EPRU Working Paper Series

Economic Policy Research Unit
Institute of Economics
University of Copenhagen
Studiestræde 6
DK-1455 Copenhagen K
DENMARK
Tel: (+45) 3532 4411
Fax: (+45) 3532 4444
Web: http://www.econ.ku.dk/epru/

Capital Taxation, Growth, and Non-renewable Resources

Christian Groth and Poul Schou

2004-16

Capital Taxation, Growth, and Non-renewable Resources

Christian Groth* and Poul Schou**

* Institute of Economics, University of Copenhagen; Chr.Groth@econ.ku.dk
** Danish Rational Economic Agents Model (DREAM); psu@dreammodel.dk

Corresponding author:

Christian Groth, Institute of Economics, University of Copenhagen,
Studiestraede 6, DK-1455, Copenhagen K, Denmark
tel: (+ 45) 35 32 30 28, fax: (+ 45) 35 32 30 00

July 8, 2004

Abstract

The conventional view within the endogenous growth literature is that interest income taxes impede economic growth and investment subsidies promote economic growth. The present paper lays out a simple framework to see whether this is still true when non-renewable resources enter the "growth engine" in an essential way. It is not! The framework allows a rich set of determinants of long-run growth, including some fiscal policy measures, but interest income taxes and investment subsidies are not among these.

The results not only contrast with the modern literature on taxes and endogenous growth, but also with observations in the literature from the 1970's on non-renewable resources and taxation - observations which were not based on general equilibrium considerations.

Keywords: Non-renewable resources, endogenous growth, greenhouse effect, taxes, subsidies.

JEL Classification: H2, O4, Q3.

—————

2

1 Introduction

The conventional view within the endogenous growth literature is that interest income taxes impede economic growth and investment subsidies promote economic growth (Lucas 1990, King and Rebelo 1990, Rebelo 1991, Jones et al. 1993, Barro and Sala-i-Martin 1995, Stokey and Rebelo 1995, Milesi-Ferretti and Roubini 1998, Aghion and Howitt 1998b). There may be disagreement as to the size of the effects, but not their sign.

The present paper lays out a simple framework to see whether the conventional view goes through when non-renewable resources enter the "growth engine" in an essential way. It does not! Indeed, the framework allows a rich set of determinants of long-run growth, including some fiscal policy measures, but interest income taxes and investment subsidies are not among these.

The conventional view rests on growth models that ignore non-renewable natural resources. Some contributions (Robson 1980, Takayama 1980, Jones and Manuelli 1997, Aghion and Howitt 1998a (pp. 163-64), Scholz and Ziemes 1999, and Schou 2000, 2002) do examine endogenous growth in a framework where non-renewable resources are present, but in these models the natural resources do not appear in the sector constituting the growth engine of the model. This is a crucial feature and at the same time it is unrealistic. Most production sectors, including educational institutions and research labs, use fossil fuels for heating and transportation purposes, or minerals and oil products for machinery, computers, etc.

Our analysis is based on a simple extension of the one-sector model of Stiglitz (1974a, 1974b, 1975). In contrast to Stiglitz we focus on the case of increasing returns at the aggregate level with respect to labor, "broad" capital, and the resource taken together. This case may arise as a result of the "learning-by-investing" effect hypothesized by Arrow (1962) or the non-rivalness of technical knowledge as emphasized by Romer (1990). In fact, constant or even increasing returns to capital alone are not excluded. Further, in order to face an interesting taxation problem we allow for a negative externality of resource use (like the "greenhouse effect", say). This externality is modelled in the simplest possible way, in line with Suzuki (1976) and Sinclair (1992, 1994). While the focus of these authors was on sustainability of the *level* of consumption and on emission abatement by a specific tax (a carbon tax), respectively, the present paper focuses on growth effects of different kinds of taxes and subsidies.

Conventional endogenous growth models rely on exactly constant returns to capital in the sector that drives growth. Slightly increasing returns lead to explosive growth

(infinite output in finite time!)[1], whereas slightly decreasing returns eventually lead to a zero growth rate. As shown by Groth and Schou (2002), if an essential natural resource enters the growth generating sector this knife-edge problem is alleviated: The need to save increasingly on resource use counteracts the potentially explosive effects of capital accumulation implied by increasing returns. But a stable steady growth path exists only if there is population growth.

In a similar vein the present paper shows how other endogenous growth features are overturned when the presence of non-renewable resources in the macro production function is taken into account. It is demonstrated that an interest income tax and a capital investment subsidy affect neither consumption growth nor the speed of resource depletion in the long run. A policy directed at influencing these variables must change the returns to conservation of the nonrenewable resource, and this is what a tax on capital gains on the resource stock as well as a credibly announced declining tax on resource use do (though in opposite directions). The key to our results is that any balanced growth path of the economy has to comply with the linear differential equation describing resource extraction,

$$\dot{S} = -uS$$

where S is the stock of the resource, and u is the (proportionate) depletion rate. Hence, policies acting on the depletion rate become important for growth, that is for future consumption possibilities.

Our results not only contrast with the modern literature on taxation and growth, but also with observations in the older literature on non-renewable resources (Stiglitz 1975, Dasgupta and Heal 1979, Ch. 12). In fact, these observations were concerned only with taxation in a partial equilibrium framework.

Uhlig and Yanagawa (1996) also present an analysis arguing against the conventional view on interest income taxation and growth. While the conventional view is based on representative agent models, they use an OLG framework to demonstrate that a rise in the tax on interest income may raise growth when the interest elasticity of savings is sufficiently low and the revenue is used to lower labor taxes (leaving the young generation with a larger disposable income from which they can make savings). By taking into consideration the - realistic - presence of natural resources in the production function, we show that also in a representative agent model, interest taxes need not affect growth negatively in the long run.

---

[1]Cf. Solow (1994).

The organization of the paper is as follows. The next section describes the elements of the model. In Section 3 we study existence and stability of balanced growth paths and the effects of an investment subsidy and of taxes on interest income, capital gains, and resource use. Section 4 describes a first-best policy. It turns out, in the setup we consider, that a declining tax on resource use and a capital investment subsidy are necessary ingredients of a first-best policy, while capital-gains and interest income should not be taxed. A summary of the conclusions is given in the final section.

2 The model

2.1 Technology and firms' behavior

In order that the problem should be interesting, the non-renewable resource should be necessary for production, but should not *a priori* rule out sustainable (non-decreasing) consumption. Therefore, we follow Stiglitz (1974a, 1974b) and assume a production function of Cobb-Douglas form. Let $Y_i(t)$ be firm i's output, $K_i(t)$ its capital input, $N_i(t)$ its labour input, and $R_i(t)$ its input of the non-renewable resource, all at time t. Then firm i produces according to:

$$Y_i(t) = A(t)K_i(t)^{\alpha_1}N_i(t)^{\beta}R_i(t)^{\gamma}, \qquad \alpha_1, \beta, \gamma > 0, \quad \alpha_1 + \beta + \gamma = 1, \qquad (1)$$

where $A(t)$ is total factor productivity, given by

$$A(t) = e^{\theta t}K(t)^{\alpha_2}S(t)^{\lambda}, \qquad \theta, \alpha_2, \lambda \geq 0.$$

As in Suzuki (1976) and Sinclair (1992, 1994), total factor productivity is gradually decreased in line with the extraction and use of the resource. This is interpreted as the result of degradation of environmental quality associated with pollution from the use of fossil fuels etc., the stock of pollution being proxied inversely by the remaining resource stock $S(t)$ (think of the greenhouse effect[2]). There is CRS with respect to the three inputs that the *firm* can control. But if $\alpha_2 > 0$, aggregate capital, $K(t)$, has a positive external effect on productivity (the "learning-by-investing" effect hypothesized by Arrow 1962). This gives rise to increasing returns at the aggregate level and possibly "endogenous growth"; this is where we depart from the model in Sinclair (1994).[3] Increasing returns to scale at the aggregate level or at least in the

---

[2]This interpretation ignores the regeneration ability of the atmosphere and is (at best) applicable only for a limited span of time, namely as long as emissions from resource use is much higher than the regeneration ability.

[3]Empirical evidence furnished by, e.g., Hall (1990) and Caballero and Lyons (1992) suggests that there are quantitatively significant increasing returns to scale or external effects in U.S. and European

sector(s) constituting the "growth engine" of the economy is standard in models of semi-endogenous growth (as Jones 1995) as well as models of strictly endogenous growth (as Romer 1990).[4] Though we name K just 'capital', one may interpret K as 'broad capital' including technical knowledge and human capital.

There may also be an irreducibly exogenous element in technology growth, represented by the parameter θ. By allowing both α_2 and θ to be positive, we are able to demonstrate whether and when the *source* of growth - exogenous or endogenous - matters for the results.

There is a large number of similar firms and each of them takes the aggregate capital and resource stocks as given. Assuming perfect competition and using current output as the *numeraire*, profit maximization leads to:

$$\alpha_1 \frac{Y_i}{K_i} = (1 - \sigma)(r + \delta), \quad 0 \leq \sigma < 1, \delta \geq 0, \tag{2}$$

$$\beta \frac{Y_i}{N_i} = w, \tag{3}$$

$$\gamma \frac{Y_i}{R_i} = (1 + \tau_u)p, \tag{4}$$

where $r + \delta$ is the capital cost (rate of interest plus rate of capital depreciation), w is the real wage, and p is the real price of a unit flow of the resource (we ignore the time argument of the variables when not needed for clarity). The parameter σ represents a subsidy to absorb part of the cost of buying capital services (for brevity an "investment subsidy"), and τ_u is a (possibly time-dependent) tax on resource use (like a carbon tax).

Since all firms hire factors in the same proportions aggregate output can be written

$$Y \equiv \sum_i Y_i = e^{\theta t} K^\alpha N^\beta R^\gamma S^\lambda, \qquad \alpha \equiv \alpha_1 + \alpha_2, \tag{5}$$

where $K \equiv \sum_i K_i, N \equiv \sum_i N_i$, and $R \equiv \sum_i R_i$. Assuming market clearing, K, N, and

---

manufacturing. Antweiler and Trefler (2002) examine trade data for goods-producing sectors and find evidence for increasing returns to scale.

[4]The term "semi-endogenous growth models" refers to models where, first, per capita growth is driven by some internal mechanism (in contrast to exogenous technology growth). Second, unlike "strictly endogenous growth", sustained per capita growth requires the support of some growing exogenous factor, typically the labor force. While strictly endogenous growth requires $\alpha > 1$ (when $\gamma > 0$), semi-endogenous growth may be an attractive alternative, requiring only $\alpha + \beta > 1$. For most of the results of the present paper the distinction is not important.

R can also be interpreted as the aggregate supplies, and (2), (3), and (4) imply

$$r = \frac{\alpha_1}{1 - \sigma}\frac{Y}{K} - \delta, \tag{6}$$

$$w = \beta\frac{Y}{N}, \tag{7}$$

$$p = \frac{\gamma}{1 + \tau_u}\frac{Y}{R}. \tag{8}$$

2.2   Households

There is a fixed number of infinitely-lived households (families), all alike. For convenience we let the number of households be one, the representative household. It has N members, each supplying one unit of labor inelastically. We let household size grow at a constant exogenous rate $n \geq 0$, i.e., $N = N(0)e^{nt}, N(0) > 0$. The household consumes and saves, and savings can be either in loans, physical capital or the resource stock. At the aggregate level loans and deposits sum to zero (closed economy, no government debt). Let $\hat{p}$ denote the price of a unit of stock of the (not yet extracted) resource. Then, assuming no extraction costs we have in equilibrium $\hat{p} = p$, and financial wealth, V, satisfies

$$V = K + pS, \tag{9}$$

where S is the resource stock owned by the household.

We assume utilitarian preferences with iso-elastic instantaneous utility and a constant rate of time preference ρ. The intertemporal utility function then is

$$U_0 = \int_0^\infty \frac{c^{1-\varepsilon} - 1}{1 - \varepsilon}Ne^{-\rho t}dt, \qquad \varepsilon > 0, \rho > n \geq 0, \tag{10}$$

where ε is the (constant) numerical value of the elasticity of marginal utility[5]. The assumption $\rho > n$ is introduced to ease convergence of the integral.

The household has perfect foresight and chooses a path $(c, S)_{t=0}^\infty$ to maximize U_0 subject to $c \geq 0, S \geq 0$, and

$$\dot{V} = (1 - \tau_r)r(V - pS) + (1 - \tau_{cg})\dot{p}S + wN - T - cN, \qquad V(0) \text{ given}, \tag{11}$$

$$\lim_{t \to \infty} Ve^{-(1-\tau_r)\int_0^t r(s)ds} \geq 0. \tag{12}$$

Here $\tau_r < 1$ represents a constant rate of tax on interest income (if $\tau_r < 0$ one should think of a subsidy to interest income), $\tau_{cg} < 1$ is a constant rate of tax (subsidy

---

[5]In case $\varepsilon = 1$, the expression $\frac{c^{1-\varepsilon}-1}{1-\varepsilon}$ should be interpreted as $\ln c$.

if negative) on capital gains, and T is a lump sum tax (amounting to a transfer, if negative); finally, (12) is the no-Ponzi-game condition. We abstract from wage income taxes and consumption taxes because their effects are trivial in a model without utility of leisure.

Taxation of capital gains on resources in the ground are rarely seen in the real world[6]. However, τ_{cg} together with τ_r can be seen as particular representations of a global tax on all types of capital income. In the present model where the household can invest in two different kinds of physical assets (the resource and physical capital), a comprehensive capital income tax corresponds to the special case $\tau_r = \tau_{cg}$. But in order to understand fully the differential roles of taxes on the two different assets, we allow τ_{cg} and τ_r to differ (including the limiting case $\tau_{cg} = 0$).

Existence of an interior solution to the household decision problem implies, first, the Keynes-Ramsey Rule

$$\frac{\dot{c}}{c} = \frac{1}{\varepsilon}\left[(1 - \tau_r)r - \rho\right],\tag{13}$$

second, the (tax-adjusted) Hotelling Rule

$$\frac{(1 - \tau_{cg})\dot{p}}{p} = (1 - \tau_r)r,\tag{14}$$

and, third, a transversality condition implying that (12) holds with equality. The Hotelling Rule is a no-arbitrage condition between investing in the resource (leaving it in the ground) and investing in ordinary financial assets. In case of a comprehensive tax on capital income, $i.e.$, $\tau_{cg} = \tau_r$, the condition reduces to the well-known, simple Hotelling rule, $\dot{p}/p = r$.

2.3 Government

At any time the government balances its budget by adjusting T so that

$$\tau_u pR + \tau_r rK + \tau_{cg}\dot{p}S + T = \sigma(r + \delta)K.\tag{15}$$

The only public expense is the subsidy σ paid out to firms to reduce their capital costs. On the revenue side we have the tax τ_u on resource use (the "carbon tax") imposed on firms, while the interest income tax τ_r, the capital gains tax τ_{cg}, and the lump-sum tax T (or transfer) are imposed on households.

There is a given finite resource stock to extract from, hence, what matters for resource extraction is not the level of the tax τ_u, but its rate of change, as pointed

---

[6]Dasgupta & Heal (1979, p. 368) discuss existence and partial equilibrium consequences of such taxes.

out by Sinclair (1992, 1994). We assume that the government credibly announces

$$\dot{\tau}_u = -(1 + \tau_u)\psi, \qquad \lim_{t\to\infty} \psi = \bar{\psi} \geq 0, \tag{16}$$

where ψ may generally be time-dependent (in a smooth way), but is constant in the limit to ensure compatibility with balanced growth in the long run. We shall call ψ the postponement stimulus, since $\psi > 0$ implies a declining tax rate, thereby stimulating postponement of extraction.[7]

3   Economic development

Output is used for consumption and for investment in capital goods, so that

$$\dot{K} = Y - C - \delta K, \qquad K(0) > 0, \tag{17}$$

where $C \equiv cN$ is total consumption. The resource stock S diminishes with resource extraction:

$$\dot{S} = -R, \qquad S(0) > 0. \tag{18}$$

The definitional non-negativity condition on S implies, from (18), the restriction

$$\int_0^\infty R(t)dt \leq S(0), \tag{19}$$

showing the finite upper bound on cumulative extraction of the resource over the infinite future. Obviously, from this restriction it follows that resource use must approach zero for $t \to \infty$.

The transversality condition of the household is that (12) holds with equality; this implies, by (9) and (14),

$$\lim_{t\to\infty} K(t)e^{-(1-\tau_r)\int_0^t r(s)ds} = 0, \quad \text{and} \tag{20}$$

$$\lim_{t\to\infty} S(t)e^{\frac{\tau_{cg}}{1-\tau_{cg}}(1-\tau_r)\int_0^t r(s)ds} = 0. \tag{21}$$

The last condition requires (when $\tau_{cg} > 0$ and $\lim_{t\to\infty} r(t) > 0$) not only that no finite part of the resource stock will be left unused forever, but also that the resource stock diminishes at a sufficient speed.

The system characterized by the technology (1), the intertemporal utility function (10), and the dynamic resource conditions (17) and (18), will be called an *economic*

---

[7]Solving the differential equation (16) gives $\tau_u(t) = (1 + \tau_u(0))e^{-\int_0^t \psi(s)ds} - 1$, where we assume $\tau_u(0) > -1$. Hence, we allow $\tau_u(t)$ to be (or become) negative.

system. A *viable* economic system is a system where $C, Y, K, R,$ and S are (strictly) positive for all $t \geq 0$ ("no collapse").

The quadruple $(\tau_r, \tau_{cg}, \sigma, (\psi)_{t=0}^{\infty})$ of tax and subsidy instruments will be called a *policy.* Given the policy $(\tau_r, \tau_{cg}, \sigma, (\psi)_{t=0}^{\infty})$, an *equilibrium* of a viable economic system is a path for prices and quantities such that: (i) households maximize discounted utility, taking the time paths of the interest rate, the resource price, and the wage rate as given; (ii) firms maximize profits choosing inputs of capital, labor, and the resource, taking the prices of these inputs as given; (iii) the government adjusts lump sum taxes T so that the budget constraint (15) is satisfied at any t; and (iv) in each market, the supply is equal to the demand.

Let the output-capital ratio, the consumption-capital ratio, and the resource depletion rate be denoted $z, x,$ and u, respectively, i.e.,

$$z \equiv \frac{Y}{K}, \qquad x \equiv \frac{C}{K}, \qquad u \equiv \frac{R}{S},$$

These ratios turn out to be central to the analysis. Let g_m denote the growth rate of a variable $m \ (> 0)$, that is $g_m \equiv \dot{m}/m$. Then, we may write (17) as

$$g_K = z - x - \delta. \tag{22}$$

Similarly, by (18),

$$g_S = -u. \tag{23}$$

In an equilibrium (5) holds, implying, by logarithmic differentiation, using (23),

$$g_Y = \alpha g_K + \beta n + \gamma g_R + \theta - \lambda u. \tag{24}$$

Similarly, (6), (8), (14), and (16) lead to

$$g_Y - g_R + \psi = \frac{1 - \tau_r}{1 - \tau_{cg}} \left(\frac{\alpha_1}{1 - \sigma} z - \delta \right), \tag{25}$$

and (6) and (13) yields

$$g_C = g_c + n = \frac{1}{\varepsilon} \left[(1 - \tau_r)(\frac{\alpha_1}{1 - \sigma} z - \delta) - \rho \right] + n. \tag{26}$$

Note that *any* equilibrium satisfies (22), (23), (24), (25), and (26).

3.1 Balanced paths

A path $(C, Y, K, R, S)_{t=0}^{\infty}$ will be called a *balanced growth path* (henceforth abbreviated BGP) if $C, Y, K, R,$ and S change with constant relative rates for all $t > 0$ (some or all

these rates may be negative). An equilibrium path $(C, Y, K, R, S)_{t=0}^{\infty}$ which is a BGP will be called a *balanced growth equilibrium* (abbreviated BGE)[8].

Lemma 1 *For any BGE* $(C, Y, K, R, S)_{t=0}^{\infty}$ *the following holds:*

(i) $g_C = g_Y = g_K \equiv g^*$, *a constant;*

(ii) $g_R = g_S = -u = -u^*$, *where* u^* *is some positive constant;*

(iii) z *and* x *are positive constants, and* $\psi = \bar{\psi}$;

(iv) g^* *and* u^* *satisfy*

$$(1 - \alpha)g^* + (\gamma + \lambda)u^* = \beta n + \theta, \tag{27}$$

$$[\varepsilon - (1 - \tau_{cg})]\, g^* - (1 - \tau_{cg})u^* = \varepsilon n - \rho + (1 - \tau_{cg})\bar{\psi}; \tag{28}$$

(v) $\tau_{cg}(g^* + \bar{\psi}) < (1 - \tau_{cg})u^*$.

Proof See Appendix.

The necessity of (v) is due to the transversality conditions (20) and (21).

From the equations (27) and (28) immediately follows:

Proposition 1 *The capital gains tax* τ_{cg} *and the postponement stimulus* $\bar{\psi}$ *can affect growth in a BGE, but the interest income tax* τ_r *and the investment subsidy* σ *cannot.*

This result can be explained in the following way. Equation (27), linking g^* and u^* independently of policy parameters, is dictated by mere technical feasibility, given the aggregate production function (5). In contrast, (28) represents the effects of the market mechanism. These effects are such that τ_r and σ are decoupled from the determination of growth. Indeed, profit maximization implies (8), hence, in a BGE, $\dot{p}/p = g^* + u^* + \bar{\psi}$, and then net returns on saving, $(1 - \tau_r)r$, equals $(1 - \tau_{cg})(g^* + u^* + \bar{\psi})$, by the Hotelling rule, whatever the size of τ_r. The fact that the consumer can choose to save in different assets (physical capital or the resource stock) implies that the net returns on saving in physical capital must adjust to the net returns on saving in the resource. Therefore, $(1 - \tau_{cg})(g^* + u^* + \bar{\psi})$ can be substituted for $(1 - \tau_r)r$ in the Keynes-Ramsey rule. In this way τ_r as well as σ (and α_1), through (6), are excluded from the subsystem of zero order in the causal structure. In short, an increase in τ_r leads to a portfolio adjustment, implying less demand for capital so that K/N goes down and r goes up until $(1 - \tau_r)r$ is as before. And an increase in σ increases demand for capital so that K/N goes up and z goes down until $r = \alpha_1 z/(1 - \sigma) - \delta$ is as before.

---

[8]The values taken by the variables along a BGE are marked by *.

3.2 Existence and stability

Let D be the determinant of the linear system (27) and (28):

$$D \equiv (1 - \tau_{cg})(1 - \alpha - \gamma - \lambda) + (\gamma + \lambda)\varepsilon. \tag{29}$$

Given $D \neq 0$, by solving the system (27) and (28) we find the growth rate of output and the depletion rate, respectively, along a BGE:

$$g^* = \frac{(1 - \tau_{cg})(\beta n + \theta) + \left[\varepsilon n - \rho + (1 - \tau_{cg})\bar{\psi}\right](\gamma + \lambda)}{D}, \tag{30}$$

$$u^* = -g_R^* = \frac{\left[(\alpha + \beta - 1)n + \theta\right]\varepsilon - (1 - \tau_{cg})(\beta n + \theta) + \left[\rho - (1 - \tau_{cg})\bar{\psi}\right](1 - \alpha)}{D}. \tag{31}$$

Now, by (25), (22), and (27) we find

$$z^* = \frac{1 - \sigma}{\alpha_1}\left[\frac{1 - \tau_{cg}}{1 - \tau_r}(g^* + u^* + \bar{\psi}) + \delta\right], \quad \text{and} \tag{32}$$

$$x^* = \frac{\left[(1 - \sigma)\frac{1 - \tau_{cg}}{1 - \tau_r}(1 - \alpha - \gamma - \lambda) + (\gamma + \lambda)\alpha_1\right]u^* + \left[(1 - \sigma)\frac{1 - \tau_{cg}}{1 - \tau_r} - \alpha_1\right](\beta n + \theta)}{(1 - \alpha)\alpha_1}$$

$$+ (1 - \sigma)\frac{1 - \tau_{cg}}{1 - \tau_r}\frac{\bar{\psi}}{\alpha_1} + \frac{1 - \sigma - \alpha_1}{\alpha_1}\delta, \tag{33}$$

where the formulas for g^* and u^* can be inserted (the resulting formulas, given in Appendix, are volumnious and not particularly illuminating).

Let π denote the vector $(\alpha_1, \beta, \gamma, \alpha_2, \lambda, \theta, \alpha, \delta, n, \varepsilon, \rho, \sigma, \bar{\psi}, \tau_r, \tau_{cg})$, and let P be the set of $\pi \in R^{15}$ such that $\alpha_1, \beta, \gamma, \alpha_2, \lambda, \theta, \delta, n, \varepsilon, \rho, \sigma, \bar{\psi} \in R_+$, $\tau_r < 1$, $\tau_{cg} < 1$, $\alpha = \alpha_1 + \alpha_2$, and the parameter inequalities stated in (1), (2), and (10) are satisfied. Let $P^* \subset P$ be the subset satisfying the requirements: $D \neq 0$, u^*, z^*, and x^* are strictly positive, and (v) of Lemma 1 is satisfied. This subset P^* will be called the *BGE supporting set.* We have:

Lemma 2 The set $P^* \subset P$ has a non-empty interior.

Proof See Appendix.

Proposition 2 *Let the parameter vector $\pi \in P$. Assume $D \neq 0$, and let $\psi(t) = \bar{\psi}$ for all $t \geq 0$. Then:*

    (i) *There exists a BGE if and only if $\pi \in P^*$.*

    (ii) *For any given $\pi \in P^*$, the BGE (g^*, u^*, z^*, x^*) is unique.*

Proof See Appendix.

Remark 1. In the Stiglitz case, $\alpha + \beta + \gamma = 1$, $\lambda = 0$, and $\tau_{cg} = \tau_r = \sigma = \bar{\psi} = 0$, the existence requirements $z^* > 0$ and $x^* > 0$ are automatically satisfied, and the only thing to check is whether the parameters are consistent with $u^* > 0$.

Remark 2. In case $D = 0$, a BGE can exist only in the knife-edge case $\beta n + \theta = -\left[\bar{\psi} + (\varepsilon n - \rho)/(1 - \tau_{cg})\right](\gamma + \lambda)$, and in this case existence implies a continuum of BGE's.

From now we consider π as given and belonging to the BGE supporting set P^*. Generally, we allow ψ to be time-dependent[9], except in the limit. Therefore, a BGE (g^*, u^*, z^*, x^*) may be realizable only asymptotically. We shall use the phrase "there exists a BGE ..." as a shorthand for "there exists, at least asymptotically, a BGE ...".

The dynamics of the model can be reduced to a three-dimensional system in u, z, and x. In view of the assumption $\psi(t) \to \bar{\psi}$ for $t \to \infty$, the system is asymptotically autonomous. The associated steady state (u^*, z^*, x^*) is a BGE with $g^* = z^* - x^* - \delta$. We call the BGE saddle-point stable if there exists a unique solution converging to the steady state for $t \to \infty$. And we call the BGE totally unstable if all three eigenvalues of the associated Jacobian have positive real part. In order not to endanger saddle-point stability the assumption $\lambda/2 < 1 - \gamma$ is convenient (and quite innocent since, empirically, γ is likely to be quite low, say less than .05).

Proposition 3 *If $D > 0$ and $\lambda/2 < 1 - \gamma$, then a BGE is saddle-point stable. On the other hand, if $D < 0$ and in addition*

$$\sigma \leq 1 - \alpha_1/\alpha, \quad and \quad \tau_{cg} \approx \tau_r, \tag{A1}$$

then (at least for λ "small") a BGE is totally unstable.

Proof See Appendix.

The policy assumption (A1) is invoked in order to have a clear-cut instability implication of $D < 0$. The assumption is "natural" in the sense that its first part says that the investment subsidy does not overcompensate the positive external effect of investment, and its second part says that the capital gains tax is not very different from other capital income taxes.[10]

---

[9]This is because, as we shall see (Section 4), in the first-best solution ψ is only asymptotically constant.

[10]The qualifier referring to "smallness" of the externality λ is explained in the Appendix.

3.3 Economic growth

We shall concentrate on the case $D > 0$ which seems also the most realistic case since $D \leq 0$ requires a considerable amount of increasing returns (i.e., $\alpha + \gamma \geq 1 - \lambda + \frac{(\gamma + \lambda)\varepsilon}{1 - \tau_{cg}}$ > 1 for λ "small"). The *per capita* growth rate in a BGE is

$$g_c^* = g^* - n = \frac{(1 - \tau_{cg})\left[(\alpha + \beta + \gamma + \lambda - 1)n + \theta\right] - \left[\rho - (1 - \tau_{cg})\bar{\psi}\right](\gamma + \lambda)}{D}. \quad (34)$$

Now, when is stable *positive* per capita growth possible in spite of the diminishing input of the resource? To answer this, we first introduce a purely technological condition that is necessary for $g_c^* > 0$.

Lemma 3 *Assume $D > 0$. A BGE with $g_c^* > 0$ can exist only if $(\alpha + \beta - 1)n + \theta$ > 0.*

Proof Consider a BGE. From Lemma 1, $u^* > 0$. Hence, by (27), $(1 - \alpha)g^* < \beta n + \theta$, and since $g^* = g_c^* + n$, this implies $(1 - \alpha)g_c^* < (\alpha + \beta - 1)n + \theta$. □

Proposition 4 *Assume $D > 0$. Then a BGE has $g_c^* > 0$, if and only if the parameters satisfy*

$$(\alpha + \beta - 1)n + \theta > \max\left[(\frac{\rho}{1 - \tau_{cg}} - \bar{\psi} - n)(\gamma + \lambda), 0\right]. \quad (35)$$

When $\theta = 0$ (the case of no exogenous technical progress), $g_c^* > 0$ if and only if

$$\alpha + \beta > 1 \ and \ n > \max\left[(\frac{\rho}{1 - \tau_{cg}} - \bar{\psi})\frac{\gamma + \lambda}{\alpha + \beta + \gamma + \lambda - 1}, 0\right]. \quad (36)$$

Proof Assume $D > 0$. Then, (35) follows immediately from (34) and Lemma 3. If $\theta = 0$, then, (35) is equivalent to (36) since, by Lemma 3, $g_c^* > 0 \wedge \theta = 0 \Rightarrow \alpha + \beta > 1$ $\wedge \ n > 0$. □

When $\alpha + \beta = 1$ or $n = 0$, the right hand side of (35) gives a lower bound for the rate of *exogenous* technical progress required to compensate for the growth drag resulting from non-renewable resources. The capital gains tax τ_{cg} tends to increase this bound, while the postponement stimulus $\bar{\psi}$ (acting similarly to a decrease in the rate of time preference) tends to decrease it. If there is increasing returns with respect to capital and labor, *endogenous* growth may occur. We define endogenous growth to be present if $g_c^* > 0$ even when $\theta = 0$. As the last part of the proposition shows, endogenous per capita growth requires not only increasing returns with respect to capital and labor, but also a *sufficient* amount of population growth to let the increasing returns come into action, given the preferences of the representative household. These features are

14

needed to offset the effects of the inevitable decline in resource use.[11] If on the other hand the opposite of (35) is true, the long-run perspective is famine and a Malthusian check on population − if not doomsday.

Notice that population growth may be important not only for existence of endogenous growth, but also for stability. If θ is large enough to generate growth without the assistance of increasing returns, then the BGE may be stable even if there is no population growth. But if growth is endogenous, then, under laissez-faire, stability requires population growth.[12] Indeed, if $\theta = n = \bar{\psi} = \tau_{cg} = 0$, then $g_c^* = g^* > 0$ requires $D < 0$, by (30), and this implies instability. The interpretation is *not* that population growth stabilizes an otherwise unstable BGE. Stability-instability is governed by the sign of D, independently of n. Rather, given $D > 0$, letting n decrease from a level above the critical value in (36) to a level below, changes g_c^* from positive to negative, i.e., growth comes to an end. This is in contrast to standard endogenous growth models with non-renewable resources (such as Schou 2000) where population growth is not necessary for stable growth. The difference is explained by the fact that the resource does not enter the "growth engine" of these models.

The *source* of growth matters for the question whether population growth is good or bad for growth. Equation (34) shows that in the Stiglitz-case where $\theta > 0$ is combined with constant returns to scale with respect to K, N and R, and no externalities, population growth either does not affect g_c^* (if $\lambda = 0$) or affects g_c^* negatively (if $\lambda > 0$). But if there are increasing returns, population growth affects g_c^* positively[13]. This is the net result of three different effects: There is a direct positive effect on output growth because a growing population means a growing labor force, magnifying the effects of increasing returns to scale. In addition, there is an indirect effect because population growth also affects resource extraction as seen from (31), though the sign of this effect is ambiguous without further specification. Thirdly, higher population growth naturally means more mouths to feed, and this implies a drag on growth in *per capita* consumption possibilities.

---

[11]A numerical example is: $\alpha_1 = .60, \alpha = .90, \beta = .30, \gamma = .015, \lambda = .005, n = .01, \theta = 0, \delta = .07,$ $\varepsilon = 2.00, \rho = .02, \sigma = .33, \bar{\psi} = .007,$ and $\tau_{cg} = \tau_r = 0$; then $D > 0$ (stability), $g_c^* = .016$ and $u^* = .02$.

[12]In Groth and Schou (2002) we explore further this fact and its relation to the knife-edge property of conventional endogenous growth models without non-renewable resources.

[13]This trait should not be seen as a prediction about individual countries in an internationalized world, but rather as pertaining to larger regions, perhaps the global economy.

3.4 Effects of taxes and subsidies

To prepare the ground for the analysis of tax and subsidy effects we make some observations on the rate of interest. Of course, if $g_c^* > 0$, then, by the Keynes-Ramsey Rule, $r > \rho/(1 - \tau_r)$, which is positive. More generally, in view of (6) and (32),

$$r^* = \frac{1 - \tau_{cg}}{1 - \tau_r}(g^* + u^* + \bar{\psi}), \tag{37}$$

from which follows that, as a rule, $r^* > 0$. To be precise, let $\bar{P}$ be the set of all parameter vectors $\pi = (\alpha_1, \beta, \gamma, \alpha_2, \lambda, \theta, \alpha, \delta, n, \varepsilon, \rho, \sigma, \bar{\psi}, \tau_r, \tau_{cg})$ in the BGE-supporting set P^* such that if δ is substituted by zero, then the new vector is also in P^*. Clearly, $\bar{P}$ is non-empty[14].

Lemma 4 For all $\pi \in \bar{P}, r^* > 0$.

Proof Consider a given $\pi \in \bar{P}$. Fix all coordinates of π except δ. Consider π as a function $\pi(\delta)$ of δ. Let r^* in the BGE corresponding to $\pi(0) \in P^*$ be called r_0^*. Since $z^* > 0$ in a BGE, $r_0^* > 0$, by (6). Then for any $\delta > 0$ such that $\pi(\delta) \in P^*$, $r^* = r_0^* > 0$, by (37), since g^* and u^* are independent of δ. $\square$

The only policies capable of affecting long-run growth are the capital gains tax τ_{cg} and the postponement stimulus $\bar{\psi}$. Assuming $D > 0$ and $r^* > 0$, we have, from (34),

$$\frac{\partial g_c^*}{\partial \tau_{cg}} = -\frac{(1 - \tau_r)(\gamma + \lambda)}{(1 - \tau_{cg})D}r^* < 0, \tag{38}$$

$$\frac{\partial g_c^*}{\partial \bar{\psi}} = \frac{(1 - \tau_{cg})(\gamma + \lambda)}{D} > 0. \tag{39}$$

Thus, an increase in the capital gains tax impedes growth. The explanation is that taxing capital gains on leaving the resource in the ground fuels resource extraction. This creates a tendency to faster exhaustion of the resource stock, hence faster decline in resource use, implying that mere sustainability of per capita consumption takes up a larger share of the ongoing capital accumulation, leaving less aside for growth. An announced declining tax on resource use ($\bar{\psi} > 0$) has the opposite effects. Indeed, the declining tax implies a lower required before-tax return on leaving the marginal resource in the ground. This defers resource extraction, and thereby growth is enhanced.[15]

---

[14]The example in footnote 11 shows this.

[15]Introducing a tax rate on capital gains of .3, with the parameters from footnote 11, g_c^* in a BGE will fall with almost one fifth to about .013. Removal of the postponement stimulus reduces g_c^* further to .012. Whether growth is exogenous or endogenous does not matter for the qualitative effects of the policies (as long as $\alpha < 1$).

The exact effects on the depletion rate of the two policies are, from (31),

$$\frac{\partial u^*}{\partial \tau_{cg}} = (1-\alpha)\frac{1-\tau_r}{(1-\tau_{cg})D}r^* > 0, \quad \text{when } \alpha < 1, \tag{40}$$

$$\frac{\partial u^*}{\partial \bar{\psi}} = -\frac{(1-\tau_{cg})(1-\alpha)}{D} < 0, \quad \text{when } \alpha < 1,$$

respectively.[16]

As already commented on above, the interest income tax rate τ_r and the capital subsidy rate σ affect neither growth nor resource extraction in a BGE. These policy parameters affect only *levels*. Assuming $D > 0$, and $r^* > 0$, the level effects of all four policy parameters are, using (26), (22) (38), and (13):

$$\frac{\partial z^*}{\partial \tau_{cg}} = -\frac{(1-\sigma)\varepsilon(\gamma+\lambda)}{(1-\tau_{cg})\alpha_1 D}r^* < 0, \tag{41}$$

$$\frac{\partial x^*}{\partial \tau_{cg}} = -\frac{(1-\sigma)\varepsilon - (1-\tau_r)\alpha_1}{(1-\tau_{cg})\alpha_1}\frac{\gamma+\lambda}{D}r^* \lesseqgtr 0 \text{ for } \varepsilon \gtreqless \frac{(1-\tau_r)\alpha_1}{1-\sigma}, \tag{42}$$

$$\frac{\partial z^*}{\partial \bar{\psi}} = \frac{(1-\sigma)(1-\tau_{cg})\varepsilon(\gamma+\lambda)}{(1-\tau_r)\alpha_1 D} > 0 \tag{43}$$

$$\frac{\partial x^*}{\partial \bar{\psi}} = (\frac{(1-\sigma)\varepsilon}{(1-\tau_r)\alpha_1} - 1)\frac{(1-\tau_{cg})(\gamma+\lambda)}{D} \gtreqless 0 \text{ for } \varepsilon \gtreqless \frac{(1-\tau_r)\alpha_1}{1-\sigma}, \tag{44}$$

$$\frac{\partial z^*}{\partial \tau_r} = \frac{\partial x^*}{\partial \tau_r} = \frac{1-\sigma}{(1-\tau_r)\alpha_1}r^* > 0, \tag{45}$$

$$\frac{\partial z^*}{\partial \sigma} = \frac{\partial x^*}{\partial \sigma} = -\frac{1}{1-\sigma}z^* < 0. \tag{46}$$

The counterpart of the dampening effect on growth of a higher capital gains tax is a lower rate of interest, that is, a lower marginal and average product of capital as seen by (41). For reasonable values of the desire for consumption smoothing, ε, also the consumption-capital ratio is diminished, cf. (42). The effects of a higher postponement stimulus go the opposite way. As to the effect of a change in τ_r, whatever the size of τ_r, net returns on saving, $(1-\tau_r)r^*$, equals, by the Hotelling rule, $(1-\tau_{cg})(g^*+u^*+\bar{\psi})$, where g^* and u^* are independent of τ_r. That is, if τ_r goes up, r^* goes up also, leaving $(1-\tau_r)r^*$ unchanged. The increase in r^* is reflected in the increase in the marginal and average product of capital shown in (45). Finally, a higher capital subsidy rate σ stimulates demand for capital services and twists the capital intensity upwards and the output-capital ratio downwards, as seen by (46).[17]

---

[16]On the other hand, if $\alpha > 1$, an increase in τ_{cg} as well as a *decrease* in $\bar{\psi}$ diminishes the depletion rate in the BGE. These counter-intuitive effects are due to the fact that if $\alpha > 1$, then changes in capital accumulation are self-enforcing and creates divergence if not counterbalanced by changes in the depletion rate in the *same* direction.

[17]It may be added that the level effects of τ_r and σ can endanger *existence* of a BGE and thereby indirectly affect growth. For example with $\sigma > .53$, keeping the other parameter values in footnote 11 unchanged, we get $x^* < 0$, implying non-existence of a BGE.

The results here are in contrast to traditional endogenous growth models without natural resources (e.g., Lucas 1990, King and Rebelo 1990, Rebelo 1991, Jones et al. 1993, Barro and Sala-i-Martin 1995, Stokey and Rebelo 1995, Milesi-Ferretti and Roubini 1998, Aghion and Howitt 1998b) where taxation of interest income as well as investment subsidies influence long-run growth. In these models, what stimulates saving and investment stimulates growth. However, when the growth engine depends on a non-renewable resource this alternative stock variable causes policy instruments directed towards capital accumulation as such to be unimportant for the growth rate. But policies affecting resource extraction (the bottleneck of the economy) are assigned a central role.

In general the key to having policy affecting long-run growth is the presence of a linear differential equation in the model (as noted by Romer 1995). In the present framework the resource extraction relation, $\dot{S} = -uS$, is such an equation. Inspection of the growth accounting relation (27) confirms that generally (i.e., when $\alpha \neq 1$) only policies affecting u^* can matter for long-run growth.[18]

3.5   A special case: Constant returns to capital

In the case $\alpha = 1$ we have a kind of AK model augmented with an explicit role for both the labor force and natural resources in production. The growth rate and the depletion rate simplify to

$$g_c^* = \frac{(1 - \tau_{cg})\left[(\beta + \gamma + \lambda)n + \theta\right] - \left[\rho - (1 - \tau_{cg})\bar{\psi}\right](\gamma + \lambda)}{D},$$
$$u^* = \frac{\beta n + \theta}{\gamma + \lambda},$$

where in this case $D = (\varepsilon - 1 + \tau_{cg})(\gamma + \lambda)$. It is interesting that under laissez-faire ($\tau_{cg} = \bar{\psi} = 0$), only an elasticity of intertemporal substitution ($1/\varepsilon$) below 1 is compatible with stability ($D > 0$). But a positive τ_{cg} stabilizes, by making the condition $D > 0$ more likely to occur.

This is the only case where, given $D > 0$, a decrease in τ_{cg} or an increase in $\bar{\psi}$ promotes growth *without* affecting resource extraction in the BGE. This counter-intuitive feature is due to the "growth accounting" relation (27) saying that, when $\alpha = 1$, the balance between g_Y and g_K requires unchanged u.

For the case of no exogenous technical progress ($\theta = 0$) we see that $\alpha = 1$ combined with population growth may generate stable endogenous per capita growth. This con-

---

[18]Note that this conclusion would not change by the addition of extraction costs to the model. Such costs leave the fundamental linearity, $\dot{S} = -uS$, unaffected.

trasts with the model without non-renewable resources ($\gamma = 0$), where a situation with $\alpha = 1$, $\beta > 0$, and population growth is not compatible with a steady state, but implies explosive growth. In the present model, population growth is not only compatible with, but necessary for positive stable growth. One of the models examined in Aghion & Howitt (1998a, pp. 162-63) is an AK model with a non-renewable resource. In that model, however, labor does not appear in the production function. Because of this specification, long-run growth is not possible in that model.

4   First-best solution

For the case of constant returns to scale and exogenous technical progress, Sinclair (1994) showed that the negative resource externality leads to too slow growth in the long run and that its correction requires a *declining* tax on resource use. In this section we show that this and related results come true also if growth is endogenous and when alternative tax and subsidy instruments are available.

Consider the problem of maximizing (10) subject to the conditions (5), (17), (18), and the usual non-negativity constraints. We call the solution to this problem (when it exists) an *optimal allocation* or the *social planner's rule*, indexed by "SP". Given $D^{SP} \equiv 1 - \alpha + (\varepsilon - 1)\gamma \neq 0$, the GDP growth rate and the depletion rate in a steady state of the optimal allocation are[19], respectively,

$$g^{*SP} = \frac{\beta n + \theta + (\varepsilon n - \rho)\gamma}{D^{SP}}, \quad \text{and} \tag{47}$$

$$u^{*SP} = \frac{\gamma}{\gamma + \lambda} \frac{[(\alpha + \beta - 1)n + \theta]\varepsilon - (\beta n + \theta) + (1 - \alpha)\rho}{D^{SP}}. \tag{48}$$

As also Sinclair (1994) observed, the resource externality λ has no effect on the optimal growth rate, but only on the rate of resource depletion, which is smaller the larger is λ. The explanation is that a higher cost in terms of lower productivity in the future implies a lower required return on leaving the marginal resource in the ground. Thereby resource extraction is stretched out, offsetting a higher λ such that the drag on output growth stemming from the exhaustible resource, i.e., $(\gamma + \lambda)u$, by (27), is unaffected. Then a larger λ shows up only as a smaller resource depletion rate.[20]

We want to compare the optimal allocation with the market equilibrium. There are two distortions, the negative externality from resource use and the positive externality from aggregate capital. Define "laissez-faire" as the policy $(\tau_r, \tau_{cg}, \sigma, (\psi)_{t=0}^\infty) =$

---

[19]For derivation, see Appendix.
[20]Also, the steady state values of z^{SP} and x^{SP} are independent of λ.

$(0, 0, 0, 0)$, and let $D(0)$ denote the value of D for $\tau_{cg} = 0$, i.e., $D(0) \equiv 1 - \alpha - \gamma - \lambda + (\gamma + \lambda)\varepsilon$, cf. (29). We have:

Proposition 5 *Assume the parameters $\alpha_1, \beta, \gamma, \alpha, \lambda, \theta, \delta, n, \varepsilon, \rho$ are such that a BGP exists under both laissez-faire and the social planner's rule. Assume $D(0) > 0$.*

(i) If $\lambda > 0$, then laissez-faire implies $g^* < g^{*SP}$.

(ii) If $\lambda = 0$, but $\alpha_1 < \alpha$, then laissez-faire implies $g^* = g^{*SP}$, but $z^* > z^{*SP}$.

Proof See Appendix.

Hence, irrespective of whether growth is exogenous or endogenous, under laissez-faire the negative resource externality leads to too slow growth in the long run; in the "normal" case where $\alpha < 1$ this reflects a too fast resource depletion, cf. (27). Further, the positive capital externality entails too little capital investment so that z^* and r^* become too high and too little scope for consumption in the long run is provided even if the growth rate is appropriate.[21]

Are the available tax and subsidy instruments adequate for correcting these distortions? It seems almost trivial that the capital externality can be compensated by subsidizing investment by $\sigma = 1 - \alpha_1/\alpha$. But what about the negative resource externality and the resulting drag on growth? On the face of it, both a declining tax on resource use and a *negative* capital gains tax could deliver the required premium on delaying extraction and making scope for optimal growth. It turns out, however, that only the first policy is adequate. Indeed:

Proposition 6 *Whenever an optimal allocation exists, it can be implemented as an equilibrium allocation if and only if the policy: $\sigma = 1 - \alpha_1/\alpha$, $\psi = \lambda u/\gamma$, and $\tau_{cg} = \tau_r = 0$, is applied.*

Proof See Appendix.

The social planner's Hotelling Rule

$$\frac{d(\frac{\partial Y}{\partial R})}{dt} = (\frac{\partial Y}{\partial K} - \delta)\frac{\partial Y}{\partial R} - \frac{\partial Y}{\partial S} \tag{49}$$

is helpful in understanding this result. The rule states that along an interior optimal path the return (capital gain) on leaving the marginal unit of the resource in the

---

[21] Existence of an optimal allocation is guaranteed for any parameter vector $(\alpha, \beta, \gamma, \lambda, \delta, n, \theta, \varepsilon, \rho)$ such that D^{SP} as well as u^{*SP}, z^{*SP}, and x^{*SP} are strictly positive, provided $\alpha + \gamma + \lambda \leq 1$ (footnote 11 shows an example). For the case $\alpha + \gamma + \lambda > 1$, which violates concavity of the maximized Hamiltonian, we have not been able to prove existence.

ground must equal the marginal return on the alternative asset, capital, minus the marginal extraction cost in terms of lower productivity in the future. Under laissez-faire ($\psi \equiv 0$) this cost is not internalized, resulting in a too high required private return on delaying extraction, thereby speeding up extraction. The proper correction comes into sight when we divide through by $\partial Y / \partial R$ in (49) and use the Cobb-Douglas specification to get

$$g_Y - g_R = \alpha z - \delta - \frac{\lambda}{\gamma} u.$$

The corresponding relation for the market economy is the no-arbitrage condition (25). Inserting $\sigma = 1 - \alpha_1/\alpha$ and $\tau_{cg} = \tau_r = 0$ into this condition we see that the postponement stimulus should be set at $\psi = \lambda u / \gamma$, i.e., proportional to the current depletion rate, whether the system is in steady state or outside.

As to the roles of τ_{cg} and τ_r our results are in contrast to Dasgupta and Heal (1979) who state (on p. 368) that a capital gains tax when accompanied by an equally high interest income tax does not distort resource extraction at all. Also, Stiglitz (1975, p. 78) maintains, from inspecting the Hotelling rule, that an economy will pursue an excessively conservationist resource extraction policy when the tax on capital gains is lower than the tax on interest income. However, though these observations may be true in partial equilibrium, they do not hold in general equilibrium where the rate of interest is endogenous. Whatever the value of τ_r, a positive capital gains tax tends to incite too little conservation and impede growth.

5   Conclusion

Based on a Cobb-Douglas one-sector model, allowing for increasing returns to scale and an essential non-renewable resource, this paper has studied the influence of various policy instruments on long-run growth. Contrary to the predictions of standard endogenous growth theory neither a tax on interest income nor a subsidy to capital accumulation affect the long-run growth rate. However, policies directed towards the returns to resource conservation do influence growth. For example, taxing the capital gains, due to the rising price of the resource as its scarcity grows, makes extraction too favorable − to the detriment of long-run growth possibilities. A tax on resource use (like a carbon tax) matters if it is time-varying. When it keeps declining over time, it favours conservation and growth, and this is desirable if externalities like a greenhouse effect are present. Hence, the conclusion from the model is that policies concerned with natural resources ultimately mean more for welfare than do traditional capital taxes and subsidies.

The qualitative effects of taxes and subsidies are in most cases independent of whether the source of growth is exogenous or endogenous. But the source of growth matters for the role of population growth. In the endogenous growth case (tantamount to increasing returns to scale) population growth is needed for the existence of stable positive per capita growth, and higher population growth leads to higher per capita growth.

A problem left for future research is whether and how the results are modified when resource extraction depends on capital and labor as inputs. Productivity increases in the extractive industries may (besides discoveries of new deposits) be one of the reasons why, contrary to the prediction of the model, the data for the last century do not indicate a rising resource price trend (Nordhaus 1992).

6   Appendix

Proof of Lemma 1. Consider a BGE $(C, Y, K, R, S)_{t=0}^{\infty}$. (i) By definition of a BGE, g_C, g_Y, and g_K are constant. By constancy of g_C, z is a constant, in view of (26). This implies, first, that $g_Y = g_K$, and, second, that x is constant, in view of (22). Therefore, $g_C = g_K$. The common constant value of g_C, g_Y, and g_K is called g^*. (ii) By definition of a BGE, g_S is constant. Hence, by (23), u is a constant, say u^*, implying $g_R = g_S = -u^*$, and therefore $R(t) = R(0)e^{-u^*t}$. In view of $R(0) > 0$ we have $u^* > 0$ since otherwise (19) would be violated. (iii) and (iv) That z and x are constants has already been proved; by definition of a BGE they must also be positive. From (24), with $g_Y = g_K = g^*$ and $g_S = -u^*$, follows (27). Inserting (25) in (26) gives $g^* = \frac{1}{\varepsilon}[(1 - \tau_{cg})(g^* - g_R^* + \psi) - \rho] + n$, which is a contradiction unless ψ is constant, and by (16) this constant must be $\bar{\psi}$. Inserting $g_R = -u^*$ and reordering gives (28). (v) In view of (25), $(1 - \tau_{cg})(g^* - g_R^* + \psi) = (1 - \tau_r)r$, since, by (6), $r = \alpha_1 z/(1 - \sigma) - \delta$, where z is constant in the BGE. Inserting $u^* = -g_R^*$ and $\psi = \bar{\psi}$ gives

$$(1 - \tau_{cg})(g^* + u^* + \bar{\psi}) = (1 - \tau_r)r. \tag{50}$$

Now, the desired conclusion follows from the transversality conditions (20) and (21), by Lemma A below. □

Lemma A. *Let* $\lim_{t\to\infty} g_K = g^*, \lim_{t\to\infty} g_S = -u^*$, *and* $\lim_{t\to\infty} r(t) = r$, *where* r *satisfies (50). Then the transversality conditions (20) and (21) taken together are equivalent with the inequality in (v) of Lemma 1.*

Proof We have

$$\begin{aligned}
(20) \quad &\Leftrightarrow \quad g^* - (1 - \tau_r)r < 0 \Leftrightarrow g^* < (1 - \tau_{cg})(g^* + u^* + \bar{\psi}) \quad \text{(from (50))} \\
&\Leftrightarrow \quad \tau_{cg}g^* < (1 - \tau_{cg})(u^* + \bar{\psi}); \tag{51} \\
(21) \quad &\Leftrightarrow \quad -u^* + \frac{\tau_{cg}(1 - \tau_r)}{1 - \tau_{cg}}r < 0 \Leftrightarrow -(1 - \tau_{cg})u^* + \tau_{cg}(1 - \tau_r)r < 0 \\
&\Leftrightarrow \quad \tau_{cg}(1 - \tau_{cg})(g^* + u^* + \bar{\psi}) < (1 - \tau_{cg})u^* \quad \text{(from (50))} \\
&\Leftrightarrow \quad \tau_{cg}(g^* + \bar{\psi}) < (1 - \tau_{cg})u^*.
\end{aligned}$$

The last inequality ensures (51) since, by (16), $\bar{\psi} \geq 0$. $\square$

The output-capital and consumption-capital ratios. Inserting (30) and (31) into (32) gives

$$\begin{aligned}
z^* &= \frac{1 - \sigma}{\alpha_1}\frac{1 - \tau_{cg}}{1 - \tau_r}\cdot\frac{[(\alpha + \beta + \gamma + \lambda - 1)n + \theta]\varepsilon + (\gamma + \lambda)\varepsilon\bar{\psi} + (1 - \alpha - \gamma - \lambda)\rho}{D} \\
&\quad + \frac{1 - \sigma}{\alpha_1}\delta. \tag{52}
\end{aligned}$$

Inserting (31) into (33) gives

$$\begin{aligned}
x^* &= \frac{(1 - \sigma)\frac{1 - \tau_{cg}}{1 - \tau_r}\left\{[(\alpha + \beta + \gamma + \lambda - 1)n + \theta]\varepsilon + (1 - \alpha - \gamma - \lambda)\rho + (\gamma + \lambda)\varepsilon\bar{\psi}\right\}}{\alpha_1 D} \\
&\quad - \frac{(1 - \tau_{cg})(\beta n + \theta) + (\gamma + \lambda)\left[\varepsilon n - \rho + (1 - \tau_{cg})\bar{\psi}\right]}{D} + \frac{1 - \sigma - \alpha_1}{\alpha_1}\delta. \tag{53}
\end{aligned}$$

Proof of Lemma 2. The example in footnote 11 shows that P^* has a non-empty interior. $\square$

Proof of Proposition 2. Let $\pi \in P$ and assume $D \neq 0$. (i) As to the "if"-part we can construct a BGE in the following way. Given $\pi \in P^*$, let u^*, z^*, and x^* be defined as in (31), (52), and (53), respectively. By definition of P^* we know u^*, z^*, and x^* are strictly positive, and (v) of Lemma 1 is satisfied. Let $Q = (C, Y, K, R, S)_{t=0}^{\infty}$ be a path satisfying $u \equiv R/S = u^*$, $z \equiv Y/K = z^*$, and $x \equiv C/K = x^*$ for all t. This path has $g_R = g_S = -u^*$, from (23). We calculate g^* from (30). By construction, u^* and g^* satisfy (27) and (28). By (27) and (24), g^* is the common value of g_Y and g_K along the path Q, given $g_R = -u^*$. Further, given g^* and u^*, z^* satisfies (32), which implies the Hotelling rule (25). Now, combining (28) and (25) shows that g^* satisfies (26); hence, with $r = \frac{\alpha_1}{1 - \sigma}z^* - \delta$, and $g_c = g^* - n$ the Keynes-Ramsey rule (13) is satisfied. Finally, with $r = \frac{\alpha_1}{1 - \sigma}z^* - \delta$, (25) implies (50); hence, in view of (v) of Lemma 1 the

transversality conditions are satisfied, by Lemma A , and the path Q *is* a BGE. On the other hand, since $D \neq 0$, $\pi \notin P^*$ implies, by definition of P^*, that at least one of z^*, x^*, and u^* is non-positive or (v) of Lemma 1 is not satisfied. Hence, no BGE exists. (ii) Given $\pi \in P^*$, uniqueness of the BGE (g^*, u^*, z^*, x^*) follows from (30), (31), (52), and (53). $\square$

Proof of Proposition 3. Using (22), the identities $z \equiv Y/K$ and $x \equiv C/K$ imply

$$\dot{z} = (g_Y - z + x + \delta)z. \tag{54}$$

$$\dot{x} = (g_C - z + x + \delta)x, \tag{55}$$

Define the relative taxation index

$$\xi \equiv \frac{1 - \tau_r}{1 - \tau_{cg}} > 0.$$

Inserting (22) and the Hotelling Rule, (25), into (24) yields

$$g_Y = \frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma}z - \frac{\alpha}{1-\gamma}x - \frac{\lambda}{1-\gamma}u + \frac{\gamma}{1-\gamma}\psi + \frac{\beta n + \theta + (\xi\gamma - \alpha)\delta}{1-\gamma}. \tag{56}$$

Inserting this expression into (54), we find

$$\begin{aligned}
\dot{z} &= \left[(\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma} - 1)z + \frac{1-\alpha-\gamma}{1-\gamma}x - \frac{\lambda}{1-\gamma}u \right. \\
&\quad \left. + \frac{\gamma}{1-\gamma}\psi + \frac{\beta n + \theta + (1-\alpha-\gamma+\xi\gamma)\delta}{1-\gamma} \right] z.
\end{aligned} \tag{57}$$

Inserting $g_C = g_c + n$ and the Keynes-Ramsey Rule, (26), into (55) gives

$$\dot{x} = \left[(\frac{(1-\tau_r)\alpha_1}{(1-\sigma)\varepsilon} - 1)z + x + n - \frac{\rho}{\varepsilon} + (1 - \frac{1-\tau_r}{\varepsilon})\delta \right] x. \tag{58}$$

Differentiating the identity $u \equiv R/S$ with respect to time and using (18), we get

$$\dot{u} = (g_R + u)u. \tag{59}$$

By (25) and (56), this gives

$$\dot{u} = (\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1}{1-\gamma}z - \frac{\alpha}{1-\gamma}x + \frac{1-\gamma-\lambda}{1-\gamma}u + \frac{1}{1-\gamma}\psi + \frac{\beta n + \theta + (\xi - \alpha)\delta}{1-\gamma})u. \tag{60}$$

The dynamics of z, x, and u are completely described by the system (57), (58), and (60). Along a BGE this system is in steady state, i.e., $\dot{z} = \dot{x} = \dot{u} = 0$, by Lemma 1. We form the Jacobian evaluated in a steady state:

$$J = \begin{bmatrix} \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial u} \\ \frac{\partial \dot{x}}{\partial z} & \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial u} \\ \frac{\partial \dot{u}}{\partial z} & \frac{\partial \dot{u}}{\partial x} & \frac{\partial \dot{u}}{\partial u} \end{bmatrix} = \begin{bmatrix} (\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma} - 1)z^* & \frac{1-\alpha-\gamma}{1-\gamma}z^* & -\frac{\lambda}{1-\gamma}z^* \\ (\frac{(1-\tau_r)\alpha_1}{(1-\sigma)\varepsilon} - 1)x^* & x^* & 0 \\ \frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1}{1-\gamma}u^* & -\frac{\alpha}{1-\gamma}u^* & \frac{1-\gamma-\lambda}{1-\gamma}u^* \end{bmatrix}. \tag{61}$$

The determinant of J is

$$\det J = -\frac{\xi D}{(1-\sigma)(1-\gamma)\varepsilon} a_1 z^* x^* u^*, \tag{62}$$

which is negative if and only if $D > 0$. The trace of J is

$$
\begin{aligned}
\operatorname{tr} J \;&=\; \left(\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma} - 1\right)z^* + x^* + \frac{1-\gamma-\lambda}{1-\gamma}u^* \\
&\geq\; \frac{1-\xi\gamma}{1-\gamma}\frac{\alpha_1}{1-\sigma}z^* - z^* + x^* + \frac{1-\gamma-\lambda}{1-\gamma}u^* \quad \left(\text{since } \frac{\alpha_1}{1-\sigma} \leq \alpha,\ \text{by (A1)}\right) \\
&=\; \frac{1-\xi\gamma}{1-\gamma}\left(\frac{g^* + u^* + \bar\psi}{\xi} + \delta\right) - g^* - \delta + \frac{1-\gamma-\lambda}{1-\gamma}u^* \quad \text{(by (32) and (22))} \\
&=\; \frac{(1-\xi)g^* + (1-\xi\gamma)(u^* + \bar\psi + \xi\delta)}{\xi(1-\gamma)} + \frac{1-\gamma-\lambda}{1-\gamma}u^* - \delta \\
&\approx\; \left(2 - \frac{\lambda}{1-\gamma}\right)u^* + \bar\psi > 0 \quad \text{(since } \xi \approx 1,\ \text{by (A1)},\ \lambda/2 < 1-\gamma,\ \text{and } \bar\psi \geq 0).
\end{aligned}
$$

This shows that the three eigenvalues cannot all be negative (or have negative real part). Hence, if $D > 0$, i.e., $\det J < 0$, there is one negative and two positive eigenvalues. Now, even though z, x, and u are all 'jump variables', by substituting uS for R in the production function (5) we get $z = e^{\theta t}K^{\alpha-1}N^\beta u^\gamma S^{\gamma+\lambda}$, showing that, given K, N, and S, the values of z and u are not independent. In view of the implied boundary value condition one negative eigenvalue and two positive eigenvalues implies saddle-point stability.

Now, assume on the contrary $D < 0$. Then $\det J > 0$, and on the face of it there are two possible cases: Either all three eigenvalues are non-negative or there is one positive eigenvalue and two eigenvalues with non-positive real part. However, there exists a number $\bar\lambda > 0$ such that for $\lambda < \bar\lambda$ this last-mentioned case can be excluded. Indeed, if $\lambda = 0$, then J is block-triangular, and $u^* > 0$ is an eigenvalue. And since both the determinant and the trace of the upper left 2×2 sub-matrix of J is positive (at least given the policy assumption (A1)), the other two eigenvalues are also positive. By continuity, all three eigenvalues will be positive also if $\lambda \in (0, \bar\lambda)$ for some sufficiently small positive number $\bar\lambda$. $\square$

Optimal allocation. The current-value Hamiltonian for the optimal allocation problem, described in Section 4, is

$$H = \frac{c^{1-\varepsilon} - 1}{1 - \varepsilon}N + \mu_1(Y - cN - \delta K) - \mu_2 R, \tag{63}$$

where $Y = e^{\theta t}K^\alpha N^\beta R^\gamma S^\lambda$, and μ_1 and μ_2 are the co-state variables associated with

physical capital and the resource stock, respectively. Necessary first order and transversality conditions for an interior solution are:

$$c^{-\varepsilon} = \mu_1, \tag{64}$$

$$\mu_1 \gamma \frac{Y}{R} = \mu_2, \tag{65}$$

$$\dot{\mu}_1 = \rho \mu_1 - (\alpha \frac{Y}{K} - \delta)\mu_1, \tag{66}$$

$$\dot{\mu}_2 = \rho \mu_2 - \lambda \frac{Y}{S} \mu_1, \tag{67}$$

$$\lim_{t \to \infty} e^{-\rho t} \mu_1 K = 0, \tag{68}$$

$$\lim_{t \to \infty} e^{-\rho t} \mu_2 S = 0. \tag{69}$$

From (64) and (66),

$$g_c = \frac{1}{\varepsilon}(\alpha z - \delta - \rho). \tag{70}$$

Similarly, (65), (66), and (67) lead to

$$g_Y - g_R = \alpha z - \delta - \frac{\lambda}{\gamma} u. \tag{71}$$

The equations (22), (24), (54), (55), and (59) are still valid. If an optimal allocation is a BGP, g_C, g_Y, g_K, g_R, and g_S are constant, by definition. Hence, along a BGP also z, x, and u are constant, $g_C = g_c + n = g_Y = g_K = g^{*SP}$, and $g_R = g_S = -u^{*SP}$. Inserting into (71) and using (70) gives

$$(\varepsilon - 1) g^{*SP} - \frac{\gamma + \lambda}{\gamma} u^{*SP} = \varepsilon n - \rho. \tag{72}$$

Combining this with the growth accounting relation (27) gives (47) and (48), assuming $D^{SP} \neq 0$. The values of z and x along a BGP are

$$z^{*SP} = \frac{[(\alpha + \beta + \gamma - 1)n + \theta]\varepsilon + (1 - \alpha - \gamma)\rho}{\alpha D^{SP}} + \frac{\delta}{\alpha},$$

$$x^{*SP} = \frac{[(\alpha + \beta + \gamma - 1 - \alpha\gamma)n + \theta]\varepsilon - \alpha(\beta n + n) + (1 - \alpha)(1 - \gamma)\rho}{\alpha D^{SP}} + \frac{(1 - \alpha)\delta}{\alpha}.$$

Transitional dynamics. Inserting (22) and (71) into (24) yields

$$g_Y = \alpha z - \frac{\alpha}{1 - \gamma} x + \frac{\beta n + \theta - (\alpha - \gamma)\delta}{1 - \gamma}. \tag{73}$$

Inserting this expression into (54), we find

$$\dot{z} = \left[(\alpha - 1)z + \frac{1 - \alpha - \gamma}{1 - \gamma} x + \frac{\beta n + \theta + (1 - \alpha)\delta}{1 - \gamma} \right] z. \tag{74}$$

26

Inserting $g_C = g_c + n$ and (70) into (55) yields

$$\dot{x} = \left[(\frac{\alpha}{\varepsilon} - 1)z + x + n - \frac{\rho}{\varepsilon} + (1 - \frac{1}{\varepsilon})\delta \right] x. \tag{75}$$

By (71) and (73), (59) gives

$$\dot{u} = (\frac{-\alpha}{1 - \gamma}x + \frac{\gamma + \lambda}{\gamma}u + \frac{\beta n + \theta + (1 - \alpha)\delta}{1 - \gamma})u. \tag{76}$$

The dynamics of z, x, and u are completely described by the system (74), (75), and (76). Along a BGP the system is in steady state. The Jacobian evaluated in steady state is

$$J^{SP} = \begin{bmatrix} (\alpha - 1)z^{*SP} & \frac{1-\alpha-\gamma}{1-\gamma}z^{*SP} & 0 \\ (\frac{\alpha}{\varepsilon} - 1)x^{*SP} & x^{*SP} & 0 \\ 0 & -\frac{\alpha}{1-\gamma}u^{*SP} & \frac{\gamma+\lambda}{\gamma}u^{*SP} \end{bmatrix},$$

which is block-triangular, hence one of the eigenvalues is $\frac{\gamma+\lambda}{\gamma}u^{*SP} > 0$. The determinant of the upper left 2×2 sub-matrix of J^{SP} is

$$\det J^{SP} = -\frac{\alpha}{(1 - \gamma)\varepsilon}D^{SP}z^{*SP}x^{*SP}.$$

When $D^{SP} > 0$ this is negative so that there is one negative and two positive eigenvalues, implying saddle-point stability.

Let the parameters $\alpha, \beta, \gamma, \lambda, \delta, n, \theta, \varepsilon, \rho$ be such that D^{SP} as well as u^{*SP}, z^{*SP}, and x^{*SP} are strictly positive (footnote 11 shows an example). Then a unique BGP exists, and there is (at least locally) a unique feasible path $(C, Y, K, R, S)_{t=0}^{\infty}$, fulfilling the first order conditions, such that the associated triple $(z, x, u)_{t=0}^{\infty} \to (z^*, x^*, u^*)$ for $t \to \infty$. To show that this path *is* optimal, at least when $\alpha + \gamma + \lambda \leq 1$, two steps remain.

Step 1. The path satisfies the two transversality conditions. Indeed, using successively (66) and (70),

$$\lim_{t \to \infty}(\frac{\dot{K}}{K} + \frac{\dot{\mu_1}}{\mu_1} - \rho) = g^{*SP} + \lim_{t \to \infty}(\delta - \alpha z) = g^{*SP} - \lim_{t \to \infty}(\varepsilon\frac{\dot{c}}{c} + \rho)$$
$$= (1 - \varepsilon)g^{*SP} + \varepsilon n - \rho < 0,$$

where the inequality follows from (72), since $u^{*SP} > 0$. Hence, (68) is satisfied. Similarly, using (67) and (65),

$$\lim_{t \to \infty}(\frac{\dot{S}}{S} + \frac{\dot{\mu_2}}{\mu_2} - \rho) = -\frac{\gamma + \lambda}{\gamma}u^{*SP} < 0,$$

showing that (69) is satisfied. In view of (64) and (65), $\mu_1 > 0$ and $\mu_2 > 0$ for all $t \geq 0$. Hence, the path $(C, Y, K, R, S)_{t=0}^\infty$, which is our candidate for an optimal solution, satisfies

$$\lim_{t\to\infty} \left[e^{-\rho t} \mu_1 (\tilde{K} - K) + e^{-\rho t} \mu_2 (\tilde{S} - S) \right] \geq 0,$$

for all feasible paths $(\tilde{C}, \tilde{Y}, \tilde{K}, \tilde{R}, \tilde{S})_{t=0}^\infty$.

Step 2. We check whether the Hamiltonian is jointly concave in the state variables (K, S) after the controls c and R have been substituted by their maximizing values. We get from (64) and (65)

$$
\begin{aligned}
c &= \mu_1^{-\frac{1}{\varepsilon}}, \\
R &= (\gamma e^{\theta t} N^\beta \frac{\mu_1}{\mu_2})^{\frac{1}{1-\gamma}} S^{\frac{\lambda}{1-\gamma}} K^{\frac{\alpha}{1-\gamma}} \equiv A_1 S^{\frac{\lambda}{1-\gamma}} K^{\frac{\alpha}{1-\gamma}}.
\end{aligned}
$$

Inserting this into (63) gives the maximized Hamiltonian

$$
\begin{aligned}
\hat{H} &= \frac{\mu_1^{-\frac{1-\varepsilon}{\varepsilon}} - 1}{1 - \varepsilon} N + \mu_1 e^{\theta t} K^\alpha N^\beta S^\lambda A_1^\gamma S^{\frac{\lambda\gamma}{1-\gamma}} K^{\frac{\alpha\gamma}{1-\gamma}} - \mu_1^{1-\frac{1}{\varepsilon}} N - \mu_1 \delta K - \mu_2 A_1 S^{\frac{\lambda}{1-\gamma}} K^{\frac{\alpha}{1-\gamma}} \\
&\equiv A_2 + A_3 S^{\frac{\lambda}{1-\gamma}} K^{\frac{\alpha}{1-\gamma}} - \mu_1 \delta K,
\end{aligned}
$$

where

$$
\begin{aligned}
A_2 &\equiv \left(\frac{\mu_1^{-\frac{1-\varepsilon}{\varepsilon}} - 1}{1 - \varepsilon} - \mu_1^{1-\frac{1}{\varepsilon}} \right) N, \\
A_3 &\equiv (\mu_1 e^{\theta t} N^\beta \gamma)^{\frac{1}{1-\gamma}} \mu_2^{\frac{-\gamma}{1-\gamma}} (\gamma^{-1} - 1) > 0 \quad \text{since } \gamma < 1.
\end{aligned}
$$

It follows that $\hat{H}$ is concave in (K, S) if and only if $\frac{\lambda+\alpha}{1-\gamma} \leq 1$, i.e.,

$$\alpha + \gamma + \lambda \leq 1. \tag{77}$$

From Step 1 and Step 2 follows that when $\alpha + \gamma + \lambda \leq 1$, the path $(C, Y, K, R, S)_{t=0}^\infty$ is optimal, cf. Theorem 14 in Seierstad and Sydsæter, 1987, p. 236.

Proof of Proposition 5. Let the parameter vector $(\alpha_1, \beta, \gamma, \alpha, \lambda, \theta, \delta, n, \varepsilon, \rho)$ be such that a BGP exists both under laissez-faire and the social planner's rule. Assume $D(0) > 0$ and consider the laissez-faire BGE. By Proposition 6, a first-best BGE has $\bar{\psi} = \lambda u^{*SP}/\gamma > 0$ and $\sigma = 1 - \alpha_1/\alpha$. (i) Assume $\lambda > 0$. Then, a shift from first-best to laissez-faire affects g^* only through the decrease of $\bar{\psi}$ to zero, and this effect is negative, since $\partial g^*/\partial \bar{\psi} = \partial g_c^*/\partial \bar{\psi} > 0$, by (39). (ii) Assume $\lambda = 0$, $\alpha_1 < \alpha$. Then, a shift from first-best to laissez-faire changes only σ, which decreases from $1 - \alpha_1/\alpha > 0$ to zero. This decrease in σ does not affect g^*, but increases z^* in view of (46). $\square$

Proof of Proposition 6. The dynamic system, (57), (58), and (60), for the market economy is seen to be identical to that of the optimal allocation, (74), (75), and (76), if and only if

$$\sigma = 1 - \alpha_1/\alpha, \quad \psi = \frac{\lambda}{\gamma}u, \quad \text{and} \quad \tau_{cg} = \tau_r = 0.$$

This proves the proposition. $\square$

References

Aghion, Philippe, and Peter Howitt, 1998a: *Endogenous Growth Theory*, MIT Press, Cambridge, MA.

Aghion, Philippe, and Peter Howitt, 1998b: Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth, *Journal of Economic Growth, 3,* 2, 111-30.

Antweiler, Werner, and Daniel Trefler, 2002: Increasing Returns and All That: A View from Trade, *American Economic Review, 92, 1, 93-119.*

Arrow, Kenneth J., 1962: The Economic Implications of Learning by Doing, *Review of Economic Studies, 29*, 155-173.

Barro, Robert J., & Xavier Sala-i-Martin, 1995: *Economic Growth.* McGraw-Hill, New York.

Caballero, Ricardo J., and Richard K. Lyons, 1992: The Case of External Economies, in A. Cukierman, Z. Hercowitz, and L. Leiderman (eds.), *Political Economy, Growth, and Business Cycles,* MIT Press, Cambridge, MA.

Dasgupta, Partha, & Geoffrey Heal, 1979: *Economic Theory and Exhaustible Resources.* Nisbet/Cambridge University Press.

Groth, Christian, and Poul Schou, 2002: Can Nonrenewable Resources Alleviate the Knife-edge Character of Endogenous Growth? *Oxford Economic Papers,* 54, 386-411.

Hall, Robert E., 1990: Invariance Properties of Solow's Residual, in P. Diamond (ed.), *Growth/Productivity/Unemployment. Essays to Celebrate Bob Solow's Birthday,* MIT Press, Cambridge, MA.

Jones, Charles I., 1995: R & D-Based Models of Economic Growth. *Journal of Political Economy*, 103, 759-784.

Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi, 1993: Optimal Taxation in Models of Endogenous Growth. *Journal of Political Economy*, 101, 485-517.

King, Robert G., and Sergio Rebelo, 1990: Public Policy and Economic Growth: Developing Neoclassical Implications. *Journal of Political Economy*, 98, no. 5, pt. 2, S126-S150.

Lucas, Robert E., 1990: Supply-Side Economics: An Analytical Review, *Oxford Economic Papers,* 42.

Milesi-Ferretti, Gian M., and Nouriel Roubini, 1998: Growth Effects of Income and Consumption Taxes. *Journal of Money, Credit, and Banking*, 30, 721-44.

Nordhaus, William, 1992: Lethal Model 2: The Limits to Growth Revisited, *Brookings Papers on Economic Activity*, 2, 1-43.

Rebelo, Sergio, 1991: Long- Run Policy Analysis and Long-Run Growth. *Journal of Political Economy*, 99, 3, 500-21.

Robson, A. J., 1980: Costly Innovation and Natural Resources. *International Economic Review*, 21, 17-30.

Romer, Paul M., 1990: Endogenous Technological Change. *Journal of Political Economy*, 98, S71-S102.

Romer, Paul M., 1995, Comment to Srinivasan, in *Growth Theories in the Light of the East Asian Experience*, ed. by T. Ito and A.O. Krueger, University of Chicago Press, Chicago.

Scholz, Christian M., and Georg Ziemes, 1999: Exhaustible Resources, Monopolistic Competition, and Endogenous Growth, *Environmental and Resource Economics*, 13, 169-185.

Schou, Poul, 2000: Polluting Nonrenewable Resources and Growth. *Environmental and Resource Economics*, 16, 211-27.

Schou, Poul, 2002: When Environmental Policy is Superfluous: Growth and Polluting Resources. *Scandinavian Journal of Economics*, 104, 605-620.

Sinclair, Peter, 1992, High Does Nothing and Rising Is Worse: Carbon Taxes Should Keep Declining to Cut Harmful Emissions, *The Manchester School*, 60, 41-52.

Sinclair, Peter, 1994, On the Optimum Trend of Fossil Fuel Taxation, *Oxford Economic Papers*, 46, 869-877.

Solow, Robert M., 1994, Perspectives on Growth Theory, *Journal of Economic Perspectives*, 8, 45-54.

Stiglitz, Joseph, 1974a: Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths. *Review of Economic Studies*, Symposium Issue, 123-137.

Stiglitz, Joseph, 1974b: Growth with Exhaustible Natural Resources: The Competitive Economy. *Review of Economic Studies*, Symposium Issue, 139-52.

Stiglitz, Joseph, 1975: The Efficiency of Market Prices in Long-run Allocations in the Oil Industry, in G.M. Brannon (ed.), *Studies in Energy Tax Policy*. Ballinger Publ. Comp., Cambridge (Mass.), 55-99.

Stokey, Nancy L., and Sergio Rebelo, 1995, Growth Effects of Flat-Rate Taxes, *Journal of Political Economy*, 103, 519-50.

Takayama, Akira, 1980: Optimal Technical Progress with Exhaustible Resources, in M.C. Kemp and N.V. Long (eds.), *Exhaustible Resources, Optimality, and Trade*. North-Holland, Amsterdam.

Uhlig, Harald, and Noriyuki Yamagawa, 1996: Increasing the capital income tax may lead to faster growth, *European Economic Review*, 40, 1521-1540.