Efficiency in the Provision of Public and Private Capital in 17 OECD countries.

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Abstract

In this paper we have used a sample of 17 OECD countries to test efficiency conditions in the provision of public and private capital. Our estimations with quarterly data for the 1970-1995 period indicate that both capitals have followed criteria of efficient provision. This result is common to the seventeen OECD countries in our sample, no matter that we estimate the Euler equations with a general or a partial equilibrium perspective. We find that relative investment good prices are an important factor (usually not considered in the analysis) to evaluate investment decisions by public or private agents. In contrast to other results in the literature, we do not find excessive returns to public investment, neither important and/or persistent differences between the rates of return of both types of capital. Nevertheless, it is still possible to appreciate certain periods of lack or excess of public investment.

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I. Introduction.

An influential stream of the recent economic literature has emphasized that actual levels of public capital, and more specifically infrastructures, may be sub-optimal compared to those that prevailed two or three decades ago. If public capital exercises a positive impact on private production, the concern in many economies has been that productivity or cost performance of private firms may have suffered from a shortage of public infrastructures. One of the reasons that made some economists come to such conclusion was the fact that in many industrialized countries, although not all of them, there coincided a slowdown in productivity growth with an important decline in the ratio of public capital to output.

Although in its own it is controversial to quantify the strength of the effect of public capital on productive performance of countries or regions, there seems to be some consensus that public capital is a productive input into private production. Probably, initial estimates of production functions by Aschauer (1989a,b) or Munnell (1990) where far too optimistic about the contribution of public capital to private production. Later studies, as Holtz-Eakin (1994) and Garcia-Mila, McGuire and Porter (1996), questioned both the magnitude and, inclusively, the basic underlying question whether public capital is a productive input or not. Despite the fact that there is lot of research in the last decade which either supports or questions the early conclusions reached by Aschauer and other authors (see the excellent surveys by Gramlich (1994) or de la Fuente (1996)), it seems pointless to conclude that public capital is not productive.

In this paper we are going to put the emphasis in issues like adequacy and efficiency of public capital provision. Taking as starting point that public capital is a productive input into private production, our aim is to provide some evidence as to whether governments of advanced industrialized countries have followed efficiency criteria in the provision of public capital. Our theoretical and empirical device is taken from Otto and Voss (1998), although we will discuss some dimensions of their theoretical framework and propose an alternative strategy to arrive to similar efficiency conditions, though with different theoretical and empirical implications.

The most usual approach to analyze the effects of public capital on private productive performance has been to estimate production functions (usually in Cobb Douglas form) or to make growth accounting exercises. None of these approaches is entirely satisfactory to conclude about the adequacy of public capital provision, because

they are not able to identify on solid statistical or econometric grounds if a change in public capital endowment is sub-optimal or not[1]. To answer to these questions one needs to resort to models of intertemporal optimization, which allow defining a criterion of optimal or efficient behavior as a benchmark. One possibility is to use a dual approach based on the optimization of aggregate cost or benefit functions, incorporating theoretical restrictions derived from the envelope conditions (some examples are Berndt and Hanson (1992) or Morrison and Schwartz (1996)). A different possibility is the one pursued by Otto and Voss (1998) that is a natural extension of the consumption-asset pricing literature on the empirical analysis of Euler equations. In this setting investment returns are modeled using production variables, in a way closely related to Cochrans' (1991) production-based asset pricing model.

Our approach in this paper is also based in the estimation of Euler equations. It has some advantages that are worth mentioning. First, as pointed out by Otto and Voss (1998) these conditions derive from models of intertemporal behavior where the maintained hypothesis is that capital provision (public or private) has been efficient. So, we will test if this hypothesis is rejected by the data. Second, as we will see later, the parameters to estimate are the same output elasticities of private and public capital that can be recovered from production functions. Third, this approach uses information about the dynamics of the relative prices of private and public investment goods, which is not processed while estimating production functions. Finally, although usually public capital is considered as an external input into private production, the efficiency conditions given by the Euler equations treat the government decision of public investment as endogenous.

In this paper we extend the work by Otto and Voss (1998) in two directions. First, we estimate their model extending the sample to seventeen OECD countries, to see if their results and conclusions for Australia are also valid for other advanced countries. Second, we reinterpret their theoretical model that we will define rather imprecisely as a general equilibrium model, giving it a partial equilibrium dimension that allows us to test Euler equations that have different, although not necessarily mutually exclusive, theoretical and empirical implications.

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[1] In addition, as Finn (1993), Balmaseda (1996) or Gramlich (1996) have pointed out the very high rates of return that are implicit in many production function estimates have to be seen with caution. These supernormal rates of return of public capital arrive to 100% and are many times 70% higher than private rates of return. ¿Why governments are so myopic as to leave unexploited these investment possibilities?

After identifying the conditions for efficient resource allocation for an economy with both a private and a public capital sector and testing whether or not these conditions are satisfied, we find that both capitals have followed criteria of efficient provision. This result is common to the seventeen OECD countries in our sample, no matter that we estimate the Euler equations with the general or the partial equilibrium perspective. The results for Australia by Otto and Voss (1997) are confirmed in a broader sample, giving full support to these authors claim that relative investment good prices are an important factor (usually not considered in the analysis) to evaluate investment decisions by public or private agents. Consequently, we find that there have not been excessive rates of return to public investment, nor systematic or significant differences between both private and public rates of return.

An added dimension of our analysis is that all along we estimate quite reasonable levels of both rates of return and output elasticities[2]. Nevertheless, in this case there are differences between the general and the partial equilibrium approaches. In the first case, we estimate investment returns for both types of capital around 9% per annum for the average OECD country, and output elasticities around 23% and 8% for private and public capital, respectively. In the second case, these figures are 5%, 19% and 5% for both rates of return, the private and the public output elasticities, respectively. Although we are not able to discriminate on statistical grounds between both approaches, we present some casual evidence that reconciles these figures, assuming different values for the ex-ante real interest rate relevant for investment decisions. In any case, both approaches differ substantially in their implications on the dynamics of the implied optimal stock of capital. The general equilibrium approach does not allow recognizing certain episodes of excess or shortage of public or private capital, while the partial equilibrium approach allows identifying those periods.

In this paper we have used a sample of 17 OECD countries with data for the 1970-1995 period. The structure of the paper is as follows. In section 2 we present the theoretical model and discuss the testable efficiency conditions under different assumptions. Section 3 presents the data, that are taken or elaborated from the OECD *Business Sector Database* (BSDB), and makes some econometric considerations. The last two sections present, respectively, the empirical results and the conclusions.

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[2] As emphasized by Poterba (1997) the relative rates of return of both types of capital are crucial to evaluate the adequacy of public investment efforts.

II. Efficiency conditions in the provision of capital.

Consider an economy that produces output with two inputs, capital and labor. There are two capital sectors in the economy, private and public, according to the ownership of the input. Both capital sectors obey similar, although not necessarily identical, processes of accumulation and depreciation. The framework we are using is the one by Arrow and Kurz (1970) followed also by Otto and Voss (1998), where public ownership of capital is warranted given that services of public capital may suffer, for instance, from large fixed costs or from congestion. Additionally, public investment will be considered as endogenous as private investment, given that both will be determined according to similar optimization processes.

In this simplified world with two capital sectors we can define a different rate of return to each capital good. To do this let p'_{it} be the price of the investment good i ($i=1,2$) at time t and p_t the price of output. One additional unit of good i invested at time t costs p'_{it} and provides in $t+1$ an increase in output given by $p_{t+1} f_{it+1}$ (where $f_{it+1} = \partial Y_{t+1}/\partial K_{it+1}$ is the marginal productivity of capital i) and an increase in the capital stock of $(1-d_i) p'_{it+1}$. It follows that in this case the nominal rate of return, r^N_{it+1}, is given by:

$$r^N_{it+1} = \frac{p_{t+1} f_{it+1} + (1-d_i) p'_{it+1} - p'_{it}}{p'_{it}} \qquad [1]$$

Now, if we define p_{it} as the relative price of the investment good and output, and express the rate of growth of output prices as $\hat{p}$, we can rewrite previous expression as:

$$r^N_{it+1} = \frac{1+\hat{p}_t}{p_{it}} \left[f_{it+1} + (1-d_i) p_{it+1} \right] - 1 \qquad [2]$$

To transform the nominal rate of return of investment into a real measure, given that we work in discrete time, we use the standard expression

$$1 + r^R_{it+1} = \frac{1+r^N_{it+1}}{1+\hat{p}_t} \qquad [3]$$

so that the real net return, $R_{it+1} = 1 + r^R_{it+1}$, for investment in sector i is[3]:

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[3] We define this real rate of return as <u>net</u>, to distinguish it from the gross real rate of return, which is the name given in this literature to the marginal productivity of capital.

$$R_{it+1} = \frac{1}{p_{it}} \left[f_{it+1} + (1 - d_i) p_{it+1} \right] \qquad [4]$$

To compute the (net) real return of private and public capital defined in expression [4] we need three ingredients. First, data about the relative prices of investment goods and output; second the depreciation rates of both types of capital and third, a procedure to compute the marginal productivities of private and public capital. As we will see later, we have available series of relative prices and depreciation rates. To estimate the marginal productivities of capital we need to define a concrete functional form for the aggregate production function. The most standard function used in the literature is the Cobb-Douglas production function with constant returns to scale in all factors:

$$Y = K_1^{r_1} K_2^{r_2} N^{1-r_1-r_2} \qquad [5]$$

where Y is total aggregated output, K_1 the stock of private capital, K_2 the stock of public capital, and N is total labor in both sectors. Contrary to other studies that analyze the effect of public capital, our measure of output will be total aggregated output instead of production of the private sector[4]. Given that public capital contributes to both private and public output, it seems more adequate to recover the rates of return considering total output instead of private output alone. Additionally, we do not include a measure of technical efficiency, because we are interested in the relative sectoral returns and these can be expressed independently of the usual specifications of technical progress, as a Hicks neutral technology.

The rates of return derived in previous paragraphs present some advantages respect to other measures in evaluating the optimality of capital provision. First of all, our rates of return take explicitly into account the variation in the relative prices between the (private or public) investment good and output. Second, the rates of return of both types of capital can be directly compared, so that it can be investigated if there are what Finn (1993) and Gramlich (1994) defined as supernormal returns to public investment.

To make the rates of return empirically operative it is necessary to establish a benchmark and to make explicit an estimation strategy to compute the necessary

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[4] Although our empirical results are obtained using total GDP as the output measure, we did not find remarkable differences when we employed other output measures that exclude public sector output.

parameters to calculate the returns. The idea is that if public capital provision has fallen today to suboptimal levels, the reason has to be that the rates of return are today higher than some years ago, given a fixed opportunity cost of investment. Alternatively, if the opportunity cost of investment has grown, today's capital stock will be suboptimal if the rate of return has grown at a lower rate. In addition, for a given opportunity cost of investment, public investment may be more or less profitable than private investment, if the rate of return of both types of capital diverge. In this sense, for an economy with two capital sectors we can define the conditions for efficient resource allocation in each sector as[5]:

$$E_t\left(\frac{1}{1+i_{t+1}^r}R_{it+1}\right)=1, \qquad i=1,2. \qquad [6]$$

where E_t is the expectations operator conditional on information available at time t, and $\frac{1}{1+i_{t+1}^r}$ is the inverse of the gross real interest rate relevant to investment decisions.

There are different strategies to estimate the efficiency conditions defined by equation [6]. The crucial question is to determine the relevant gross real interest rate to investment decisions. One imaginative possibility is the one proposed by Otto and Voss (1998), that take the common stochastic discount factor, derived from an optimization model of intertemporal consumption, as a good approximation to the relevant real interest rate. The key idea is that $E_t \, m_{t+1} = E_t \frac{1}{1+i_{t+1}^r}$, where[6]:

$$E_t \, m_{t+1} = b \, E_t \frac{u'(c_{t+1})}{u'(c_t)} \qquad [7]$$

The stochastic discount factor (m_{t+1}) is the intertemporal marginal rate of substitution for a representative consumer with an intertemporally separable utility function, c_t is per capita real consumption[7] and b is the subjective discount factor,

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[5] As can be seen the conditions for efficient resource allocation given in expression [6] are the Euler conditions that can be derived from a standard optimization problem (in two periods) for a representative firm. As we argued before, notice that public capital is a given input for the firm that is provided by an optimizing public sector, that is taking private capital as given.

[6] See Hall (1978).

[7] We express consumption in per capita terms given that we have a representative agent optimal consumption model with aggregated data. This is a standard practice in the empirical literature of aggregate consumption (see Deaton (1992)).

supposed constant. Additionally, we assume the usual constant relative risk aversion utility function (CRRA):

$$u(c) = \frac{c^{s}}{s}$$ [8]

with $s \neq 1$. Notice that $1/(1-s)$ is the intertemporal elasticity of substitution and $1-s$ the coefficient of relative risk aversion.

Then, the conditions for efficient resource allocation can be expressed as:

$$E_t\left(m_{t+1} R_{it+1}\right) = 1, \qquad i = 1,2.$$ [9]

or, using [4] and [7], as:

$$E_t\left[b\left(\frac{c_{t+1}}{c_t}\right)^{s-1}\left(\frac{1}{p_{it}}\right)[\ r_i\frac{Y_{t+1}}{K_{it+1}} + (1-d_i)\, p_{it+1}\ ]\right] = 1, \ i = 1,2$$ [10]

where we have made use of the fact that in a Cobb-Douglas production function the marginal productivity of capital is $f_{it+1} = r_i\dfrac{Y_{t+1}}{K_{it+1}}$. The efficiency conditions summarized in expression [10] can be estimated directly[8]. As a result we get estimates of the output elasticities of both types of capital, r_i, without estimating directly the aggregate production function.

The most straightforward alternative strategy to the estimation of Euler equations is to use an available ex-ante real interest rate, as the relevant measure for investment decisions[9]. In this case combining equations [4] and [6] the efficiency conditions become:

$$E_t\left[\frac{1}{1+i_{t+1}^{r}}\left(\frac{1}{p_{it}}\right)[\ r_i\frac{Y_{t+1}}{K_{it+1}} + (1-d_i)\, p_{it+1}\ ]\right] = 1, \ i = 1,2$$ [11]

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[8] In the next section we will explicit both the estimation technique and some additional assumptions necessary to estimate [10].

[9] We have computed real ex–ante interest rates for all the countries in our sample. Details about the procedure we have used are presented in Appendix 1.

Both equation [10] and [11] can be defined following Cochrane (1991) as the efficiency conditions emerging from a production-based asset pricing model[10]. Testing such models requires taking the stochastic discount factor as given (or the ex-ante real interest rate) and analyzing if the return to investment behaves according to it.

There are some interesting theoretical questions that are worth mentioning, when interpreting the results of estimating equation [10] or [11]. Both equations are closely related to models of investment with adjustment costs, where instead of modeling explicitly an adjustment cost in terms of investment expenditure and capital stocks, we assume that the cost arises in the transformation of the output good into the capital good. Then, the adjustment cost is assumed to show up in the dynamics of the relative prices, given that these are measuring the cost of the i investment good in terms of aggregate output. Nevertheless, it is a matter of fact that we could also model explicitly the adjustment costs[11], because these costs can arise independently that there is a unit relative price or not. For purposes of comparison with Otto and Voss (1998) we prefer to keep on with this interpretation, because we have the added dimension of estimating the Euler equation in the form of [11].

To clarify further the interpretation of equations [10] and [11], notice that the strategy of modeling the relevant real interest rate with the stochastic discount factor may be considered as a general equilibrium approach. The reason is that we are taking the intertemporal marginal rate of substitution for a representative consumer, which is a parameter for the whole economy, as a benchmark to determine if capital has been provided efficiently. In this sense, given that the canonical consumption model establishes that consumption smoothing obeys to changes in the real interest rate, any discrepancy between the stochastic discount factor and the real interest rate may be interpreted in two ways. Either there are adjustment costs, or whatever other element not taken into account (e.g. tax rates) when modeling the investment rate of return, or there are liquidity restrictions or precautionary saving in the sphere of consumption[12].

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[10] Notice the analogy with the standard intertemporal consumption model. In such case the Euler equation is:

$$E_t\left[b\left(\frac{c_{t+1}}{c_t} \right)^{s-1} (1 + i_{t+1}^r) \right] = 1$$

The test of the permanent income hypothesis checks if consumption behaves according to the evolution of the real interest rate.

[11] This is the approach followed by Cochrane (1991).

[12] In other words, equation [10] allows testing for efficiency, but does not permit discerning the sources of the gap between the stochastic discount factor and the real interest rate.

On the other hand, taking an available or constructed real interest rate as a benchmark to check efficiency in capital provision may be considered as a partial equilibrium approach[13]. Verifying the efficiency conditions in this framework implies that both sectors have decided investment optimally, but does not fully clarify neither if there are adjustment costs nor if we are dealing with a framework of pure capital demand, with no investment function.

III. Data and econometric issues.

Most of the variables used in this work have been taken from the OECD *Business Sector Database* (BSDB)[14]. The statistical information in the BSDB is available on a quarterly basis, with a high degree of homogeneity and it is expressed at constant prices, although the base year is not the same for all countries. From the sample of countries at our disposal, we have ruled out some of them because of lack of some of the relevant variables, different from capital, in part or all of the period of analysis. So, from the BSDB we have taken the series of GDP, gross fixed capital formation of the private and public sectors, final private consumption, long run nominal interest rates[15] and output and investment deflators. On the other hand, the quarterly series of population have been obtained interpolating the corresponding annual series taken from the OECD *National Accounts.*

The construction of the private and public capital stocks has been specially difficult and hard working. In the BSDB there exist official data (or estimations) for some countries, but only for the gross private productive capital stock. In any case, there does not exist such kind of data for the net private productive capital stock and for the public capital stock. Given that the relevant capital stock is the net stock[16], it has been necessary to use other statistical sources when it was possible, or to construct the stocks

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[13] We are aware that we are using rather imprecisely the concepts of general and partial equilibrium. To this respect, see the excellent discussion by Cochrane (1991), page 212.

[14] For a general description of this database see Keese and Salou (1991).

[15] As stated previously, the appropriate measure for the interest rate is some ex-ante real long run rate. See Appendix 1 for details in the construction of this variable.

[16] See Ward (1976) for an exact definition of the gross and net capital stocks. In the calculation of the gross capital stock a unit of productive equipment disappears only at the end of its assumed life period. In contrast, when the net capital stock is constructed the equipment is assumed to depreciate all along its life period.

from the public and private investment flows from the BSDB[17]. To sum up, we have used official capital stock data from the OECD publication *Flows and Stocks of Fixed Capital*, for the countries where these series were available. Concretely, we have taken from this source the net productive private stock for Australia, Belgium, Canada, Germany, Denmark, Finland, France, Sweden and the United States[18]. With regard to public capital, we have used the official OECD series for Australia, Belgium, Canada, Denmark, Finland, France and Sweden. For the remaining countries the series of public and private capital have been constructed using the perpetual inventory method[19].

With regard to the estimation of the efficiency conditions in the provision of capital (expressions [10] and [11]), it is necessary assuming an expectations formation mechanism. If expectations are rational, the 2×1 error vector, u_{t+1}, has the usual asymptotic properties and it should not be correlated with any variable known in period t. Because of this, the model needs to be estimated by instrumental variables techniques, to guarantee the consistency of the estimators. In practice, estimation has been done by the Generalized Method of Moments (see Hansen (1982)). Then, if z_t is a q×1 vector of instrumental variables, there are $2q$ orthogonality conditions that should be satisfied:

$$E_t \left[u_{t+1} \otimes z_t \right] = 0$$

Because of these orthogonality conditions it is necessary to be very careful in choosing the instruments. The real interest rate is an obvious candidate to be used as instrument, given its presumably high relationship with investment returns. Our first set of instruments contains a constant and different lags of the real interest rate. Besides, we use a second set of instruments, adding to the previous one, the price weighted output-capital ratios and the gross growth rates of relative investment prices. Although the first set of instruments is applied homogeneously to all countries in the sample, the second set varies across countries.

With respect to the dating of the instruments, we do not introduce in the regression analysis any lag before the second for any of the instruments. In this way we

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[17] Private and public investments flows in the BSDB are perfectly coherent with the same series in other OECD official statistics, as the *National Accounts* or the *Economic Outlook*.

[18] In the cases where the lapse of time of these series was not complete, we have extrapolated them backwards or forwards using the perpetual inventory method (see Appendix 2).

[19] Both the official capital stocks and the stocks constructed from the investment flows have been obtained on an annual basis. In a second step, we generated the quarterly series by interpolation. Obviously, we checked that the base year of the official stocks coincided with the base year of the BSDB flows. For more details about the procedure, see Appendix 2.

avoid the well-known problems due to temporal aggregation of consumption data, that produces a first order MA structure in the error term (see Ermini (1988) and Christiano, Eichenbaum and Marshall (1991)) and the possibility of serial correlation. In any case, we check the results with the Sargan test of the overidentifying restrictions imposed by each model.

Finally, as is obvious looking at equations [10] and [11] the model is non-linear. Given the biases generated by the usual linearization procedures, we have chosen to discard the use of any of them, estimating the model non-linearly and jointly for both capital sectors. On the other hand, we have only estimated the production function parameters, setting the consumption parameters at concrete values. The values given to the subjective discount rate, b, and to the coefficient of relative risk aversion, σ, are the same considered by Otto and Voss (1998). b is set at 0.99, implying an annual discount rate of 4% given the quarterly periodicity of the data, and s is set at $!1.0$, implying a coefficient of relative risk aversion of 2.0 (and an elasticity of intertemporal substitution of 0.5). These numbers are coherent with the values considered in the empirical consumption literature[20]. Finally, when we use official OECD data, we set the depreciation rates according to the consumption of fixed capital implicit in these series, and at annual rates of 4.8% and 6.8% for public and private capital, respectively, if the series have been constructed (see Appendix 2).

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[20] The estimation of the elasticity of intertemporal substitution is not the aim of this work, especially when there is an important debate in the literature about the subject. To this respect see Hall (1988) and Attanasio and Weber (1989).

IV. Empirical results.

The estimation results for the efficiency conditions when the Euler equations incorporate the stochastic discount factor (equation [10]) are presented in Table 1. As can be appreciated both sets of instruments provide quite similar estimates of the output elasticities of private and public capital and of the rates of return. Concretely, with the second set of instruments the private capital output elasticity is 0.234 on average, while the public capital elasticity is 0.078. The average (net) rates of return are approximately 8.7% and 9.3% per annum for both private and public investment, respectively. Notably, there are not striking differences in these numbers across countries, and also there are not important discrepancies between private and public rates of return in any country of the sample. These estimates seem quite reasonable for at least three reasons. First, the capital output elasticities imply an output elasticity of labor around 0.69 for the average country, which matches pretty well with labor income shares in many advanced countries. Second, these figures imply much lower estimates of public capital output elasticities than the ones obtained in initial studies (notably Aschauer (1989) or Munnell (1990)) and, also, with much less variance across countries than is appreciable in the literature. Finally, we do not perceive any supernormal profitability to either public or private capital, nor important differences across countries.

The orthogonality conditions imposed by the Euler equations are safely accepted by the data in all countries. The results from Sargan's test of the overidentifying restrictions point unambiguously to the acceptance of the efficiency conditions for capital provision in the seventeen countries. Moreover, we can qualify the results of these tests as outstanding given that only in the cases of Australia, Canada, Germany, Spain, France, Greece and Japan marginal significance levels are less than 50% for the estimations with the first set of instrument. On the other hand, significance levels are in almost every country higher than 80% for the estimations with the second set of instruments. Notice additionally, that the inclusion of investment prices in the second set of instruments generates better results of the test in all countries[21] (with the exception of Belgium). These results give additional support to the basic argument in

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[21] For the average OECD economy marginal significance levels increase more than 30% when investment prices are included in the set of instruments.

Table 1: Estimation Results. General Equilibrium Approach.

Countries	First set of instruments					Second set of instruments				
				Rate of Return					Rate of Return	
	r_1	r_2	Sargan Test	PRI	PUB	r_1	r_2	Sargan Test	PRI	PUB
Australia	0.247 (0.013)	0.064 (0.003)	16.743 (0.402)	1.075	1.080	0.270 (0.006)	0.068 (0.002)	36.992 (0.963)	1.089	1.088
Austria	0.276 (0.018)	0.081 (0.006)	15.200 (0.510)	1.095	1.094	0.287 (0.009)	0.091 (0.003)	35.583 (0.975)	1.102	1.112
Belgium	0.227 (0.006)	0.056 (0.002)	8.603 (0.929)	1.072	1.082	0.246 (0.003)	0.064 (0.001)	28.559 (0.807)	1.088	1.101
Canada	0.262 (0.011)	0.044 (0.002)	19.096 (0.264)	1.059	1.077	0.253 (0.010)	0.043 (0.002)	32.301 (0.452)	1.052	1.073
Switzerland	0.190 (0.011)	0.089 (0.006)	14.550 (0.558)	1.065	1.058	0.181 (0.006)	0.112 (0.003)	42.825 (0.902)	1.057	1.089
Germany	0.247 (0.010)	0.067 (0.004)	18.672 (0.286)	1.095	1.093	0.221 (0.004)	0.059 (0.001)	43.529 (0.888)	1.075	1.074
Denmark	0.199 (0.017)	0.067 (0.008)	12.745 (0.691)	1.062	1.063	0.217 (0.013)	0.075 (0.006)	24.336 (0.832)	1.076	1.075
Spain	0.233 (0.012)	0.047 (0.003)	22.158 (0.138)	1.089	1.103	0.247 (0.005)	0.042 (0.001)	42.228 (0.877)	1.100	1.087
Finland	0.271 (0.020)	0.088 (0.006)	11.934 (0.749)	1.094	1.101	0.254 (0.009)	0.096 (0.003)	37.963 (0.927)	1.084	1.114
France	0.204 (0.006)	0.058 (0.002)	24.869 (0.072)	1.082	1.087	0.208 (0.005)	0.059 (0.002)	34.480 (0.717)	1.086	1.088
United Kingdom	0.246 (0.013)	0.140 (0.010)	12.224 (0.728)	1.101	1.102	0.228 (0.006)	0.156 (0.005)	41.185 (0.931)	1.088	1.121
Greece	0.188 (0.009)	0.061 (0.003)	17.114 (0.378)	1.110	1.096	0.198 (0.003)	0.061 (0.001)	37.757 (0.931)	1.120	1.095
Ireland	0.258 (0.019)	0.085 (0.007)	14.231 (0.582)	1.117	1.118	0.270 (0.008)	0.097 (0.003)	35.540 (0.961)	1.125	1.141
Italy	0.221 (0.009)	0.064 (0.003)	14.437 (0.566)	1.095	1.100	0.213 (0.005)	0.061 (0.001)	39.497 (0.899)	1.089	1.094
Japan	0.290 (0.008)	0.120 (0.004)	15.730 (0.472)	1.106	1.104	0.291 (0.005)	0.128 (0.003)	31.936 (0.662)	1.107	1.114
Sweden	0.212 (0.018)	0.059 (0.006)	12.921 (0.679)	1.080	1.066	0.201 (0.013)	0.052 (0.004)	32.185 (0.863)	1.072	1.052
USA	0.193 (0.006)	0.062 (0.003)	14.073 (0.593)	1.068	1.068	0.188 (0.004)	0.059 (0.002)	37.287 (0.960)	1.064	1.062
Average	0.233	0.074		1.086	1.088	0.234	0.078		1.087	1.093

Notes to Table 1:

The period considered for all the countries goes from the first quarter of 1970 to:
- the fourth quarter of 1994 in Austria.
- the fourth quarter of 1995 in Switzerland, Spain, Finland, Greece and Ireland.
- the fourth quarter of 1996 for Belgium, Denmark, Italy and Sweden.
- the second quarter of 1997 for Australia, Canada, Germany, France, United Kingdom, Japan and United States.

First set of instruments (identical in all countries): constant and lags 4th to 11th for r_t.

Second set of instruments (different across countries): constant and lags 4th to 11th of r_t; lags 4th to 12th of p_{it-1}/p_{it-2} (except Belgium, Canada, Denmark and Japan: lags 4th to 7th of p_{it-1}/p_{it-2}, France and Sweden: lags 4th to 9th of p_{it-1}/p_{it-2}); $(Y_{t-1}/K_{1t-1})/p_{1t-2}$ in the case of Australia, Spain and United States, $(Y_{t-1}/K_{2t-1})/p_{2t-2}$ in the case of Austria and Sweden and both in the case of Belgium, Switzerland, Germany, United Kingdom and Japan.

Standard errors and significance levels for the Sargan test in brackets.

Otto and Voss (1998), that relative investment good prices play a key role (usually not considered) when evaluating the efficiency of capital provision[22].

To have a better understanding of what is going on with our estimations, in Figure 1 we have depicted the time profile of private and public rates of return for the average OECD country[23]. A few things are worth mentioning. First, both rates of return seem to be stationary around a mean of approximately 9% per annum[24]. Second, both rates of return move close to each other over the sample period, being quite difficult to appreciate prolonged periods where both returns diverge substantially. This pattern of returns is quite reasonable if we take into account that the Euler equations we are estimating implicitly use the average over the sample period of the stochastic factor as the benchmark for the optimal levels of public and private capital. If the overidentifying restrictions are not rejected by data in any country, as is the case, then the average returns to both types of investment have to equal the average discount factor in every country in the sample. This fact is confirmed looking at Figure 2, where we present the differences between the rates of return and the stochastic discount factor. Both series are stationary around a zero mean and there is no way to find periods of excess or shortage of either private or public capital.

Summing up, the general picture that emerges from our results in Table 1 is that public and private capital provision has followed efficiency criteria, and that both the estimated output elasticities and the investment returns seem quite reasonable. Nevertheless, there are certain aspects of this story that deserve a more careful scrutiny. In Table 2 we present the averages of the stochastic discount factor and of our estimated real gross interest rate for the seventeen countries in our sample. For the average country there is a difference of four percentage points between both magnitudes, given that the ex-ante real interest is 5% on average. Having these numbers in mind, it seems surprising that along a period of 25 years there has been such a big discrepancy between the rates of return of private and public investment (8.7% and 9.3%, respectively) and

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[22] We also estimated the same model without prices (the benchmark Cobb-Douglas model in Otto and Voss terminology) finding uniformly worst results, that confirm the important role of relative prices.

[23] We prefer to present results for the average economy (the simple average across countries), because it resembles the pattern across countries, with the only caution that the average investment returns for a given country can be higher or lower than for the average OECD economy, as is evident in Table1. Additionally, we are going to present always the results for the second instrument set.

[24] Recall that the rates of return depend on the evolution of the output capital ratios that display some trend behavior in many countries, the depreciation rates that are constant and the investment output relative prices. Obviously, the stationarity of the return series is due to the inclusion of the price series in the model.

the real ex-ante interest rate (5%). Notice that, as we discussed in the theoretical section, the discrepancy between the real interest rate and the stochastic discount factor may be capturing not modeled aspects of the investment process (adjustment costs, tax wedge, etc.) or aspects of the consumption decision (liquidity restrictions, precautionary saving, etc.). In other words, we can not be sure that the relevant gross interest rate for investment decisions is the stochastic discount factor and not some other measure, like ours, of the ex-ante real interest rate[25].

Figure 1: Average Rates of Return

General Equilibrium Model
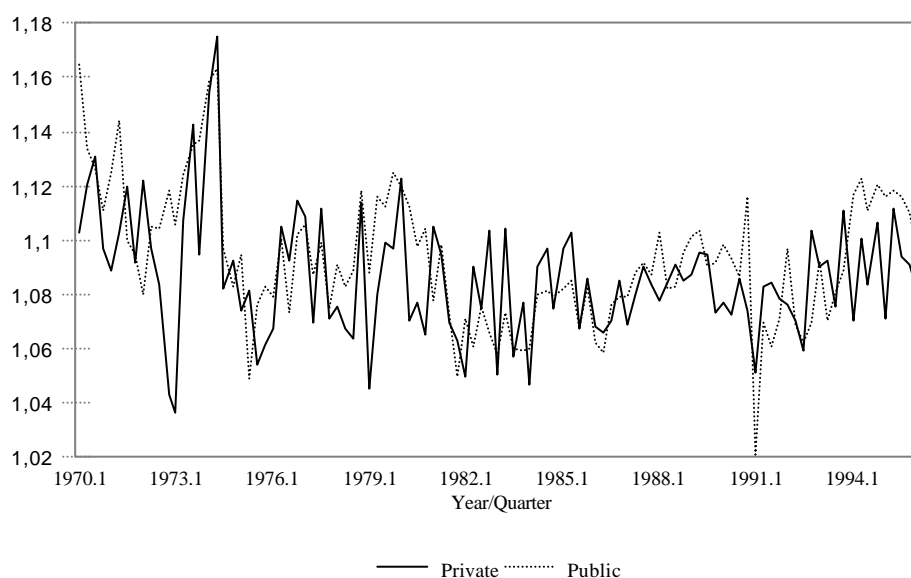
The most straightforward way to shed some light on these questions is estimating the Euler equations, taking the real interest rate as the benchmark for efficient capital provision. The estimation results of equation [11] are presented in Table 3. As can be seen, this model fits also very well with the data in almost every country in the sample[26]. The overidentifying restrictions are not rejected by the data, giving full support to the efficiency hypothesis. As was the case with the model estimated previously, the inclusion of the relative prices in the second set of instruments improves

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[25] It could be thought that the four-percentage points difference between the interest rate and the stochastic discount factor is due to the poor quality of our estimated real interest rates. We do not think that this is a reasonable possibility, given that it seems rather implausible that the average of the ex-ante real interest rates in the OECD has been significantly higher than 5% in the 1970-95 period. Additionally, as discussed in Appendix 1, our method corrects explicitly the negative values that are obtained in the first half of the 70's when using other techniques.

[26] The case of Greece can be considered as atypical. The average real interest rate in this country is 1.9%. Although the Sargan test does not reject the overidentifying restrictions, the values of the output elasticities and of the rates of return seem rather implausible.

greatly marginal significance levels of Sargan tests in all countries, pointing again to the relevance of investment prices in determining if capital provision has been optimal.

Figure 2: R. of Return - Disc. Factor
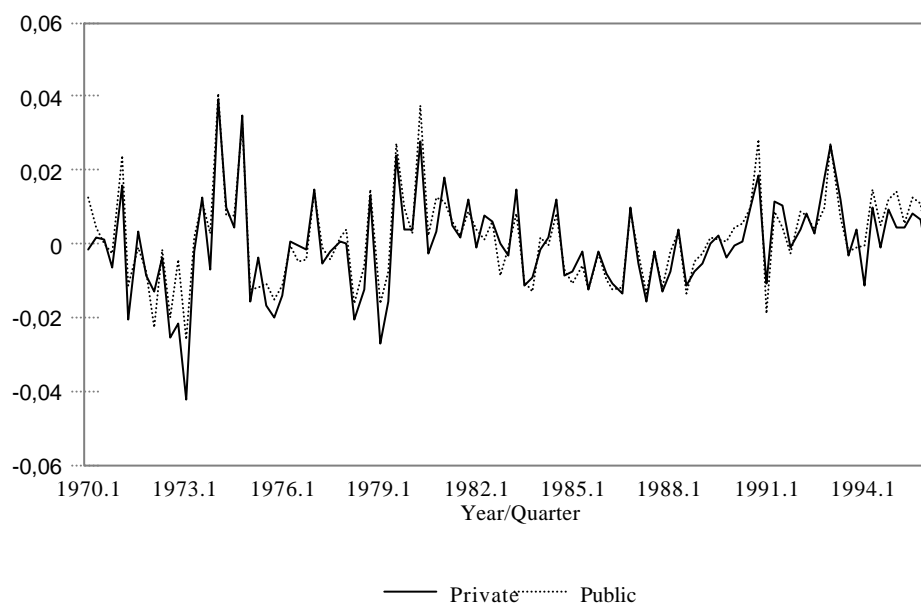General Equilibrium Model

Table 2: Stochastic Discount Factor and Real Interest Rates.
Average values. 1970-1995.

Country	Gross Real Interest Rate	Stochastic Discount Factor
Australia	1.060	1.083
Austria	1.045	1.110
Belgium	1.058	1.089
Canada	1.068	1.086
Switzerland	1.015	1.068
Germany	1.047	1.090
Denmark	1.074	1.086
Spain	1.058	1.088
Finland	1.058	1.091
France	1.056	1.087
U. Kingdom	1.048	1.096
Greece	1.019	1.096
Ireland	1.064	1.108
Italy	1.045	1.100
Japan	1.050	1.107
Sweden	1.047	1.067
USA	1.042	1.083
Average	1.050	1.090

Table 3: Estimation Results. Partial Equilibrium Approach.

Country	First set of instruments					Second set of instruments				
				Rate of Return					Rate of Return	
	r_1	r_2	Sargan Test	PRI	PUB	r_1	r_2	Sargan Test	PRI	PUB
Australia	0.226 (0.010)	0.059 (0.002)	23.875 (0.092)	1.062	1.071	0.204 (0.004)	0.048 (0.001)	42.270 (0.876)	1.049	1.049
Austria	0.208 (0.007)	0.055 (0.003)	14.795 (0.540)	1.050	1.049	0.203 (0.003)	0.055 (0.001)	35.642 (0.975)	1.047	1.049
Belgium	0.197 (0.004)	0.047 (0.002)	21.164 (0.172)	1.049	1.060	0.206 (0.002)	0.049 (0.001)	35.507 (0.492)	1.056	1.065
Canada	0.260 (0.009)	0.042 (0.002)	25.660 (0.059)	1.058	1.070	0.248 (0.008)	0.040 (0.001)	34.166 (0.364)	1.049	1.064
Switzerland	0.133 (0.008)	0.048 (0.004)	17.418 (0.359)	1.019	1.005	0.156 (0.003)	0.056 (0.002)	43.869 (0.880)	1.037	1.015
Germany	0.178 (0.003)	0.044 (0.002)	19.547 (0.241)	1.043	1.044	0.170 (0.001)	0.043 (0.001)	43.733 (0.883)	1.038	1.040
Denmark	0.201 (0.007)	0.067 (0.003)	19.079 (0.265)	1.064	1.062	0.203 (0.006)	0.068 (0.003)	31.461 (0.494)	1.065	1.064
Spain	0.213 (0.007)	0.044 (0.002)	21.603 (0.156)	1.074	1.096	0.177 (0.003)	0.025 (0.001)	42.871 (0.862)	1.047	1.036
Finland	0.231 (0.011)	0.068 (0.005)	16.417 (0.424)	1.070	1.068	0.239 (0.006)	0.076 (0.002)	36.789 (0.945)	1.075	1.081
France	0.189 (0.004)	0.049 (0.001)	24.407 (0.081)	1.060	1.065	0.185 (0.003)	0.051 (0.001)	34.329 (0.723)	1.065	1.070
United Kingdom	0.192 (0.010)	0.107 (0.007)	17.276 (0.368)	1.063	1.063	0.190 (0.006)	0.118 (0.003)	39.376 (0.955)	1.061	1.075
Greece	0.080 (0.007)	0.014 (0.004)	24.067 (0.088)	1.008	0.994	0.083 (0.004)	0.012 (0.002)	35.978 (0.956)	1.010	0.991
Ireland	0.211 (0.011)	0.065 (0.004)	12.171 (0.732)	1.081	1.081	0.207 (0.005)	0.061 (0.002)	35.917 (0.956)	1.077	1.074
Italy	0.184 (0.007)	0.051 (0.002)	22.960 (0.115)	1.067	1.069	0.152 (0.004)	0.030 (0.001)	46.261 (0.698)	1.043	1.022
Japan	0.207 (0.004)	0.077 (0.003)	23.299 (0.106)	1.051	1.048	0.219 (0.003)	0.088 (0.002)	39.059 (0.334)	1.058	1.063
Sweden	0.197 (0.012)	0.053 (0.003)	17.129 (0.377)	1.068	1.053	0.210 (0.010)	0.059 (0.003)	34.969 (0.771)	1.079	1.067
USA	0.157 (0.012)	0.045 (0.002)	17.445 (0.357)	1.032	1.034	0.160 (0.001)	0.047 (0.001)	41.739 (0.888)	1.035	1.039
Average	0.192	0.055		1.054	1.055	0.189	0.054		1.052	1.051

Notes to Table 3:

The period considered for all the countries goes from the first quarter of 1970 to:
- the fourth quarter of 1994 in Austria.
- the fourth quarter of 1995 in Switzerland, Spain, Finland, Greece and Ireland.
- the fourth quarter of 1996 for Belgium, Denmark, Italy and Sweden.
- the second quarter of 1997 for Australia, Canada, Germany, France, United Kingdom, Japan and United States.

First set of instruments (identical in all countries): constant and lags 4th to 11th for r_t.

Second set of instruments (different across countries): constant and lags 4th to 11th of r_t; lags 4th to 12th of p_{it-1}/p_{it-2} (except Belgium, Canada, Denmark and Japan: lags 4th to 7th of p_{it-1}/p_{it-2}, France and Sweden: lags 4th to 9th of p_{it-1}/p_{it-2}); $(Y_{t-1}/K_{1t-1})/p_{1t-2}$ in the case of Australia, Spain and United States, $(Y_{t-1}/K_{2t-1})/p_{2t-2}$ in the case of Austria and Sweden and both in the case of Belgium, Switzerland, Germany, United Kingdom and Japan.

Standard errors and significance levels for the Sargan test in brackets.

The important differences between both models show up in the estimated elasticities and in the rates of return. Consequently with the new benchmark for efficiency, the average rates of return across countries are reduced to values around 5.2% and 5.1% for private and public investment, respectively. In Figure 3 we have depicted both rates of return and the gross real interest rate for the average economy. From the observation of this figure it is apparent that both rates of return move close to each other and that the means of the three series are similar. Again, in this model the maintained hypothesis of efficient provision, implicit in the Euler equations, imposes restrictions on the estimated parameters. As a consequence, the average values for the output elasticities settle down to values of 0.189 and 0.054 for private and public capital, respectively. These are also reasonable values, both looking at the average OECD values and at the cross-country differences, which are not quite important.

Figure 3: R. of Return and Int. Rates
Partial Equilibrium Model
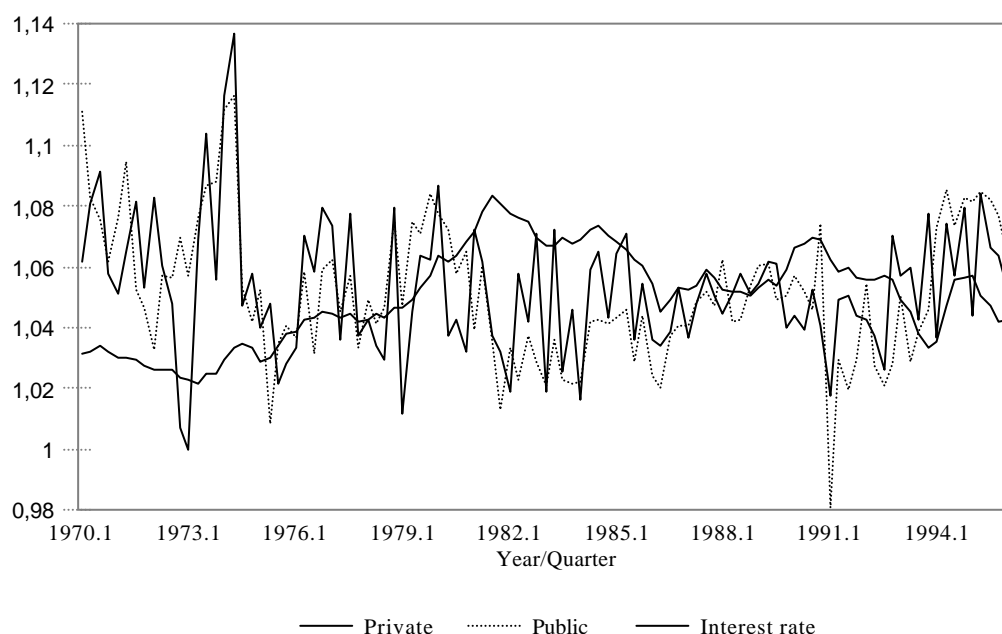
If we look separately at our two estimated models, both seem to perform rather well. Conditions for efficient resource allocation can be safely accepted in both cases. The reason is that in those countries where public investment has been reduced along the sample period, suggesting to many researchers the possibility of under-investment in the public sector, the price of public investment goods has risen relative to the price of aggregate output. All along, in these countries the prices of private investment goods

have become less expensive relative to output prices. Thus, it is not surprising that in those countries where the cost of public investment goods relative to private goods has risen the result is a decline in the public to private capital intensity. Although this is not necessarily the pattern in all of the countries in our sample, in all of them the relative variation of investment prices seems to account for the movements of the capital intensities.

At this point, the important question to address is which is the preferred model. Unfortunately it is difficult to answer to this question for al least three reasons. First, both models perform rather well across countries and display, compared with the amazing disparity in the literature, rather reasonable, although different, values of both the rates of return and the capital output elasticities. Second, the theoretical implications of both models are not mutually exclusive. The general equilibrium approach can be interpreted as a model of investment with adjustment costs (implicit in the transformation of output into investment expenditure using relative prices), but it can account also for other not modeled factors in the spheres of consumption and/or investment. Our partial equilibrium approach can also be interpreted as a model of investment with adjustment costs, but it can be also derived from a model of pure capital demand with no investment function. Third, the non-linear models we are estimating are non-nested and we can not relay on a specific econometric test to discriminate between them.

Nevertheless, it is still possible to make some additional important considerations about our results. The first one has to do with the behavior of the capital stocks along the sample period. In this sense, it is quite noticeable that with the partial equilibrium approach we can recognize periods of time where capital stocks have been above or below their optimal levels, although these discrepancies compensate if we consider the whole period. To see this, in Figure 4 we present the differences between the rates of return and the stochastic discount factor for the average economy. Both series have a zero mean, but, in contrast to the general equilibrium approach (see Figure 2), these series are not stationary around the mean. Moreover, the series display a clear cyclical behavior that matches well with our knowledge about the business cycle in OECD countries. If there is a story of adjustment costs going on, it seems more reliable the pattern observed in Figure 4, than the picture emerging from Figure 2. If firms face adjustment costs, it is reasonable that it takes some time to correct a given gap between desired and observed capital. In other words, the (net) return can be higher or lower than

the opportunity cost (the real interest rate) of investing, for the time that takes firms to adjust effective to optimal capital levels. This is captured in the partial equilibrium model, because the benchmark for efficient resource allocation is given by the gross real interest rate and this is a variable that moves along the business cycle of the economies. On the other hand, the stochastic discount factor is a stationary series that does not display cyclical movements.

Figure 4: R. of Return - Interest Rate
Partial Equilibrium Model
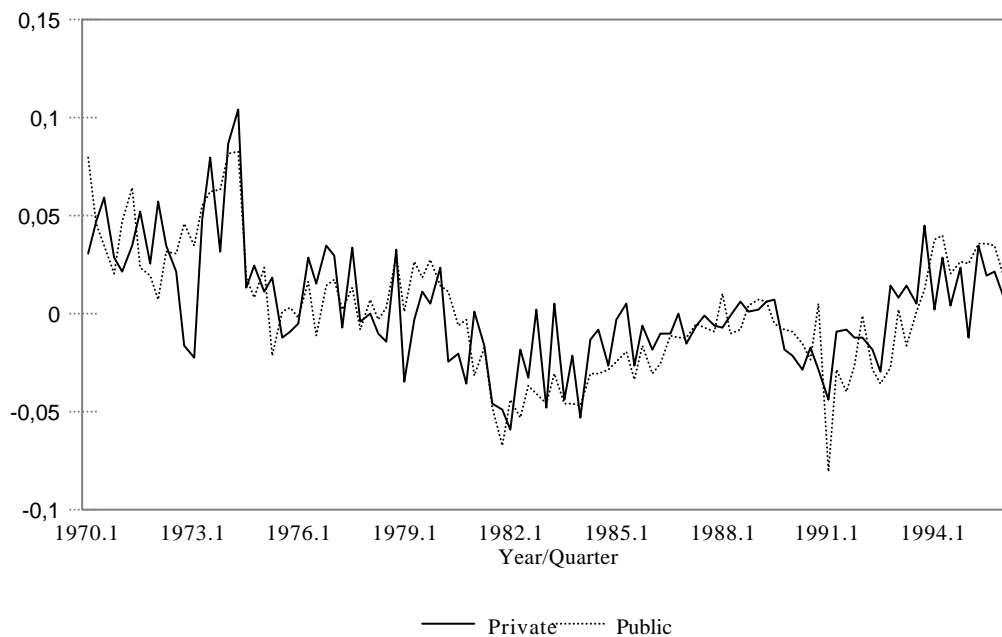
Finally, it also remains the open question of which are the most reliable values for the rates of return and the capital output elasticities. Apparently both approaches to the Euler equations seem irreconcilable at this respect, but it is not difficult to amend the partial equilibrium equation to obtain similar figures. To do this, assume that we introduce a lump-sum tax on corporate incomes in our analysis. This would be equivalent to paying each year a higher price for the investment good or, alternatively getting a lower price for the output good that the firms sells in the market. We can introduce this in the Euler equation [11] in a very simple way, because this lump-sum tax would be equivalent to having a higher interest rate. In Table 4 we perform this exercise, adding each time a percentage point to the real interest. As can be seen, augments of both the elasticities and the rates of return accompany each uniform increase in the interest rates. Concretely, the last row of this table shows that when we

add four percentage points to our interest rates, so that the average value is now around 9%, we get almost identical figures for the elasticities and the rates of return than the ones we obtain estimating the general equilibrium equation. Although this is only a crude approximation, it highlights possible explanations for the differences in the estimated values between both models. In any case, both of our models support the hypothesis of efficient capital provision.

Table 4. Rates of return and elasticities for different interest rates. Average values. 1970-1995.

	First set of instruments		Rates of Return		Second set of instruments		Rates of Return	
	r_1	r_2	PRI	PUB	r_1	r_2	PRI	PUB
r_t^r	0.192	0.055	1.054	1.055	0.189	0.054	1.052	1.051
$r_t^r + 1$	0.205	0.060	1.064	1.065	0.202	0.060	1.062	1.061
$r_t^r + 2$	0.218	0.066	1.074	1.075	0.216	0.067	1.073	1.074
$r_t^r + 3$	0.232	0.071	1.084	1.085	0.229	0.072	1.083	1.084
$r_t^r + 4$	0.245	0.076	1.094	1.095	0.243	0.077	1.093	1.094

V. Conclusions.

In this work, we analyze the provision of both public and private capital, through the estimation of efficiency conditions derived from processes of optimization of firms. Our empirical exercises cover a sample of seventeen OECD countries along the 1970-95 period. To perform the empirical analysis, it was necessary to elaborate series of public capital for all the countries in our sample and also of private capital for some of them. We applied the perpetual inventory method to our initial capital stocks estimates, using official OECD investment flows from the Business Sector Database.

Our approach in this paper to the issue of efficiency is similar to the one pursued by Otto and Voss (1998) that is a natural extension of the consumption-asset pricing literature on the empirical analysis of Euler equations. In this setting investment returns are modeled using production variables, in a way closely related to Cochrans' (1991) production-based asset pricing model. The efficiency conditions derive from models of intertemporal behavior where the maintained hypothesis is that capital provision (public or private) has been efficient. This approach uses information about the dynamics of the relative prices of private and public investment goods that is usually denied in the literature. We have reinterpreted the theoretical model by Otto and Voss (1998), which we have defined as a general equilibrium approach, giving it a partial equilibrium dimension allowing us to test Euler equations that have different, although not necessarily mutually exclusive, theoretical and empirical implications.

As a general conclusion, we can point out that independently of the approach we consider, we do not find either evidence of excessive returns to public capital, nor systematic or significant differences between both private and public rates of return. This finding contrasts with usual results obtained in the literature, fundamentally those based on the estimation of production functions and/or on growth accounting exercises. In this sense, for the most part of the countries, when we consider that the investment decisions are taken in a general equilibrium approach, the estimated public and private rates of return present values around 9%, while the estimated output elasticities are 23% and 8% for private and public capital, respectively. Alternatively, considering that the investment decisions are taken in a partial equilibrium approach, the estimated public and private rates of return present values around 5%, and the output elasticities around 19% and 5% in the cases of private and public capital, respectively. These numbers seem quite reasonable, given that there is also not much variation across countries.

More remarkably, from the empirical analysis with both approaches we conclude that the provision of capital (public or private) in the OECD has been done according to efficiency criteria. This is an important result that survives to the notable differences between the values of the rates of return and the output elasticities obtained with both approaches. Given that differences in these parameters are important to assess properly the impact of investment decisions, trying to discriminate between the two models is a very important issue. To address this question, considering that the theoretical implications of both approaches are not mutually exclusive and that it is not possible to discriminate econometrically between them, we have examined some indirect pieces of empirical evidence.

First, from the comparison of the temporal behavior of the rates of return obtained with both approaches, we can conclude that the partial equilibrium approach seems to be the most suitable, if we think that relative prices are capturing the existence of adjustment costs. This approach allows identifying periods of excess or shortage of capital that have a clear cyclical evolution, contrary to the general equilibrium approach.

Second, we have checked that the differences across models between the rates of return and the output elasticities are a direct consequence of the gap between the stochastic discount factor and the real interest rate, which are the two benchmarks for efficient resource allocation we are considering. Nevertheless, we performed a simple exercise of augmenting each time one-percentage point the interest rates, finding that it is possible to reconcile the values obtained in both models for the elasticities and the rates of return. Although this is a very unsophisticated exercise, it highlights possible explanations for the differences in the estimated values between both models. The tentative conclusion is that the partial equilibrium approach performs better describing the time pattern of investments and that we have two reasonable bounds for the elasticities and the rates of return.

In any case, the most important conclusion of this work is that, independently of the approach considered in the analysis, the provision of both capitals displays high levels of efficiency in a big sample of OECD countries, confirming previous results for Australia obtained by Otto and Voss (1998). At the same time, the analysis of capital provision through Euler equations derived from optimization processes, seems to give more reasonable estimates for the rates of return and the output elasticities than other traditional strategies.

Appendix 1: Estimation of the ex-ante real interest rates.

We have followed the method proposed in Boscá, Doménech and Taguas (1999) to calculate the long run real interest rates. In summary, our estimated ex-ante real interest rate is given by the following expression:

$$\hat{r}_{it}^e = \overline{r}_i + \hat{u}_{it}$$

where $\hat{u}_{it}$ is the residual from a linear regression of the ex-post real interest rate in each country[27] regressed on a constant and the inflation rate, and $\overline{r}_i$ is the mean of the ex-post real interest rate for the 1985-95 period. We have chosen this period because it constitutes, approximately, an entire economic cycle and there are not atypical fluctuations of the inflation rate along it, as is the case in the 70's with the oil shocks.

This method produces values for the real interest rates very similar to the ex-post interest rates, except in the second half of the 70's, where, generally, the former are positive. This is a very important feature, given that the ex-post real interest rates have been negative in many countries along the 1973-79 period, because of the very high inflation rates due to the supply-side shocks. For this reason, it is not a good idea to use the ex-post interest rate as an approximation to the long run rate relevant to investment decisions. Additionally, other alternative techniques (filtering the inflation rate to get its trend component, adjusting an ARIMA model for it, etc..) do not either solve the problem of getting several years in the middle of the 70's where the estimated real interest rates are negative. We consider that long run inflation expectations in these years, where taking into account that the unusual inflation rates were going to return, sooner or later, to more reasonable levels (at least if we think that the relevant period for investment decisions is a long one). In this situation, long run inflation expectations could be notably lesser than observed inflation rates, producing a negative correlation between current inflation and the difference between ex-ante and ex-post interest rates. Summing up, our approach is based on the assumption of a lack of correlation between ex-ante real interest rates and inflation rates.

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[27] The nominal interest rate used in this work is the long run interest rate available in the BSDB; which coincides with the same series in the OECD *Economic Outlook*. It is a rate for long run assets relatively homogeneous across countries, with an approximated maturity of 10 years.

Appendix 2: Estimation of the capital stocks.

As is well known, the basic problem using the *perpetual inventory method* is the need to have an initial stock of capital as reliable as possible, given that as time goes by, and for given depreciation rates, the value of the accumulated stock is more and more realistic. To solve this problem, among the different solutions in the literature, we have chosen Doménech and Taguas (1999) proposal. In fact, under some assumptions, it is possible to estimate an initial stock of capital from the basic accumulation equation. So, this equation can be written as:

$$g_{Ki,t} = \frac{\dot{K}_i}{K_{it}} = \frac{i_{it}Y_t}{K_{it}} - d_i, \qquad i = 1,2$$

where d_i is the depreciation rate (assumed to be 0.048 for public capital and 0.068 for private capital), i_{it} is the investment rate, Y_t is the output level and K_{it} is the stock of capital to be estimated. To obtain an initial stock of capital from this equation, we only need to assume that the growth rate of investment is a good approximation for the growth rate of the capital stock. Nevertheless, and to improve this approximation, given that investment flows are highly volatile, we have filtered the investment series using the Hodrick and Prescott filter, with a smoothing parameter l=10. Because of the problems of this filter in the extremes of the sample, and given that the investment series start at the beginning of the 60's in most of the countries, we have used 1965 as the base year to calculate the initial stocks of capital[28]:

$$K_{i,65} = \frac{\bar{I}_{i,65}}{(d_i + g_{i,\bar{I}60-70})}$$

In this expression $\bar{I}_{i,65}$ is trend investment obtained with the Hodrick and Prescott filter and $g_{i,\bar{I}60-70}$ is the trend growth rate in the 60's. Once we have the base year capital stocks, we apply the perpetual inventory method to estimate public and private capital stocks for the remaining years according to:

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[28] In some cases, we have used as the base a different year around 1965, given that it produced better results, attending to the ratios of private to public capital, public to private investment, investment to output and both capitals to output.

$$K_{i,65+j} = \sum_{s=1}^{j} (1-\boldsymbol{d}_i)^{s-1} I_{i,65+j-s} + (1-\boldsymbol{d}_i)^j K_{i,65}$$

As Doménech and Taguas (1999) argue, this method to generate capital stocks has several advantages with respect to others, because it takes into account that the economy can be out of the steady state. In a different standard approximation (see Coe and Helpman (1996)) the initial stocks are obtained dividing observed investment in the base year (instead of trend investment) by the depreciation rate plus the mean growth rate of observed investment for the whole period (instead of the trend growth rate). Given that growth rates have settled down in many OECD economies, probably because they were out of their steady states at the beginning of the period, and that current investment could be affected by transitory shocks, it seems much appropriate to use the trend growth rate for a shorter period (1960-70, in our case), assigning it to the central year.

To check the validity of our estimated capital stocks, we have done a battery of proves. First, we have checked that in most countries the resulting depreciation series are very similar (almost identical, in many cases) to the series of fixed capital consumption in the National Accounts. Second, we have also checked that the temporal profiles of the ratios of both investments to total investment and of both stocks of capital to the total stock are coherent. Third, we have analyzed the time evolution of the ratio of public to private capital, and we have found that it is coherent with public to private investment ratios. In Table A.2 we present the means for the capital-output and investment-output ratios. The capital-output ratio average for all the countries is 1.88 (1.32 and 0.56 for public and private capital, respectively), although there are important differences across countries. The same happens with the investment rates, with average values of 14% and 4% for private and public investment, respectively.

In any case, although investment flows are in real terms (assuring their intertemporal comparability), comparisons across countries will require correcting the investment and output series with the different purchasing power parities. Nevertheless, in our case this is not necessary, because we have estimated the efficiency conditions separately for each country.

Table A.2: Ratios Capital-Output and Investment-Output.
Averages 1970-1995

Countries	K/Y	K1/Y	K2/Y	I1/Y	I2/Y
Australia	2,25	1,70	0,55	0,16	0,04
Austria	2,12	1,52	0,60	0,17	0,05
Belgium	1,70	1,26	0,44	0,11	0,03
Canada	1,63	1,29	0,34	0,12	0,03
Switzerland	2,06	1,27	0,79	0,13	0,07
Germany	1,88	1,39	0,49	0,14	0,04
Denmark	2,14	1,43	0,71	0,14	0,04
Spain	1,67	1,33	0,34	0,15	0,03
Finland	2,49	1,79	0,70	0,17	0,04
France	1,59	1,16	0,42	0,14	0,04
Un. Kingdom	2,36	1,47	0,88	0,14	0,06
Greece	1,70	1,12	0,58	0,13	0,05
Ireland	1,77	1,15	0,62	0,14	0,05
Italy	1,80	1,34	0,46	0,13	0,04
Japan	2,05	1,29	0,76	0,18	0,09
Sweden	1,50	1,09	0,41	0,11	0,03
USA	1,32	0,84	0,48	0,10	0,04
Average	1,88	1,32	0,56	0,14	0,04

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