Estimating a Demand System with Seasonally Differenced Data

Ardian Harri
Mississippi State University

Andrew Muhammad
Mississippi State University

John D. Anderson
Mississippi State University

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Abstract

Researchers estimating demand systems have often used annual data even though monthly or quarterly data are available. Monthly data may be avoided because with monthly data it becomes more difficult to specify seasonality, autocorrelation is more likely to be significant, and there is a greater chance of finding significant dynamics in demand. This paper shows how to obtain consistent and asymptotically efficient estimates of a demand system using seasonal differenced data. It also shows that several alternative estimators are either inefficient or implausible for demand systems.

Key Words: demand system, seasonal differences, autocorrelation, Monte Carlo

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1. Introduction

Researchers estimating demand systems have often used annual (Chavas 1983), Duffy (1987), Brown, Lee and Seale (1995), Eales, Durham and Wessells (1997), Brown and Lee (2000), Seale and Marchant (2003), Seale, Marchant and Basso (2003), Muhammad (2007), (Moschini and Meilke; Alston and Chalfant; Eales and Unnevehr; and Mutondo and Henneberry) even though monthly or quarterly data are available. Monthly data may be avoided because with monthly data it becomes more difficult to specify seasonality, autocorrelation is more likely to be significant, and there is a greater chance of finding significant dynamics in demand.

Seasonality is commonly assumed to be present in budget shares in the estimation of demand systems. A common assumption is that seasonality is deterministic and thus is accounted for by the use of seasonal dummies. However, the use of dummy variables to account for seasonality may be inappropriate. As noted by Fraser and Moosa (2002), “assuming seasonality is deterministic when it is actually stochastic will yield a misspecified model” (p. 83).

With deterministic seasonality, the intercepts as well as the parameters for the dummy variables are assumed constant. However, changes in tastes and preferences may cause these parameters to change over time. The changes in the parameters may be sudden or gradual over time. This means that assuming deterministic seasonality may lead to models that are misspecified and fail the tests of structural stability.

Therefore, another alternative is to estimate the general model used by Fraser and Moosa (2002) that nests the deterministic and stochastic seasonality. Yet, the assumption of stochastic seasonality is not without limitations. The reader is referred to Fraser and Moosa (2202) for a discussion of these limitations.

The conclusion from the discussions so far is that there is no agreement on the appropriate form of seasonality in the estimation of demand systems. Moreover, each form is not without limitations. Researchers therefore let the data determine the form and locations of seasonality components (Arnade, Pick, and Gehlhar, 2004).

As an alternative and a mean to eliminate altogether of dealing with seasonality, a number of researchers have used seasonal difference models. These models let the researchers use the higher frequency data, do not require specifying the form of seasonality, and are not likely to show significant dynamic effects in demand. But, as we show, such models are autocorrelated with the degree of autocorrelation depending on the level of seasonal differencing.

The reason for this is that the use of annual differences when quarterly or monthly data are available leads to the problem of overlapping data. The econometric problem resulting from the use of overlapping data is the moving average (MA) autocorrelation which results in inefficient estimates and biased hypothesis tests. Harri and Brorsen (2007) compare different estimators used with overlapping data in the context of the univariate equation model. They show that when lagged values of the dependent variables are not included as explanatory variables, the GLS estimator is the appropriate estimator. The covariance matrix for the GLS transformation can be derived analytically in the case of overlapping data.

In this paper, we show how to obtain consistent and asymptotically efficient estimates of a demand system using seasonal differenced data. Specifically, we propose a GLS estimator for estimating a system of equations with overlapping data. Monte Carlo simulations are used to compare the properties of the GLS estimator with overlapping data (annual differences) and the conventional SUR estimator with disaggregate data (monthly observations). Alternative estimators are also considered like an SUR estimator using non-overlapping and the maximum likelihood estimator developed by Beach and MacKinnon (1979).

The rest of the paper is organized as follows. Section two derives the GLS estimator. Section three discusses the Monte Carlo simulation. Section four provides an empirical application to the case of US meat demand. Section five concludes.

2. The Model

We start with the following system of M equations:

$$\boldsymbol{w_m} = \boldsymbol{\alpha} + \boldsymbol{Z_m \beta_m} + \boldsymbol{D_m \gamma_m} + \boldsymbol{\varepsilon_m}, \quad \boldsymbol{m = 1,\dots,M} \qquad (1)$$

where $\boldsymbol{w}_m$ is a (T * 1) vector of the values of the dependent variable, where T is the length of time series, $\boldsymbol{Z}_m$ is a (T * l_m) matrix of the values of the explanatory variables, $\boldsymbol{D}_m$ is a (T * p_m) matrix of the values of the p dummy variables with p = 11 for monthly data and p = 3 for quarterly data, $\boldsymbol{\beta}_m$ and γ_m are respectively a (l_m * 1) and a (p_m * 1) vectors of regression coefficients, and $\boldsymbol{\varepsilon}_m$ is a (T * 1) vector of the disturbances. We assume that $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_1', \boldsymbol{\varepsilon}_2', \dots, \boldsymbol{\varepsilon}_M']'$ has $E[\boldsymbol{\varepsilon}] = 0$ and $E[\boldsymbol{\varepsilon\varepsilon'}] = \boldsymbol{\Sigma}$. We further assume that disturbances are uncorrelated across observations, but have contemporaneous covariance $\boldsymbol{V}$. In other words, $E[\boldsymbol{\varepsilon}_{mt}\boldsymbol{\varepsilon}_{ns}] = \boldsymbol{\sigma}_{mn}$, if $t = s$ and zero otherwise. Therefore, we can also write $\boldsymbol{\Sigma} = \boldsymbol{V \Theta I_T}$.

We will refer to the system in (1) as the disaggregate model which, depending on the available data, can be estimated with either monthly differences or quarterly differences. If one instead uses annual differences, these annual differences represent an aggregation of level k=12 for monthly differences or k=4 for quarterly differences. The system with the aggregated variables can be represented as:

$$\boldsymbol{y}_m = \boldsymbol{X}_m \boldsymbol{\beta}_m + \boldsymbol{u}_m, \quad \boldsymbol{m} = \mathbf{1}, \ldots, \boldsymbol{M} \qquad (2)$$

where

$$y_{tm} = \sum_{j=t}^{t+k-1} w_{jm}, \; X_{tm} = \sum_{j=t}^{t+k-1} Z_{jm}, \; u_{tm} = \sum_{j=t}^{t+k-1} \varepsilon_{jm} \qquad (3)$$

where, k is as previously defined. Given the size of the original sample, T, the new sample size is T-k+1. Note also that the seasonal dummy variables no longer appear in (2). The aggregation of the variables in (3) induces an MA process of order k-1 in the error term $\boldsymbol{u}_m$ in (2).

From the assumption that the original error terms were uncorrelated with zero mean, it follows that:

$$E[u_{tm}] = E[\sum_{j=0}^{k-1} \varepsilon_{(t+j)m}] = \sum_{j=0}^{k-1} E[\varepsilon_{(t+j)m}] = 0 \qquad (4)$$

Also, since the successive values of ε_{jm} are homoskedastic and uncorrelated, the unconditional variance of u_{tm} is:

$$\text{var}[u_{tm}] = \sigma_u^2 = E[\varepsilon_{tm}^2] = k\sigma_{\varepsilon_m}^2 \qquad (5)$$

Based on the fact that two different error terms, u_{tm} and $u_{(t+s)m}$, (t = 1, ..., T and s = t+1, ..., T) have $k - s$ common original error terms, ε_m, for any $k - s > 0$, the covariances between the error terms in (2) are:

$$\text{cov}[u_{tm}, u_{(t+s)m}] = E[u_{tm}, u_{(t+s)m}] = (k-s)\sigma_{\varepsilon_m}^2 \qquad \forall (k-s) > 0 \qquad (6)$$

Similarly, the contemporaneous covariances between the error terms in (2) are:

$$\text{cov}[u_{tm}, u_{sn}] = E[u_{tm}, u_{sn}] = E[\sum_{j=0}^{k-1} \varepsilon_{(t+j)m}, \sum_{j=0}^{k-1} E[\varepsilon_{(s+j)n}] = k\sigma_{mn} \quad \forall t = s \quad (7)$$

Dividing (6) by (5) we get the correlations between two different error terms, u_{tm} and $u_{(t+s)m}$ as follows:

$$corr\ [u_{tm}, u_{(t+s)m}] = \frac{k-s}{k} \quad \forall (k-s) > 0 \qquad (8)$$

Collecting terms we have the correlation matrix of each $\boldsymbol{u}_m$, $\boldsymbol{\Omega}$ (T-k+1 * T-k+1) as:

$$\Omega = \begin{bmatrix} 1 & \frac{k-1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{1}{k} & 0 & \ldots & 0 \\ \frac{k-1}{k} & 1 & \frac{k-1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{1}{k} & \ldots & 0 \\ \ldots & \frac{k-1}{k} & 1 & \frac{k-1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{1}{k} & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \frac{1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{k-1}{k} & 1 & \frac{k-1}{k} & \ldots \\ 0 & \ldots & \frac{1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{k-1}{k} & 1 & \frac{k-1}{k} \\ 0 & \ldots & 0 & \frac{1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{k-1}{k} & 1 \end{bmatrix}$$

With $\boldsymbol{\Omega}$ defined as above, we can express the covariance matrix for

$\boldsymbol{u} = [\boldsymbol{u}_1', \boldsymbol{u}_2', \ldots, \boldsymbol{u}_M']'$ as $E[\boldsymbol{uu'}] = \boldsymbol{\Sigma_u} = k\boldsymbol{V\Theta\Omega}$.

To obtain efficient estimates, the generalized least squares (GLS) parameter estimates can be derived as follows:

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X'\Sigma_u}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X'\Sigma_u}^{-1}\boldsymbol{y}$$

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X'}(\frac{\mathbf{1}}{\boldsymbol{L}+\mathbf{1}}\boldsymbol{V}^{-1} \otimes \boldsymbol{\Omega}^{-1})\boldsymbol{X})^{-1}\boldsymbol{X'}(\frac{\mathbf{1}}{\boldsymbol{L}+\mathbf{1}}\boldsymbol{V}^{-1} \otimes \boldsymbol{\Omega}^{-1})\boldsymbol{y}$$

or

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X'V^{-1} \otimes \Omega^{-1} X})^{-1} \boldsymbol{X'V^{-1} \otimes \Omega^{-1}})\boldsymbol{y} \qquad (9)$$

Let $\boldsymbol{P'} = \boldsymbol{C\Lambda}^{-\frac{1}{2}}$, where $\boldsymbol{C}$ is the matrix of the eigenvectors of $\boldsymbol{\Omega}$ and $\boldsymbol{\Lambda}$ is the diagonal matrix containing the eigenvalues of $\boldsymbol{\Omega}$. Then, $\boldsymbol{\Omega}^{-1} = \boldsymbol{P'P}$. Substituting this into (9) and rearranging we obtain:

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X'P'(I \otimes V^{-1})PX})^{-1} \boldsymbol{X'P'(I \otimes V^{-1})Py} \qquad (10)$$

where $\boldsymbol{I} = \boldsymbol{I}_{T\_L}$. Let $\boldsymbol{X^*}=\boldsymbol{PX}$ and $\boldsymbol{y^*}=\boldsymbol{Py}$ we get:

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X^*(I \otimes V^{-1})X^*})^{-1} \boldsymbol{X^*(I \otimes V^{-1})y^*} \qquad (11)$$

which is the conventional seemingly unrelated equations (SUR) estimator with an unknown contemporaneous covariance matrix $\boldsymbol{V}$ with the transformed variables $\boldsymbol{X^*}$ and $\boldsymbol{y^*}$.

Similarly the variance-covariance matrix of the GLS estimates from (11) is:

$$\boldsymbol{Var[\hat{\beta}]} = \sigma_{\varepsilon}^{2}(\boldsymbol{X^{*\prime}(I \otimes V^{-1})X^*})^{-1} \qquad (12)$$

Alternative Estimators

Among alternative estimators, an obvious estimator is the one that uses non-overlapping data. In other words only the *k*-th observation from (2) will be used in the estimation. This will eliminate the issue of MA autocorrelation, but the estimator is inefficient since it does not use all available information.

Another alternative estimator is the maximum likelihood estimator developed by Beach and MacKinnon. This estimator (from hereon referred to as the AR(1) estimator) imposes the same AR(1) parameter for all *m* equations. In the general case considered by Beach and MacKinnon the AR(1) parameter needs to be estimated. However, in our case this parameter can be derived analytically. It is (k-1)/k, which is the first off-diagonal term in the Ω matrix. In other words, the AR(1) model in this case uses a form of Ω where only the first off-diagonal term is positive while the others are set to zero. This estimator is therefore inefficient too since it does not account fully for the autocorrelation present in the error term.

Finally, the seasonal difference model of Box and Jenkins (1970), which is called a seasonal unit root model in more recent literature, uses data which are in some sense overlapping, but do not create an overlapping data problem if correctly specified. For annual data, the seasonal unit root model is

$$\begin{aligned} \omega_t &= a\kappa_t + \eta_t \\ \eta_t &= \eta_{t-12} + \xi_t \end{aligned} \qquad (13)$$

where ξ_t is i.i.d. normal. In this case, the disaggregate model

$$\omega_t - \omega_{t-12} = \alpha(\kappa_t - \kappa_{t-12}) + \xi_t \qquad (14)$$

has no autocorrelation. In this example, twelfth differencing leads to a model that can be estimated using overlapping data and ordinary least squares. Seasonal unit roots have largely been used when the research objective was forecasting (e.g. Clements and Hendry 1997). One problem with the seasonal unit root model is that it is often rejected in empirical work (e.g. McDougall 1995). Another is that it implies that each month has its own independent unit root process and so each month’s price can wonder aimlessly away from the prices of the other months. Such a model seems implausible for most economic time series. Hylleberg et al. suggest that the seasonal unit roots may be cointegrated, and in the case of the demand systems the adding up condition would impose some type of cointegration which can overcome the criticism of one month’s price moving aimlessly away from another month’s price. Wang and Tomek

(2007) present another challenge to the seasonal unit root model since they argue that commodity prices should not have any unit roots. While a seasonal unit root model may be an unlikely model, if it is the true model, it does not create an overlapping data problem.

In this section we showed how to obtain consistent and asymptotically efficient estimates of a demand system using seasonal differenced data. We also showed that two of the alternative estimators are inefficient while the seasonal difference model of the Box-Jenkins type seems implausible for demand systems.

3. Monte Carlo Simulation

In this section we discuss the Monte Carlo study used to compare the properties of the proposed estimator and alternative estimators. We generate the data according to (1). We use a system of three equations and thus $\boldsymbol{Z}_m$ consists of three correlated log prices series, P_1, P_2, P_3, and an exogenous variable representing the log of the ratio of expenditures on the price index, $ln(X/P)$. $\boldsymbol{D}_m$ consists of four quarterly or twelve monthly fixed dummy variables that satisfy the following conditions:

$$\sum_{j=1}^{k} d_j = 0 \quad and \quad \sum_{i=1}^{m} d_i = 0$$

where, k and m are as previously defined. The second condition is to impose the adding up restriction. In addition, to ensure the adding up restriction we impose these three other conditions:

$$\sum_{i=1}^{m} \alpha_i = 1, \quad \sum_{i=1}^{m} w_i = 1, \quad and \quad \sum_{i=1}^{m} \varepsilon_i = 0$$

where α_i represent the intercept for the i^{th} equation. In case one of the three shares is negative then that system observation is regenerated with a different draw of correlated random errors until all three shares are positive. Finally, the homogeneity and symmetry restrictions are imposed on the parameters of the system.

We generate 1000 samples of 60 and 120 observations according to (1). We obtain aggregate observations according to (2) using two different levels of aggregation, k=12 for monthly observations and k=4 for quarterly observations. We estimate both (1), from now on to be referred as the disaggregate model, and (11), from now on to be referred as the overlapping model, for each sample. We also estimate the model using nonoverlapping (to be referred as the NON model) observations by using only the k^{th} aggregate observations. Finally, we obtain the maximum likelihood estimates for the AR(1) model in (2) by imposing the same AR(1) parameter for each equation equal to (k-1)/k.

3. Monte Carlo Results

The actual slope parameters and the means of their Monte Carlo estimates and standard errors from all the models are presented in Table 1. We report results only for one equation, since the results are very similar. Three main findings are to be noted from the results in table 1. First, slope estimates from all models are consistent as expected. Second, the slope estimates and their standard errors are exactly the same for the disaggregate model and the aggregate model with the proposed GLS estimator. This finding is consistent with the theoretical results presented above. Third, the standard deviations of both the model estimated with non-overlapping data and the AR(1) model are larger than those of the disaggregate model and the aggregate model with GLS. On average, the standard errors of the AR(1) model are 18 to 30 percent larger, while those of the model with non-overlapping data are from 2 to 4.65 times larger.

Table 2 reports the number of rejections of the hypothesis that estimated parameters are equal to the actual values for the significance level of 5 percent. The number of rejections is twice as large as the nominal level for the AR(1) model. It is also almost twice as large for the model with non-overlapping data when the ratio of sample size to aggregation level is small. In the meantime the rejection rates for the aggregate model with GLS (and the disaggregate model which are not reported as they are the same as the ones for the aggregate model) are very close to the nominal level.

4. Empirical Application

We estimate the U.S. meat demand to compare the empirical performance of the different estimation models discussed above. Data are monthly observations from January 1989 to August 2007. Per capita beef, pork, and poultry quantities and retail prices were obtained from USDA' *Livestock and Poultry Situation and Outlook Reports*. Per capita fish quantities and retail prices were derived using the approach in Schmitz and Capps (p. 10) and Kinnucan et al. (1997). Bryant and Davis (2008) using the Bayesian Averaging of Classical Estimates (BACE) approach find that the first differenced Almost Ideal Demand (FDAID) model outperforms the other models considered in their analysis. Therefore, we use FDAID as our functional form. The fish equation is dropped from the estimation. Finally, since the test of the symmetry hypothesis does not reject it we impose the symmetry.

Results of the U.S. meat demand are reported in table 3. Table 3 reports parameter estimates and their standard errors for the four different models and for the three equations of beef, pork and poultry. Parameter estimates and their standard errors for the disaggregate model and the aggregate model with GLS are very similar for the three estimated equations. Results for the AR(1) model and the model that uses nonoverlapping data confirm their inefficiency, as shown by higher standard errors and lower significance levels.

5. Conclusions

Estimation of demand systems with seasonal (annual or quarterly) differenced data leads to models which are autocorrelated with the degree of correlation depending on the level of differencing. Ignoring this autocorrelation results in inefficient estimates and biased hypothesis tests. The Beach and MacKinnon estimator, used in some previous works, is also inefficient in this case and so is the estimator that uses nonoverlapping data.

We show how to obtain consistent and asymptotically efficient estimates of a demand system using seasonal differenced data. Monte Carlo simulations confirm the theoretical derivation that a GLS estimator using an analytically derived correlation matrix produces consistent and efficient estimates.

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Table 1. Monte Carlo Simulation Results

Sample Size	Aggregation Level	Variable	Actual Parameter Values	Disaggregate Model		Overlapping Model		Nonoverlapping Model		AR(1) Model	
				Parameter Estimates	Standard Error	Parameter Estimates	Standard Error	Parameter Estimates	Standard Error	Parameter Estimates	Standard Error
60	4	P1	0.02	0.0200	0.0141	0.0200	0.0141	0.0208	0.0320	0.0204	0.0176
		P2	0.03	0.0311	0.0205	0.0311	0.0205	0.02899	0.0468	0.0308	0.0252
		P3	-0.05	-0.0514	0.0290	-0.0514	0.02895	-0.0491	0.0656	-0.0516	0.0360
		ln(X/P)	0.025	0.0250	0.0092	0.0250	0.0092	0.0246	0.0211	0.0248	0.0112
60	12	P1	0.02	0.01997	0.0153	0.01997	0.0153	-	-	0.0195	0.0191
		P2	0.03	0.0314	0.0223	0.0314	0.0223	-	-	0.0318	0.0270
		P3	-0.05	-0.0516	0.0313	-0.0516	0.0313	-	-	-0.0510	0.0379
		ln(X/P)	0.025	0.0251	0.00997	0.0251	0.00997	-	-	0.0252	0.0122
120	4	P1	0.02	0.0197	0.00996	0.0197	0.00996	0.0201	0.0204	0.0199	0.0122
		P2	0.03	0.0304	0.0140	0.0304	0.0140	0.0305	0.02999	0.0304	0.0172
		P3	-0.05	-0.0498	0.0199	-0.0498	0.0199	-0.0504	0.042	-0.0503	0.0241
		ln(X/P)	0.025	0.0248	0.006	0.0248	0.006	0.0245	0.0134	0.0247	0.0075
120	12	P1	0.02	0.0196	0.00998	0.0196	0.00998	0.0205	0.0452	0.0197	0.0120
		P2	0.03	0.0305	0.0145	0.0305	0.0145	0.0306	0.0674	0.0304	0.0173
		P3	-0.05	-0.0502	0.0206	-0.0502	0.0206	-0.0506	0.0951	-0.0501	0.0242
		ln(X/P)	0.025	0.0248	0.0065	0.0248	0.0065	0.0237	0.0304	0.0247	0.0077

Table 2. Rejection levels of the hypothesis that estimated parameters are equal to their actual values.

Sample Size	Aggregation Level	Nominal Level	Variable	Rejection Level: Overlapping Model	Nonoverlapping Model	AR(1) Model
60	4	0.05	P1	0.051	0.057	0.114
		0.05	P2	0.054	0.046	0.118
		0.05	P3	0.044	0.049	0.118
		0.05	Ln(X/P)	0.054	0.043	0.116
60	12	0.05	P1	0.056		0.122
		0.05	P2	0.056		0.102
		0.05	P3	0.047		0.109
		0.05	Ln(X/P)	0.048		0.108
120	4	0.05	P1	0.046	0.039	0.129
		0.05	P2	0.043	0.038	0.107
		0.05	P3	0.041	0.035	0.104
		0.05	Ln(X/P)	0.052	0.034	0.1
120	12	0.05	P1	0.047	0.091	0.109
		0.05	P2	0.044	0.088	0.099
		0.05	P3	0.043	0.089	0.096
		0.05	Ln(X/P)	0.046	0.071	0.098

Table 3. Parameter Estimates for the U.S. Meat Demand

Variable	Beef Equation				Pork Equation				Poultry Equation			
	Disagg. Model	GLS Model	AR(1) Model	NON Model	Disagg. Model	GLS Model	AR(1) Model	NON Model	Disagg. Model	GLS Model	AR(1) Model	NON Model
PBeef	-0.2380*	-0.2361*	-0.1375**	-0.0592								
	(0.0397)	(0.040)	(0.054)	(0.067)								
PPork	0.0805*	0.0817*	0.1461**	0.0497	-0.2247*	-0.2238*	-0.0532	-0.084**				
	(0.0289)	(0.029)	(0.063)	(0.036)	(0.033)	(0.033)	(0.090)	(0.038)				
PPoultry	0.1575*	0.1544*	0.2012**	0.0095	-0.144*	-0.142*	-0.336*	-0.035	-0.3011*	-0.2959*	-0.135*	-0.0442
	(0.0411)	(0.041)	(0.043)	(0.069)	(0.031)	(0.031)	(0.061)	(0.044)	(0.059)	(0.059)	(0.032)	(0.087)
PFish	-0.00003	-0.00004	-0.0123	0.00008	-0.0002	-0.0002	-0.0237	0.0003*	0.0004***	0.0004***	0.0358	0.00001
	(0.0003)	(0.0003)	(0.096)	(0.0001)	(0.0003)	(0.0003)	(0.137)	(0.0001)	(0.0002)	(0.0002)	(0.071)	(0.0002)
Expend	0.2921*	0.2920*	0.2737*	0.1757*	0.1639*	0.1638*	-0.426*	0.165*	-0.4557*	-0.4557*	0.1522*	-0.3400*
	(0.008)	(0.008)	(0.008)	(0.035)	(0.006)	(0.006)	(0.012)	(0.022)	(0.011)	(0.011)	(0.006)	(0.041)
Feb	0.01934*				0.0417*				-0.0611*			
	(0.004)				(0.003)				(0.006)			
Mar	-0.0435*				0.0101*				0.0334*			
	(0.004)				(0.003)				(0.006)			
Apr	0.0059				0.0227*				-0.0285*			
	(0.04)				(0.003)				(0.006)			
May	-0.010**				-0.0005				0.0100***			
	(0.004)				(0.003)				(0.006)			
Jun	0.0029				0.0215*				-0.0244*			
	(0.004)				(0.003)				(0.006)			
Jul	-0.0042				0.025*				-0.0209*			
	(0.004)				(0.003)				(0.006)			
Aug	-0.0277*				0.0208*				0.0069			
	(0.004)				(0.003)				(0.006)			
Sep	-0.0025				0.0399*				-0.0373*			
	(0.004)				(0.003)				(0.006)			
Oct	-0.0334*				0.0200*				0.0135**			
	(0.004)				(0.003)				(0.006)			
Nov	-0.0045				0.0370*				-0.0324*			
	(0.005)				(0.003)				(0.007)			
Dec	0.0096**				0.0336*				-0.0432*			
	(0.004)				(0.003)				(0.006)			

Note: *, **, and *** denote respectively significance at 1%, 5% and 10%.