A Model of the Learning Process with Local Knowledge Externalities Illustrated with an Integrated Graphical Framework

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Abstract

We present a unified graphical framework accounting for the nature and impact of spillover effects. The dynamics of the learning process with a specific spillover transfer mechanism can be illustrated by recurring to this four-quadrant picture. In particular, a whole cycle of technological learning is explained with help of such a graphical representation of the basic learning process in the presence of knowledge spillovers.

We hypothesize two different functional specifications of spillover exchanges among firms within a local innovation system. Each conceivable shape for the knowledge transfer relationship among firms expresses a possible mode and intensity of information processing arising from technology spillovers. A general proposition regarding the relative efficiency of the two alternative formal models with spillovers effects is derived. The basic models with spillover effects are then extended in several relevant directions

Keywords: Learning; knowledge; technology spillovers; knowledge externalities; local innovation systems

1. Introduction

In this paper we present a theoretical modeling of the learning process with knowledge externalities to R&D and other learning inputs within a local innovation system, be it a region, a technological district, an industry or a technological cluster with fast rates of accumulation of new technological knowledge. As there are several definitions of localized technological knowledge and learning opportunities (according to stressing for instance the technical space, or to the regional space of firms), we can therefore find several possible applications to the basic modeling. The analysis of the learning firm interacting with a specific region in the production of new technological knowledge is just one of those.

The analytical model we develop is amenable to a graphical representation. Thus we provide a unifying graphical framework, consisting of a four-quadrant picture, to analyze the process of knowledge accumulation by learning firms located and operating in a specific region or industry, which simultaneously stresses the nature of the basic learning process and the importance of true knowledge spillovers in the generation of new knowledge.

We adopt the following approach to constructing stocks or pools of available knowledge arising from spillover effects. First, the magnitude or state of aggregate knowledge available within a region or industry can be reconstructed through historic accumulation of flows of knowledge. Thus, the aggregate level of knowledge can always be updated after every learning loop is completed, or at every point in discrete time, once a time unit of measurement is fixed at the outset of our analysis. Secondly, every firm within a region or industry is treated symmetrically regarding both spillover effects and magnitude of external knowledge available. Such statement meaning that the amount of aggregate knowledge borrowed from any available source, the region or industry under analysis, or even some other distant region or industry, is regarded as being the same by every firm. And finally, we model both the loss of appropriation of benefits from innovation and the distance between different technological bases or regional sources in terms of single parameters, weighting respectively the connectivity and the absorptive intensities associated to new flows of technological knowledge by firms.

This paper is structured as follows. In the next section, the nature of a local innovation system is briefly characterized and contrasted with a neoclassical view of technology transfer. Then, in Section 3, a unified graphical framework accounting for the nature of a specific spillover transfer mechanism is presented and commented. After, in Section 4, we move to the mathematical modeling of spillover effects assuming in turn the spillover transfer function presented earlier and an alternative way of specifying the transfer technological knowledge. The main results are then analytically derived and presented (Section 5) and later commented (Section 6). Our basic models are compared to other relevant models with spillover effects (Section 7) and then extended in several directions (Section 8). Finally, Section 9 concludes the paper.

2. On the nature of a Local Innovation System

Growing literature refers to the localized nature of the learning process, a property of the learning process explaining the emergence of mutations in the economic system. Technological knowledge implies the competence and capability necessary to use information, within the specific context of each agent, as well as to participate in communication and, eventually, to generate additional information. Technological knowledge is 'localized' in tacit learning processes that are embedded into the background and experience of each innovator and hence highly idiosyncratic. In particular, technological knowledge tends to be localized in well-defined technical, institutional, regional and industrial situations. By being specific to each industry, region or firm, technological knowledge becomes therefore costly to use elsewhere, increases its appropriability and reduces its spontaneous circulation in the economic system (Antonelli, 1999).

Technological innovation takes place within a particular structure, a specific context of industrial products and production processes. Analysis of the conditions and context for effective technological communication to take place in turn lies at the heart of the innovation system approach (Antonelli, 2001).

We would like to know: What are the basis conditions for localized technological knowledge to become collective and for external increasing returns in the production of knowledge to take place?

Within communication networks, we see that the magnitude and impact of the effective flow of information which is both emitted and received by each agent can perceived to be the outcome of the interaction between to classes of events: (1) the connectivity event, according to which the flows of effective communication and the exchange of information take place; (2) the receptivity event, according to which the results of the research and learning efforts of each firm in the system are effectively assimilated.

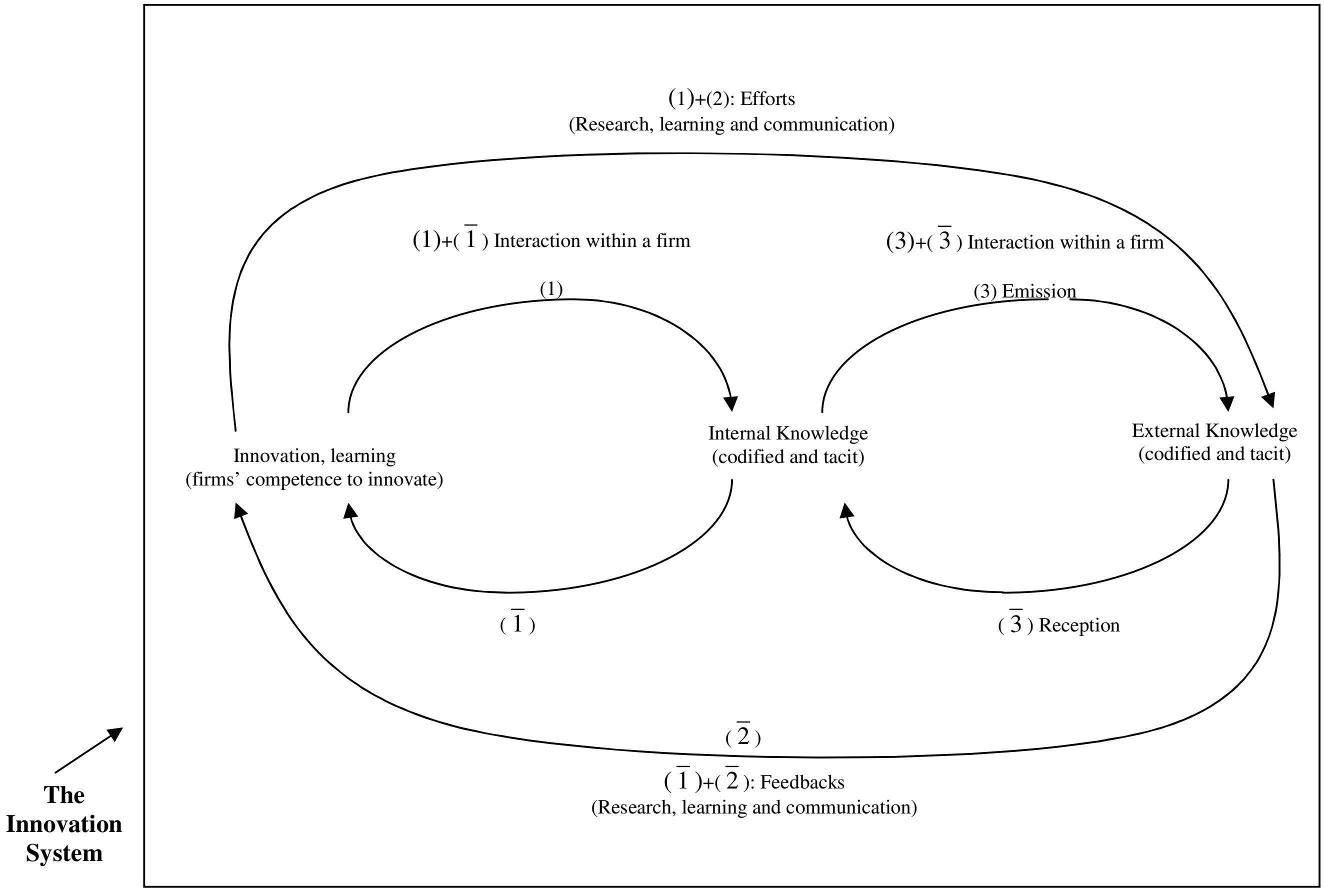

Figure 1: The basic innovation process and the innovation capability of the firm

This picture, by making explicit a mode of interference between the process of technological change and the local context, is also clearly inspired in insights from evolutionary theory. To this approach, what matters is the complex interaction between technology and local contexts. In turn this means that a local context is an entity playing a role in the process of creation and diffusion of technologies through specific learning mechanisms that mostly rely on the specific institutional framework of the local entity under consideration.

And such an emphasis on the complex interaction between the process of technological change and the local context leads us in turn to the main difference between this approach and other theoretical approaches like neoclassical economics. While neoclassical theory reduces the impact of technology on the local context to a simple, quantitative, difference in the speed of diffusion of technology, evolutionary theory stresses the role of different local contexts due to different institutional frameworks have on processes of innovation exhibiting qualitative differences.

We consider now two extreme cases of modeling spillover effects which basically arise by neglecting from our analysis relevant contextual conditions for technology transfer. Either case can be structured within a neoclassical theoretical framework, as Romer (1990) actually does. In fact Romer (1990) assumes that the output of new designs produced by researcher *j* can be written as a continuous, deterministic function of the inputs applied. Romer makes also the extreme assumption that anyone engaged in research has free access to the entire stock of knowledge. All researchers can therefore take advantage of *A*, the stock of knowledge, at the same time. Summing across all people engaged in research, the aggregate stock of designs evolves according to $\dot{A} = \delta H_A A(t)$, where δ is a productivity parameter and H_A stands for total human capital employed in research. The figure below illustrates exactly how to reach this very conclusion.

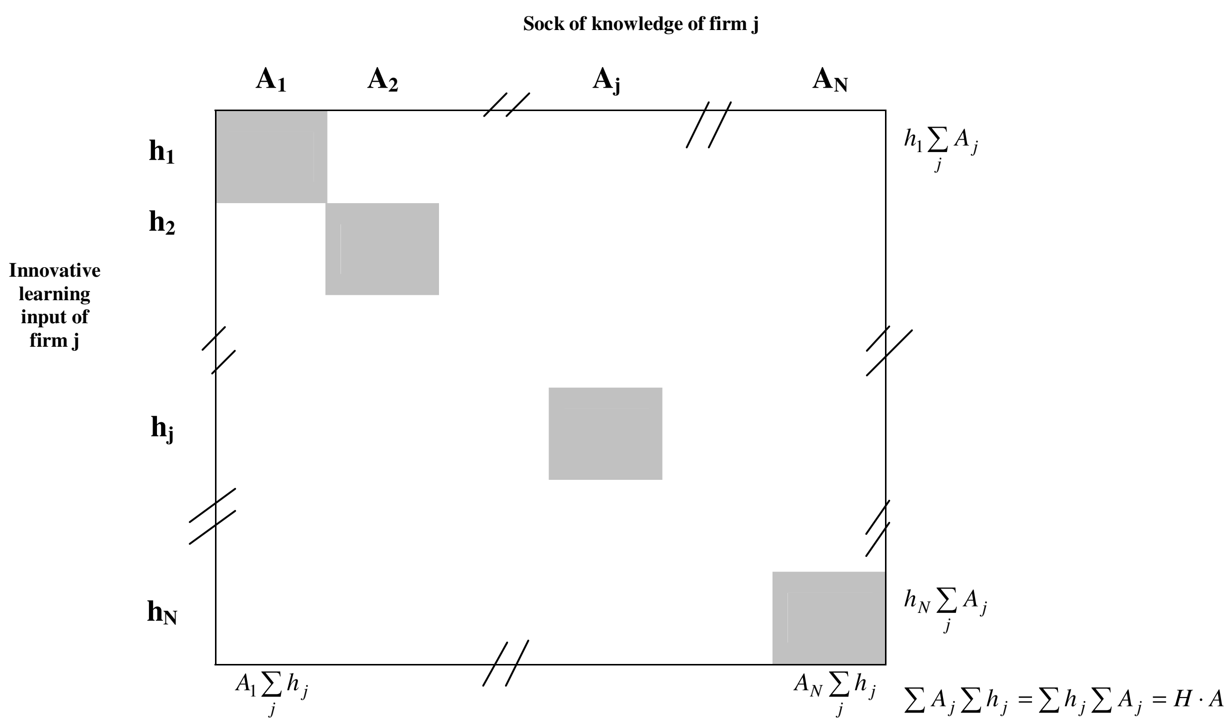

Figure 2: Horizontal accumulation of knowledge – the two extremes

This view of technology transfer and diffusion, according to which firms are supposed to adopt a new technology more or less instantaneously, in the sense that no diffusion lags exist, is currently spread in standard microeconomic textbooks. To contrast with this extreme view, we also consider the other polar case referred to in Romer (1990): the no technology diffusion case. Obviously, quite opposite assumptions about secrecy and property rights would have now to be considered.

Thus, an individual researcher *j* possessing an amount of human capital h_j and having access to a portion A_j of the total stock of knowledge implicit in previous designs, will produce new designs at the rate of $\dot{A}_j = \delta h_j A(t)$.

Check on this regard the main diagonal of the figure above where graphical measures of flows of knowledge for each and every individual researcher are depicted. The aggregate output of researchers under this extreme scenario is therefore the smallest possible of all.

3. Four-quadrant graphical representation

The dynamics of the learning process with spillover effects can be illustrated by recurring to a four-quadrant picture as follows. Such particular explanation of the evolution of *A(t)*, the aggregate stock of knowledge over time, will be called Model I in the next section.

We begin to trace a chronological succession of a few learning loops around the four quadrants, each one of them starting and ending in the same quadrant and respecting a logical order of learning events and moves. The underlying causality nexus of the process of knowledge accumulation is expected to reveal the interaction and potential synergies between flows and stocks of research and development expenditures and other learning inputs over time.

The analytical modeling tools supporting our four-quadrant graphic representation below consist basically of four fundamental functions, each one of them plotted in a specific plan or quadrant, and applied to a local innovation system with, by hypothesis, *N* identical firms. They are: a standard function of knowledge production, or accumulation, one for every single learning firm; an external increasing returns function, which is relevant to the extent that the production of new knowledge is also the result of knowledge externalities; a connectivity, or leakage, function of flows of knowledge, linking every firm with its outer competitive environment or context, reflecting the extent of transaction costs supported by each firms in technology communication; and a receptivity, or absorptive, function, representing each firm's capability of absorption of external technological knowledge. Contextual parameters α and β, main representatives of the last two functions, are both positive constants, strictly less than one. The higher they get, *ceteris paribus*, the more intense knowledge spillovers between firms turn out to be.

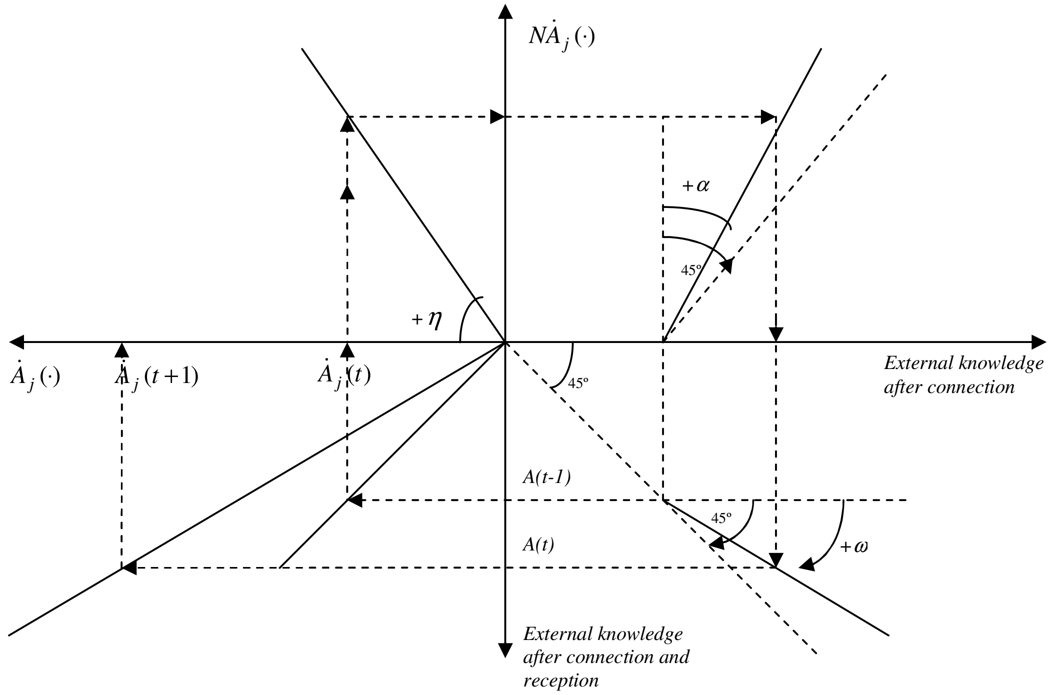

Figure 3: The cycle of horizontal accumulation of knowledge

We explain the cycle of technological learning with help of this graphical representation of the basic learning process in the presence of knowledge spillovers as follows. We start in the 3rd quadrant at the level *A(t–*1*)* of external knowledge after connection and reception. The generation of new knowledge by one learning firm depends not only on the level of the available pool of knowledge but also on the internal R&D effort and other learning inputs purchased by it. We thus depict in this quadrant a knowledge accumulation function, by stressing both the state of aggregate knowledge available to every firm in the region or industry, and a firm's specific R&D efforts and knowledge stock (these latter elements accounting for the slope of the curve depicted). The incremental amount of new knowledge generated by the learning firm will in turn be added to its own specific stock of knowledge.

We then plot in the 2nd quadrant an external increasing returns function, according to which individual flows of knowledge can be aggregated to yield the aggregate stock of knowledge available. The underlying rationale is that once spilled over, new flows of knowledge can eventually be borrowed by firms within a specific region or industry. It shows as well that the size of the stock of external knowledge is a function of the number of economic agents within the region or industry, with which each firm is able to (effectively) communicate. It is expected that the larger the number of firms spilling over (some fraction of) their knowledge flows, the larger the level of aggregate knowledge available to generate new knowledge.

True, not all of this knowledge is immediately understood and applicable to the generation of new knowledge by any particular firm in the region or industry under study. And thus we plot in the 1st quadrant a connectivity function, representative of the fraction or the extent to which the gains from innovation are not uniquely appropriated by the learning firm. Communication and appropriation of the gains from innovation by the learning firm is partial and imperfect in general.

Now, the absorption of new knowledge borrowed from some distant region or industry can in fact be subject to quite long lags. We depict in the 4th quadrant an absorptive function to precisely characterize such phenomenon. This weighting function is supposed to be interpreted as representing the fraction of available knowledge effectively received by a learning firm and later used to generate new knowledge. And we are back to the 3rd quadrant where we have started in the first place, the level of external knowledge after connection and reception being higher however, that is, *A(t)*. And therefore we close one cycle of the learning process.

4. Mathematical modeling of spillover effects

Any economic and technology study of spillover effects involves implicit or explicit assumptions about the way technological knowledge and innovations, once originated in some points of the economic system, spread over the system itself. We hypothesize two different functional specifications of spillover exchanges among firms within a local innovation system. Each specific shape of the knowledge transfer relationship among firms expresses a possible form and intensity of information processing of technology spillovers.

A distant source of inspiration for these two possible specializations in our research arose after reviewing the jargon emerged from the diffusion studies tradition, which is now well established, and the measurement problems and solutions in diffusion and adoption studies. Some of the elements of these analyses will be used in one way or another in the remainder of this paper. For instance, established ideas such as: at the inter-firm level, the existence of diffusion lags; and at intra-level firm, the gradualism of internal adoption of one innovation.

The outcome of the theoretical approach adopted in this paper is clearly that we are dealing with two alternative full formal models of technological spillovers, hereon labeled Model I and Model II. We begin our mathematical modeling with the common specification of an individual production function of new localized knowledge. Then we address the modeling of alternative technological spillover functions.

The production function of new localized technological knowledge of any individual firm *i* is therefore assumed to be given by the following differential equation (where a dot hereon stands for time derivate):

$$\dot{A}_i(t) = \delta h A_i^{\gamma}(t) A_A^{\mu}(t)\,, \tag{1}$$

where δ a productivity parameter of the research department (and other learning-based departments), h is firm *i*'s amount of own research efforts and learning inputs, $A_i(t)$ is the stock of knowledge of firm *i* at time *t* and $A_A(t)$ is the pool of knowledge available at time *t* to every identical firm within the local innovation system. Parameters γ and μ measure the marginal productivity of each respective component. Each stock of knowledge component of this production function is indispensable given the hypothesis of imperfect substitution of knowledge sources assumed in the model.

In accounting for spillover effects upon the production of new knowledge in our modeling, it is assumed that the level of productivity achieved by one specific firm depends not only on its own research efforts and learning inputs but also on the level of the pool of knowledge accessible to it. Since the productivity of a firm's own research is affected by the size of the pool, or pools, of knowledge it can draw upon, there is an interaction between the size of individual firm and aggregate R&D (research and development) efforts and learning inputs.

This specific functional form of production of new knowledge follows both an analogous function in Romer (1990) and a related production function of an individual firm in Griliches (1995). Regarding Romer's modeling, we have simply added another multiplicative term representing the pool of knowledge available to each and every firm within the innovation system. Focusing primarily on measuring the contribution of industrial R&D through econometric studies, Griliches develops a simple model of within-industry spillovers effects:

$Y_i = B(X_i)^{(1-\lambda)}(K_i)^{\lambda}(K_A)^{\mu}$,

where Y_i is the output of the *i*-th firm which depends on the level of conventional inputs X_i, its specific knowledge capital K_i, and on the state of aggregate knowledge in this industry K_A.

We begin to address now the mathematical modeling of a plausible technological spillover function. Let us start then with the distinctive characteristic of Model I. The individual production functions can then be aggregated to yield:

$$\dot{A}_A(t) = \sum_{i=1}^{N} \alpha\beta\dot{A}_i(t) = \alpha\beta N\dot{A}_i(t)\,. \tag{2}$$

Recall that we have tried to give some intuition of the evolution of $A_A(t)$ over time with help of Figure 3 above. An alternative way of aggregation of knowledge exchanges, consisting of adding up a constant fraction of all $A_i(t)$ instead of their time derivatives, gives rise to the following spillover function characterizing Model II:

$$\dot{A}_A(t) = \sum_{i=1}^{N} \alpha\beta A_i(t) = \alpha\beta N A_i(t)\,. \tag{3}$$

This time the connectivity and receptivity parameters characteristic of local innovation systems are related to each firm's stock of knowledge, not to each firm's flow of knowledge. However, later on when establishing comparisons of relative efficiency of communication systems, and therefore of dynamic efficiency of different local innovation systems, we will assume for the sake of simplicity that the levels of these parameters do not change with fundamental changes in the technology exchange environment.

5. Analytical derivation of results

It is time to derive some analytical results for our basic models. We begin with by presenting the common methodology employed in our work, which is based on expressing relevant technology variables in terms of steady rates of growth of technology. Then we address the mathematical derivation of results in Model I and later in Model II.

Technological progress occurs when A, a technology variable, increases over time. We make the assumption that A is growing at a constant rate:

$$\dot{A}(t)/A(t) = g \Leftrightarrow A(t) = A(0)e^{gt}\,, \tag{4}$$

where g is a parameter representing the growth rate of technology, and $A(0) > 0$ is the initial condition of the differential equation (4).

We define a steady state as a situation in which the various relevant quantities grow at constant rates. Hence, in a steady state the growth rate of A_i is equal to the growth rate of A_A, that is, these two values grow at the same rate g.

Partly because of its empirical appeal, an economic situation in which capital, output, population, and technology are growing at constant rates is often analyzed in growth models. According to one particular stylized fact describing some feature of the U.S. economy in the long run, the average growth rate of output per person has been positive and relatively constant over time – i.e., the United States exhibits steady, sustained per capita income growth. Unfortunately, we are not aware of the existence of any empirical data showing that steady growth rates of technological progress have been taken place over decades within regional and technological systems of innovation.

However, there is evidence of sustained growth rates of localized technological knowledge and technological change. The evidence about the regional and technological clustering of innovative activities, together with high rates of growth of total factor productivity, can be interpreted as a confirmation that relevant increasing returns are at work in the production of technological knowledge and technological change.

Let's further assume in our modeling that $\gamma + \mu = 1$. This simplifies the analysis greatly. We are now able to derive analytically the constant value of the rate of growth of knowledge in various theoretical set-ups following well-known technical procedures. We actually believe that the differential equation for the rate of growth of the stock of knowledge of an individual firm must necessarily take in the constant returns functional restriction just shown above in order for the models to have a steady state with constant growth rates.

This formulation of the spillover phenomenon is, most certainly, rather simplistic and based on untenable assumptions. To stress one in particular, note again that constant returns to scale are assumed with respect to A_i and A_A.

We start by analyzing Model I of spillover effects. What is the growth rate in this model along a steady state growth path? Differential equation (1) together with differential equation (2) can be solved simultaneously for the steady-state growth rate g. First rewrite the equations as:

$$\begin{cases} \dot{A}_i / A_i = \delta h A_A^{\mu} / A_i^{1-\gamma} \\ \dot{A}_A / A_A = \alpha\beta N \delta h A_i^{\gamma} / A_A^{1-\mu}. \end{cases} \tag{5}$$

In a steady state, the rate of growth of knowledge is constant. Hence, if we take logs and differentiate both these equations in (5) with respect to time, we obtain that the steady-state rate of growth of the stock of knowledge is:

$$g = \delta h (\alpha\beta N)^{\mu}, \tag{6}$$

once we assume that the constants of integration satisfy $A_A(0) = \alpha\beta N A_i(0)$.

Thus growth is now endogenous. In fact g is a function of parameters of the model alone.

Given that these firms are all identical, we have:

$$A_A(t) = \alpha\beta N A_i(t). \tag{7}$$

To sum up, by have taken the technology parameter A_A to be equal to the total stock of accumulated knowledge which is available and borrowed within the innovation system, we have the corresponding steady-state growth rate given in (6).

We begin to work on Model II now. With such purpose in mind we can summarize our Model II of spillover effects by representing the evolution over time of the two variables A_i and A_A according to the system of differential equations (1) and (3). We want then to find the solution to this system.

To solve this system of differential equations, we apply the following analytical procedure. First, integrating (3), and recalling the exponential rule of integration which is necessary as $A_i(t)$ is growing at the constant rate g, yields (after ignoring the constant of integration)

$$A_A(t) = (\alpha\beta N)(1/g)A_i(t). \tag{3'}$$

Substituting this analytical expression for $A_A(t)$ into equation (1) yields (after using the equality $\gamma = 1 - \mu$)

$$\dot{A}_i(t) = \delta h(\alpha\beta N / g)^{\mu} A_i \ . \tag{1'}$$

Dividing both sides of this equation by $A_i(t)$ and rewriting it in the form

$$\dot{A}_i / A_i = \delta h(\alpha\beta N / g)^{\mu},$$

which, by definition, stands for the growth rate of $A_i(t)$, allow us to derive the value of g after some algebraic manipulations:

$$g = \left[\delta h(\alpha\beta N)^{\mu}\right]^{1/(1+\mu)}. \tag{8}$$

The unknowns of integration $A_A(0)$ and $A_i(0)$ here associated are assumed to satisfy the equation $A_A(0) = (\alpha\beta N/(\delta h))^{1/(1+\mu)} A_i(0)$. It is a bit of coincidence, however, that the level of A_A at time zero is precisely the value that one would obtain after multiplying $\alpha\beta N A_i(0)$ by g^{-1}.

And finally the aggregate stock of knowledge available within the innovation system is given by the equation

$$A_A(t) = [\alpha\beta N/(\delta h)]^{1/(1+\mu)} A_i(t) \,. \tag{9}$$

Comparing to corresponding equation (7) of Model I, one difference catch immediately the eye: the coefficient of $A_i(t)$ is no longer $\alpha\beta N$ alone. Of course the growth rate embedded in $A_i(t)$ is also different.

6. Comments on the analytical results

We begin to comment and make sense of the main results of Model I. Then we move on to Model II and finally we put both results together and try to relate them in a meaningful way. We would like to understand, among other things, under what conditions the maximum efficiency of innovation systems can be achieved due to selection of an appropriate communication network structure and technology.

To recall, first of all, the growth rate of knowledge in Model I is given by equation (6): $g = \delta h(\alpha\beta N)^{\mu}$. As a consequence, the model is characterized by a (mitigated, as $\mu < 1$) scale effect. The larger the number of firms N doing research and development, the more externalities there will be in generating new technological knowledge within the innovation system and therefore the faster the innovation system will grow. In other words, the growth rate should be positively correlated with the scale of the economy, as measured by the number of firms N. This scale effect turns out to be a common feature of most endogenous growth models as well.

The implication of our model that there are scale effects associated to the research process with knowledge spillovers is bound to be criticized once data shows that the doubling of firms doing research within an innovation system has never doubled growth rates, or been close to it. To our knowledge, such data has not been gathered yet.

It is time comment on Model II's main results. Observe that equation (3) states that the acceleration of $A_A(t)$ is proportional to the velocity of $A_i(t)$. To see this just differentiate (3) with respect to time to yield

$$\ddot{A}_A(t) = \sum_{i=1}^{N} \alpha\beta \dot{A}_i(t) = \alpha\beta N \dot{A}_i(t) \,. \tag{10}$$

Compare this modeling option to equation (2) from Model I, where in practice a relation between the velocity of $A_A(t)$ and the velocity of $A_i(t)$ is established instead.

If we rather wanted to include the effect of exogenous, time-related, technological progress at the outset, then the differential equation governing the evolution of $A_A(t)$ in time (10) would be selected. Let's show why this is so. To begin with, integrate both sides of equation (10) with respect to time, which yields

$$\dot{A}_A(t) = \alpha\beta N A_i(t) + c\,, \tag{10'}$$

where c is an arbitrary constant of integration. Integrating (10') yields

$$A_A(t) = (\alpha\beta N)(1/g)A_i(t) + ct\,. \tag{10''}$$

It is clear the existence of a time trend translating exogenous technological progress in equation (10''), so long as c is a positive constant. Had we adopted this line of justification for the basic Model II, we would proceed by setting $c = 0$ at once.

What lessons can now be learnt by confronting both results drawn from Models I and II? It is assumed throughout this paper that the effective flow of information within the innovation system is determined by the structure and efficiency of the communication system in place. We have devised two alternative communication models with different modes of assessing to external technological knowledge. Now is time to undertake some comparative analysis and establish if possible a number of relevant results.

We begin to present a definition of efficiency of a communication structure related to the ultimate dynamic efficiency of a local innovation system; and then we present our main conclusions regarding the relative efficiency of the two communication system labeled Model I and Model II under the implicit assumption that the values of the main parameters and constants of integration remain the same in the two alternative theoretical settings.

DEFINITION: The relative efficiency of any two communication systems is assessed by the associated values of $A_A(t)$. The most efficient communication system is the one such that $A_A(t)$ is highest at every time t.

PROPOSITION: Regarding the relative efficiency of the two communication systems Model I and Model II,

(i) $A_A(t)$ in Model I > (<) $A_A(t)$ in Model II ⇔ g in Model I > (<) g in Model II;

(ii) g in Model I > (<) g in Model II ⇔ g in Model I > (<) 1;

(iii) g in Model I > (<) 1 ⇔ either δh or $\alpha\beta N$ (or both) is (are) sufficiently large (small).

The main result regarding the relative efficiency of Models I and II can be presented in terms of the following figure. One possible way to interpret result (ii) in the Proposition above is: whenever it is possible to set up in place at a low installation cost a structure of communication which relies essentially on exchanging every new flow of knowledge in real time (the case of g in Model I > 1), one should do it. Otherwise, when real time transfers of information are inefficient (the case of g in Model I < 1), on should rely basically on transferring in a diffuse way part of the pool of accumulated knowledge.

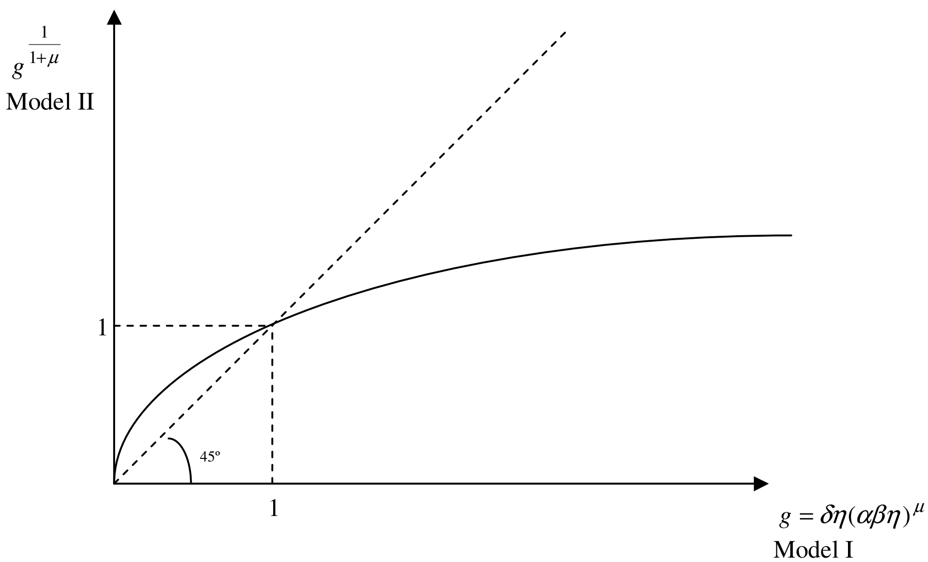

Figure 4: The relative efficiency of two alternative communication systems: Model I and Model II

The meaning of "sufficiently" large (or small) in the Proposition above is better understood by looking at the next picture. There it is depicted a frontier line demarking two regions of highest dynamic efficiency, a line which is defined in terms of two sets of parameters and variables: those internal to any firm; and those characterizing the access to external technological knowledge.

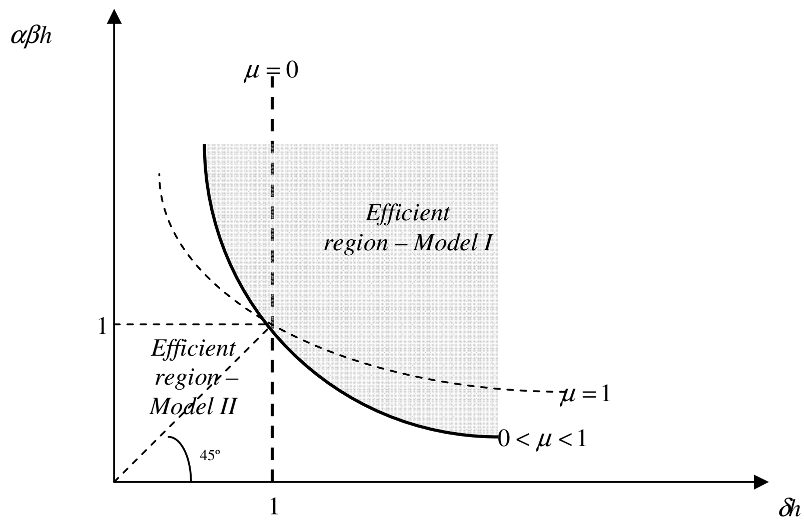

Figure 5: Frontier line and regions of relative efficiency with two alternative communication systems: Model I and Model II

It is clear that the location of the frontier of equal communication efficiency is directly affected by the parameter μ. Firms which operate with production functions of localized knowledge with high μ (that is, close to one), ceteris paribus, will be more likely to be sensitive to changes in their internal parameters and variables (δh) when selecting the most efficient model of communication. On the contrary, firms which operate with a production function characterized by low values for μ (that is, close to zero) will less sensitive to given changes in their external environment parameters and variables ($\alpha\beta N$).

Our next move is to establish some comparisons of what we have got derived so far with general models addressing technological spillovers.

7. Comparison to other models with spillovers

It is an interesting exercise to compare some assumptions and results just derived with general representations of aggregate pool of knowledge and knowledge production function made in the context of stylized modeling of spillover effects by Griliches (1995) and Antonelli (1999). To be concrete, we would like to compare equation (9) (or equation (7) above for that same matter) with the analytical expression for aggregate level of knowledge capital assumed in Griliches to begin with. Subsequently we plan to examine the expressions for the production of localized knowledge at the innovation system level in Antonelli by comparing it with equations (9) (or (7)) as well and also with equations (8) (or (6)).

Firstly, Griliches (1995) interprets also the index in K_i in his model of spillover effects as referring to industries rather than to firms. If the pools of knowledge differ for different industries or areas, the aggregate level of knowledge capital of the i-th industry should be defined as:

$K_{Ai} = \sum_j w_{ij} K_j$,

rather than a simple sum of all specific industry research and development capital levels. That is, K_{Ai} is the amount of aggregate knowledge borrowed by the *i-th* industry from all available sources. K_j measures the levels of available knowledge in these sources, while w_{ij}, labeled as the 'weighting' function, is interpreted as the effective fraction of knowledge in *j* borrowed by industry *i*. Plausibly w_{ij} becomes smaller as the 'distance', in some sense, between *i* and *j* increases. The econometric procedure recommended requires then finding an additional distributed (lag) over space function to construct a measure of the stock of borrowed knowledge.

It should, however, be clear that w_{ij} and K_j are conceptually interrelated given that K_j reflects the cumulative nature of knowledge, namely because two-way knowledge transfers between any two different contiguous industries occur over time. Reflecting the dynamic reality of the learning process, the rate of growth of knowledge deducted in our models (equations (6) and (8)) after aggregating the individual production functions is reduced by exactly taking into account the weighting $\alpha\beta < 1$.

As to Antonelli (1999), the point we would like to make has got to do with the general expression chosen for the production function of localized technological knowledge at the innovation system level. Antonelli defines the maximum attainable efficiency of innovation systems as follows:

$LTK_s = f(R\&D_s, LEARNING_s)^{(gIP)}$,

where LTK_s is the localized technological knowledge produced in the innovations system, $R\&D_s$ and $LEARNING_s$ are the aggregate R&D expenditures and learning activities conducted in the system, *g* is the average effect of external technological knowledge actually communicated (that I, delivered and received by each firm in the innovation system) on its own innovation efficiency, and *IP* the communication capability given by the interaction probability of the system.

Apparently, our functional form for the aggregate stock of knowledge available to firms within the innovation system, as given by equations (9) (or (7)) after mathematical derivation using each firm's production function of knowledge (equation 1), is different in a fundamental way. In fact the endogenous growth rates of knowledge in our modeling, as given by equations (8) (or (6)), are expressed as multiplicative functions of analogues of the aggregate amount of R&D expenditures and learning activities, the average effect of external technological knowledge actually communicated, together with the communication capability.

8. Three extensions to the basic models

We consider now three plausible extensions to our models, each one of them being illustrated and embedded within a different basic model. A first extension deals with the possibility of receptivity being a function of firms' own R&D and learning inputs. A second extension addresses the possibility of time lags in spillovers effects.

A first plausible extension, to be addressed by using Model I, is to assume that receptivity, rather than being given by a parameter β, is better represented as a function of the firm's own R&D and learning inputs, say a linear one βh *(< 1)*. In some models, like Schankerman (1979, ch.5), the amount borrowed depends also on the level of own research expenditures, thus allowing for an interaction and potential synergy between the internal and external flows of research expenditures. The growth rate then becomes:

$$g = \delta(h)^{1+\mu}(\alpha\beta N)^{\mu} . \qquad (11)$$

We immediately see in (11) that growth increases with the size of the firm as measured by the firm's labor supply devoted to R&D and learning activities. The productivity parameter of h in the rate of growth of knowledge is therefore greater $(1 + \mu)$ after aggregating the individual production functions than at the micro level (1), reflecting not only the private but also the social returns to research and development and learning activities.

To sum up our comments on the productivity parameter μ in g, made on several occasions above. If there are significant externalities to R&D and learning activities within an innovation system or industry, then the computed rate of growth should be higher at the innovation system or industry than at the firm level.

As to the second extension to be dealt with next, it consists of modifying our basic Model II by allowing technological spillovers to be subject to (quite long) lags. Contrasting with this new interpretation of Model II, one could put forward a view of Model I as basically real-time information processing, which derives its networking efficiency from employing electronic-based communications technologies.

The hypothesis of R&D and learning inputs spillovers does not really require the assumption that these effects are contemporaneous across firms and industries. In fact it is unlikely, though possible, that real technological spillovers are contemporaneous.

Let the parameter lag revealing the diffuse nature of spillover effects be denoted by *d*. It is measured in calendar time (say number of years). Thus the stock of aggregate knowledge effectively available to every firm at time t consists of part of flows of knowledge produced just up to time $t' = t - d$.

It can be shown that the new aggregate stock of knowledge available within the innovation system can be set in terms of the aggregate stock of knowledge of the basic Model II as follows:

$$A_{A-d}(t) = A_A(t-d). \tag{12}$$

Moreover it can be shown that the constant growth rate in this extension, *g*, satisfies the following equation:

$$g^{1+\mu} \cdot e^{dg\mu} = \delta h(\alpha\beta N)^{\mu}. \tag{13}$$

To recall, the right-hand side of this equation gives the growth rate in Model I. As a result, and relating to the conclusions already drawn by comparing growth rates in Models I and II, the next figure depicts a smaller region where the growth rate in modified Model II is strictly higher than the growth rate in Model I. That is, as long as we have $d > 0$, the curve depicted in the figure crosses the 45 degree ray to the left of coordinates (1, 1). One communication technology is therefore loosing the capability of promoting dynamic efficiency within the innovation system as compared to the alternative communication method as *d* gets larger and larger *ceteris paribus*.

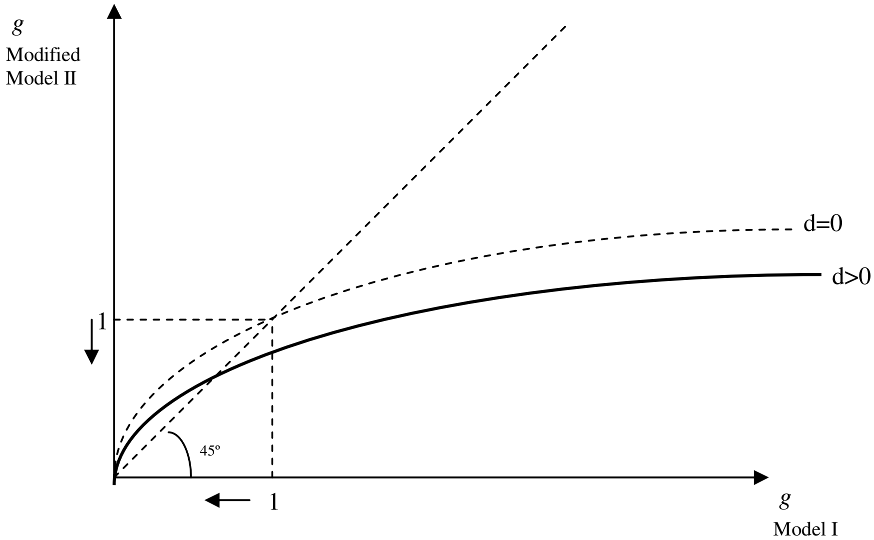

Figure 6: The relative efficiency of a new pare of communication systems: Model I and modified Model II

Finally, the third and last extension made to our two models. It intends to reflect the complexity of technological spillovers in real world concerning both their origins and differing, long time lags. Next figure shows a possible taxonomy of connectivity and reception parameters.

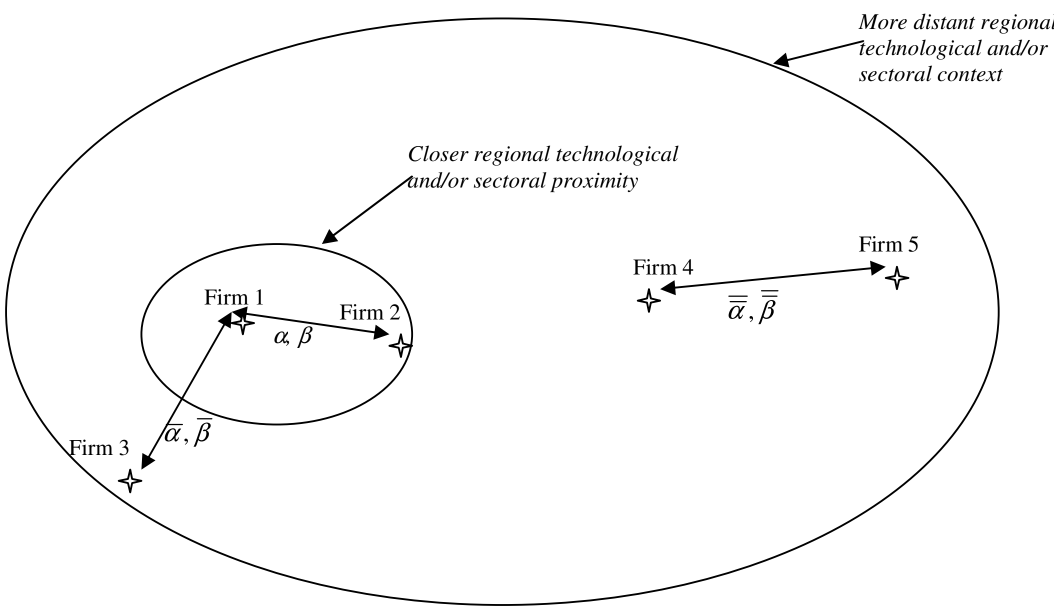

Figure 7: Communication links and innovation systems: determinants of the connectivity and reception parameters

It becomes clear that two new sets of parameters of communication arise, $\{\overline{\alpha}, \overline{\beta}\}$ and $\{\overline{\overline{\alpha}}, \overline{\overline{\beta}}\}$, dependent on the nature of various interactions observed between firms. New communication links can be established with the outer regional, technological and/or sectoral context of any firm, the efficiency of which is plausibly different from that of communication links within the closer regional or technological proximity (again from the perspective of a firm).

9. Conclusions

Real-world situations arise with many different communication technologies. Thus one would expect to reach different dynamic efficiencies in a local innovation system depending on what communication is allowed. We have devised two distinct models of structuring communication networks. We have also assessed the dynamic efficiency of each network in terms of some fundamental parameters and variables. Afterwards we have determined the relative efficiency of each communication technology and established a major result expressed in terms of a proposition. In the end we have advanced and developed three important extensions to the basic models.

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