^{CP}_{t+4},,LC^{FP}_{t+7}}} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \frac{Q^{DGS}E[\delta_{t+4}]}{\sigma^2_{LC^{FP}_{t+7}}} \geq 0$$

$$\frac{\partial f^{LC^*}}{\partial \sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}} = \frac{\partial f^{C^*}}{\partial \sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}}} = \frac{-1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \frac{Q^{DGS}E[\delta_{t+4}]\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} \gtrless 0$$

$$\frac{\partial f^{C^*}}{\partial \sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})} * \frac{Q^{DGS}\left(E[\delta_{t+4}]\right)}{\sigma^2_{C^{FP}_{t+4}}} \geq 0$$

Correlation of Market Access Probability and Futures Market Prices

$$\frac{\partial f^{LC^*}}{\partial \sigma_{\delta_{t+4},LC^{FP}_{t+7}}} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{LC^{FP}_{t+7}}} * \left\{Q^{DGS}E[DGS^{CP}_{t+4}] - Q^{C}_{DGS}E[C^{CP}_{t+4}]\right\} \gtrless 0$$

$$\frac{\partial f^{LC^*}}{\partial \sigma_{\delta_{t+4},C^{FP}_{t+4}}} = \frac{\partial f^{C^*}}{\partial \sigma_{\delta_{t+4},LC^{FP}_{t+7}}} = \frac{\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} * \left\{Q^{C}_{DGS}E[C^{CP}_{t+4}] - Q^{DGS}E[DGS^{CP}_{t+4}]\right\} \gtrless 0$$

$$\frac{\partial f^{C^*}}{\partial \sigma_{\delta_{t+4},C^{FP}_{t+4}}} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{C^{FP}_{t+4}}} * \left\{Q^{DGS}E[DGS^{CP}_{t+4}] - Q^{C}_{DGS}E[C^{CP}_{t+4}]\right\} \gtrless 0$$

Expected Probability of Market Access

$$\frac{\partial f^{LC^*}}{\partial E(\delta_{t+4})} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} * \left\{\begin{array}{l} Q^{C}_{DGS}\left[\sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7}C^{FP}_{t+4}} - \sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}\right] \\ + Q^{DGS}\left[\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}} - \sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}\sigma_{LC^{FP}_{t+7}C^{FP}_{t+4}}\right] \end{array}\right\} \gtrless 0$$

$$\frac{\partial f^{C^*}}{\partial E(\delta_{t+4})} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} * \left\{\begin{array}{l} Q^{C}_{DGS}\left[\sigma_{C^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}} - \sigma_{C^{CP}_{t+4},C^{FP}_{t+4}}\sigma^2_{LC^{FP}_{t+7}}\right] \\ + Q^{DGS}\left[\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}}\sigma^2_{LC^{FP}_{t+7}} - \sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}}\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}}\right] \end{array}\right\} \gtrless 0$$

DGS Use Intensity

$$\frac{\partial f^{LC}}{\partial Q^{DGS}} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} * \left\{\begin{array}{l} +\left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},,LC^{FP}_{t+7}} + E[DGS^{CP}_{t+4}] * \sigma_{\delta_{t+4},LC^{FP}_{t+7}}\right)\sigma^2_{C^{FP}_{t+4}} \\ -\left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}} + E[DGS^{CP}_{t+4}] * \sigma_{\delta_{t+4},C^{FP}_{t+4}}\right)\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}} \end{array}\right\} \gtrless 0$$

$$\frac{\partial f^{C}}{\partial Q^{DGS}} = \frac{1}{(1-\rho^2_{LC^{FP}_{t+7},C^{FP}_{t+7}})\sigma^2_{LC^{FP}_{t+7}}\sigma^2_{C^{FP}_{t+4}}} * \left\{\begin{array}{l} \left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},C^{FP}_{t+4}} + E[DGS^{CP}_{t+4}] * \sigma_{\delta_{t+4},C^{FP}_{t+4}}\right)\sigma^2_{LC^{FP}_{t+7}} - \\ \left(E[\delta_{t+4}]\sigma_{DGS^{CP}_{t+4},LC^{FP}_{t+7}} + E[DGS^{CP}_{t+4}] * \sigma_{\delta_{t+4},LC^{FP}_{t+7}}\right)\sigma_{LC^{FP}_{t+7},C^{FP}_{t+4}} \end{array}\right\} \gtrless 0$$

Table 2. Descriptive Statistics of Variables (January 1996 – December 2007)*

	Mean	*Minimum*	*Maximum*	*Standard Deviation*
CASH PRICES[a]				
DGS^{CP}	100.97	67.00	189.17	26.05
C^{CP}	2.52	1.63	4.98	0.69
LC^{CP}	75.18	58.02	100.91	11.26
FUTURES MARKET PRICES[a]				
C_t^{FP}	2.56	1.78	4.94	0.67
C_{t+4}^{FP}	2.61	1.90	4.37	0.59
LC_t^{FP}	75.60	59.79	98.30	10.83
LC_{t+7}^{FP}	75.54	61.02	100.89	9.90
HISTORICAL VARIANCES[b]				
$\sigma^2_{LC_{t+7}^{FP}}$	24.68	1.90	97.12	23.13
$\sigma^2_{LC_{t+7}^{CP}}$	40.39	3.31	124.85	39.32
$\sigma^2_{C_{t+4}^{FP}}$	0.26	0.09	0.33	0.04
$\sigma^2_{C_{t+4}^{CP}}$	0.51	0.09	0.69	0.10
$\sigma^2_{DGS_{t+4}^{CP}}$	771.64	456.12	1,003.89	135.88
HISTORICAL CORRELATIONS[b]				
$\sigma_{LC_{t+7}^{FP}, C_{t+4}^{FP}}$	-0.39	-0.58	0.19	0.16
$\sigma_{LC_{t+7}^{CP}, LC_{t+7}^{FP}}$	0.96	0.92	0.99	0.02
$\sigma_{C_{t+4}^{CP}, LC_{t+7}^{FP}}$	-0.39	-0.58	0.14	0.16
$\sigma_{LC_{t+7}^{CP}, C_{t+4}^{FP}}$	-0.32	-0.51	0.21	0.15
$\sigma_{C_{t+4}^{CP}, C_{t+4}^{FP}}$	0.98	0.96	1.00	0.01
$\sigma_{DGS_{t+4}^{CP}, C_{t+4}^{FP}}$	0.81	0.60	0.97	0.08
$\sigma_{DGS_{t+4}^{CP}, LC_{t+7}^{FP}}$	-0.34	-0.47	0.03	0.10

*Variable names are defined on pages 5 and 6 as they appear in equations 2 and 3.

[a] Distiller's grains (DGS), corn (C), and live cattle (LC) prices are in \$/ton, \$/bushel, and \$/cwt units, respectively.

[b] Historical variance and correlation values are long-run valuations derived using all data available at the time of the forecast, beginning with May 1995 observations.

Table 3. Assumed Range of Possible Valuations of Unknown Variables*

	Q^{DGS}	Q^{C}_{DGS}	$Q^{C}_{NO\_DGS}$
Value #1	0.00	0.00	55.00
Value #2	0.18	7.50	47.50
Value #3	0.36	15.00	40.00
Value #4	0.54	22.50	32.50
Value #5	0.72	30.00	25.00
	$E[\delta_{t+4}]$	$\sigma_{\delta_{t+4},LC^{FP}_{t+7}}$	$\sigma_{\delta_{t+4},C^{FP}_{t+4}}$
Value #1	0.00	-1.00	-1.00
Value #2	0.25	-0.50	-0.50
Value #3	0.50	0.00	0.00
Value #4	0.75	0.50	0.50
Value #5	1.00	1.00	1.00

*Variable names are defined on pages 5 and 6 as they appear in equations 2 and 3.

Appendix A:

The decomposition of covariance terms associated with market access risk (equations 8.1-8.4) stem from Bohrnstedt and Goldberger. For instance, let $\sigma_{\delta_{t+4}C^{CP}_{t+4}LC^{FP}_{t+7}} = \text{cov}(\delta_{t+4}C^{CP}_{t+4}, LC^{FP}_{t+7})$. Following Bohrnstedt and Goldberger (see equation 12), $\text{cov}(\delta_{t+4}C^{CP}_{t+4}, LC^{FP}_{t+7})$ can be expanded to:

$$\text{cov}(\delta_{t+4} * C^{CP}_{t+4}, LC^{FP}_{t+7}) = E[\delta_{t+4}C^{CP}_{t+4} - E(\delta_{t+4}C^{CP}_{t+4})][LC^{FP}_{t+7} - E(LC^{FP}_{t+7})]$$

$$= E(\delta_{t+4})\,\text{cov}(C^{CP}_{t+4}, LC^{FP}_{t+7}) + E(C^{CP}_{t+4})\,\text{cov}(\delta_{t+4}, LC^{FP}_{t+7}) + E[\Delta\delta_{t+4} * \Delta C^{CP}_{t+4} * \Delta LC^{FP}_{t+7}]$$

where $\Delta\delta_{t+4} = \delta_{t+4} - E[\delta_{t+4}]$, $\Delta C^{CP}_{t+4} = C^{CP}_{t+4} - E[C^{CP}_{t+4}]$,and $\Delta LC^{FP}_{t+7} = LC^{FP}_{t+7} - E[LC^{FP}_{t+7}]$. Bohrnstedt and Goldberger show that under the assumption of multivariate normality $E[\Delta\delta_{t+4} * \Delta C^{CP}_{t+4} * \Delta LC^{FP}_{t+7}] = 0$. A weaker, but sufficient assumption of $E[\Delta\delta_{t+4}] = E[\Delta C^{CP}_{t+4}] = E[\Delta LC^{FP}_{t+7}] = 0$ also yields $E[\Delta\delta_{t+4} * \Delta C^{CP}_{t+4} * \Delta LC^{FP}_{t+7}] = 0$. Either assumption results in the following decomposition:

$\text{cov}(\delta_{t+4} * C^{CP}_{t+4}, LC^{FP}_{t+7}) = E(\delta_{t+4})COV(C^{CP}_{t+4}, LC^{FP}_{t+7}) + E(C^{CP}_{t+4})COV(\delta_{t+4}, LC^{FP}_{t+7})$. The same technique can be utilized to derive equations (8.2)-(8.4).

Appendix B

This pertains to the changes in covariance terms when market access is known with certainty:

If $E[\delta_{t+4}] = 1$ THEN: $\sigma_{\delta_{t+4}C^{CP}_{t+4}, C^{FP}_{t+4}} = E[\delta_{t+4}]\sigma_{C^{CP}_{t+4}, C^{FP}_{t+4}} + E[C^{CP}_{t+4}]\sigma_{\delta_{t+4}, C^{FP}_{t+4}} + E[\Delta\delta_{t+4}, \Delta C^{CP}_{t+4}, \Delta C^{FP}_{t+4}]$.

Imposing the assumption of multivariate normality ($E[\Delta\delta_{t+4}, \Delta C^{CP}_{t+4}, \Delta C^{FP}_{t+4}] = 0$) or the weaker assumption of $E[\Delta\delta_{t+4}] = E[\Delta C^{CP}_{t+4}] = E[\Delta C^{FP}_{t+4}] = 0$, and noting $E[\delta_{t+4}] = 1$, results in:

$\sigma_{\delta_{t+4}C^{CP}_{t+4}, C^{FP}_{t+4}} = \sigma_{C^{CP}_{t+4}, C^{FP}_{t+4}}$.