# A Note on Blanchard & Kiyotaki (1987)*

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**Abstract**

This note reconsiders the private losses and welfare effects of a monetary expansion obtained in the seminal Blanchard & Kiyotaki (1987) article. In the original article it is argued that the welfare "dependence is a complex one". Therefore, the authors only present some numerical examples. On the contrary, this note argues that the dependence is relatively simple. It is even possible to derive unambiguous comparative static results. Furthermore, it is shown that figures on both private losses and welfare effects reported in Table 1 and 2 of the article are wrong and new figures are reported.

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# 1 Introduction

A seminal article within New Keynesian economics is Blanchard & Kiyotaki (1987) which is reprinted in the first volume on "*New Keynesian Economics*" edited by Mankiw and Romer, 1991. This note reconsiders the private losses (minimum required menu costs) and welfare effects of a monetary expansion obtained in the original article. Section 2 shows that the menu costs required for a single worker not to adjust the wage are nearly 3 times as small as stated in the original article. Section 3 examines the welfare effects of a monetary expansion. The authors argue that the welfare "dependence is a complex one". Therefore, they present only some numerical examples.[1] On the contrary, Section 3 shows that it is just as easy to derive a second order Taylor approximation of the welfare effects as it is to derive the second order Taylor approximation of private losses. Secondly, it is possible to derive unambiguous comparative static results. Amongst other things, this shows the intuitive result that the welfare effect of a monetary expansion decreases when the marginal disutility of work increases; yet Table 2 of the original article indicates the opposite relationship. Thirdly, the figures reported in Table 2 of the original article are wrong and new figures reveal that the original article overestimates the welfare effects.[2] Thus, correcting for the errors in the article implies that price rigidity is more likely but that the welfare consequences of price rigidity are smaller. Fourth, the authors do not report the relative importance of consumption and money in the utility function (the parameter $\gamma$ in the article) though it is important when deriving the welfare effects. The reason is that price setters and wage setters have a negative externality on the household's utility of real money balances. Section 3 presents some examples of the importance of this externality.

[1] The paper is on reading lists in many places. However, the normal procedure when lecturing on the welfare effects seems to be to present Table 2 of the original article on a slide.

[2] The Appendix contains the derivation of the exact formulas for both the required menu costs and the welfare consequences. Using these formulas to derive the tables in the text reveals a very close resemblance to the new approximations, thus documenting the correctness of the new figures.

# 2 Private Losses and Menu Costs

It is easy to reproduce the private loss of a firm not adjusting the price in Table 1 part (a) of the original article. However, it is less easy to reproduce the private loss of a worker not adjusting the wage in part (b) of Table 1. The formula used for deriving the table is given at p. 657 in the original article and rewritten here

$$\Phi(\sigma, \theta, \beta, \alpha, \delta) = \frac{(\beta - 1)^2 (\sigma - 1)}{2(1 + \sigma(\beta - 1))} \left(\frac{\theta - 1}{\theta\alpha}\right) \left((1 + \delta)^\alpha - 1\right)^2, \tag{1}$$

where $\sigma$ is the elasticity of substitution between the different types of labor in production, $\alpha$ is the inverse of the degree of returns to scale, $\theta$ is the elasticity of substitution between goods in utility, $\beta - 1$ is the elasticity of marginal disutility of labor, and $\delta \equiv \frac{M_1 - M_0}{M_0}$ is the proportional change in the stock of money equal to the relative difference between the money stock after the expansion, $M_1$, and the initial money stock, $M_0$. The term inside the last bracket denotes the proportional change in employment, $\frac{N_1 - N_0}{N_0} = (1 + \delta)^\alpha - 1$.

The private loss to a worker from not adjusting the wage after a monetary expansion is shown in Table 1 below for the parameter constellations examined in the original article. The figures without brackets are the new ones computed from (1) whereas the ones in brackets are taken from part (b) of Table 1 in the original article. Table 1 reveals that the losses are nearly 3 times as small as stated in Blanchard & Kiyotaki (1987). The exact losses are calculated in the Appendix and are almost identical to the new figures.

Table 1 - Menu Costs

| $\theta = 5$, | $\alpha = 1.1$ | $M_1/M_0 = 1.05$ | $M_1/M_0 = 1.10$ |
|---|---|---|---|
| $\beta$ | $\sigma$ | Loss (%) | Loss (%) |
| 1.2 | 5 | .009 (.025) | .036 (.100) |
| 1.4 | 5 | .024 (.066) | .095 (.265) |
| 1.4 | 2 | .010 (.027) | .039 (.111) |
| 1.4 | 20 | .037 (.105) | .150 (.418) |
| 1.6 | 5 | .040 (.112) | .160 (.451) |

## 3 Welfare Effects

This section shows that it is easy to derive a second order Taylor approximation of the welfare gain from a monetary expansion. The formula is simple and the dependence of each parameter is unambiguous. To make the derivation, we need the following equations from the original article: (1), (A2), (A13), (A14), (A16), and (A17). They are rewritten below

$$Y_i = \left( \sum_{j=1}^{n} N_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)\alpha}} \tag{2}$$

$$M_j' = (1-\gamma) I_j \tag{3}$$

$$\frac{P_i}{P} = \left( n^{\frac{1}{1-\sigma}} m^{1-\alpha} \frac{\theta\alpha}{\theta-1} \frac{W}{P} Y^{\alpha-1} \right)^{\frac{1}{1+\theta(\alpha-1)}} \tag{4}$$

$$U_j = \mu I_j - N_j^{\beta} \tag{5}$$

$$\frac{W_j}{W} = \left( \frac{\sigma}{\sigma-1} \frac{\beta}{\mu} n^{1-\beta} \frac{P}{W} N^{\beta-1} \right)^{\frac{1}{1+\sigma(\beta-1)}} \tag{6}$$

$$Y = \frac{\gamma}{1-\gamma} \frac{M}{P} \tag{7}$$

For later purposes, we derive aggregate employment and aggregate production by evaluating the above equations in symmetric equilibrium where $N_{ij} = \frac{N}{mn}\ \forall i,j$, $P_i = P\ \forall i$, and $W_j = W\ \forall j$. The production function (2) yields

$$Y \equiv \sum_{i=1}^{m} Y_i = m^{\frac{\alpha-1}{\alpha}} n^{\frac{1}{(\sigma-1)\alpha}} N^{\frac{1}{\alpha}}. \tag{8}$$

Combine this equation with (4) and (6) to get

$$N = n^{\frac{1+\alpha(1-\sigma+\beta\sigma-\beta)}{(\sigma-1)(\alpha\beta-1)}} m^{\frac{1-\alpha}{1-\beta\alpha}} \left( \frac{\theta\alpha}{\theta-1} \frac{\sigma}{\sigma-1} \frac{\beta}{\mu} \right)^{\frac{\alpha}{1-\alpha\beta}}, \tag{9}$$

which is the aggregate employment in symmetric general equilibrium without any nominal rigidities. Insert the aggregate employment into the production function to obtain

$$Y = m^{\frac{\beta(1-\alpha)}{1-\beta\alpha}} n^{\frac{1+\beta\sigma-\sigma}{(\sigma-1)(\alpha\beta-1)}} \left( \frac{\theta\alpha}{\theta-1} \frac{\sigma}{\sigma-1} \frac{\beta}{\mu} \right)^{\frac{1}{1-\alpha\beta}}, \tag{10}$$

which is the aggregate production (GNP) in symmetric general equilibrium without any nominal rigidities. The aggregate utility function of the households is derived from (5)

$$U = \mu I - n\left(\frac{N}{n}\right)^{\beta} \qquad , \qquad I = \sum_{j=1}^{n} I_j,$$

which gives aggregate utility as a function of aggregate wealth, $I$, and aggregate employment, $N$. Using (3), (7), and (8), we obtain

$$U = \frac{\mu}{\gamma} Y - n\left(m^{1-\alpha} n^{\frac{\sigma}{1-\sigma}}\right)^{\beta} Y^{\alpha\beta}. \tag{11}$$

The gain in aggregate welfare following a monetary expansion is (like the private losses) approximated by a second order Taylor expansion around the initial equilibrium

$$\begin{aligned} \Delta U \;\approx\; & \frac{\mu}{\gamma}\Delta Y - \alpha\beta n\left(m^{1-\alpha} n^{\frac{\sigma}{1-\sigma}}\right)^{\beta} Y_0^{\alpha\beta-1}\Delta Y \\ & -\alpha\beta\frac{\alpha\beta-1}{2} n\left(m^{1-\alpha} n^{\frac{\sigma}{1-\sigma}}\right)^{\beta} Y_0^{\alpha\beta-2}\left(\Delta Y\right)^2, \end{aligned}$$

where $\Delta U \equiv U_1 - U_0$ and $\Delta Y \equiv Y_1 - Y_0$ measure the differences between the new equilibrium and the initial equilibrium. Measured relative to initial consumption/GNP the gain equals

$$\frac{\Delta U}{\mu Y_0} \approx \frac{1}{\gamma}\frac{\Delta Y}{Y_0} - \alpha\beta\frac{n}{\mu}\left(m^{1-\alpha} n^{\frac{\sigma}{1-\sigma}}\right)^{\beta} Y_0^{\alpha\beta-1}\left(\frac{\Delta Y}{Y_0} + \frac{\alpha\beta-1}{2}\left(\frac{\Delta Y}{Y_0}\right)^2\right). \tag{12}$$

Using (10) to derive the term $\alpha\beta\frac{n}{\mu}\left(m^{1-\alpha} n^{\frac{\sigma}{1-\sigma}}\right)^{\beta} Y_0^{\alpha\beta-1}$ and (7) to derive the term $\frac{\Delta Y}{Y_0}$, we get

$$\frac{\Delta U}{\mu Y_0} \approx \Omega\left(\gamma,\sigma,\theta,\beta,\alpha,\delta\right) = \frac{1}{\gamma}\delta - \left(\frac{\sigma-1}{\sigma}\frac{\theta-1}{\theta}\right)\left(\delta + \frac{\alpha\beta-1}{2}\delta^2\right), \tag{13}$$

where $\delta \equiv \frac{M_1-M_0}{M_0}$. Equation (13) yields the welfare effects of a monetary expansion provided that $\delta$ is sufficiently small. The equation does not seem very complex; it is even possible to sign the derivatives:

$$\Omega_{\gamma}\left(\gamma,\sigma,\theta,\beta,\alpha,\delta\right) = -\frac{1}{\gamma^2}\delta < 0,$$

$$\Omega_\sigma(\gamma,\sigma,\theta,\beta,\alpha,\delta) = -\frac{1}{\sigma^2}\frac{\theta-1}{\theta}\left(\delta+\frac{\alpha\beta-1}{2}\delta^2\right) < 0,$$

$$\Omega_\theta(\gamma,\sigma,\theta,\beta,\alpha,\delta) = -\frac{1}{\theta^2}\frac{\sigma-1}{\sigma}\left(\delta+\frac{\alpha\beta-1}{2}\delta^2\right) < 0,$$

$$\Omega_\beta(\gamma,\sigma,\theta,\beta,\alpha,\delta) = -\frac{\alpha}{2}\frac{\sigma-1}{\sigma}\frac{\theta-1}{\theta}\delta^2 < 0,$$

$$\Omega_\alpha(\gamma,\sigma,\theta,\beta,\alpha,\delta) = -\frac{\beta}{2}\frac{\sigma-1}{\sigma}\frac{\theta-1}{\theta}\delta^2 < 0.$$

The welfare gain of a monetary expansion is decreasing in all parameters. Note, that Table 2 of the original article indicates that $\Omega_\beta(\gamma,\sigma,\theta,\beta,\alpha,\delta)$ should be positive whereas it is unambiguously negative according to the above formula. Intuitively, it is also hard to see how it can be positive: A monetary expansion equal to $\delta\%$ yields an increase in employment equal to $\alpha\delta\%$ implying a larger utility loss to households if the elasticity of marginal disutility with respect to work $\beta-1$ is large. Another striking thing is the dependence on $\gamma$ (the consumption share of income). When deriving their Table 2, the authors do not report the value of $\gamma$ although the results depend on this parameter. The above equation shows that a low value of $\gamma$ increases the welfare gain. The reason is that producers and households do not take into consideration how they influence the utility of real money balances when making their price and wage decisions. This negative externality on household utility implies that real money balances are inefficiently low in the decentralized economy even with price taking behavior in all markets.[3] Obviously, a monetary expansion reduces this inefficiency by increasing real money balances and the effect is largest when $\gamma$ is small; i.e., when money balances have relatively more weight in the utility function.

The welfare effects of a monetary expansion are shown below for the parameter constellations examined in the original article. The figures without brackets are the new ones computed from (13) whereas the ones in brackets are taken from Table 2 in the original article. The new figures are calculated for $\gamma$ equal to one as this gives results relatively close to the ones reported in

[3] This is also noted in footnote 11 of the original article.

the original article. The table does not reveal large differences - but systematic differences. First, the figures reported in the original article overestimate the welfare gain. Second, the welfare gain depends positively on $\beta$ in the original article whereas the true dependence is negative. The exact welfare gains are calculated in the Appendix and are almost identical to the new figures in Table 2. Table 2 contains also new aggregate private losses (minimum requirement for menu costs) and new welfare-menu costs ratios. The minimum requirements for menu costs are substantially lower than obtained in the original article because of the error when calculating the worker's losses from not adjusting. The general implication of correcting the errors are that price rigidity is more likely but that the welfare consequences of price rigidity are a little more modest. The welfare-menu costs ratios are larger in almost all parameter constellations.

Table 2 - Menu Costs and Welfare Effects ($\gamma = 1$)

| | | | $\frac{M_1}{M_0} = 1.05$ | | | $\frac{M_1}{M_0} = 1.10$ | | |
|---|---|---|---|---|---|---|---|---|
| $\sigma, \theta$ | $\alpha$ | $\beta$ | Menu (%) | Welfare (%) | Ratio | Menu (%) | Welfare (%) | Ratio |
| 5 | 1.1 | 1.2 | .01 (.03) | 1.77 (1.79) | 148 (60) | .05 (.11) | 3.50 (3.54) | 71 (32) |
| | | 1.4 | .03 (.07) | 1.76 (1.83) | 65 (26) | .11 (.28) | 3.43 (3.60) | 32 (13) |
| | | 1.6 | .04 (.11) | 1.74 (1.91) | 40 (17) | .17 (.46) | 3.36 (3.72) | 19 (8) |
| | 1.2 | 1.2 | .02 (.04) | 1.77 (1.82) | 88 (45) | .08 (.15) | 3.46 (3.57) | 44 (24) |
| | | 1.4 | .04 (.08) | 1.75 (1.87) | 49 (24) | .14 (.33) | 3.38 (3.67) | 23 (11) |
| | | 1.6 | .05 (.13) | 1.73 (1.98) | 32 (15) | .22 (.53) | 3.31 (3.85) | 15 (7) |
| 10 | 1.1 | 1.2 | .02 (.03) | 0.92 (0.94) | 44 (31) | .08 (.11) | 1.77 (1.86) | 22 (17) |
| | | 1.4 | .04 (.06) | 0.90 (1.02) | 22 (17) | .17 (.23) | 1.68 (1.93) | 10 (8) |
| | | 1.6 | .06 (.09) | 0.87 (1.11) | 14 (12) | .25 (.36) | 1.59 (2.05) | 6 (6) |
| | 1.2 | 1.2 | .03 (.04) | 0.91 (0.99) | 29 (25) | .13 (.16) | 1.72 (1.87) | 14 (12) |
| | | 1.4 | .05 (.07) | 0.88 (1.07) | 16 (16) | .22 (.29) | 1.63 (2.01) | 7 (7) |
| | | 1.6 | .08 (.11) | 0.86 (1.27) | 11 (12) | .32 (.44) | 1.53 (2.24) | 5 (5) |

As mentioned, the authors do not report their choice of $\gamma$ when deriving their Table 2. Table 3 gives an example of the dependence on $\gamma$ by reconsidering the above welfare effects of $\Delta = 5\%$ for different values of $\gamma$. It reveals that one may obtain substantially larger welfare effects if households obtain more utility from holding money.

Table 3 - Welfare Effects of $M_1/M_0 = 1.05$

| $\sigma = \theta$ | $\alpha$ | $\beta$ | $\gamma = 0.5$ | $\gamma = 0.7$ | $\gamma = 0.9$ |
|---|---|---|---|---|---|
| 5 | 1.1 | 1.2 | 6.774 | 3.917 | 2.330 |
| | | 1.4 | 6.757 | 3.900 | 2.312 |
| | | 1.6 | 6.739 | 3.882 | 2.295 |
| | 1.2 | 1.2 | 6.765 | 3.908 | 2.320 |
| | | 1.4 | 6.746 | 3.888 | 2.301 |
| | | 1.6 | 6.726 | 3.869 | 2.282 |
| 10 | 1.1 | 1.2 | 5.918 | 3.060 | 1.473 |
| | | 1.4 | 5.895 | 3.038 | 1.451 |
| | | 1.6 | 5.873 | 3.016 | 1.429 |
| | 1.2 | 1.2 | 5.905 | 3.048 | 1. 461 |
| | | 1.4 | 5.881 | 3.024 | 1.437 |
| | | 1.6 | 5.857 | 3.000 | 1.412 |

# References


[1] Blanchard, O. J. & Kiyotaki, N., "Monopolistic Competition and Aggregate Demand," *American Economic Review*, September 1987, 77, pp. 647-666.

[2] Mankiw, N. G. & Romer, D. (eds.), "*New Keynesian Economics*, vol. 1, *Imperfect Competition and Sticky Prices.*" Cambridge: MIT Press, 1991.

# A Appendix

## A.1 Exact calculation of the values in Table 1

This section derives the exact formula for $\Phi(\sigma, \theta, \beta, \alpha, \delta)$ in (1). The formula is used to calculate the exact values for the parameter constellations examined in Table 1 instead of using the second order Taylor approximation. Without loss of generality, we normalize the initial money stock ($M_0$), number of goods ($m$), and number of workers ($n$) to 1. The indirect utility function of the worker is

$$U_j = \mu \frac{W_j}{P} N_j - N_j^{\beta}.$$

The utility loss from not adjusting the wage $W_j$ after a monetary expansion measured relative to initial consumption/GNP is[4]

$$\frac{\Delta U_j}{\mu Y_0} = \frac{U_{j2} - U_{j1}}{\mu Y_0} = \Phi(\sigma, \theta, \beta, \alpha, \delta) \equiv \frac{\left(\mu \frac{W_{j2}}{P} N_{j2} - N_{j2}^{\beta}\right) - \left(\mu \frac{W_{j1}}{P} N_{j1} - N_{j1}^{\beta}\right)}{\mu Y_0},$$

where subscript 1 denotes the values of the variables if the worker does not adjust the wage, subscript 2 denotes the values of the variables if the worker does adjust the wage, and $Y_0$ equals initial production determined by (10). Using (8) to substitute $N_j$ with $Y$ yields

$$\Phi(\sigma, \theta, \beta, \alpha, \delta) = \frac{\left(\mu \frac{W_{j2}}{P} Y_2^{\alpha} - Y_2^{\alpha\beta}\right) - \left(\mu \frac{W_{j1}}{P} Y_1^{\alpha} - Y_1^{\alpha\beta}\right)}{\mu Y_0}. \tag{14}$$

In the no adjustment case the wage is identical to the wage in the initial equilibrium which is obtained from (2), (6), and (10):

$$\frac{W_{j1}}{P} = \frac{\sigma}{\sigma - 1} \frac{\beta}{\mu} Y_0^{\alpha(\beta-1)} = \frac{\sigma}{\sigma - 1} \frac{\beta}{\mu} \left(\frac{\theta\alpha}{\theta - 1} \frac{\sigma}{\sigma - 1} \frac{\beta}{\mu}\right)^{\frac{\alpha(\beta-1)}{1-\alpha\beta}}. \tag{15}$$

The production in the case of no adjustment is derived from (7)

$$Y_1 = Y_0 (1 + \delta). \tag{16}$$

[4]Note, that the production of a single firm is identical to initial consumption/GNP because of the normalization of $m$ and $n$.

The last term in (14) may now be obtained from (15) and (16):

$$\frac{W_{j1}}{P}\frac{Y_1^{\alpha}}{Y_0}-\frac{1}{\mu}\frac{Y_1^{\alpha\beta}}{Y_0}=\frac{\theta-1}{\theta\alpha}\left((1+\delta)^{\alpha}-(1+\delta)^{\alpha\beta}\frac{\sigma-1}{\sigma}\frac{1}{\beta}\right). \tag{17}$$

Combing (8) and (6) gives

$$\frac{W_{j2}}{W}=\left(\frac{\sigma}{\sigma-1}\frac{\beta}{\mu}\frac{P}{W}Y_{j2}^{\alpha(\beta-1)}\right)^{\frac{1}{1+\sigma(\beta-1)}}. \tag{18}$$

Equation (8) in the original article states

$$N_{j2}=Y_2^{\alpha}\left(\frac{W_{j2}}{W}\right)^{-\sigma}=Y_2^{\alpha}\left(\frac{\sigma}{\sigma-1}\frac{\beta}{\mu}\frac{P}{W}Y_2^{\alpha(\beta-1)}\right)^{-\frac{\sigma}{1+\sigma(\beta-1)}}, \tag{19}$$

where the last equality follows from (18). As aggregate production is independent of the adjustment of a single worker, it follows that $Y_2=Y_1=Y_0\,(1+\delta)$. The first term in (14) may now be derived from (18) and (19). After some manipulation one obtains

$$\frac{\mu\frac{W_{j2}}{P}Y_2^{\alpha}-Y_2^{\alpha\beta}}{\mu Y_0}=\frac{\theta-1}{\theta}\frac{1+\sigma\beta-\sigma}{\alpha\sigma\beta}\,(1+\delta)^{\frac{\alpha\beta}{1+\sigma(\beta-1)}}. \tag{20}$$

Inserting (17) and (20) into (14) yields

$$\Phi\left(\sigma,\theta,\beta,\alpha,\delta\right)=\frac{\theta-1}{\alpha\theta}\left(\frac{1+\sigma\beta-\sigma}{\sigma\beta}\,(1+\delta)^{\frac{\alpha\beta}{1+\sigma(\beta-1)}}-(1+\delta)^{\alpha}+(1+\delta)^{\alpha\beta}\frac{\sigma-1}{\sigma\beta}\right) \tag{21}$$

Using this equation, it is possible to calculate the exact loss of a worker that does not adjust the wage after a monetary expansion. This is done in Table 1b below where the figures in brackets are the values obtained in Table 1 when using the second order approximation. It reveals that the second order approximation yields results very close to the exact values.

Table 1b - Menu Costs

| $\theta=5,$ | $\alpha=1.1$ | $M_1/M_0=1.05$ | $M_1/M_0=1.10$ |
|---|---|---|---|
| $\beta$ | $\sigma$ | Loss (%) | Loss (%) |
| 1.2 | 5 | 0.0088 (0.009) | 0.032 (0.036) |
| 1.4 | 5 | 0.0235 (0.024) | 0.086 (0.095) |
| 1.4 | 2 | 0.0099 (0.010) | 0.036 (0.039) |
| 1.4 | 20 | 0.0370 (0.037) | 0.134 (0.150) |
| 1.6 | 5 | 0.0398 (0.040) | 0.145 (0.160) |

## A.2 Exact calculation of the welfare effects in Table 2

This section derives the exact formula for $\Omega(\sigma, \theta, \beta, \alpha, \delta)$ in (13). The formula is used to calculate the exact values for the parameter constellations examined in Table 2 instead of using the second order Taylor approximation. Without loss of generality we normalize the initial money stock ($M_0$), number of goods ($m$), and number of workers ($n$) to 1. The utility gain of a monetary expansion measured relative to initial consumption/GNP is derived from the indirect utility function (11) and (10)

$$\frac{U_1 - U_0}{\mu Y_0} = \Omega(\gamma, \sigma, \theta, \beta, \alpha, \delta) \equiv \frac{1}{\gamma}\left(\frac{Y_1}{Y_0} - 1\right) - \frac{1}{\mu}\left(\frac{Y_1^{\alpha\beta}}{Y_0} - Y_0^{\alpha\beta-1}\right),$$

where $Y_0$ is initial aggregate production whereas $Y_1$ is aggregate production after the monetary expansion. Equation (7) implies

$$\frac{Y_1}{Y_0} = \frac{M_1}{M_0} = 1 + \delta,$$

$\Rightarrow$

$$\Omega(\gamma, \sigma, \theta, \beta, \alpha, \delta) = \frac{\delta}{\gamma} - \frac{1}{\mu}\left(\left((1+\delta)^{\alpha\beta} - 1\right) Y_0^{\alpha\beta-1}\right).$$

Inserting $Y_0$ from (10) gives

$$\Omega(\gamma, \sigma, \theta, \beta, \alpha, \delta) = \frac{\delta}{\gamma} - \left((1+\delta)^{\alpha\beta} - 1\right)\frac{1}{\alpha\beta}\frac{\theta - 1}{\theta}\frac{\sigma - 1}{\sigma}.$$

Using this equation, it is possible to calculate the exact welfare consequence of a monetary expansion. This is done in Table 2b below where the figures in brackets are the values obtained in Table 2 when using the second order approximation. It reveals that the second order approximation is very close to the exact values.

Table 2b - Welfare Effects

| $\gamma = 1$ | | | $M_1/M_0 = 1.05$ | $M_1/M_0 = 1.10$ |
|---|---|---|---|---|
| $\sigma = \theta$ | $\alpha$ | $\beta$ | Welfare (%) | Welfare (%) |
| 5 | 1.1 | 1.2 | 1.7747 (1.774) | 3.4998 (3.498) |
| | | 1.4 | 1.7571 (1.757) | 3.4298 (3.427) |
| | | 1.6 | 1.7394 (1.739) | 3.3587 (3.357) |
| | 1.2 | 1.2 | 1.7651 (1.765) | 3.4617 (3.459) |
| | | 1.4 | 1.7459 (1.746) | 3.3846 (3.382) |
| | | 1.6 | 1.7265 (1.726) | 3.3064 (3.306) |
| 10 | 1.1 | 1.2 | 0.9180 (0.918) | 1.7732 (1.770) |
| | | 1.4 | 0.8957 (0.895) | 1.6845 (1.681) |
| | | 1.6 | 0.8734 (0.873) | 1.5946 (1.592) |
| | 1.2 | 1.2 | 0.9059 (0.905) | 1.7250 (1.722) |
| | | 1.4 | 0.8815 (0.881) | 1.6274 (1.625) |
| | | 1.6 | 0.8570 (0.857) | 1.5284 (1.527) |