

## Review of Philip H. Wicksteed's *Essay on the Co-ordination of the Laws of Distribution*

by

Alfred W. Flux

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[p.308]

Recent economic studies have displayed a marked tendency towards a complete recasting of the "Theory of Distribution". With these words, Mr. Wicksteed's begins the prefatory note to his "Essay on the Co-ordination of the Laws of Distribution." Those who have followed the course of discussions on the Theory of Rent will remember that several writers endeavoured, and with some success, a few years since, to demonstrate that the Ricardian doctrines with regard to payment for the use of land might be equally applied to the determination of the remuneration of any other of the factors of production. In a certain sense, General Walker may be said to have suggested these discussions, since the scant references to the principle by Mill, Senior and others whom Professor Marshall mentions (*Principles*, p.453) did not produce much fruit till General Walker elaborated on the doctrine in reference to the profits of *entrepreneurs*. Some three years ago, a number of the *Quarterly Journal of Economics* consisted almost entirely of expositions of these views.<sup>1</sup> It was but natural to expect that some writers would set forth the view that, so far from the remuneration of every industrial [p.309] agent being of the nature of rent, even the payment for the hire of land may be shown to lack those characteristics which connect it with the form in which the doctrine is commonly expounded. The Duke of Argyll has argued that the hire of land has no special claim to be treated as a different problem from the hire of other means of production. Mr. Wicksteed in his essays takes up this view very strongly, and backs his opinions by a display of mathematical symbols through which his readers will find it necessary to thread their way with care and no small patience. Two chief objects are aimed at by the author. The first is to prove that, if P be the total produce from L units of land, with appropriate labour and capital, and P be expressed as a function of L and variables representing this labour and capital, the rent per unit of area of the land is  $\frac{\partial P}{\partial L}$ .

This means that the payment for the use of land is measured by its marginal productivity, exactly as in the case of other industrial agents.

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<sup>1</sup> [p.308] The view of rent here referred to is that expressed in (i.) below.

The next point is to show that, when each of the factors of production has received remuneration at the rate measured by its marginal productivity, the whole product is exactly distributed. Of the importance of a demonstration of this last theorem there can be little doubt, and the attention which he directs to the problem, even more than the form of the solution, must be counted as the chief merit of Mr. Wicksteed's latest work.

Before stating his conclusions and the applications of his doctrines, we will venture to give a brief *résumé* of the analysis by means of which the above propositions are proved (pp.24-31). We will adhere tolerably closely to the symbolism of the author though denuding his proof of most of its elaboration and reducing it to the smallest possible compass.

We shall suppose the conditions of industry to remain steady for a sufficient time to enable the entire round of productive operations to be completed, i.e., take no account of change in progress; time will not be one of the variables with which we are concerned. In fact we shall deal with the produce per unit of time or in any given time.

We shall also, for the sake of brevity, assume for the time that all other factors of production than land can be reduced to a single common measure, and that L units of land have C units of capital-and-labour employed on them, producing a product P.

$$\text{Let } C : L = x : 1 = 1 : z.$$

Then taking

$$F(x) = \text{Product per unit of land, when } x \text{ units of capital-and-labour are profitably employed on each unit of land}$$

we have

$$F'(x) = \text{rate or return to capital-and-labour, i.e. the marginal return per unit.}$$

According to the ordinary theory of rent, the amount of rent per unit of area of land is

$$F(x) - xF'(x) \quad \dots\dots\dots(i.)$$

**[p.310]**

Now if  $x$  units of capital-and-labour are provided for each unit of land,  $z$  units of land will be provided for each unit of capital-and-labour, since  $z = \frac{1}{x}$ .

And  $z \cdot F(x)$  is the product per unit of capital-and-labour, when  $z$  units of land are profitably employed with it.

$$\text{Calling this product } \Phi(z), \text{ we have } \Phi(z) = z \cdot F(x) \quad \dots\dots\dots(ii.)$$

By differentiation with respect to  $z$ , remembering that  $\frac{dx}{dz} = -\frac{1}{z^2} = -x^2$ , since  $x \cdot z = 1$ ,

$$\Phi'(z) = F(x) - x \cdot F'(x) \quad \dots\dots\dots(\text{iii.})$$

Therefore the ordinary formula for the rent of land gives us  $\Phi'(z)$  as an expression for it, per unit of area.

Now  $\Phi(z)$  is the product per unit of capital-and-labour,  
 $\therefore C \cdot \Phi(z)$  is the product of  $C$  units of capital-and-labour,  
*i.e.*, of these  $C$  units and the  $L$  units of land associated with them; or  $P = C \cdot \Phi(z)$ .

Now  $z = L/C$  when  $C$  is regarded as fixed and  $L$  variable,  $\therefore \partial L = C \cdot \partial z$

$$\therefore \frac{\partial P}{\partial L} = \frac{C \cdot \partial \Phi}{C \cdot \partial z} = \frac{\partial \Phi}{\partial z} = \Phi'(z),$$

or, the rent per unit of area is  $\frac{\partial P}{\partial L}$  which is the first point to be proved.

In like manner we have  $\frac{\partial P}{\partial C} = F'(x)$  and therefore equation (iii) gives

$$\frac{\partial P}{\partial L} = F(x) - \frac{C}{L} \cdot \frac{\partial P}{\partial C}$$

or

$$L \cdot \frac{\partial P}{\partial L} + C \cdot \frac{\partial P}{\partial C} = L \cdot F(x) = P$$

since  $F(x)$  is the product per unit of  $L$ , which proves the second point.

It is to be noted that the assumption has been made that with  $z$  times each of the factors of production,  $z$  times the product is obtained, and an exactly similar assumption is made elsewhere in the above proof as well as in establishing the relation (ii.).

This assumption is similarly made, though not quite so obviously, by Mr. Wicksteed, who discusses its justice. The conclusion at which he arrives is that it is not correct to make this assumption in the case of monopolies generally. He argues that it is true in many, even in most other cases, at any rate when we consider a business which deals with but a small proportion of the total trade in its product. If we consider, not the exchange value of the product, but its amount, or even [p.311] the total satisfaction accruing to the community, consumers and producers together, from the processes of industry, he also considers the assumption justifiable as applied to the whole of an industry, and not merely to that small part of it which concerns any single manufacturer.

If we make this assumption, *i.e.*, confine our view to the cases where it is justifiable, we obtain a very much more simple proof of the second proposition, which also involves the proof of the first. In this method of proof, we have the additional advantage of avoiding the difficulty created by imagining that a large number of heterogeneous factors of production can be dealt with by taking some single representative of all.

If, A, B, C, ..., represent the amounts of these different industrial agents which are required to cooperate in the production of a product P, we write

$$P = \Psi(A, B, C, \dots).$$

The form of  $\Psi$  is such that, if the amount of all the agents of production be increased in the same proportion, the product is increased in that same ratio, or we have

$$kP = \Psi(kA, kB, kC, \dots)$$

whatever  $k$  may be, provided it be independent of A, B, C, &c. We may, for the sake of closer adherence to actual conditions, limit  $k$  to values near unity. Now the form of  $\Psi$  which permits this is clearly a homogeneous function of the first degree in A, B, C, ..., and Euler's equation gives us at once the result

$$P = A \cdot \frac{\partial P}{\partial A} + B \cdot \frac{\partial P}{\partial B} + C \cdot \frac{\partial P}{\partial C} + \dots \quad (\text{iv.})$$

here seems no need for delaying to prove a relation so well known as this, as Mr. Wicksteed does.

If A denotes the agent *Land*, the ordinary Ricardian theory gives as the rent of the A units of area which are required in conjunction with B, C, etc. units of other agents to produce the product P,

$$P - B \cdot \frac{\partial P}{\partial B} - C \cdot \frac{\partial P}{\partial C} - \dots$$

which by the above is  $A \cdot \frac{\partial P}{\partial A}$ , or the rent of land is  $\frac{\partial P}{\partial A}$  per unit area.

These are the chief parts of Mr. Wicksteed's new contribution to economic theory. The relation of the equation (iv.) to different actual cases, and the representation of such factors of production as "travelling" and "risk-taking" among the variables are dwelt upon. The characteristic of economic equilibrium is pointed out, *viz.*, that  $\frac{\partial P}{\partial K}$  from the point of view of the owner of the industrial agent denoted by K is equal to the value of  $\frac{\partial P}{\partial K}$  where P denotes the satisfaction afforded, not [p.312] to the said individual, but to the community. A want of equality between these would lead to an alteration in the allocation of capital, etc., to different productive purposes.

Having stated formally the theses to establish which is the main purpose of the book, we are in a position to consider their importance and the criticisms of the current views which are based on them.

We may first observe that, for the due establishment of our equations it is quite unnecessary to examine the nature of the functions dealt with either when  $x = 0$ , or  $z = 0$  (*e.g.* in equation (iii.)). We are not immediately concerned with the question of what amount of product is obtained by capital and labour without land or by land without capital and labour. Whether either of these vanishes is a matter of indifference to the theory, and we may content ourselves with the reflection that they are either zero or finite and positive, certainly neither infinite nor negative.

Next we may observe that the fact expressed in equation (iii.) is one which, so far from weakening the position of those who regard rent as a surplus, by showing that the use of land is paid for in accordance with the marginal utility of the service rendered by it, shows what is indeed Mr. Wicksteed's object to prove, that the two views are essentially contained, each in the other. What is brought into relief is the underlying assumption involved in the expression of rent in the form (i.), viz. that we suppose varying quantities of capital and labour, duly proportioned, to be employed upon land, till the rate of increase of the product is reduced to the bare amount necessary to secure the co-operation of the other necessary factors, and that, as soon as there is no excess above this amount, we have reached the position which marks the due apportionment of land to the other agents in the state of industry with which we are concerned.

To show that the payment for land may be expressed in the form of the marginal productivity of land does not destroy the value of the conception of it as a surplus. The essential feature which distinguishes the treatment of land and some other agents, and makes it useful to regard their earnings as a surplus, is that, even if the circumstances of society be such as to render their marginal usefulness very great indeed, either a considerable period must elapse before a changed supply of these agents modifies the excessive demands made on the existing supply, or else the supply is practically incapable of any change. It is surely natural, in such a case, to take the point of view which regards the more readily changeable agents as changeable, and the more definitely fixed ones as fixed absolutely. If no action directly affecting land itself can alter its marginal utility, it is obviously more useful to fix attention on the changeable factors of production, to see if, through action on them, the marginal utility of the scarce agent may not be reduced, and thus the proportion of the product which is normally assigned to the owners of the scarce agent be reduced at the same time.

Students of General Walker's writings must be familiar with the [p.313] two aspects of the doctrine of rent as applied to managing ability which that author takes, treating it now as a differential gain, now as a payment in harmony with other wages, *i.e.* in accordance with marginal productivity. In fact, it seems as if the establishing of this double view and the necessary consistency of this view was one of the chief results reached by General Walker. And again in Professor Marshall's writings, the necessary identity of the two views of this payment seems never to be doubted. This does not lead the Cambridge Professor to stigmatize the question, "Does rent enter into cost of production?" as foolish. Neither is the question foolish, nor does any consideration adduced by Mr. Wicksteed give more than plausibility to the view that he expresses. He concentrated his attention on the

expression of rent as  $\frac{\partial P}{\partial L}$  and, once that is established, appears to forget the fact that it was established by means of the formula (i.) which is not one whit less true because it admits of expression in a different form.

Some very interesting diagrams accompany the book, which illustrate the analysis, though they are hardly as necessary, nor is the form of the extremities of such vital importance as their author appears to think. Figure (5) is perhaps the most interesting, as it combines in one diagram two curves, the common ordinate of which is divided by the curves so as to show the apportionment of the product between land and other agents, thus representing these two portions in a similar form, and not, as is usually done, representing one by a line while leaving the other to be deduced from a curvilinear area.

In printing a book containing so many algebraic symbols, and especially one containing so many which might easily be confused, a number of misprints are inevitable. They are especially dangerous in such a case, and it is to be hoped that both those which have already been corrected in the copy which reached us, and those which have escaped notice, will be corrected before the book is finally issued to the public, if it be not already too late to effect this.

A.W. Flux