Local Public Goods, Debt and Migration*

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Abstract

Migration raises a potential free rider problem for the provision of durable local public goods if the late-comers can enjoy the public good without paying for it. Allowing communities to finance public goods by debt mitigates this problem, since future immigrants have to share the burden of the debt. However, in equilibrium there will be over-accumulation of local debt. There may be more or less public good than in first best, but *conditional on the inefficiently high level of debt* there will be too few public goods. A competitive market for land reduces but does not in general eliminate the inefficiencies.

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1 Introduction

This paper analyzes the interplay between local public goods provision, debt, and migration in a two-community model. By increasing its production of local public goods, a district will attract new immigrants and thereby experience more congestion. This free riding problem causes the district to undersupply the local public good. However, if the local public good is financed by local public debt, then future immigrants must pay for the public good by sharing the debt burden. This mitigates the free riding problem and makes the current residents more willing to provide durable public goods. On the other hand, since accumulation of debt makes the district less attractive for immigrants and may even cause emigration, it imposes a negative externality on the other district (increased congestion). Because of this externality, debt will be over-used in equilibrium.

The link between local public goods, local public debt and migration is illustrated by the experience of the Faroe Islands, a district in Denmark with a large amount of autonomy. Citizens of the Faroe Islands are free to move within Denmark. Throughout the late eighties many public goods were built in the Faroe Islands: schools, hospitals, roads, tunnels, all financed by local public debt. As the fishing industry suffered from the effects of overfishing in the early nineties repaying the debt became very cumbersome, and around 15 percent of the population emigrated in the period 1989-94 (see Det Raadgivende Udvaelg Vedroerende Faeroerne, 1996, 1998). The districts in our model should be thought of as sub-national communities such as the Faroe Islands, or states in a federation where migration is fairly easy. The model may also become more important from a European perspective if the coming unification of Europe leads to more migration than hitherto seen. Areas with abundant local goods provision or well established welfare states may then see a large inflow of immigrants from other parts of Europe, while areas with large amounts of local public debt will see an outflow.

In our model there are two symmetric districts, and two periods. In the first period, the representative inhabitant of each district sets the levels of local public good and local public debt in order to maximize his own life time utility. Thus, the strategy-spaces are two-dimensional. The public good is with congestion and can be enjoyed in both periods. Between the two periods, citizens of the two districts can freely and without cost move between the districts.[1] The second period population of a district enjoys the public good of the district but has to repay whatever debt the district incurred in the first period. Citizens who move become responsible for their share of the debt in the new district, but have no responsibilities in the district they left. When the period one policy of a district is chosen, the representative citizen takes into account the fact that both he and other people may migrate.

If in equilibrium both districts desire more immigrants, then if one more person accidentally were to move from district a to district b, district b would become more attractive than district a. This would induce more migration to district b, so such an equilibrium would not be stable. We focus on stable

[1] Since migration only takes place between the periods, a period 1 tax cut in a district is enjoyed only by the current inhabitants, while a higher debt burden reduces immigration in the next period. Our results would not change if we allowed migration *before* policy decisions are made in period 1, as long as agents are not allowed to migrate again until period 1 is over. Our results would change, however, if agents were allowed to switch districts *instantaneously* as a response to period 1 policy decisions. In this case there would be no strategic benefit of using debt instead of taxes to finance the public good, since a tax-cut today would cause instantaneous immigration. But such a frictionless scenario would be unrealistic. Decisions to migrate need to be planned ahead of time, so that potential migrants are to some extent locked into their current district.

rational expectations equilibria.[2] In a stable equilibrium there must be at least one district which on the margin dislikes immigration. In a symmetric equilibrium this will be true for both districts. A district which dislikes immigration tries to reduce immigration by under-providing the public good and over-accumulating debt (we assume it is not feasible to restrict migration directly by quotas and so on). The accumulation of debt implies that first period citizens have extra money to spend in the first period, and some of this money will surely be spent on the public good. In addition, fewer immigrants are attracted if they have to share the burden of the debt. For these two reasons, local debt stimulates the production of the local public good. In fact, the tendencies to over-use one instrument (debt) and under-use the other (public goods) will counter balance each other, and we cannot predict how the final outcome will compare to the first best. We give an example where the equilibrium level of public goods equals the first best level. However, *conditional on the level of debt* there will be too few public goods. Conversely, conditional on the level of public goods, there is too much debt. Thus, the equilibrium is inefficient. The excessive borrowing suggests that a federal government should introduce debt limits. In fact, for more than 100 years, the debt of local jurisdictions in the United States has been restricted by the state governments (see Epple and Spatt, 1987). However, in our model there are too few public goods in equilibrium, and a debt limit will in general cause the district to provide even fewer public goods. This

[2] It does not seem possible to prove the general existence of equilibrium in our model, although in the examples we have looked at, an equilibrium always exists. Since immigration has conflicting effects (it increases congestion but reduces the per capita debt level), we cannot *in general* say whether immigration will be good or bad for the current inhabitants. As suggested by Konishi, Le-Breton and Weber (1997), such non-monotonicity may cause existence failure.

negative effect counters the positive direct effect on welfare from a debt limit. A debt limit is Pareto improving if the fall in the level of public goods is not too big. More precisely, if a dollar reduction in debt causes the district to cut public goods provision by *less* than a dollar, then a debt limit is Pareto improving.

The fact that local public debt may be over-accumulated if the burden of the debt can be shifted onto future generations was discussed by Daly (1969), Oates (1972) and Bruce (1995) (although they did not discuss the provision of public goods). Daly and Oates argued that in fact the burden of the debt cannot be shifted, since the debt will be reflected in reduced local property values. In Section 6 we address this important issue by introducing land into the model. In general equilibrium the land price is determined by the marginal utility from land. Taxes may be levied on income or on land. If some revenue is raised through income taxes, then the capitalization of the debt burden in land prices is not complete. Qualitatively, the model then behaves in the same way as the model without land. We cannot even say that the inefficiencies are lower with land, although in a specific example, land mitigates the free riding problem. If *all* taxation is on land, then the debt level will be fully capitalized in land prices, and issuing debt does not reduce immigration. Because of this effect, it seems plausible that taxing land instead of income will lower the public goods provision. We will give an example where this is true. In any case, we argue that the *method* of taxation determines whether or not debt is capitalized in land prices. In contrast, public goods are never capitalized in land prices for any tax-system since in our model the level of public goods does not change the marginal utility of land. Thus, a high level of public goods always attracts immigration, the free-rider effect persists, and there will be under-provision of public goods in equilibrium.

Epple and Spatt (1987) analyze a model where *defaults* impose a negative externality across districts: if one district defaults the credit rating of all districts is lowered. To prevent defaults, a federal government may want to restrict local borrowing. In our model the possibility of default is not the problem. Instead it is migration across districts which leads to the wrong marginal incentives to accumulate debt and produce public goods. There is a large literature on migration decisions and local public goods when agents differ in their willingness to pay for public goods (Tiebout, 1956, Atkinson and Stiglitz, 1980, Epple et al, 1984, Epple and Romer, 1991), but this literature does not discuss debt versus tax financing. Bucovetsky and Wilson (1991) have a model with two-dimensional strategy space: different jurisdictions can tax mobile capital and non-mobile labor. There is no debt. If capital-income is taxed with a source-based tax rather than a residence-based tax, capital flees taxation. This introduces a distortion, and jurisdictions under provide public goods and use inefficiently low rates of capital-income tax relative to the (non-distortionary) wage-income tax. If capital-income is subject to a residence-based tax, the distortions disappear. Jensen and Toma (1991) consider a two-period, two-district model with a non-durable local public good. Local debt may be issued, but it plays a different role than in our model. Capital is perfectly mobile in both periods, which leads to tax competition (households never move). A high local debt level is a commitment to setting high taxes tomorrow. If taxes in the two districts are strategic complements, then debt will be issued in order to reduce the tax competition problem. The local public good is under-provided in period two, but not necessarily in period one. If instead taxes are strategic substitutes, each district will accumulate a surplus in period one. The local public good is under-provided in period one, but not necessarily in period two.

Schultz and Sjöström (1997) investigate the outcomes of *local elections* in a model where mayors decide on debt levels, migration can take place, but there is no public good. Voters will vote for a mayor with a low discount factor who will borrow a lot, because this will induce other districts to decrease their debt level (debt is a strategic substitute). Therefore, the debt levels will be higher than they would be if the voters could vote directly on the size of the local debt (as in Switzerland).

2 The model

There are two periods and two districts, a and b, each with a continuum of citizens normalized to size one. All agents are identical. Each agent has an endowment of the private good ("income") in each period which we normalize to 1. The utility function is $u(c, y, s)$, where $c \geq 0$ and $y \geq 0$ denote consumption of private and public goods, respectively, and $s \in [0, 2]$ is the number of inhabitants in the district (reflecting the congestion effects). For simplicity we assume there is no discounting of future utility, so lifetime utility is the sum of the utilities in the two periods. (Discounting would not change our results). We make the following assumption.

Assumption A. $u(c, y, s) = -\infty$ if $c = 0$ or $y = 0$ or $s = 2$; and $u(c, y, s) > -\infty$ otherwise.

Thus, if the private or public consumption level falls to zero, or the population size reaches two, then the district becomes a very undesirable place to live in. For $c > 0$, $y > 0$ and $s < 2$, the utility function $u(c, y, s)$ is differentiable and concave, with $u_c > 0$, $u_y > 0$, and $u_s < 0$ (where u_c, u_y and u_s denote partial derivatives with respect to the three arguments).

In the first period, each district decides how much local public good to produce. This good lasts for two periods. Each unit of the public good costs one unit of the private good. It is not possible to produce more public goods in the second period. A district can finance the production of the public good either by taxes or by debt. We assume that there is a capital market external to the economy where a district can borrow at zero interest rate.

After period one, the agents can move costlessly between districts. At the time of migration, each agent knows the debt and public goods levels in both districts. The number of citizens living in district a in period two is denoted n, where $0 \leq n \leq 2$. In district b the population size is $2 - n$. The local debt in a district has to be repaid in the second period by the second period citizens of the district, subject to the constraint that the consumption of the private good cannot be negative (limited liability). Suppose district a decides to produce the amount $y_a \geq 0$ of the public good and to borrow $d_a \geq 0$. The first period per capita consumption of private and public goods in district a will be $1 - y_a + d_a$ and y_a, respectively (recall that each individual has an "income" of 1 in each period). Non-negativity requires $0 \leq y_a \leq 1 + d_a$. In period two, the public goods level in district a is the same as in period one, y_a. The per capita debt is d_a/n. Per capita consumption of private goods in district a in period two is $1 - d_a/n$ if $n > d_a$, and zero if $n \leq d_a$. More concisely, consumption is $\max\{0, 1 - \frac{d_a}{n}\}$.[3] The case where the local debt d_a exceeds the district's total income, n, may be considered a case of default.

This paper will study the marginal incentives of inhabitants of local districts to build public goods and incur debt, but it will not contribute to the literature analyzing the incentives to default on local debt (as in Epple

[3]If $d_a = 0$ then per capita debt is zero, and private consumption in period 2 is 1 regardless of n.

and Spatt, 1987). Assumption A implies that if district b is not expected to default, then district a does not want to default either. The reason is that if district a defaults, the second period inhabitants of district a get zero private consumption and utility $-\infty$. Under such conditions, nobody wants to live in district a, and emigration to district b will take place until utility in district b is also $-\infty$, which happens when the second period population reaches 2. Hence, default does not pay. Since the districts are symmetric, neither district wants to default if it thinks the other district will not default. However, there is a coordination problem: Assumption A does not rule out the possibility that both districts default simultaneously. Suppose district a expects district b to borrow an amount that it can never possibly repay. Then district a will realize that by *not* defaulting, district a will attract all of district b's inhabitants in period 2, and so have a population of 2 and a utility of $-\infty$. In this case district a might just as well default, and the same is true for district b. That is a coordination problem which we shall not address in this paper. We shall not consider "bad" equilibria with utility equal to $-\infty$ in both districts, but instead we focus on equilibria where the utility in each district is finite. Of course, the "bad equilibrium" with default would in any case be impossible if lenders have rational expectations, since rational lenders will restrict the amount of borrowing to ensure repayment[4].

We end this section by considering the symmetric first best solution, assuming equal Pareto-weights to all individuals. As long as u is concave,

[4] In a previous version of this paper we explicitly modelled the behavior of the lenders, and showed that this eliminated the possibility of default in equilibrium. A more reduced-form solution to the problem of default would be to assume a debt limit of $1 - \varepsilon$ per district, where $\varepsilon > 0$ is some arbitrarily small number. The total income in the whole economy is 2, so if total borrowing does not exceed $2(1 - \varepsilon)$ then it is impossible for both districts to default simultaneously.

the first best involves $n = 1$, and (y_j, d_j) for $j \in \{a, b\}$ is chosen to maximize

$$u(1 - y_j + d_j, y_j, 1) + u(1 - d_j, y_j, 1) \tag{1}$$

This implies $d_j = y_j/2$ (perfect consumption smoothing). Substituting $d_j = y_j/2$ into (1) we find that the first best level $y_j = y^{opt}$ is determined by the first order condition

$$2u_y(1 - \frac{y^{opt}}{2}, y^{opt}, 1) = u_c(1 - \frac{y^{opt}}{2}, y^{opt}, 1)$$

3 Equilibrium

We consider a non-cooperative game where the two players are a representative period one citizen of district a, and a representative period one citizen of district b. We refer to these two players as "district a" (or player a) and "district b" (or player b). The objective of player $j \in \{a, b\}$ is to choose first period debt and public goods levels, (d_j, y_j), so as to maximize his own expected lifetime utility. It is assumed that d_j and y_j are always non-negative.

Following any first period history (d_a, y_a, d_b, y_b), each of the infinitely many agents will make a migration decision which maximizes his utility. Therefore, the second period population level n of district a will be a function of the history, $n = n(d_a, y_a, d_b, y_b)$, called a *migration function.* This *function* will be denoted $n(.)$. Let

$$\Gamma(n \mid d_a, y_a, d_b, y_b) \equiv u\left(\max\{0, 1 - \frac{d_a}{n}\}, y_a, n\right) - u\left(\max\{0, 1 - \frac{d_b}{2-n}\}, y_b, 2 - n\right)$$

denote the increased payoff from living in district a rather than b in period two, when debt and public goods levels are (d_a, y_a, d_b, y_b) and the population level in district a is n. We will say that migration decisions are *rational* if and only if (2) and (3) are both satisfied, for all (d_a, y_a, d_b, y_b):

$$\Gamma(n(d_a, y_a, d_b, y_b) \mid d_a, y_a, d_b, y_b) > 0 \quad \Rightarrow \quad n(d_a, y_a, d_b, y_b) = 2 \tag{2}$$

$$\Gamma(n(d_a, y_a, d_b, y_b) \mid d_a, y_a, d_b, y_b) < 0 \quad \Rightarrow \quad n(d_a, y_a, d_b, y_b) = 0 \tag{3}$$

Here (2) states that if living in district a is strictly more desirable, then all agents live in district a, and (3) states that if living in district b is strictly more desirable, then all agents live in district b. Notice that, as utility is intertemporally separable and there are no costs of moving, all agents are in a symmetric position after the first period and the decision of where to locate in the second period is not influenced by where the agent lived in the first period.

An *equilibrium* is a configuration $(d_a^*, y_a^*, d_b^*, y_b^*, n(.))$ such that migration decisions are rational, and each player j's first period choice (d_j^*, y_j^*) maximizes the expected *life time* utility of player j, given the first period choice of the other district and migration function $n(.)$. Note that we assume rational expectations: (1) in equilibrium, each player $j \in \{a, b\}$ can correctly predict the other player's first period choice; and (2) for each first period choice of (d_a, y_a, d_b, y_b), all agents correctly predict district a's second period population level $n = n(d_a, y_a, d_b, y_b)$.

Let $V^j(d_a, y_a, d_b, y_b, n(.))$ denote the expected lifetime utility of player j, given first period choices (d_a, y_a, d_b, y_b) and migration function $n(.)$.

Definition 1 *The list* $(d_a^*, y_a^*, d_b^*, y_b^*, n(.))$ *is an* equilibrium *if and only if: (a) migration decisions are rational, i.e.,* $n(.)$ *satisfies (2) and (3); (b)* $V^a(d_a^*, y_a^*, d_b^*, y_b^*, n(.)) \geq V^a(d_a, y_a, d_b^*, y_b^*, n(.))$ *for all* $(d_a, y_a) \geq 0$*; and (c)* $V^b(d_a^*, y_a^*, d_b^*, y_b^*, n(.)) \geq V^b(d_a^*, y_a^*, d_b, y_b, n(.))$ *for all* $(d_b, y_b) \geq 0$.

Let $(d_a^*, y_a^*, d_b^*, y_b^*, n(.))$ be any equilibrium, and let $n^* \equiv n(d_a^*, y_a^*, d_b^*, y_b^*)$ denote the equilibrium second period population of district a. For utility to be finite in equilibrium, we must have

$$0 < n(d_a^*, y_a^*, d_b^*, y_b^*) < 2 \tag{4}$$

and

$$d_a^* < n(d_a^*, y_a^*, d_b^*, y_b^*) \tag{5}$$

and

$$d_b^* < 2 - n(d_a^*, y_a^*, d_b^*, y_b^*) \tag{6}$$

To see this, notice that if (80) is violated, then all agents live in the same district in period 2 and they all get a utility of $-\infty$ by Assumption A. On the other hand, if (80) holds, then (2) and (3) imply that both districts are equally pleasant in period 2. Then, if (81) is violated, second period consumption in district a is zero so utility is $-\infty$ for anybody who lives in district a. Since district b is no more pleasant, utility is $-\infty$ in both districts. Similarly, one can show that utility in both districts is $-\infty$ if (6) is violated. We conclude that in order to rule out a situation where all agents get a utility of $-\infty$, (80), (81) and (6) must hold. In particular, there is no default, and both districts are equally pleasant in period 2:

$$\Gamma(n(d_a^*, y_a^*, d_b^*, y_b^*) \mid d_a^*, y_a^*, d_b^*, y_b^*) \equiv 0 \tag{7}$$

By the implicit function theorem, if

$$\frac{d}{dn}\,\Gamma(n \mid d_a^*, y_a^*, d_b^*, y_b^*)|_{n=n^*} \neq 0$$

then there exists a neighborhood of $(d_a^*, y_a^*, d_b^*, y_b^*)$ and a *unique* differentiable function $n(.)$ such that $n(d_a^*, y_a^*, d_b^*, y_b^*) = n^*$, and for all (d_a, y_a, d_b, y_b) in the neighborhood,

$$\Gamma(n(d_a, y_a, d_b, y_b) \mid d_a, y_a, d_b, y_b) \equiv 0 \tag{8}$$

If in fact

$$\frac{d}{dn}\,\Gamma(n \mid d_a^*, y_a^*, d_b^*, y_b^*)|_{n=n^*} > 0 \tag{9}$$

then the equilibrium is *unstable.* Starting at the equilibrium level of $n = n^*$, if (9) holds then a small increase in n would make district a more attractive relative to district b. This would make more people want to move to district a, which would drive the system further away from n^*. We shall not be interested in the case where (9) holds, as such equilibria would be unlikely to ever happen in the real world.

Definition 2 *The equilibrium* $(d_a^*, y_a^*, d_b^*, y_b^*, n(.))$ *is a* stable regular equilibrium *if it is an equilibrium such that the utility in each district is finite,*

$$\frac{d}{dn}\,\Gamma(n \mid d_a^*, y_a^*, d_b^*, y_b^*)|_{n=n^*} < 0 \tag{10}$$

and in some neighborhood of $(d_a^*, y_a^*, d_b^*, y_b^*)$ *the migration function is the unique differentiable function satisfying (8). (Note that in (10),* $n^* \equiv n(d_a^*, y_a^*, d_b^*, y_b^*)$*).*

4 Some properties of stable regular equilibria

Suppose $(d_a^*, y_a^*, d_b^*, y_b^*, n(.))$ is a stable regular equilibrium. We are interested in the partial derivatives of the migration function $n(.)$, evaluated at the equilibrium point $(d_a^*, y_a^*, d_b^*, y_b^*)$. We use the simplified notation

$$\frac{\partial n}{\partial y_a} \equiv \frac{\partial n}{\partial y_a}(d_a^*, y_a^*, d_b^*, y_b^*)$$

etc. To find these partial derivatives at a stable regular equilibrium, totally differentiate equation (8), noticing that

$$\Gamma(n \mid d_a, y_a, d_b, y_b) = u\left(1 - \frac{d_a}{n}, y_a, n\right) - u\left(1 - \frac{d_b}{2-n}, y_b, 2-n\right).$$

This differentiation yields

$$u_y^a(2) + \Delta^a \frac{\partial n}{\partial y_a} = -\Delta^b \frac{\partial n}{\partial y_a} \tag{11}$$

and

$$\Delta^a \frac{\partial n}{\partial y_b} = -\Delta^b \frac{\partial n}{\partial y_b} + u_y^b(2) \tag{12}$$

Here we have defined

$$\Delta^a \equiv \frac{d}{dn}\ u(1 - \frac{d_a^*}{n}, y_a^*, n)\bigg|_{n=n^*} = u_c^a(2)\frac{d_a^*}{(n^*)^2} + u_s^a(2) \tag{13}$$

and

$$\Delta^b \equiv \frac{d}{d(2-n)}\ u(1 - \frac{d_b^*}{2-n}, y_b^*, 2-n)\bigg|_{n=n^*} = u_c^b(2)\frac{d_b^*}{(2-n^*)^2} + u_s^b(2)$$

We use the notation $u_c^j(t)$, $u_y^j(t)$ and $u_s^j(t)$ to denote the partial derivatives in district j in period $t \in \{1, 2\}$, evaluated at the equilibrium. That is, $u_c^a(1) \equiv u_c(1 - y_a^* + d_a^*, y_a^*, 1)$, $u_y^a(2) \equiv u_y(1 - \frac{d_a^*}{n^*}, y_a^*, n^*)$, etc. Notice that Δ^j is the change in second period utility in district j if, starting at the equilibrium, one more person were to move to district j. Δ^j consists of two terms, corresponding to the dilution in the debt burden and the increased congestion. The first term is positive (when $d_j^* > 0$) and the second negative.

Equations (11) and (12) yield

$$\frac{\partial n}{\partial y_a} = \frac{-u_y^a(2)}{\Delta^a + \Delta^b} \tag{14}$$

and

$$\frac{\partial n}{\partial y_b} = \frac{u_y^b(2)}{\Delta^a + \Delta^b} \tag{15}$$

Similarly,

$$\frac{\partial n}{\partial d_a} = \frac{u_c^a(2)/n}{\Delta^a + \Delta^b} \tag{16}$$

and

$$\frac{\partial n}{\partial d_b} = \frac{-u_c^b(2)/(2-n)}{\Delta^a + \Delta^b} \tag{17}$$

Stability implies

$$\frac{d}{dn}\,\Gamma(n \mid d_a^*, y_a^*, d_b^*, y_b^*)|_{n=n^*} = \Delta^a + \Delta^b < 0 \tag{18}$$

so that

$$\frac{\partial n}{\partial y_a} > 0 \ , \ \frac{\partial n}{\partial d_a} < 0 \ , \ \frac{\partial n}{\partial y_b} < 0 \ , \ \text{ and } \frac{\partial n}{\partial d_b} > 0. \tag{19}$$

Thus, in a stable regular equilibrium, a larger public goods provision attracts immigration while a larger debt gives rise to emigration.

Immigration leads to more congestion, but also reduces the per capita debt in the district. Stability does not necessarily imply the net effect is negative: more immigration can be desirable to a district. One extra person moving to district a could make a a better place to live, as long as it makes b a better place too, and the second effect dominates. (Such migration would be Pareto improving.) What stability rules out is a situation where more immigration is desired in *both* districts.

Proposition 1 *Suppose Assumption A holds. In any stable regular equilibrium, at least one district dislikes immigration, i.e. either* $\Delta^a < 0$ *or* $\Delta^b < 0$ *or both. If the equilibrium is symmetric then both districts dislike immigration.*

Proof. This follows from (18). ■

Consider district a's problem of choosing (d_a, y_a) to maximize $V^a(d_a, y_a, d_b^*, y_b^*, n(.))$, taking (d_b^*, y_b^*) and $n(.)$ as given. We are interested only in interior solutions with a finite utility, so we can analyze the first order conditions. To calculate the change in an individual's expected utility for small changes in the choice variables, we may assume that he never leaves his first period home, for if he moves, he gets the same utility by (8). Thus, if district a chooses (d_a, y_a) and district b chooses (d_b, y_b), the representative period one inhabitant of district a gets life time payoff

$$V^a(d_a, y_a, d_b, y_b, n(.)) \equiv u(1-y_a+d_a, y_a, 1)+u(1-\frac{d_a}{n(d_a, y_a, d_b, y_b)}, y_a, n(d_a, y_a, d_b, y_b))$$

For maximization of this expression with respect to d_a and y_a, the necessary first order conditions are

$$u_c^a(1) - \frac{1}{n^*}u_c^a(2) + \frac{\partial n}{\partial d_a}\Delta^a = 0 \quad (20)$$

and

$$-u_c^a(1) + u_y^a(1) + u_y^a(2) + \frac{\partial n}{\partial y_a}\Delta^a = 0 \quad (21)$$

Without migration we would have $n^* = 1$ and $\frac{\partial n}{\partial d_a} = \frac{\partial n}{\partial y_a} = 0$, and (20) and (21) would simply determine the first best allocation. But with migration, it can be seen that the districts' decisions diverge from the first best.

For future reference, we now derive some simple consequences of the first order conditions. Using (14) - (17) in the first order conditions (20) and (21), we get

$$u_c^a(1) - u_y^a(1) = \frac{\Delta^b}{\Delta^a + \Delta^b}u_y^a(2) \quad (22)$$

$$u_c^a(1) = \frac{\Delta^b}{\Delta^a + \Delta^b}\frac{u_c^a(2)}{n^*} \quad (23)$$

Therefore,

$$\frac{u_c^a(1)}{u_c^a(2)/n^*} = \frac{u_c^a(1) - u_y^a(1)}{u_y^a(2)} \quad (24)$$

or

$$u_c^a(1)u_y^a(2) = \frac{1}{n^*}\left(u_c^a(1) - u_y^a(1)\right)u_c^a(2) \quad (25)$$

Since analogous first order conditions hold for district b we have, corresponding to (22), (23) and (25):

$$u_c^b(1) - u_y^b(1) = \frac{\Delta^a}{\Delta^a + \Delta^b} u_y^b(2) \tag{26}$$

$$u_c^b(1) = \frac{\Delta^a}{\Delta^a + \Delta^b} \frac{u_c^b(2)}{2 - n^*} \tag{27}$$

$$u_c^b(1) u_y^b(2) = \frac{1}{2 - n^*} \left(u_c^b(1) - u_y^b(1) \right) u_c^b(2) \tag{28}$$

5 Economic Policy

When evaluating the effects of economic policy, we can restrict attention to two types of individuals: those who spend their whole life in district a and those who spend their whole life in district b. (A person who migrates after period one has the same level of welfare as one of these two types because second period utility is equalized.) By Proposition 1, in any stable regular equilibrium, symmetric or not, some district, say district a, dislikes immigration. District b sets y_b and d_b such that the first order condition is satisfied in district b. Therefore, by definition, a small increase in y_b or decrease in d_b would only have a second order effect on a person who lives all his life in district b. However, since these policies reduce the second period population in district a (by (19)), they have a first order positive effect on a person who lives all his life in district a. The first-period citizens of district a could pay a small compensation to the first-period citizens of district b, to make everybody strictly better off. Therefore, the equilibrium is Pareto inefficient. From a social point of view, district b provides too little public good and accumulates too much debt. In a *symmetric* equilibrium *both* districts dislike immigration, so both districts under-produce public goods and over-accumulate debt. To summarize the discussion:

Proposition 2 *Suppose Assumption A holds. In any stable regular equilibrium, there exists a district j such that a Pareto improvement could be achieved by a small decrease in d_j or a small increase in y_j. Moreover, in a symmetric equilibrium, this statement holds for* both *districts.*

Proposition 2 implies that there is a district that under-produces public goods in the sense that, given the equilibrium debt level, an increased production of public goods would raise social welfare. However, as the district also over-accumulates debt, the equilibrium public goods level is not necessarily lower than *the first best level.* The highly indebted district controls a large amount of resources in the first period, and this will normally stimulate both public and private consumption. These types of arguments have important consequences for economic policy.

Consider whether it would be Pareto improving to implement a debt limit. Given an equilibrium $(d_a^*, y_a^*, d_b^*, y_b^*, n(.))$, consider forcing each district j to lower its debt level to $d_j^* - \gamma$ where $\gamma \geq 0$. Both districts have to reduce their borrowing by the same amount. They can still choose the public good freely. Let $y_j(\gamma)$ denote district j's public goods level in the new equilibrium (facing the new debt limit). By construction, $y_j(0) = y_j^*$. Now, the community which is identified in Proposition 2 as having over-accumulated debt also under-provides the public good, and a debt limit can make things worse overall if the community reacts by cutting local public goods by a lot, i.e. if $y_j(\gamma)$ is much smaller than y_j^*. Before the debt limit, in the first period community j has $1 + d_j^*$ to spend on private and public goods. After the debt limit, they have only $1 + d_j^* - \gamma$ to spend on private and public goods, and they would now be expected to reduce their consumption of both types of good in period one. Private consumption before the debt limit is $1 + d_j^* - y_j^*$ and after the debt limit it is $1 + d_j^* - \gamma - y_j(\gamma)$. The debt limit will reduce the consumption of the private good iff $y_j(\gamma) \geq y_j^* - \gamma$. For infinitesimal γ, this is equivalent to

$$\frac{dy_j(0)}{d\gamma} \geq -1 \tag{29}$$

If (29) holds for $j = a, b$, then we say preferences are *normal*. In other words, preferences are normal if and only if, when forced to reduce the debt by one dollar, each community $j \in \{a, b\}$ reduces private consumption in period one, and cuts public goods provision by *at most one dollar*. One expects this to be the usual case.

Proposition 3 *Suppose Assumption A holds, and consider a stable regular equilibrium where preferences are normal. Forcing both districts to reduce their debt levels by the same small amount increases the welfare of all individuals.*

Proof. Consider forcing each district j to reduce its debt from d_j^* to $d_j^* - \gamma$, where $\gamma > 0$ is very small. The new public goods level, optimally chosen by each district j in the new equilibrium, is denoted $y_j(\gamma)$. It suffices to consider two types of individuals: those who spend their whole life in district a and those who spend their whole life in district b. A person who lives all his life in district a gets

$$\begin{aligned}
&V^a(d_a^* - \gamma, y_a(\gamma), d_b^* - \gamma, y_b(\gamma), n(.)) \equiv u(1 - y_a(\gamma) + d_a^* - \gamma, y_a(\gamma), 1) \\
&+u(1 - \frac{d_a^* - \gamma}{n(d_a^* - \gamma, y_a(\gamma), d_b^* - \gamma, y_b(\gamma))}, y_a(\gamma), n(d_a^* - \gamma, y_a(\gamma), d_b^* - \gamma, y_b(\gamma)))
\end{aligned}$$

Differentiating this expression and evaluating at $\gamma = 0$, we find

$$\begin{aligned}
&\frac{d}{d\gamma} V^a(d_a^* - \gamma, y_a(\gamma), d_b^* - \gamma, y_b(\gamma), n(.))\bigg|_{\gamma=0} \\
= \quad & -u_c^a(1) + \frac{1}{n} u_c^a(2) - \Delta^a \frac{\partial n}{\partial d_a} + (-u_c^a(1) + u_y^a(1) + u_y^a(2) + \Delta^a \frac{\partial n}{\partial y_a}) \frac{dy_a(0)}{d\gamma}
\end{aligned}$$

$$
\begin{aligned}
&+\Delta^a\left(\frac{\partial n}{\partial y_b}\frac{dy_b(0)}{d\gamma}-\frac{\partial n}{\partial d_b}\right)\\
=&\ \Delta^a\left(\frac{\partial n}{\partial y_b}\frac{dy_b(0)}{d\gamma}-\frac{\partial n}{\partial d_b}\right)=\frac{\Delta^a}{\Delta^a+\Delta^b}\left(u_y^b(2)\frac{dy_b(0)}{d\gamma}+\frac{u_c^b(2)}{2-n}\right)\\
=&\ \left(u_c^b(1)-u_y^b(1)\right)\frac{dy_b(0)}{d\gamma}+u_c^b(1) \qquad (30)
\end{aligned}
$$

In this equation, the second equality uses the envelope theorem, the third equality uses (14) - (17), and the final equality uses (26) and (27). We claim the expression in (30) is strictly positive if preferences are normal.

From (28), $u_c^b(1)-u_y^b(1)>0$, so (30) is certainly strictly positive if $\frac{dy_b(0)}{d\gamma}\geq 0$. If instead

$$-1\leq\frac{dy_b(0)}{d\gamma}\leq 0 \qquad (31)$$

then rewrite (30) as

$$u_c^b(1)\left(1+\frac{dy_b(0)}{d\gamma}\right)-u_y^b(1)\frac{dy_b(0)}{d\gamma} \qquad (32)$$

But (31) implies that (32) is strictly positive. Thus, the expression in (30) is strictly positive as long as $\frac{dy_b(0)}{d\gamma}\geq -1$. Under this condition, people living all their lives in district a are made better off by the debt limit. A similar argument shows that if $\frac{dy_a(0)}{d\gamma}\geq -1$ then people living all their lives in district b are made strictly better off. ■

Proposition 3 is fairly general in the sense that it applies at any equilibrium, whether symmetric or not, and it does not place further restrictions on the utility function. If in fact the equilibrium is symmetric, then $\Delta^a=\Delta^b$ and $n^*=1$ so (22) becomes

$$u_c^a(1-y_a(\gamma)+d_a^*-\gamma,y_a(\gamma),1)=u_y^a(1-y_a(\gamma)+d_a^*-\gamma,y_a(\gamma),1)+\frac{1}{2}u_y^a(1-d_a^*,y_a(\gamma),1)$$

Differentiating and evaluating at $\gamma=0$ yields:

$$
\begin{aligned}
&(-u_{cc}^a(1)+u_{cy}^a(1))\frac{dy_a(0)}{d\gamma}-u_{cc}^a(1)\\
=&\ (-u_{yc}^a(1)+u_{yy}^a(1)+\frac{1}{2}u_{yy}^a(2))\frac{dy_a(0)}{d\gamma}-(u_{yc}^a(1)-\frac{1}{2}u_{yc}^a(2))
\end{aligned}
$$

So:

$$\frac{dy_a(0)}{d\gamma} = \frac{u^a_{cc}(1) - u^a_{yc}(1) + \frac{1}{2}u^a_{yc}(2)}{u^a_{cc}(1) + u^a_{yy}(1) + \frac{1}{2}u^a_{yy}(2) - 2u^a_{yc}(1)}$$

Then we have

$$-1 \leq \frac{dy_a(0)}{d\gamma} \leq 0$$

if (33)- (35) hold:

$$u^a_{yc}(1) + \frac{1}{2}u^a_{yc}(2) \geq u^a_{yy}(1) + \frac{1}{2}u^a_{yy}(2) \tag{33}$$

$$u^a_{cc}(1) - u^a_{yc}(1) + \frac{1}{2}u^a_{yc}(2) \leq 0 \tag{34}$$

$$u^a_{cc}(1) + u^a_{yy}(1) + \frac{1}{2}u^a_{yy}(2) - 2u^a_{yc}(1) \leq 0 \tag{35}$$

The following corollary to Proposition 3 follows from (33) through (35).

Corollary 1 *Suppose Assumption A holds. If the utility function is separable in y and c, then in any stable regular equilibrium which is* symmetric, *forcing both districts to reduce their debt levels by the same amount strictly increases the welfare of all individuals.*

6 Introducing Land

6.1 The model

An interesting question is whether introducing a competitive market for land will eliminate the inefficiencies of the non-cooperative equilibrium. Oates (1972, page 156) have argued that "bond finance does not permit the shifting of costs to future residents relative to tax finance, since the deferred tax payments are reflected in reduced local property values". Oates did not specify the kind of tax to be used to service the debt, but Daly (1969, p. 48) argued that it does not matter: "The critical factor is not the *type* of tax but the basis on which a person is ruled liable for the tax. So long as this basis is the individual's residence in the particular community then the burden of the debt will not be shifted to future generations". The question of whether capitalization of public goods in land prices affects the efficiency of public goods provision is also treated in the literature (see Wildasin, 1987), although in these models there is no debt financing. [5]

Suppose there is one unit of divisible land in each district, which initially is evenly distributed among the period one citizens of the district. Between periods one and two, land is traded in a competitive market. Let the utility function be $u(c, y, s, \ell)$, where $\ell \geq 0$ is the amount of land owned, and $c \geq 0$, $y \geq 0$, $s \in [0, 2]$ have the same meaning as before. The appropriate modification of Assumption A would be:

Assumption A'. $u(c, y, s, \ell) = -\infty$ if $c = 0$ or $y = 0$ or $\ell \leq 1/2$, and $u(c, y, s, \ell) > -\infty$ otherwise.

Notice that if the population of a district reaches two, then the average amount of land in the district is one half, which explains Assumption A'. (Alternatively, we could assume $u(c, y, s, \ell) = -\infty$ when $s = 2$.) Unfortunately our model becomes quite complicated when land is introduced, and we have only been able to solve the following special case. Assume there is a strictly increasing and concave function $\hat{u} : \mathbf{R} \to \mathbf{R}$ such that

$$u(c, y, s, \ell) = \hat{u}(c + f(y, s) + v(\ell)) \tag{36}$$

Here the function $v(\ell)$ is strictly increasing and concave and captures the utility of land. The function $f(y, s)$ is the utility derived from the public good ($f_y > 0$) with the congestion effect ($f_s < 0$). We may assume $v(\ell) = -\infty$ if

[5] It is interesting to note that in the case of the Faroe Islands discussed in the Introduction, house prices fell by around 40 percent in larger towns between 1989 and 1994 (Det Raadgivende Udvalg Vedroerende Faeroerne, 1996, 1998).

and only if $\ell \leq 1/2$, to make sure each individual wants strictly more than one half unit of land. Life-time utility is again the sum of the utilities in the two periods, and we will focus on stable regular equilibria.

Let the private good be a numeraire with a price equal to one. The functional form (36) is an increasing transformation of the quasi-linear function $c + f(y, s) + v(\ell)$, so there are no wealth effects on the demand for land. In equilibrium, the price of land must equal the marginal rate of substitution between land and private consumption, which is $v'(\ell)$. The period two inhabitants of community a each consume $1/n$ units of land. The equilibrium price of land in community a when the population size is n is, therefore,

$$p(n) = v'(\frac{1}{n}) \tag{37}$$

The concavity of $v(\ell)$ implies $p'(n) > 0$. That is, land becomes more valuable the higher is the population density. The land price in community b satisfies

$$p(2-n) = v'(\frac{1}{2-n}) \tag{38}$$

Taxes can either be levied on income (the endowment of the private good) or landholding. Section 6.2 studies the case where the only tax is an income tax. Since each person has the same per period income, the debt burden will be shared equally among all the citizens. Section 6.3 will study the case of a tax on land (a property tax), where the debt burden is shared in proportion to landownership.

6.2 Income taxes

Consider a person who is born in district a. If he does not migrate, then his second period private consumption equals his endowment, 1, less taxes, d_a/n, plus the value of his net sale of land, $p(n)\left(1 - \frac{1}{n}\right)$. He gets second period utility

$$\hat{u}\left(1-\frac{d_a}{n}-\frac{p(n)}{n}+p(n)+f(y_a,n)+v(\frac{1}{n})\right)$$

If he moves to district b he gets

$$\hat{u}\left(1-\frac{d_b}{2-n}-\frac{p(2-n)}{2-n}+p(n)+f(y_b,2-n)+v(\frac{1}{2-n})\right)$$

Notice that the value of his period one land holdings, $p(n)$, enters the same way in both expressions. In fact, because the market for land has no transactions costs, we can always assume the agent sells his one unit of land at the end of period 1 for the price $p(n)$ and, if he does not migrate, "buys back" $1/n$ units of land in district a, at a cost of $p(n)/n$ (if he does migrate, he buys $1/(2-n)$ units at a cost of $p(2-n)/(2-n)$). The agent prefers to live in district a in period 2 iff:

$$\begin{aligned} & 1-\frac{d_a}{n}-\frac{p(n)}{n}+p(n)+f(y_a,n)+v(\frac{1}{n}) \\ \geq\; & 1-\frac{d_b}{2-n}-\frac{p(2-n)}{2-n}+p(n)+f(y_b,2-n)+v(\frac{1}{2-n}) \end{aligned} \tag{39}$$

Since $p(n)$, the value of his first period land holdings, cancels from both sides of this inequality, (39) is equivalent to the condition that would make a person born in district b willing to move to district a. The fact that you already own a house in a certain district does not influence your preference over districts in the second period, given a perfect market for land and no wealth effects.

Consider a stable regular equilibrium $(d_a^*, y_a^*, d_b^*, y_b^*)$. For both districts to be populated, equality must hold in (39). Using (37) and (38) this equality can be expressed as

$$-\frac{d_a+v'(\frac{1}{n})}{n}+f(y_a,n)+v(\frac{1}{n})=-\frac{d_b+v'(\frac{1}{2-n})}{2-n}+f(y_b,2-n)+v(\frac{1}{2-n}) \tag{40}$$

This equation, which corresponds to (8), implicitly determines the population size $n=n(d_a,y_a,d_b,y_b)$. We now carry on the analysis as before.

Differentiating both sides of (40) yields

$$-\frac{1}{n^*} + \Delta_\ell^a \frac{\partial n}{\partial d_a} = -\Delta_\ell^b \frac{\partial n}{\partial d_a}$$

where

$$\Delta_\ell^a = \frac{v''(1/n^*)}{(n^*)^3} + \frac{d_a^*}{(n^*)^2} + \frac{\partial f(y_a^*, n^*)}{\partial s} \tag{41}$$

and

$$\Delta_\ell^b = \frac{v''(1/(2-n^*))}{(2-n^*)^3} + \frac{d_b^*}{(2-n^*)^2} + \frac{\partial f(y_b^*, 2-n^*)}{\partial s} \tag{42}$$

Thus,

$$\frac{\partial n}{\partial d_a} = \frac{1}{n^*} \frac{1}{\Delta_\ell^a + \Delta_\ell^b} \tag{43}$$

Similarly,

$$\frac{\partial n}{\partial y_a} = -\frac{\partial f(y_a^*, n^*)}{\partial y_a} \frac{1}{\Delta_\ell^a + \Delta_\ell^b} \tag{44}$$

In a stable equilibrium $\Delta_\ell^a + \Delta_\ell^b < 0$ so although the exact expressions have changed compared to the case with no land, it is still true that

$$\frac{\partial n}{\partial d_a} < 0 \quad \text{and} \quad \frac{\partial n}{\partial y_a} > 0 \tag{45}$$

With an income tax, the local public debt and local public goods are not capitalized directly in land prices, since from (37) and (38) land prices only depend on the population levels, not the debt or public goods levels. There is however an indirect effect: increasing d_a or reducing y_a reduces n which reduces land prices in district a (since $p'(n) > 0$). But these effects involve a change in the population size. Indeed, (45) implies that the debt and public goods levels can be used to manipulate migration flows. In a stable equilibrium, immigration is welfare reducing in at least one district. Just as before this will imply that the debt level will be too high (given the public goods level) and the public goods level will be too low (given the debt level).

We can look more closely at these effects. From (41), at a symmetric equilibrium where $n^* = 1$ we have $\Delta_\ell^a = v''(1) + d_a^* + \frac{\partial f(y_a^*,1)}{\partial s}$, where the first term is the slope of the inverse demand function for land. If the demand for land is very elastic, so $v''(1)$ is large in absolute value, then Δ_ℓ^a and Δ_ℓ^b are large in absolute value. Now, (45) implies that an increase in d_a (or fall in y_a) causes emigration from district a, and the land sold by the emigrants must be bought by the remaining residents. But if the demand is very elastic, even a small amount of emigration causes a big drop in land prices. This will quickly make the district more attractive to live in and thereby restore equilibrium, so an increase in d_a (or fall in y_a) causes only a very small amount of emigration. Formally, (43) and (44) imply that $\frac{\partial n}{\partial d_a}$ and $\frac{\partial n}{\partial y_a}$ are *small* in absolute value when Δ_ℓ^a and Δ_ℓ^b are large in absolute value. In this case, the externalities caused by migration are small. In the limiting case of an infinitely elastic demand for land, changes in d_a and y_a have no effect on n. The two communities are then effectively isolated from each other, so there are no externalities and the non-cooperative equilibrium is first best. But, arguably, this is a very special case.

We conclude that, if there is an income tax and the demand for land is not infinitely elastic, the model with land yields the same qualitative results as the model without land. The inefficiencies associated with the choice of debt and public goods provision are still present. In particular, there is always a district such that reducing debt or increasing the public goods level is welfare improving, if all other variables (including the population levels) are held constant. In a symmetric equilibrium this is true for both districts.

6.3 Property taxes

Now we assume that all taxation is on land. In contrast to Section 6.2, we will show that now local debt levels are fully capitalized in land prices. Increasing the local public debt has no effect on future immigration, since land prices fall just enough to precisely compensate for the higher future tax rates. On the other hand, the local public good is not capitalized in land prices, so a higher public good level will still attract immigrants, which leads to congestion. Therefore, the level of the public good will be inefficiently low.

Let t_j and p_j denote the second period land tax rate and land price in district j. Consider an agent who lived in district $i \in \{a, b\}$ in period one and decides to live in district a in period two (note that i may or may not be equal to a). If in period two he has ℓ units of land in district a, then he pays a tax $t_a p_a \ell$. The ℓ units of land are worth $p_a \ell$. His wealth consists of his endowment, 1, and the value of his one unit of land district i, p_i. His second period consumption is, therefore,

$$c = 1 + p_i - p_a (1 + t_a) \ell \tag{46}$$

District a's second period budget restriction implies

$$p_a t_a = d_a \tag{47}$$

since there is one unit of land in the district. Substituting (47) in (46),

$$c = 1 + p_i - (p_a + d_a) \ell \tag{48}$$

Substituting from (48) in (36) and maximizing with respect to ℓ yields

$$p_a + d_a = v'(\ell) \tag{49}$$

In equilibrium, each second period inhabitant of district a holds $1/n$ units of land. Thus, equilibrium land prices in district a are determined by

$$p_a = v'\left(\frac{1}{n}\right) - d_a \tag{50}$$

and similarly

$$p_b = v'\left(\frac{1}{2-n}\right) - d_b \tag{51}$$

From (48), the second period utility is

$$\hat{u}\left(1 - (p_a + d_a)\frac{1}{n} + p_i + f(y_a, n) + v(\frac{1}{n})\right)$$

As before, consider a stable regular equilibrium. All agents must be indifferent between the two districts, which using (50) and (51) implies

$$1 - v'\left(\frac{1}{n}\right)\frac{1}{n} + f(y_a, n) + v(\frac{1}{n}) = 1 - v'\left(\frac{1}{2-n}\right)\frac{1}{2-n} + f(y_b, 2-n) + v(\frac{1}{2-n}) \tag{52}$$

Equation (52) implicitly determines the population size $n = n(d_a, y_a, d_b, y_b)$. In fact (52) does not involve d_a and d_b, so migration decisions are independent of debt levels:

$$\frac{\partial n}{\partial d_a} = \frac{\partial n}{\partial d_b} = 0 \tag{53}$$

The local public debt is fully capitalized in the land price, as can be seen directly from (50) and (51). Consequently, the local population in period one will fully internalize the debt burden. Thus, *given the equilibrium public goods level*, the debt level will be efficient. This result is only true, however, when *all* taxation is on land in the form of a property tax. If some non-zero fraction of the debt was repaid using an income tax, (50) and (51) would be no longer be satisfied, and the local debt could again be used to fight future immigration, just as in Section 6.2. The qualitative conclusions would then be similar to the previous sections.

Turning now to the choice of public goods level, we find that

$$\frac{\partial n}{\partial y_a} = -\frac{\partial f(y_a^*, n^*)}{\partial y_a}\frac{1}{\Delta_{\ell t}^a + \Delta_{\ell t}^b} > 0 \tag{54}$$

where

$$\Delta_{\ell t}^a = -\frac{1}{(n^*)^2} + \frac{v''(1/n^*)}{(n^*)^3} + \frac{\partial f(y_a^*, n^*)}{\partial s} < 0$$

and

$$\Delta^b_{\ell t} = -\frac{1}{(2-n^*)^2} + \frac{v''(1/(2-n^*))}{(2-n^*)^3} + \frac{\partial f(y^*_b, 2-n^*)}{\partial s} < 0$$

Since immigrants cannot be made to share the burden of the debt, immigration is undesirable. The marginal utility of an extra unit of local public good is reduced since immigrants are attracted. Hence, the migration externality is still present and in equilibrium there will be under-provision of public goods. Moreover, since the local debt cannot be used to fight immigration with a land tax, we conjecture that public goods provision will always be smaller with a land tax than with an income tax. Below, we verify this conjecture in an example.

Since the results are somewhat different for different tax-systems, we may speculate on the effect of allowing each district to choose its method of taxation. We have been able to show that for a fixed level of public goods, the first period residents prefer income taxes. The reason is that they do not want the local public debt to be capitalized in the value of their property. We conjecture that a similar result holds when the public good level is variable.

7 Examples

7.1 Without land

Suppose

$$u(c, y, s) = -\exp(-c - \frac{\sqrt{y}}{s}) \tag{55}$$

Should it happen that $y = s = 0$, define per capita public goods consumption to be zero (i.e. in (55) we take $\sqrt{0}/0 \equiv 0$). This utility function yields simple calculations.

First consider the first best. Obviously, $n = 1$ is efficient. The first best levels of public good and debt, y^{opt} and d^{opt}, maximize

$$-\exp(-(1-y+d)-\sqrt{y})-\exp(-(1-d)-\sqrt{y})$$

The solution is

$$y^{opt}=1 \text{ and } d^{opt}=\frac{1}{2}$$

Proposition 4 *When utility functions are given by (55) there exists a unique symmetric regular equilibrium. This equilibrium is stable, and public goods and debt levels are*

$$y_a^* = y_b^* = 1 \tag{56}$$

$$d_a^* = d_b^* = \frac{1+\ln 2}{2} \approx 0.8 \tag{57}$$

Proof. In the first period, utility in district a is

$$u^a(1) = -\exp(-(1+d_a-y_a)-\sqrt{y_a})$$

In the second period, private consumption is $1-d_a/n$ and utility is

$$u^a(2) = -\exp(-1-\frac{\sqrt{y_a}-d_a}{n}) \tag{58}$$

Optimal migration decisions in period 2 imply both districts are equally pleasant: $u^a(2)=u^b(2)$. This implies, using (58) and the similar expression for $u^b(2)$, that

$$\frac{1}{n}=\frac{1}{2}\left(1+\frac{\sqrt{y_b}-d_b}{\sqrt{y_a}-d_a}\right) \tag{59}$$

This determines n as a function of (d_a, y_a, d_b, y_b). District a is maximizing $V^a \equiv u^a(1)+u^a(2)$, and using (59) we obtain

$$V^a = -\exp(-1)\left[\exp\left(-\left(d_a-y_a+\sqrt{y_a}\right)\right)+\exp\left(-\frac{\sqrt{y_b}-d_b}{2}\right)\exp\left(-\frac{\sqrt{y_a}-d_a}{2}\right)\right] \tag{60}$$

As $-e^{-z}$ and $-e^{-z/2}$ are concave functions of z, and $\left(d_a-y_a+\sqrt{y_a}\right)$ and $\left(\sqrt{y_a}-d_a\right)$ are concave functions of (d_a, y_a), V^a is a concave function of (d_a, y_a). Thus, to find a maximum it suffices to consider the first order necessary conditions for an interior equilibrium:

$$\frac{\partial V^a}{\partial d_a} = -u^a(1) + \frac{1}{2}u^a(2) = 0 \tag{61}$$

and

$$\frac{\partial V^a}{\partial y_a} = -(\frac{1}{2\sqrt{y_a}} - 1)u^a(1) - \frac{1}{2}\frac{1}{2\sqrt{y_a}}u^a(2) = 0$$

Combining these yields $y_a = 1$, and similarly $y_b = 1$. Substituting this result in (61) yields

$$2 - 3d_a - d_b = -2\ln 2$$

and by symmetry:

$$2 - 3d_b - d_a = -2\ln 2$$

which can be solved to get (57).

Finally, we need to check stability. Consider

$$\Gamma(n \mid d_a^*, y_a^*, d_b^*, y_b^*) = -\exp(-1 - \frac{1 - \frac{1+\ln 2}{2}}{n}) + \exp(-1 - \frac{1 - \frac{1+\ln 2}{2}}{2-n})$$

The equilibrium is indeed stable, because

$$\frac{d}{dn}\,\Gamma(n \mid d_a^*, y_a^*, d_b^*, y_b^*)|_{n=1} = -\,(1 - \ln 2)\exp(-\frac{3 - \ln 2}{2}) < 0$$

■

In this example, the provision of the public good happens to equal the first best level, but there is too much debt. *Given* the high debt level, a small increase in the public good (raising it *above* the first best level) would be welfare improving. This follows from Proposition 2 and it is a typical "second best" result.

Suppose a Federal government enforces a debt limit of $\bar{d} \leq \frac{1+\ln 2}{2}$. District a chooses y^a to maximize V^a, given by (60) with $d_a = d_b = \bar{d}$, and similarly for district b. In a symmetric equilibrium, the first order conditions imply that $y_a = y_b = y$ solves

$$\frac{y + \ln(4\sqrt{y} - 2)}{2} = \bar{d} \tag{62}$$

When the districts are forced to reduce their borrowing, they will cut their public goods provision, but by less than one dollar for each dollar reduction in debt: as $0 \leq y \leq 1$, equation (62) yields

$$0 < \frac{dy}{d\bar{d}} = 2\frac{2y - \sqrt{y}}{2y - \sqrt{y} + 1} \leq 1$$

Thus, in this example preferences are normal and a debt limit is Pareto improving by Proposition 3. In the equilibrium described in Proposition 4, welfare is -0.473. If, for example, a debt limit of $\bar{d} = 0.5$ is imposed (so that debt will be at its first best level) then the public goods level falls to approximately 0.7, but welfare increases to -0.457. Now in this case more than 70% of the cost of the public good is paid for in period 2 ($\frac{0.5}{0.7} \approx 0.71$), and given that consumption smoothing across periods is desirable, borrowing should be restricted even further. In fact, a numerical analysis shows that the optimal debt limit is $\bar{d} \approx 0.45$, yielding a public goods level of $y_a = y_b \approx 0.666$ according to (62). Thus, given the under-provision of public goods, the optimal debt limit is actually lower than the first best level. Finally, it can be shown that prohibiting debt altogether ($\bar{d} = 0$) yields a utility which is lower than -0.473. Hence, even though it would be desirable to impose a debt limit on the districts, it would be detrimental to welfare to prohibit debt altogether. After all, debt allows consumption smoothing, and in addition it mitigates the under provision of the public good.

7.2 With land

7.2.1 Utility functions

Now suppose the agents care about land, ℓ. Utility is of the form

$$u(c, y, s, \ell) = -\exp(-c - \frac{\sqrt{y}}{s} - v(\ell)) \tag{63}$$

with

$$v(\ell) = \begin{cases} \ell \ln(2 - \frac{1}{\ell}) & \text{if } \ell > 1/2 \\ -\infty & \text{if } \ell \leq 1/2 \end{cases} \tag{64}$$

Should it happen that $y = s = 0$, then replace the expression $\frac{\sqrt{y}}{s}$ by zero. Again, this functional form have been chosen in order to simplify calculations. Notice that $v(1) = 0$ and if $\ell \to 1/2$ then $v(\ell) \to -\infty$. The introduction of land does not change the first best allocation, which remains $y^{opt} = 1$ and $d^{opt} = \frac{1}{2}$ (and $n = 1$). For $\ell > 1/2$,

$$u(c, y, s, \ell) = -(2 - \frac{1}{\ell})^{-\ell} \exp(-c - \frac{\sqrt{y}}{s}) \tag{65}$$

Notice that

$$v'(\ell) = \ln\left(2 - \frac{1}{\ell}\right) + \frac{1}{\ell\left(2 - \frac{1}{\ell}\right)} > 0$$

$$v''(\ell) = -\frac{1}{\ell^3\left(2 - \frac{1}{\ell}\right)^2} < 0$$

so that v is concave for $\ell > 1/2$.

7.2.2 Income tax

First consider the case of an income tax. Compared to Section 7.1, the level of debt will turn out to be lower when a market for land is introduced, and closer to the first best level, but the level of public good has not changed: it remains at the first best level. Hence, the introduction of land (in combination with an income tax) unambiguously alleviates the distortions from the free riding problem.

Proposition 5 *Suppose utility functions are given by (63) and the only tax is an income tax. There exists a unique symmetric regular equilibrium. This equilibrium is stable, and public goods and debt levels are* $y_a^* = y_b^* = 1$ *and* $d_a^* = d_b^* = d^*$*, where* $d^* \approx 0.6$ *is the unique*[6] *root in* $[0,1]$ *for the equation*

$$g(d) \equiv \exp(1-2d) - \frac{1}{2}\left(1 + \frac{1}{2-d}\right) = 0 \tag{66}$$

Proof. The market price for land in the second period is:

$$p(n) = v'(\frac{1}{n}) = \ln(2-n) + \frac{n}{2-n} \tag{67}$$

in district a and

$$p(2-n) = \ln n + \frac{2-n}{n}$$

in district b. Equilibrium second period population levels are determined by the condition that both districts are equally pleasant:

$$\frac{\sqrt{y_a} - d_a - p(n)}{n} + v(\frac{1}{n}) = \frac{\sqrt{y_b} - d_b - p(2-n)}{2-n} + v(\frac{1}{2-n}) \tag{68}$$

It is convenient to notice that

$$nv(\frac{1}{n}) - p(n) = n\frac{1}{n}\ln(2-n) - \left(\ln(2-n) + \frac{n}{2-n}\right) = -\frac{n}{2-n} \tag{69}$$

and

$$(2-n)v(\frac{1}{2-n}) - p(2-n) = -\frac{2-n}{n} \tag{70}$$

Equation (68) implies that n is determined by:

$$\frac{1}{n} = \frac{1}{2}\left(1 + \frac{x_b}{x_a}\right) > \frac{1}{2} \tag{71}$$

[6] Since $g(0) = e - 3/4 > 0$, $g(1) = e^{-1} - 1 < 0$, and $g'(d) < 0$ for all $d \in [0,1]$, there is a unique $d^* \in [0,1]$ such that $g(d^*) = 0$.

and

$$\frac{1}{2-n} = \frac{1}{2}\left(1 + \frac{x_a}{x_b}\right) > \frac{1}{2} \tag{72}$$

where we have defined

$$x_j \equiv 1 + \sqrt{y_j} - d_j > 0$$

District a is trying to maximize the life time utility of its first period citizens:

$$\begin{aligned} V^a &= u^a(1) + u^a(2) \\ &= -\exp\left(-1 - d_a + y_a - \sqrt{y_a} - v(1)\right) - \exp(-1 - p(n) - (\frac{x_a - 1 + nv(1/n) - p(n)}{n})) \end{aligned} \tag{73}$$

Using (67), (69), (71) and (72), we obtain

$$u^a(2) = -\frac{1}{2}\left(1 + \frac{x_a}{x_b}\right)\exp(-\frac{1}{2}x_b)\exp\left(\frac{1}{2}\left(-\frac{x_a}{x_b} - x_a + \frac{x_b}{x_a}\right)\right) \equiv G(x_a)$$

The first order conditions for a maximum are:

$$\frac{\partial V^a}{\partial y_a} = \left(1 - \frac{1}{2\sqrt{y_a}}\right)u^a(1) + \frac{1}{2\sqrt{y_a}}G'(x_a) = 0 \tag{74}$$

$$\frac{\partial V^a}{\partial d_a} = -u^a(1) - G'(x_a) = 0 \tag{75}$$

where

$$G'(x_a) = -\frac{1}{2}\left(\frac{1}{x_b} - \left(1 + \frac{x_a}{x_b}\right)\left(\frac{1}{2} + \frac{1}{2x_b} + \frac{1}{2}\frac{x_b}{x_a^2}\right)\right)\exp(-\frac{1}{2}x_b)\exp\left(-\frac{1}{2}x_a - \frac{1}{2}\frac{x_a}{x_b} + \frac{1}{2}\frac{x_b}{x_a}\right)$$

Equations (74) and (75) can be solved to obtain $y_a = 1$, and by symmetry $y_b = 1$. For a symmetric equilibrium, set $d_a = d_b = d$ and $y_a = y_b = 1$ in (75) to get (66), i.e. $d_a = d_b = d^*$. Finally, when $(d_b, y_b) = (d^*, 1)$ it is straightforward to check that $G''(x_a) < 0$. Therefore, V^a is the sum of two function that are both concave in (y_a, d_a), so V^a is concave and the first order conditions yield a global maximum.

Finally, from (73) it follows that, for $d_a = d_b = d^*$ and $y_a = y_b = 1$, $u^a(2)$ is decreasing in n at $n = 1$. Since the situation in district b is symmetric, both districts dislike immigration. Thus, the equilibrium is stable. ■

7.2.3 Property tax

Now suppose all taxation is on land (a property tax). It turns out that in this case the public goods level is below the first best level $y^{opt} = 1$, but the debt levels are optimal conditional on the actual public goods level. This contrasts with the case of an income tax, where (as shown in Section 7.2.2) the equilibrium public goods level equals the first best level, while the debt is inefficiently high.

Proposition 6 *Suppose utility functions are given by (63) and the only tax is a property tax. There exists a unique symmetric regular equilibrium. This equilibrium is stable, and public goods and debt levels are*

$$y_a = y_b = \frac{33 - \sqrt{65}}{32} \approx 0.8 < 1 \tag{76}$$

and

$$d_a = d_b = \frac{1}{2}\left(\frac{33 - \sqrt{65}}{32}\right) \approx 0.4 \tag{77}$$

Proof. From (50) and (51), land prices are

$$p_a = p(n) = v'\left(\frac{1}{n}\right) - d_a = \ln(2-n) + \frac{n}{2-n} - d_a \tag{78}$$

in district a, and

$$p_b = p(2-n) = v'\left(\frac{1}{2-n}\right) - d_b = \ln n + \frac{2-n}{n} - d_b \tag{79}$$

in district b. Second period population levels are determined by (52). A straightforward calculation shows that the second period populations are

$$n(d_a, y_a, d_b, y_b) = \frac{2 + 2\sqrt{y_a}}{2 + \sqrt{y_a} + \sqrt{y_b}} \tag{80}$$

and

$$2 - n(d_a, y_a, d_b, y_b) = \frac{2 + 2\sqrt{y_b}}{2 + \sqrt{y_a} + \sqrt{y_b}} \tag{81}$$

The utility of district a's first period citizens is

$$\begin{aligned} V^a &= u^a(1) + u^a(2) \quad (82) \\ &= -\exp(-c - \sqrt{y_a} - v(1)) - \exp(-(1+p_a) + \frac{(p_a + d_a)}{n} - \frac{\sqrt{y_a}}{n} - v(\frac{1}{n})) \\ &= -\exp(-1 - d_a + y_a - \sqrt{y_a}) - \exp(-\frac{1}{2-n} - \ln(2-n) + d_a - \frac{1}{n}\sqrt{y_a}) \end{aligned}$$

Using (80) to substitute for n in (82), it is readily checked that V^a is a concave function of (d_a, y_a).[7] The first order conditions, which determine a global maximum, are

$$\frac{\partial V^a}{\partial d_a} = -u^a(1) + u^a(2) = 0$$

$$\frac{\partial V^a}{\partial y_a} = \left(1 - \frac{1}{2\sqrt{y_a}}\right) u^a(1) - \frac{1}{2n\sqrt{y_a}} u^a(2) + \frac{\partial u^a(2)}{\partial n}\frac{\partial n}{\partial y_a} = 0$$

In a symmetric equilibrium, these first order conditions can be solved to get (76) and (77). Finally, holding (d_a, y_a, d_b, y_b) fixed, equation (82) implies that $u^a(2)$ is decreasing in n when $n = 1$. The situation in district b is symmetric. Thus, both districts dislike immigration, so the equilibrium is stable. ■

Finally, we can compare the utility levels with the different tax systems studied in Sections 7.2.2 and 7.2.3. With property taxes both public goods and debt levels are lower than with an income tax, but the second effect dominates and the property tax yields higher welfare (equilibrium utility -0.57) than the income tax (-0.45).

[7] The function $-e^{-z}$ is concave in z. Straightforward calculations show that $(1+d_a-y_a+\sqrt{y_a})$ and $\left(\frac{1}{2-n(y_a,y_b)} + \ln(2-n(y_a,y_b)) - d_a + \frac{1}{n(y_a,y_b)}\sqrt{y_a}\right)$ are both concave functions in (d_a, y_a). Thus, U^a is the sum of two concave functions, hence it is concave.

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