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The general interregional price model

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Abstract
In the input-output tradition regional economic activity are related to changes in exogenous real demand, including interregional spill-over and feedback effects. However, the effects of exogenous changes in costs and prices on real economic activity have usually been neglected, despite the fact that the redistributive effects from this dual element in the intra- and interregional economy can be considerable and can have effects on economic activity which are comparable with the quantity effects.

CGE-models on the other hand have explicitly addressed this issue using non-linear functions to overcome theoretical problems related to the use of fixed coefficients, permitting for example a more satisfactory treatment of substitution between factors of production or commodities as well as the effects of changing costs on patterns of trade and other forms of interaction.

Following the input-output tradition, a static structural model for the formation of prices in a local economy, the static general interregional price model, involving price determination through local economic interaction such as commuting, shopping and interregional trade, is derived. The equations of the static general interregional price model are presented together with the solution of the model. The theoretical changes examined include a set of new geographical concepts and in the context of an interregional SAM the development of the two-by-two-by-two approach, which involves two sets of actors (production units and institutional units), two types of markets (commodities and factors) and two locations (origins and destinations).

Finally, a simultaneous solution to the combined static general interregional quantity and price model based upon the most simple link between the two is outlined.

1. Introduction

Two different general approaches to regional and interregional modelling can be identified: the Keynesian input-output tradition, which is fundamentally linear approach and the more recent CGE-tradition involving models of a more flexible form and non-linear relationships. In the input-output tradition regional and interregional spill-over and feedback effects are almost exclusively related to changes in the real economic activities. Construction of the static general quantity model is presented in Madsen & Jensen-Butler (??) However, the effects of changes in costs and prices on real economic activity have usually been neglected, despite the fact that the redistributive effects from this dual element in the intra- and interregional economy can be considerable and have effects on economic activity, which are comparable with the quantity effects.

CGE-models on the other hand have explicitly addressed this issue using non-linear functions to overcome theoretical problems related to the use of fixed coefficients, permitting for example a more satisfactory treatment of substitution between factors of production or commodities as well as the effects of changing costs on patterns of trade and other forms of interaction. Although CGE-models include substitution effects arising from changes in relative prices, determining changes in patterns of demand, CGE-models do not generally include a full description of the cost and price determination both in spatial and SAM terms. CGE-models do not in general provide a full description of the operation of the local economy. Furthermore, modelling the complexity of local interregional economy in a CGE framework rapidly leads to problems of derivation of analytical solutions and even when using numerical solution there are problems of mathematical intractability, multiple equilibria and failure to converge on a solution. In addition to these issues, the benefits of the CGE approach are often unclear except perhaps for their anchorage to micro economic theoretical foundations.

These concerns lead to renewed interests in developing models based upon the classical input-output price model. The basic elements in this approach are as follows:

The general static interregional price model, which is developed below, is the dual of the general interregional quantity model (Madsen & Jensen-Butler ??). Price formation is therefore directly linked to the structure of the production and consumption determined in the general interregional quantity model, which involves the application of a set of new geographical concepts together with established SAM concepts. The core is development of the two-by-two-by-two approach, which involves two sets of actors (production units and institutional units), two types of markets (commodities and factors) and two locations (origins and destinations).

1.1 Costs and prices in the local economy

In this paper the focus of interest concerns modelling of the redistribution in costs and prices. The redistribution can have both a SAM dimension and a regional dimension or a combination of both. From a regional and local perspective, cost and price changes are passed through the system of intra- and interregional trade, shopping and tourism as well as the interregional system of commuting. From a SAM perspective cost and price changes are passed on from one actor to another. For example cost and price changes in production are passed on to final consumers which affect their real wage, which in turn affects level of wages through labour market adjustments changing the wage costs in production.

Direct cost and price changes are transferred to the end-user. The process of transfer represents, assuming a constant level of real economic activity, a pure redistribution to the end user of the direct cost and price changes. However, the total effects of these transfers generate both direct and derived effects in the real economy.

In this paper the general static interregional model for costs and prices is examined. The main focus of this chapter is on the spillover and feedback effects in the cost price system, where the model describes the process of full or partial transfer of cost and price changes. The point of departure for the interregional cost and price model is the Leontief price model, which is extended in order to set up a general model for cost and price formation in the local economy. It also includes the dual cost and price version of the interregional Miyazawa model. Finally, an analytical solution for the general static interregional price model is derived.

Assuming that the total effects of the cost and price changes can be identified, the effects on the real economy can then be evaluated using the general static interregional quantity model. Exogenous demand in the quantity model depends on the relative prices, which are derived from the price model. Structural coefficients, such as those determining the composition of demand for commodities and the structure of spatial interaction are also derived from relative prices. A simple link between the quantity and price models related to foreign export is presented and an analytical for the combined model is derived.

Finally, an interregional cost and price model is part of the LINE-model. LINE is a local economic model for the Danish spatial economy and differences in relation to the general interregional price model are identified.

2. Modelling costs and prices in the interregional economy

For models which describe the process of transfer of cost and price changes it is normally assumed that these changes are passed on in full, reflecting perfect competition assumptions. Given the level of real economic activity changes in costs and prices in this type of model follow an adding-on principle, where changes in cost or prices are passed to the next step in the production-demand chain, which in turn are passed to the next step ending as an add-on to the price for the end user. End users usually include households at home and abroad. However, who constitutes an end user depends on the type of model.

The Leontief price model is based on this principle of passing on in full changes in costs and prices. The standard Leontief price model is presented in Miller and Blair (1985) and Bulmer-Thomas (1982). At the national and international level there are a number of studies using the Leontief price model (Polenske (1978), Moses (1974)). The Leontief price model has been included in a number of macroeconomic models at the national level (for example in the Danish case ADAM (Dam 1995)) and at the international level (for example the GTAP model (Bach et al 2000)) level to model changes in cost and prices.

However, at the regional level, studies or even models of the process of price determination are rare. Oosterhaven (1981) extends the national price model to an interregional price model, including price relations operating through intermediate consumption. In a specific model the impacts on prices from changes in consumer prices operating through changes in factor payments and producers prices are examined. The model was used in a study of the regional impacts of the 1970-1975 price increases for raw materials and crude oil. Toyomane (1986) developed an interregional Leontief price model including indirect effects through intermediate consumption. The model formulated in a Isard IRIO (Isard 1951) and Chenery-Moses (Chenery 1953, Moses 1955) pool approach was used for evaluation of the impact of transport system changes. Dietzenbacher (1997) reformulated the Ghosh (1958) supply driven model as the dual to the Leontief quantity model, the standard Leontief price model. However, this was not done in the context of an interregional model.

3. Interregional price models

3.1 The interregional Leontief price model

The point of departure is the price model of the Leontief system. In matrix notation the price model is

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + \mathbf{v}' \circ (\mathbf{i}' - \mathbf{b}_{\mathbf{IC}}') \qquad (1)$$

where

$\mathbf{p}'$: Price index by sector (row vector)

$\mathbf{A}$: The matrix of direct input coefficients

$\mathbf{v}'$: GVA cost index by sector

$\mathbf{i}$: Unity vector

$'$: Transposition

$\circ$: Element by element multiplication

$\mathbf{b_{IC}}'$: Intermediate consumption as share of gross quantity by sector

Like the Leontief quantity model the Leontief price model can be solved

$$\mathbf{p}' = \mathbf{v}' \circ (\mathbf{i}' - \mathbf{b_{IC}}')(\mathbf{I} - \mathbf{A})^{-1} \quad (2\mathbf{a})$$
$$= \mathbf{v}' \circ (\mathbf{i}' - \mathbf{b_{IC}}')(\mathbf{I} + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 + \ldots) \quad (2\mathbf{b})$$

As can be seen, initially the price model looks similar to the Leontief quantity model. However, important differences should be noted: First, the sector price vector $\mathbf{p}'$, the GVA share vector $(\mathbf{i}' - \mathbf{b_{IC}}')$ and cost index vector $\mathbf{v}'$ are row vectors, whereas the gross quantity and final demand vectors in the Leontief quantity model are column vectors. Second, the cost index vector in the Leontief price model is pre-multiplied, whereas the column vector in the Leontief quantity model is post-multiplied. Finally, the cost index vector contains indices, whereas in the Leontief quantity model final demand and quantity vectors contain values. These differences lead to different interpretations of the analytical solutions and the multipliers:

The A-matrix – and the resulting multiplier matrix $(\mathbf{I} - \mathbf{A})^{-1}$ in the Leontief quantity and Leontief price models are the same. In the Leontief quantity model the exogenous final demand column vector is post-multiplied by the Leontief inverse, showing the total direct and indirect effects on gross quantity of final demand. The sectoral quantity multiplier is the sum of the values in any column in the Leontief inverse. In the Leontief price model the exogenous GVA cost index row vector is pre-multiplied by the Leontief inverse and gives the total impacts on sector prices of changes in the exogenous GVA cost index. Here the sectoral price index is sum of the values in any row in the Leontief inverse.

3.2 The interregional Leontief price model

The one region Leontief price model can be transformed into an interregional price model. In Oosterhaven (1981) the interregional Leontief price model has been formulated as the dual price model of the Isard IRIO model (Isard 1951) In the IRIO-model equations 1 and 2 apply directly interpreting different sectors as sector by region. In Toyomane (1986) the interregional price model has been formulated as both the dual price to the Isard IRIO model and as the dual model to the MRIO-quantity model (Chenery (1953) , Moses (1955)) building on the pool method.

In the formulation of the interregional Leontief price model, Oosterhaven and Toyomane assume as in the case of the Isard and the MRIO quantity models, that the make matrix of the demanding region rather than the producing region enters into the interregional price model.

Assuming instead that the make matrix of the supplying region enters into the price model and that intra- and interregional trade is defined in commodities rather than in sectors the price model can be formulated in a more satisfactory way. Taking the formulation of Greenstreet (1987) as the point of departure, the price model can now be reformulated:

$$\mathbf{p}' = \mathbf{p}'\mathbf{DTS_{IC}B_{IC}} + \mathbf{v}' \circ (\mathbf{i}' - \mathbf{b_{IC}}') \quad (3)$$

where:

- $\mathbf{D}$: The make matrix (gross quantity by commodity as share of gross quantity, by sector by place of production)
- $\mathbf{T}$: The intra- and interregional trade matrix (sales originating from place of production as share of total sales, by place of commodity market by commodity)
- $\mathbf{S_{IC}}$: The shopping matrix in intermediate consumption (intermediate consumption at the place of commodity market as share of total intermediate consumption, by place of production by commodity)
- $\mathbf{B_{IC}}$: The use matrix (intermediate consumption by commodity as share of gross quantity, by sector by place of production)

The cost and price effects follow a sequential structure similar but in the opposite direction to that in the interregional Leontief quantity-model. Starting at the place of production, the intermediate consumption price index ($\mathbf{p}'$) together with GVA cost index ($\mathbf{v}'$) determine commodity prices at the place of production, by sector. This price index is then transformed into a price index for commodities using the make matrix ($\mathbf{D}$), all at the place of production. The commodity price index is in turn transformed from place of production into place of commodity market using the matrix for intra- and interregional trade ($\mathbf{T}$). Using the matrix for shopping for intermediate commodities ($\mathbf{S_{IC}}$) transforms the commodity price index from place of commodity market into place of production. Finally, the price index is transformed from commodities back to sectors using the intermediate consumption use matrix ($\mathbf{B_{IC}}$).

Solving the interregional price model gives the following

$$\mathbf{p}' = \mathbf{v}' \circ (\mathbf{i}' - \mathbf{b_{IC}}')(\mathbf{I} - \mathbf{DTS_{IC}B_{IC}})^{-1} \quad (4\mathbf{a})$$

$$= \mathbf{v}' \circ (\mathbf{i}' - \mathbf{b_{IC}}') \cdot (\mathbf{I} + (\mathbf{DTS_{IC}B_{IC}})^1 + (\mathbf{DTS_{IC}B_{IC}})^2 + (\mathbf{DTS_{IC}B_{IC}})^3 + \ldots) \quad (4\mathbf{b})$$

This power series expansion of the analytical solution to the interregional price model gives the sequential structure of not only the model but also of the numerical solution routine, which can be denoted the cost-price circle: First, the direct effects on prices of changes in the gross GVA cost deflator are calculated ($\mathbf{v}' \circ (\mathbf{i}' - \mathbf{b_{IC}}')$), then the first round effects from the price transformations are calculated, starting with prices of production at the place of production by sector and repeating the sequential calculation of price transformations as described above in equation 4b.

3.3 The interregional Leontief and Miyazawa price model

The price changes not only have impacts on sector prices through intermediate consumption, but also on the prices of private consumption. Assuming that changes in consumer prices are transferred to wages, this in turn will have an impact on the cost of production. This effect can be seen as the dual to the Miyazawa extended quantity model. In the following the integrated model, including the interregional price model for price effects operating through the intermediate consumption system (see above) and the dual price model of the Miyazawa real model for private consumption, is presented.

The model is as follows:

$$\begin{aligned}\mathbf{p}' &= \mathbf{p}'\mathbf{DTS_{IC}B_{IC}} + (\mathbf{v^{VAR}}' + \mathbf{v^{FIX}}') \circ (\mathbf{i}' - \mathbf{b_{IC}}') \\ &= \mathbf{p}'\mathbf{DTS_{IC}B_{IC}} + \mathbf{p}'\mathbf{DTS_{CP}B_{CP}W^{VAR}J^{VAR}G^{VAR}_{j,g}} \circ (\mathbf{i}' - \mathbf{b_{IC}}') + \mathbf{v^{FIX}_{g}}'\mathbf{G^{FIX}_{j,g}} \circ (\mathbf{i}' - \mathbf{b_{IC}}') \\ &\quad \mathbf{p}'(\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}W^{VAR}J^{VAR}G^{VAR}_{j,g}} \circ (\mathbf{i}' - \mathbf{b_{IC}}')) + \mathbf{v^{FIX}_{g}}'\mathbf{G^{FIX}_{j,g}} \circ (\mathbf{i}' - \mathbf{b_{IC}}') \text{..........................(5)}\end{aligned}$$

where:

$\mathbf{v^{VAR/FIX}}$: GVA, variable (VAR) or fixed (FIX) in relation to changes in consumer prices by place of production and by sector

$\mathbf{S_{CP}}$: Shopping at the place of commodity market as share of total demand, by place of residence and by commodity

$\mathbf{B_{CP}}$: The use matrix for private consumption (private consumption by commodity as share of GVA by place of residence)

$\mathbf{W^{VAR}}$: The ratio of change in income (for income variable to changes in consumer prices) to price change by place of residence by type of production factor

$\mathbf{J^{VAR}}$: GVA (variable or fixed to changes in consumer prices) by place of residence as share of total variable GVA by place of production by type of production factor

$\mathbf{G_{j,g}}^{\mathbf{VAR/FIX}}$: GVA (variable or fixed to changes in consumer prices) by type of production factor as share of total variable GVA, by sector and by place of production

$\mathbf{v^{FIX}_{g}}$: fixed GVA cost index (GVA unaffected by changes in consumer prices) by factor group, by place of production

In this extended model the step from consumer prices to the cost index of GVA has been included. The term $\mathbf{v}'$ (GVA cost index by sector) in equation 1 has been replaced by the last two terms in equation 5, which represent the induced cost effects and the exogenous fixed cost effects. This also represents a transformation from place of commodity market through place of residence to place of production.

Looking at equation 5 in more detail, prices are transformed in a number of steps involving a division into changes in the cost index originating from changes in prices for private consumption (variable GVA – the second term in equation 4) and changes originating from other sources (fixed GVA – the third term in equation 4). First, changes in the variable GVA cost index transform changes in consumer prices to endogenous changes in GVA assuming full transfer to factor payments. Second, changes in the fixed GVA cost index (such as productivity related changes in wage determination.) transform exogenous changes in GVA to factor payments,

Changes in the variable GVA cost index are transformed from place of residence back to place of production using a matrix for commuting ($\mathbf{J^{VAR}}$). Together with the cost index for the fixed price GVA ($\mathbf{v^{FIX}}$) and the intermediate consumption price index prices of production by sector can now be found.

The cost-price model can be solved as follows:

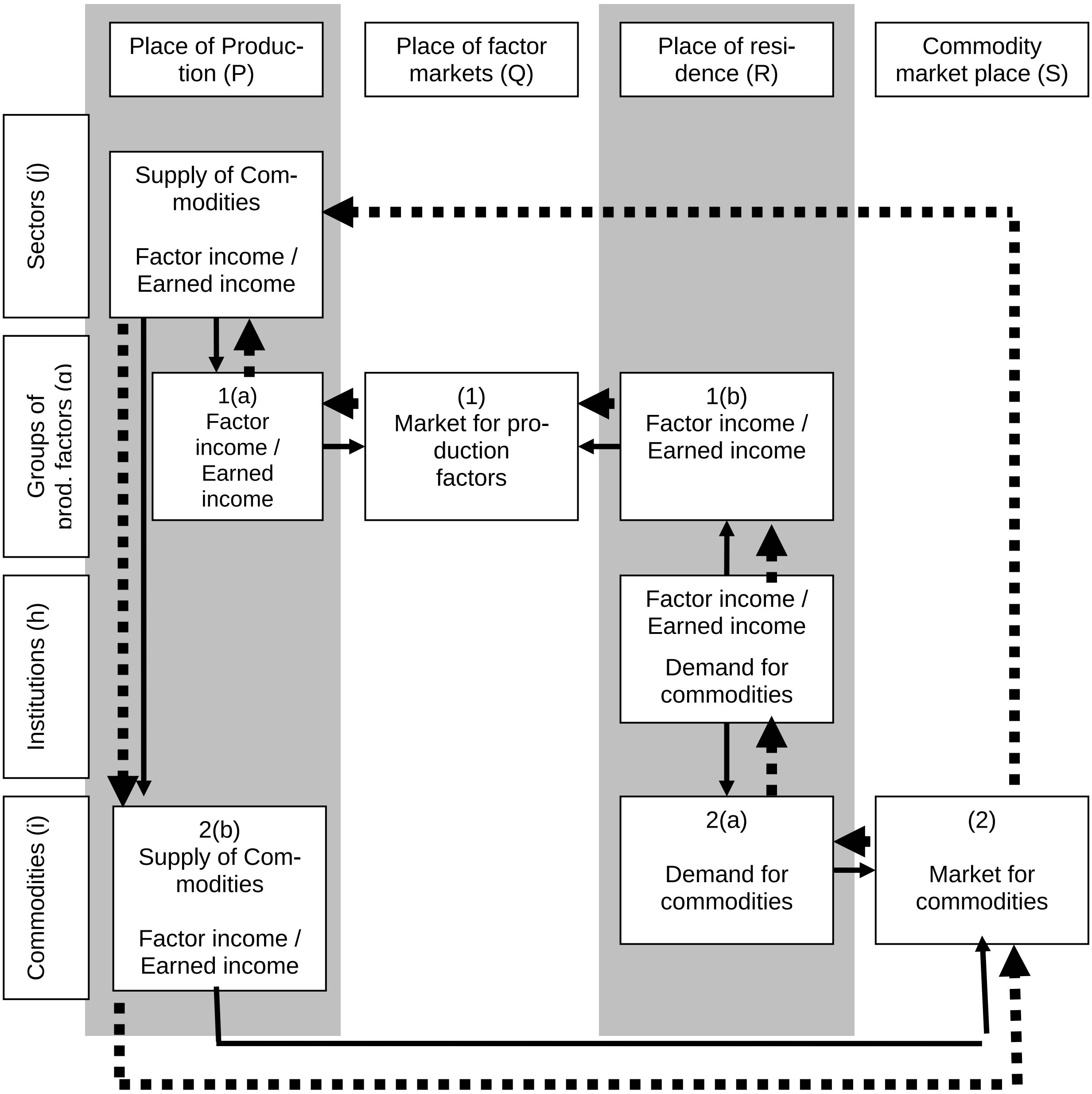
$$\mathbf{p} = \mathbf{v_g^{FIX}}'\mathbf{G_{j,g}^{FIX}} \circ (\mathbf{i}' - \mathbf{b_{IC}}')(\mathbf{I} - \mathbf{DTS_{IC}B_{IC}} - \mathbf{DTS_{CP}B_{CP}WJ^{VAR}G_{j,g}^{VAR}})^{-1} \quad (6\mathbf{a})$$

$$\begin{aligned} &= \mathbf{v_g^{FIX}}' \cdot \mathbf{G_{j,g}^{FIX}} \cdot (\mathbf{i}' - \mathbf{b_{IC}}') \cdot (\mathbf{I} + (\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}WJ^{VAR}G_{j,g}^{VAR}})^1 \\ &\quad + (\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}WJ^{VAR}G_{j,g}^{VAR}})^2 \\ &\quad + (\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}WJ^{VAR}G_{j,g}^{VAR}})^3 + \ldots\ldots\ldots) \quad (6\mathbf{b}) \end{aligned}$$

4. The general interregional price model for local and urban economies[1]

There are 3 fundamental dimensions in the general price model, following the two-by-two-by-two principle. The two-by-two-by-two modelling principle involves use of a basic spatial accounting system with three dimensions (Madsen & Jensen-Butler 2005). First, there is a balance between demand and supply involving both producers and households. Second, there is a balance between demand and supply in both the commodity market and the factor market. Third, for both demand and supply in the commodity and factor markets origins and destinations are identified and spatial equilibrium is obtained. The general model structure is presented in figure 1.

Figure 1
The conceptual basis of spatial social accounts



The price circle moves anti-clockwise and follows the structure presented in the general Leontief and Miyazawa quantity model (Madsen & Jensen-Butler ???). This model as compared with equation 6 has been extended to include the place of factor market (Q) and institutions (h) and a foreign sector.

4.1 The equations of the model

In the following the general interregional price model is presented both in equation form and graphically. In the appendix the equations of the model are presented in structural form with the partial solution[2]. The equations follow the cost and price circle as illustrated in figure 1. The model can be seen as a national model (the Leontief and Miyazawa model), where a spatial dimension (reading horizontally) and a social accounting dimension (reading vertically) have been included.

The presentation of the model commences in the upper left corner of figure 1 (cell Pj). In this cell the prices of production at the place of production and by sectors are determined. This corresponds to equation A.1 (appendix 1), where prices of production ($px_j^P{}'$) are a function of prices of intermediate consumption ($pu_{j,IC}^P{}'$) and GVA ($ph_j^P{}'$). "u" is a variable representing demand. The subscript (IC) indicates intermediate consumption and (j) the sector. The superscript shows, that the prices of production, intermediate consumption and GVA are determined at the place of production (P).

Next, the price of gross output is transformed from sector prices into commodity prices (from cell Pj to Pi). This corresponds to equation A.2. Going through the equations the sequential structure of the cost-price circle is clear and follows the graphical presentation in figure 1. The cost price circuit corresponds to the interregional Leontief and Miyazawa price model and moves anti clockwise in figure 1. Continuing in the lower left corner (cell Pi), prices of production determine foreign export prices (equation A.3) and prices for domestic trade (equation A.4). Prices are transformed from place of production (cell Pi) to place of commodity market (cell Si). Here the domestic price index is merged with the price index for foreign imports to form the composite local demand price index (equation A.5). Then the cost and price circle is divided into two. In equations A.6-A.10, the indirect effects through intermediate consumption are determined, whilst the induced income formation is determined in equations A.11-A.22.

The prices of intermediate consumption are transformed from place of commodity market (Si) to place of production (Pi) and from (Pi) commodities to sectors (Pj) closing the indirect effects sub-circle.

In the induced effects sub-circle the price index for private consumption is transformed from place of commodity market (Si) to the place of residence (Ri). Then the impacts on income by household arising from changes in consumer prices are determined, transforming prices from commodities to type of institutions (from Ri to Rh). Now follows the transformation from fixed prices to current prices and the effects on income from changes in private consumption in current prices are included. It is assumed in the general price model that only a part of the price index for private consumption $(i'-hx_{h,CP}^R{}')$ is reflected in the cost index for income $ph_h^R{}'$, where $hx_{h,CP}^R{}'$ is the share of income, which is assumed to be independent of consumption prices. Then the income cost index is transformed from type of institution to factor group (from Rh to Rg). Further, the effects on income arising from index cost index are transformed from place of residence to place of production (from Rg to Qg and to Pg) and transformed into impacts on value added by sector (from Pg to Pj). Finally, value added is added to intermediate consumption both in current prices, closing the cost and price circle.

In the general model both domestic and foreign sectors are represented in all markets. Costs and prices are thus involved in not only international trade in commodities, but also in other types of international interaction. This includes cross border commuting (commuting income flows to and from abroad), border shopping (one-day tourist expenditure) and tourists' trips in both directions being included in formation of pri-

prices. This extension is included to make the model general as most regional systems interact with »the rest of the world«.

4.2 The analytical solution for the general interregional price model

The model can now be solved by straightforward insertion. By inserting equation A.22 into equation A.21, and further down the set of equations to equation A.11 (the induced effects sub-circle). This is followed by inserting equation A.10 into equation A.9, and further down the set of equations to equation A.6 (the indirect effects sub-circle). Now, the equation can be solved for the price of gross output by sector ($px_j^P{}'$) as can be seen in equation A.24.

$$\mathbf{px_j^P}' = \begin{bmatrix} \mathbf{pz_i^{S,F}}' \circ \mathbf{d_i^{S,F}}' \circ \mathbf{S_{IC}} \circ (\mathbf{i}' - \mathbf{b_{i,IC}^{P,F}}') \cdot \mathbf{B_{IC}} \circ \mathbf{b_{j,IC}^{P}}' \\ + \mathbf{pu_{i,IC}^{P,F}}' \circ \mathbf{b_{i,IC}^{P,F}}' \cdot \mathbf{B_{IC}} \circ \mathbf{b_{j,IC}^{P}}' \\ + \mathbf{pz_i^{S,F}}' \circ \mathbf{d_i^{S,F}}' \circ \mathbf{S_{CP}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}} \circ (\mathbf{i}' - \mathbf{hx_{h,CP}^{R}}') \cdot \mathbf{H} \cdot \mathbf{J} \circ (\mathbf{i}' - \mathbf{j_g^{P,F}}') \cdot \mathbf{G} \\ + \mathbf{pu_{i,CP}^{R,F}}' \circ \mathbf{b_{i,CP}^{R,F}}' \cdot \mathbf{B_{CP}} \circ (\mathbf{i}' - \mathbf{hx_{h,CP}^{R}}') \cdot \mathbf{H} \cdot \mathbf{J} \circ (\mathbf{i}' - \mathbf{j_g^{P,F}}') \cdot \mathbf{G} \\ + \mathbf{phx_h^{R}}' \circ \mathbf{hx_{h,CP}^{R}}' \cdot \mathbf{H} \cdot \mathbf{J} \circ (\mathbf{i}' - \mathbf{j_g^{P,F}}') \cdot \mathbf{G} \\ + \mathbf{ph_g^{P,F}}' \circ \mathbf{j_g^{P,F}}' \cdot \mathbf{G} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I} - \mathbf{DT} \circ (\mathbf{i}' - \mathbf{d_i^{S,F}}') \circ \big(\mathbf{S_{IC}} (\mathbf{i}' - \mathbf{b_{i,IC}^{P,F}}') \cdot \mathbf{B_{IC}} \circ \mathbf{b_{j,IC}^{P}}' - \mathbf{S_{CP}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}} \\ \circ (\mathbf{i}' - \mathbf{hx_{h,CP}^{R}}') \cdot \mathbf{H} \cdot \mathbf{J} \cdot (\mathbf{i}' - \mathbf{j_g^{P,F}}') \circ \mathbf{G} \circ (\mathbf{i}' - \mathbf{b_{j,IC}^{P}}') \big) \end{bmatrix}^{-1} \quad ....(\mathbf{A}.24) \ \mathbf{or} \ (7)$$

From A.24 it can be seen that the price of the gross output by sector ($px_j^P{}'$) can be expressed as a product of the exogenous cost and price indices (the first part of the equation) and a cost and price multiplier (the second part of the equation). The exogenous cost and price indices include a) a cost index for income ($\mathbf{phx_h^R}'$) for the part of income which is not influenced by price changes, b) a price index for foreign imports ($\mathbf{pz_i^{S,F}}'$) and c) price indices for other commodities purchased abroad including cross border shopping ($\mathbf{pu_{i,IC}^{P,F}}'$ and $\mathbf{pu_{i,CP}^{P,F}}'$), as well a cost index for income earned by non-residents ($\mathbf{ph_g^{P,F}}'$). The cost and price multiplier consists of the usual coefficients for intermediate and private consumption.

5.The general price model – and LINE

LINE is an interregional general equilibrium model constructed for Danish municipalities (Madsen et al 2001 and Madsen & Jensen-Butler (2004)). LINE consists of a quantity and a price model. The spatial *two-by-two-by-two* principle has been the guiding principle for the construction of both the quantity and the price model. However, there are some differences between the price model in LINE and a price model based upon a pure two-by-two-by-two principle as described above.

The structure of the price model in LINE follows the basic interregional general equilibrium model shown in figure 1 with:

- Factor markets and commodity markets
- Demand and supply in both markets
- Origins and destinations in all interactions

Some simplifications and extensions are however incorporated. The general price model is adjusted as it was the case for the quantity model:

1. the market of factors has been excluded from the price model in LINE
2. only factor income from labour receives a full treatment
3. Interactions between factor groups, household and governmental sectors are included in the price model in LINE.
4. prices on consumption by institutions (households) both from a decision-making or a behavioural point of view must be divided into two nested steps.
5. prices on private consumption have been divided into local private consumption and domestic tourism.
6. different price concepts are included in the price model, reflecting the fact that different variables for economic activity use different price concepts. For goods and services, total expenditures at the place of commodity market are measured in market prices. Supply of commodities entering the goods and services market is modelled in basic prices. Basic prices are defined as the value of production at the factory, not including net commodity taxes paid by the producer. Going from basic prices to market/buyers prices in the price model involves addition of commodity taxes and trade margins, both in current prices. Changes in commodity tax and trade margins shares are exogenous variables in the price model in LINE.

6. Solving the general quantity and price model simultaneously

There are links between the quantity model and the price model. These links reflect links in the regional economy: On the one hand prices are a function of exogenous costs and prices and a system of weights originating from the quantity model. On the other hand economic activity in the quantity model depends upon price dependent demand, such as foreign export.

In the classical version of the price model, the institutional sector by sector model shown in equation 1, the exogenous cost and price variable is the GVA-deflator, whereas the weight system is the A-matrix. In the general interregional price model the exogenous cost and price variables are the GVA-deflator, foreign import prices etc., whereas the weight system is the system of transformations of quantities from demand to production or from production to demand through the interregional system of interaction.

In the formulation of the general interregional price model, the specification of the interaction between quantity system and prices in the general interregional price model is fully specified, because all weights from the quantity model have been included in the determination of prices (see equation 7 above or equation A.24) In condensed form the prices (for gross output) in the general interregional price model are determined by exogenous cost and prices and a system of weights determining the prices:

$$\mathbf{px_j^{P}}' = \text{Pr}\,\mathbf{ice}\,\text{Pr}\,\mathbf{eModel}\left[\mathbf{pz_i^{S,F}}',\mathbf{pu_{i,IC}^{P,F}}',\mathbf{pz_i^{S,F}}',\mathbf{pu_{i,CP}^{R,F}}',\mathbf{phx_h^{R}}',\mathbf{ph_g^{P,F}}'\right]\left[\mathbf{I}-\mathbf{A_{j\,to\,j}^{Pricemodel,Interreg}}\right]^{-1} \text{................}(\mathbf{A}.24\mathbf{a})\ \mathbf{or}\ (8)$$

where:

$$\mathbf{A_{j\,to\,j}^{Pricemodel,Interreg}} = \left[\mathbf{I}-\mathbf{DT}\circ(\mathbf{i}'-\mathbf{d_i^{S,F}}')\circ\left(\mathbf{S_{IC}}(\mathbf{i}'-\mathbf{b_{i,IC}^{P,F}}')\cdot\mathbf{B_{IC}}\circ\mathbf{b_{j,IC}^{P}}'-\mathbf{S_{CP}}\circ(\mathbf{i}'-\mathbf{b_{i,CP}^{R,F}}')\cdot\mathbf{B_{CP}}\circ(\mathbf{i}'-\mathbf{hx_{h,CP}^{R}}')\cdot\mathbf{H}\cdot\mathbf{J}\cdot(\mathbf{i}'-\mathbf{j_g^{P,F}}')\circ\mathbf{G}\circ(\mathbf{i}'-\mathbf{b_{j,IC}^{P}}')\right)\right]^{-1}$$

On the basis of prices for production, prices for specific demand components, such as price for export commodities can be determined adding a post model transforming prices on gross output into prices for the specific demand component:

$$\begin{aligned}\mathbf{pz_i^{P,F}}' &= \text{Pr}\,\mathbf{ice}\,\text{Pr}\,\mathbf{eModel}\left[\mathbf{pz_i^{S,F}}',\mathbf{pu_{i,IC}^{P,F}}',\mathbf{pz_i^{S,F}}',\mathbf{pu_{i,CP}^{R,F}}',\mathbf{phx_h^{R}}',\mathbf{ph_g^{P,F}}'\right]\\ &\qquad\left[\mathbf{I}-\mathbf{A_{j\,to\,j}^{Pricemodel,Interreg}}\right]^{-1}\mathbf{D^{P}}\circ\mathbf{kpz_i^{P,F}}\\ &= \text{Pr}\,\mathbf{ice}\,\text{Pr}\,\mathbf{eModel}\left[\mathbf{pz_i^{S,F}}',\mathbf{pu_{i,IC}^{P,F}}',\mathbf{pz_i^{S,F}}',\mathbf{pu_{i,CP}^{R,F}}',\mathbf{phx_h^{R}}',\mathbf{ph_g^{P,F}}'\right]\\ &\qquad\left[\mathbf{I}-\mathbf{A_{j\,to\,j}^{Pricemodel,Interreg}}\right]^{-1}\text{Pr}\,\mathbf{icePostModel}\text{................................}(9)\end{aligned}$$

In the real economy in most cases prices influence quantities, such as foreign export, which changes if foreign export prices change. However, in the quantity model there are no links between the price variables in the cost price model and the exogenous variables in the quantity model. In the quantity model economic activity, such as gross output, is explained only by quantity model variables, such as coefficients entering into the multiplier (the A-matrix in the classical institutional sector-by-sector model) and the exogenous demand (foreign export, governmental consumption and investments in the classical model). In the condensed formulation of the general interregional quantity model, gross output is determined as follows:

$$\mathbf{x_j^{P}} = \left[\mathbf{I}-\mathbf{A_{j\,to\,j}^{Interreg}}\right]^{-1}\mathbf{QuantityPostModel}\left[\mathbf{z_i^{P,F,f}},\mathbf{u_{i,IC}^{S,F,f}},\mathbf{u_{i,CP}^{S,F,f}},\mathbf{u_{i,CO}^{S,f}},\mathbf{u_{i,IR}^{S,f}},\mathbf{u_{h,CP}^{R}}\right]\text{................................}(10)$$

where:

$$A^{Interreg}_{j\ to\ j} = \Big(I - D^P T_i^{R,S,f} (i - d_i^{S,F,f}) \circ S_{i,IC}^{P,S,f} (i - b_{i,IC}^{P,F,f}) \circ B_{IC}^{P,f} b_{IC}^{P,f} - D^P T_i^{R,S,f} (i - d_i^{S,F,f}) \circ S_{i,CP}^{R,S,f} (i - b_{i,CP}^{R,F,f}) \circ B_{CP}^{R,f} (pu_{h,CP}^{R})^{-1} \circ H^R J_g^{Q,T} J_g^{P,Q} (i - j_g^{P,F}) \circ G^P ph_j^P \circ (i - b_{IC}^{P,f}) \Big)^{-1}$$

Extending the quantity model with price dependency, different formulations are possible. First, it can be assumed that prices enter into the determination of the exogenous demand, such as foreign exports. Second, it can be assumed that the coefficients in the multiplier matrix are dependent on prices.

Mathematically, linking exogenous demand to prices for exogenous demand is straightforward. Assuming a linear relation between foreign export and foreign export prices:

$$\mathbf{z}_i^{P,F,f} = \varepsilon_0 + \varepsilon_1 \mathbf{pz}_i^{S,F} \quad \text{.................................(11)}$$

Solving the combined general interregional quantity and price model gives:

$$\begin{aligned} \mathbf{x}_j^P &= \left[\mathbf{I} - \mathbf{A}^{Interreg}_{j\ to\ j}\right]^{-1} \mathbf{QuantityPostModel}\left[\mathbf{z}_i^{P,F,f}, \mathbf{u}_{i,IC}^{S,F,f}, \mathbf{u}_{i,CP}^{S,F,f}, \mathbf{u}_{i,CO}^{S,f}, \mathbf{u}_{i,IR}^{S,f}, \mathbf{u}_{h,CP}^{R}\right] \\ &= \left[\mathbf{I} - \mathbf{A}^{Interreg}_{j\ to\ j}\right]^{-1} \mathbf{QuantityPostModel}\left[\mathbf{pz}_i^{S,F}{}' \circ \varepsilon_1 + \varepsilon_0, \mathbf{u}_{i,IC}^{S,F,f}, \mathbf{u}_{i,CP}^{S,F,f}, \mathbf{u}_{i,CO}^{S,f}, \mathbf{u}_{i,IR}^{S,f}, \mathbf{u}_{h,CP}^{R}\right] \\ &= \left[\mathbf{I} - \mathbf{A}^{Interreg}_{j\ to\ j}\right]^{-1} \mathbf{QuantityPostModel}\begin{bmatrix} \mathbf{PricePreModel}\left[\mathbf{pz}_i^{S,F}{}', \mathbf{pu}_{i,IC}^{P,F}{}', \mathbf{pz}_i^{S,F}{}', \mathbf{pu}_{i,CP}^{R,F}{}', \mathbf{phx}_h^{R}{}', \mathbf{ph}_g^{P,F}{}'\right] \\ \left[\mathbf{I} - \mathbf{A}^{Pricemodel,Interreg}_{j\ to\ j}\right]^{-1} \mathbf{PricePostModel} \circ \varepsilon_1 + \varepsilon_0, \\ \mathbf{u}_{i,IC}^{S,F,f}, \mathbf{u}_{i,CP}^{S,F,f}, \mathbf{u}_{i,CO}^{S,f}, \mathbf{u}_{i,IR}^{S,f}, \mathbf{u}_{h,CP}^{R} \end{bmatrix} \text{...(12)} \end{aligned}$$

From this solution of the pure linear combined general interregional quantity and price model, assuming price dependency in the foreign export, it can be seen that production is determined by

- exogenous cost and price variables in the price model, such as GVA-deflator, foreign import prices etc.
- the weight system for transformations of quantities from demand to production and from production to demand through the interregional system of interaction
- exogenous demand, such as governmental consumption etc.
- coefficients ε_0 and ε_1 in the foreign export price equation.

Mathematically, linking the coefficients for transformation between quantities from demand to production of from production to demand through the interregional system of interaction to changes in prices is much more complicated and cannot be solved.

7. Summary

In this paper the general interregional static price model for local or urban economies has been presented. The model represents an extension and integration of the interregional Leontief price model, which involves the indirect effects and a modification of the interregional Miyazawa quantity model to include the induced effects.

The general interregional price model is based upon the two-by-two-by-two principle including a) markets for commodities and factors, b) production units and institutional units and c) origin and destination for the demand and supply in the two markets. The general interregional price model includes a foreign sector and the analytical solution to the model is presented. The general interregional price model is compared with LINE, which is a local economic model for Danish regions with a structure similar to the structure of the general interregional and local price model.

Further, the establishment of a general static model, which integrates the interregional price and quantity models through a set of links is examined. Two types of link are identified, demand price links for exogenous demand and price links determining structural coefficients. An analytical solution for the combined general static price and quantity model is presented.

Appendix

The equations for the general interregional quantity model for local and urban economies in structural form

Variables in the quantity model

The variables in the general interregional quantity model are denoted in the following way:

Variables

x: gross output by sector
px: Price index for gross output by sector
D: Make coefficient matrix
q: gross output by commodity
pq: Price index for gross output by commodity
T: Trade coefficient matrix
b: Use coefficient vector of demand
z: Trade vector
pz: Price index for trade flows
B: Use coefficient matrix of demand
pu: Price index vector for demand
hx: Share of income independent of consumer prices
phx: Wage index for income independent of consumer prices
G, H, J: Income transformation coefficient matrices
g, h, j: Income vectors
ph: Wage index for income

Superscripts

P: Place of production (regional axes)
Q: Place of factor market (regional axes)
R: Place of residence (regional axes)
S: Place of commodity market (regional axes)
D: Domestic
F: Rest of the world
f: Fixed prices

Subscripts

SAM-axes
j: Sector (SAM-axis)
g: Groups of factors (SAM-axis)
h: Type of institution (SAM-axis)
i: Commodity (SAM-axis)
IC: Intermediate consumption
CP: private consumption
CO: Governmental consumption
IR: Investments

The general interregional price model in structural form

$$\mathbf{px}_j^{P}{}' = \mathbf{pu}_{j,IC}^{P}{}' \circ \mathbf{b}_{j,IC}^{P}{}' + \mathbf{ph}_j^{P}{}' \circ (\mathbf{i}' - \mathbf{b}_{j,IC}^{P}{}') \quad \text{......} (\mathbf{A}.1)$$

$$\mathbf{pq}_i^{P}{}' = \mathbf{px}_j^{P}{}' \cdot \mathbf{D}^{P} \quad \text{......} (\mathbf{A}.2)$$ **From Pj to Pi**

$$\mathbf{pz}_i^{P,D}{}' = \mathbf{pq}_i^{P}{}' \circ \mathbf{kpz}_i^{P}{}' \quad \text{......} (\mathbf{A}.3)$$

$$\mathbf{pz}_i^{S,D}{}' = \mathbf{pz}_i^{P,D}{}' \cdot \mathbf{T}^{S,P} \quad \text{......} (\mathbf{A}.4)$$ **From Pi to Si**

$$\mathbf{pu}_i^{S}{}' = \mathbf{pz}_i^{S,D}{}' \circ (\mathbf{i}' - \mathbf{d}_i^{S,F}{}') + \mathbf{pz}_i^{S,F}{}' \circ \mathbf{d}_i^{S,F}{}' \quad \text{......} (\mathbf{A}.5)$$

$$\mathbf{pu}_{i,IC}^{S}{}' = \mathbf{pu}_i^{S}{}' \circ \mathbf{kpu}_{i,IC}^{S}{}' \quad \text{......} (\mathbf{A}.6)$$

$$\mathbf{pu}_{i,IC}^{S,D}{}' = \mathbf{pu}_{i,IC}^{S}{}' \circ \mathbf{kpu}_{i,IC}^{S,D}{}' \quad \text{......} (\mathbf{A}.7)$$

$$\mathbf{pu}_{i,IC}^{P,D}{}' = \mathbf{pu}_{i,IC}^{S,D}{}' \cdot \mathbf{S}_{IC}^{P,S} \quad \text{......} (\mathbf{A}.8)$$ **From Si to Pi**

$$\mathbf{pu}_{i,IC}^{P}{}' = \mathbf{pu}_{i,IC}^{P,D}{}' \circ (\mathbf{i}' - \mathbf{b}_{i,IC}^{P,F}{}') + \mathbf{pu}_{i,IC}^{P,F}{}' \circ \mathbf{b}_{i,IC}^{P,F}{}' \quad \text{......} (\mathbf{A}.9)$$

$$\mathbf{pu}_{j,IC}^{P}{}' = \mathbf{pu}_{i,IC}^{P}{}' \cdot \mathbf{B}_{IC}^{P} \quad \text{......} (\mathbf{A}.10)$$ **From Pi to Pj**

$$\mathbf{pu}_{i,CP}^{S}{}' = \mathbf{pu}_i^{S}{}' \circ \mathbf{kpu}_{i,CP}^{S}{}' \quad \text{......} (\mathbf{A}.11)$$

$$\mathbf{pu}_{i,CP}^{S,D}{}' = \mathbf{pu}_{i,CP}^{S}{}' \circ \mathbf{kpu}_{i,CP}^{S,D}{}' \quad \text{......} (\mathbf{A}.12)$$

$$\mathbf{pu}_{i,CP}^{R,D}{}' = \mathbf{pu}_{i,CP}^{S,D}{}' \cdot \mathbf{S}_{CP}^{R,S} \quad \text{......} (\mathbf{A}.13)$$ **From Si to Ri**

$$\mathbf{pu}_{i,CP}^{R}{}' = \mathbf{pu}_{i,CP}^{R,D}{}' \circ (\mathbf{i}' - \mathbf{b}_{i,CP}^{R,F}{}') + \mathbf{pu}_{i,CP}^{R,F}{}' \circ \mathbf{b}_{i,CP}^{R,F}{}' \quad \text{......} (\mathbf{A}.14)$$

$$\mathbf{pu}_{h,CP}^{R}{}' = \mathbf{pu}_{i,CP}^{R}{}' \cdot \mathbf{B}_{CP}^{R} \quad \text{......} (\mathbf{A}.15)$$ **From Ri to Rh**

$$\mathbf{ph}_h^{R}{}' = \mathbf{pu}_{h,CP}^{R}{}' \circ (\mathbf{i}' - \mathbf{hx}_{h,CP}^{R}{}')' + \mathbf{phx}_h^{R}{}' \circ \mathbf{hx}_{h,CP}^{R} \quad \text{......} (\mathbf{A}.16)$$

$$\mathbf{ph}_g^{R}{}' = \mathbf{ph}_h^{R}{}' \cdot \mathbf{H}^{R} \quad \text{......} (\mathbf{A}.17)$$ **From Rh to Rg**

$$\mathbf{ph}_g^{R,D}{}' = \mathbf{ph}_g^{R}{}' \circ \mathbf{kph}_g^{R,D}{}' \quad \text{......} (\mathbf{A}.18)$$

$$\mathbf{ph}_g^{Q,D}{}' = \mathbf{ph}_g^{R,D}{}' \cdot \mathbf{J}^{Q,R} \quad \text{......} (\mathbf{A}.19)$$ **From Rg to Qg**

$$\mathbf{ph}_g^{P,D}{}' = \mathbf{ph}_g^{Q,D}{}' \cdot \mathbf{J}^{P,Q} \quad \text{......} (\mathbf{A}.20)$$ **From Qg to Pg**

$$\mathbf{ph}_g^{P}{}' = \mathbf{ph}_g^{P,D}{}' \circ (\mathbf{i}' - \mathbf{j}_g^{P,F}{}') + \mathbf{ph}_g^{P,F}{}' \circ \mathbf{j}_g^{P,F}{}' \quad \text{......} (\mathbf{A}.21)$$

$$\mathbf{ph}_j^{P}{}' = \mathbf{ph}_g^{P}{}' \cdot \mathbf{G}^{P} \quad \text{......} (\mathbf{A}.22)$$ **From Pg to Pj**

The general interregional price model in reduced form

$$
\begin{aligned}
\mathbf{px}_j^{P}{}' &= \mathbf{px}_j^{P}{}' \circ \mathbf{b}_{j,IC}^{P}{}' + \mathbf{ph}_j^{P}{}' \circ (\mathbf{i}' - \mathbf{b}_{j,IC}^{P}{}') \\
&= \mathbf{px}_j^{P}{}' \cdot \underbrace{\mathbf{D}^{P} \circ \mathbf{kpz}_i^{P,D}{}' \cdot \mathbf{T}^{S,P}}_{\mathrm{DT}} \circ (\mathbf{i}' - \mathbf{d}_i^{S,F}{}') \circ \underbrace{\mathbf{kpu}_{i,IC}^{S}{}' \circ \mathbf{kpu}_{i,IC}^{S,D}{}' \cdot \mathbf{S}_{IC}^{P,S}}_{\mathrm{S_{IC}}} \circ (\mathbf{i}' - \mathbf{b}_{i,IC}^{P,F}{}') \cdot \underbrace{\mathbf{B}_{IC}^{P}}_{\mathrm{B_{IC}}} \circ \mathbf{b}_{j,IC}^{P}{}' \\
&\quad + \mathbf{pz}_i^{S,F}{}' \circ \mathbf{d}_i^{S,F}{}' \circ \mathbf{kpu}_{i,IC}^{S}{}' \circ \mathbf{kpu}_{i,IC}^{S,D}{}' \cdot \mathbf{S}_{IC}^{P,S} \circ (\mathbf{i}' - \mathbf{b}_{i,IC}^{P,F}{}') \cdot \mathbf{B}_{IC}^{P} \circ \mathbf{b}_{j,IC}^{P}{}' \\
&\quad + \mathbf{pu}_{i,IC}^{P,F}{}' \circ \mathbf{b}_{i,IC}^{P,F}{}' \cdot \mathbf{B}_{IC}^{P} \circ \mathbf{b}_{j,IC}^{P}{}' \\
&\quad + \mathbf{ph}_j^{P}{}' \circ (\mathbf{i}' - \mathbf{b}_{j,IC}^{P}{}') \text{........}(\mathbf{A}.23\mathbf{a})
\end{aligned}
$$

or

$$
\begin{aligned}
\mathbf{px}_j^{P}{}' &= \mathbf{px}_j^{P}{}' \cdot \mathbf{DT} \circ (\mathbf{i}' - \mathbf{d}_i^{S,F}{}') \circ \mathbf{S}_{IC} (\mathbf{i}' - \mathbf{b}_{i,IC}^{P,F}{}') \cdot \mathbf{B}_{IC} \circ \mathbf{b}_{j,IC}^{P}{}' \\
&\quad + \mathbf{pz}_i^{S,F}{}' \circ \mathbf{d}_i^{S,F}{}' \circ \mathbf{S}_{IC} \circ (\mathbf{i}' - \mathbf{b}_{i,IC}^{P,F}{}') \cdot \mathbf{B}_{IC} \circ \mathbf{b}_{j,IC}^{P}{}' \\
&\quad + \mathbf{pu}_{i,IC}^{P,F}{}' \circ \mathbf{b}_{i,IC}^{P,F}{}' \cdot \mathbf{B}_{IC} \circ \mathbf{b}_{j,IC}^{P}{}' \\
&\quad + \mathbf{ph}_j^{P}{}' \circ (\mathbf{i}' - \mathbf{b}_{j,IC}^{P}{}') \text{..}(\mathbf{A}.23\mathbf{a}')
\end{aligned}
$$

where:

$$\mathbf{ph_j^P}' = \mathbf{px_j^P}' \cdot \mathbf{D^P} \circ \mathbf{kpz_i^{P,F}}' \cdot \mathbf{T^{S,P}} \circ (\mathbf{i}' - \mathbf{d_i^{S,F}}') \circ \mathbf{kpu_{i,CP}^S}' \circ \mathbf{kpu_{i,CP}^{S,D}}' \cdot \mathbf{S_{CP}^{R,S}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}^R} \circ (\mathbf{i} - \mathbf{hx_{h,CP}^R})' \cdot \mathbf{H^R} \circ (\mathbf{i}' - \mathbf{j^{R,F}}') \cdot \mathbf{J^{Q,R}} \cdot \mathbf{J^{P,Q}} \cdot \mathbf{G^P}$$
$$+ \mathbf{pz_i^{S,F}}' \circ \mathbf{d_i^{S,F}}' \circ \mathbf{kpu_{i,CP}^S}' \circ \mathbf{kpu_{i,CP}^{S,D}}' \cdot \mathbf{S_{CP}^{R,S}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}^R} \circ (\mathbf{i} - \mathbf{hx_{h,CP}^R}) \cdot \mathbf{H^R} \circ (\mathbf{i}' - \mathbf{j^{R,F}}') \cdot \mathbf{J^{Q,R}} \cdot \mathbf{J^{P,Q}} \cdot \mathbf{G^P}$$
$$+ \mathbf{pu_{i,CP}^{R,F}}' \circ \mathbf{b_{i,CP}^{R,F}}' \cdot \mathbf{B_{CP}^R} \circ (\mathbf{i} - \mathbf{hx_{h,CP}^R}) \cdot \mathbf{H^R} \circ (\mathbf{i}' - \mathbf{j^{R,F}}') \cdot \mathbf{J^{Q,R}} \cdot \mathbf{J^{P,Q}} \cdot \mathbf{G^P}$$
$$+ \mathbf{phx_h^R}' \circ \mathbf{hx_{h,CP}^R} \cdot \mathbf{H^R} \circ (\mathbf{i}' - \mathbf{j^{R,F}}') \cdot \mathbf{J^{Q,R}} \cdot \mathbf{J^{P,Q}} \cdot \mathbf{G^P}$$
$$+ \mathbf{ph_g^{P,F}}' \circ \mathbf{j_g^{P,F}}' \cdot \mathbf{G^P} \text{..........} (23\mathbf{b})$$

$$\underbrace{\quad}_{DT} \qquad \underbrace{\quad}_{S_C} \qquad \underbrace{\quad}_{B_{CP}} \qquad \underbrace{\quad}_{H} \qquad \underbrace{\quad}_{J} \qquad \underbrace{\quad}_{G}$$

or

$$\mathbf{ph_j^P}' = \mathbf{px_j^P}' \cdot \mathbf{DT} \circ (\mathbf{i}' - \mathbf{d_i^{S,F}}') \circ \mathbf{S_{CP}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}} \circ \mathbf{hx_{h,CP}^R} \cdot \mathbf{H} \cdot \mathbf{J} \cdot (\mathbf{i}' - \mathbf{j_g^{P,F}}') \circ \mathbf{G}$$
$$+ \mathbf{pz_i^{S,F}}' \circ \mathbf{d_i^{S,F}}' \circ \mathbf{S_{CP}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}} \circ \mathbf{hx_{h,CP}^R} \cdot \mathbf{H} \cdot \mathbf{J} \cdot (\mathbf{i}' - \mathbf{j_g^{P,F}}') \circ \mathbf{G}$$
$$+ \mathbf{pu_{i,CP}^{R,F}}' \circ \mathbf{b_{i,CP}^{R,F}}' \cdot \mathbf{B_{CP}} \circ \mathbf{hx_{h,CP}^R} \cdot \mathbf{H} \cdot \mathbf{J} \cdot (\mathbf{i}' - \mathbf{j_g^{P,F}}') \circ \mathbf{G}$$
$$+ \mathbf{phx_h^R}' \circ (\mathbf{i}' - \mathbf{b_{h,CP}^R}') \circ \mathbf{hx_{h,CP}^R} \cdot \mathbf{H} \cdot \mathbf{J} \cdot (\mathbf{i}' - \mathbf{j_g^{P,F}}') \circ \mathbf{G}$$
$$+ \mathbf{ph_g^{P,F}}' \circ \mathbf{j_g^{P,F}}' \cdot \mathbf{G} \text{.....................} (\mathbf{A}.23\mathbf{b}')$$

where:

$$\mathbf{DT} = \mathbf{D^P} \circ \mathbf{kpz_i^{P,F}}' \cdot \mathbf{T^{S,P}}$$

$$\mathbf{S_{IC}} = \mathbf{kpu_{i,IC}^S}' \circ \mathbf{kpu_{i,IC}^{S,D}}' \cdot \mathbf{S_{IC}^{P,S}}$$

$$\mathbf{B_{IC}} = (\mathbf{i}' - \mathbf{b_{i,IC}^{P,F}}') \cdot \mathbf{B_{IC}^P} \circ \mathbf{b_{j,IC}^P}'$$

$$\mathbf{S_{CP}} = \mathbf{kpu_{i,CP}^S}' \circ \mathbf{kpu_{i,CP}^{S,D}}' \cdot \mathbf{S_{CP}^{R,S}}$$

$$\mathbf{B_{CP}} = \mathbf{B_{CP}^R} \circ \mathbf{b_{h,CP}^R}'$$

$$\mathbf{H} = \mathbf{H^R}$$

$$\mathbf{J} = \mathbf{J^{Q,R}} \cdot \mathbf{J^{P,Q}}$$

$$\mathbf{G} = \mathbf{G^P}$$

$$\mathbf{px_j^{P}}' = \begin{bmatrix} \mathbf{pz_i^{S,F}}' \circ \mathbf{d_i^{S,F}}' \circ \mathbf{S_{IC}} \circ (\mathbf{i}' - \mathbf{b_{i,IC}^{P,F}}') \cdot \mathbf{B_{IC}} \circ \mathbf{b_{j,IC}^{P}}' \\ + \mathbf{pu_{i,IC}^{P,F}}' \circ \mathbf{b_{i,IC}^{P,F}}' \cdot \mathbf{B_{IC}} \circ \mathbf{b_{j,IC}^{P}}' \\ + \mathbf{pz_i^{S,F}}' \circ \mathbf{d_i^{S,F}}' \circ \mathbf{S_{CP}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}} \circ (\mathbf{i}' - \mathbf{hx_{h,CP}^{R}}') \cdot \mathbf{H} \cdot \mathbf{J} \circ (\mathbf{i}' - \mathbf{j_g^{P,F}}') \cdot \mathbf{G} \\ + \mathbf{pu_{i,CP}^{R,F}}' \circ \mathbf{b_{i,CP}^{R,F}}' \cdot \mathbf{B_{CP}} \circ (\mathbf{i}' - \mathbf{hx_{h,CP}^{R}}') \cdot \mathbf{H} \cdot \mathbf{J} \circ (\mathbf{i}' - \mathbf{j_g^{P,F}}') \cdot \mathbf{G} \\ + \mathbf{phx_h^{R}}' \circ \mathbf{hx_{h,CP}^{R}}' \cdot \mathbf{H} \cdot \mathbf{J} \circ (\mathbf{i}' - \mathbf{j_g^{P,F}}') \cdot \mathbf{G} \\ + \mathbf{ph_g^{P,F}}' \circ \mathbf{j_g^{P,F}}' \cdot \mathbf{G} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I} - \mathbf{DT} \circ (\mathbf{i}' - \mathbf{d_i^{S,F}}') \circ \big(\mathbf{S_{IC}} (\mathbf{i}' - \mathbf{b_{i,IC}^{P,F}}') \cdot \mathbf{B_{IC}} \circ \mathbf{b_{j,IC}^{P}}' - \mathbf{S_{CP}} \circ (\mathbf{i}' - \mathbf{b_{i,CP}^{R,F}}') \cdot \mathbf{B_{CP}} \\ \circ (\mathbf{i}' - \mathbf{hx_{h,CP}^{R}}') \\ \cdot \mathbf{H} \cdot \mathbf{J} \cdot (\mathbf{i}' - \mathbf{j_g^{P,F}}') \circ \mathbf{G} \circ (\mathbf{i}' - \mathbf{b_{j,IC}^{P}}') \big) \end{bmatrix}^{-1} \text{................}(\mathbf{A}.24)$$

$$= \mathrm{Pr}\,\mathbf{ice}\,\mathrm{Pr}\,\mathbf{eModel}\left[\mathbf{pz_i^{S,F}}', \mathbf{pu_{i,IC}^{P,F}}', \mathbf{pz_i^{S,F}}', \mathbf{pu_{i,CP}^{R,F}}', \mathbf{phx_h^{R}}', \mathbf{ph_g^{P,F}}' \right]$$

$$\left[\mathbf{I} - \mathbf{A_{j\ to\ j}^{Interreg}} \right]^{-1} \text{...}(\mathbf{A}.24\mathbf{a})$$

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1. In this paper only the cost circle of the general model is presented. But a similar structure is applicable for the dual model, the real circle in the general model.

2. In another paper the equations in the real circle are shown and the mathematical solution to this model system is presented together with the simultaneous solution of the general local or urban model. This is the reason for denoting the analytical solution partial.