

Why Should a Firm Choose to Limit the Size of Its Market Area?

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We analyze a variant of the standard Dixit-Stiglitz (AER, 1977) model, adding transport costs and assuming that, in addition to price, a firm can choose the size of the market area and the quality of the product. We also modify the standard cost function, making variable costs and fixed costs increasing in both "reach" and quality. We characterize the solution of the model and we find the conditions under which a firm decides to limit the market area. Finally, we show that the firm's behavior is constrained socially optimal.

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1 Introduction

Location theory places much emphasis on the role of customers, in choosing which firm to patronize. In the Hotelling (1929) model, two competing firms are located at the edge of a unitary segment, uniformly populated by identical consumers. The consumer's choice goes to the cheapest product (gross of travel costs) and determines the extent of a firm's spatial coverage (i.e., the market area of the firm). However, in many situations, the size of the market area is not the result of the consumer's choice, but, rather, it is a voluntary decision of the firm. In fact, a firm actively chooses its "reach" when it decides to open a new store, to enter a new market, to advertise its products, to develop a distributional channel, or to cream-skim costumers.

In the Hotelling model, a firm has no advantage in limiting its spatial coverage. However, in the situations mentioned above, the choice of the market area is paramount. Why? Simply because the costs a firm sustains in expanding its "reach" may exceed its benefits. In particular, when firm's costs or benefits vary according to the proximity of a customer (or of a market), the firm may be interested in serving customers who beget low costs and high benefits and in excluding the others (i.e., consumers causing high costs and low benefits). In addition, consumers' travel costs may contribute to the decision of limiting the coverage since they narrow the demand and, hence, the additional benefits of expanding the market area.

A great deal of economic literature has been devoted to the discussion of the impact of "reach" on costs. International trade literature emphasizes that firms encounter important costs in terms of being international. For example, Rabino (1980) found that logistic and transport costs, the complexity of paperwork, problems of managing export operations, and a need to modify products to meet foreign safety and health standards were important costs, especially to small manufacturing companies. Kedia and Chhokar (1986) reported that firms encounter high costs in the understanding of export procedures

and foreign business practices, and in acquiring information on foreign prospects. These findings are confirmed in more recent studies (see, for example: McAuley, 1993, and Katsikeas and Morgan, 1994, 1998).

More recently, McCallum (1995), in analyzing the trade among regions on the U.S.-Canada border, showed that the impact of borders on trade is quite large, even if the two countries are similar in terms of culture, language, and institutions. Other studies on border effects provide many interpretations. For example, Rauch (1996) suggested that there are "search" barriers, especially for differentiated products. Anderson and van Wincoop (2001) stated that border barriers generate trade costs, which "involve real resources, such as gathering information about foreign regulations, hiring lawyers familiar with foreign laws, learning foreign languages and adjusting product designs to make them consistent with foreign customs and regulations."

In informative advertising models, the size of the market area is the fraction of people a firm would like to inform, and costs of advertising are directly related to the firm's "reach." For example, Grossman and Shapiro (1984) considered the case where oligopolistic firms and monopolistically-competitive firms are able to choose the amount of purely informative advertising (i.e., the fraction of consumers they want to inform) and assume that the costs of advertising technology are increasing and convex in terms of "reach."¹ Under these assumptions, they find that it is optimal for firms to limit their coverage.

Bandyopadhyay (1999) analyzed distribution sectors in OECD countries and showed that high distribution costs can be a barrier to trade and limit the extent of spatial coverage.

In regulation literature, public utilities encounter different costs for serving different types of customers (for example, urban and rural customers) and, in the absence of universal service obligations, it is optimal for firms to cream-skin costumers (see, for example: Laffont and Tirole, 1990).

¹ I would like to thank an anonymous referee, who suggested this example.

We have already emphasized that in the Hotelling model, consumers are only interested in the delivered price and choose to patronize the cheapest facility. This implies that market areas do not overlap: the firm located at the origin of the Hotelling line serves the left-hand-side of the segment and the firm located at the end of the line covers the right-hand-side of the segment. In this paper we will show that this result changes, however, when there is product differentiation and consumers appreciate varieties. In fact, product differentiation ensures that firms retain some market power, even under pressure from competitors. A firm with the cheapest delivered price is no longer the exclusive supplier of the consumer, instead there is a multiplicity of suppliers for each consumer. In other words, market areas overlap.²

We set the model in a context of monopolistic competition using a variant of the Dixit-Stiglitz (1977) model with transport costs.³ We assume that, in addition to the price, firms can control the size of the market area and the quality of the product. Even if we focused on the size of the market area, the extension to quality is motivated by the fact that it is an important factor in explaining trade (see, for example: Brooks, 2003).

This paper is strongly related to the stream of new economic geography literature (for a survey, see, for example: Ottaviano and Puga, 1998) and, in particular, with the core-periphery model (Krugman, 1980 and Helpman and Krugman, 1985, Chapter 10). However, our contribution is different in several ways.

² In addition to this, since customers' ordering is firm specific, firms do not exactly compete for the same markets, but their market areas do partially overlap.

³ Hart (1985) describes monopolistic competition as being identified by four main characteristics, which are included in our model. 1) There are many firms producing differentiated commodities. 2) Each firm is negligible in the sense that it can ignore its impact upon, and, hence, reactions from, other firms. 3) Free entry leads to zero profit of operating firms, and, 4) each firm faces a downward-sloping demand curve and, hence, equilibrium price exceeds marginal cost.

First, we do not set the model in a two-country framework, but, rather, in a continuous framework. Thus, transport costs and a firms' costs related to "reach" vary with the relative location of consumers and firms. Second, in the core-periphery models, it is optimal for firms to serve the entire market. Transport costs only distort the consumption in favor of home production, while in our model with increasing costs of "reach," firms can optimally choose to serve only a portion of the market.

This paper is also related to location theory (Hotelling, 1929, Christaller, 1933, and Lösch, 1944). As previously mentioned for the Hotelling model, the law of market areas states that, the market area boundary is a line along on which the delivered prices⁴ of the goods supplied by the two firms are equal.⁵ The law of market areas ensures that each firm will provide its product to some customers exclusively.

There are a few exceptions to this result. In the random utility approach of Drezner and Drezner (1996), customers, located at a certain demand point, may patronize different goods and, thus, the same place can be the catchment area of different firms (as in this paper). Recently, Anderson and de Palma (2000) have proposed a theoretical framework, integrating both space and taste for variety. Taking into consideration the value of transport costs and the importance of taste for variety, they obtain a continuum of cases comprised between pure spatial competition and pure product differentiation. Their paper presents some analogies with our model, even if our focus is on the firm's behavior and their focus is on the consumer's behavior.

Economic geography and location theory both contain a common element, which determines the spatial dimension: transport costs. Mainly, we see that the existence of positive transport costs produces exclusive and limited market areas for the latter and non-exclusive but global provision of goods for the

⁴ Usually, the delivered price is the sum of the mill price plus travel costs, which can include the cost-of-time, as in Kohlberg (1983), and the disutility of congestion, as in Wong and Yang (1999).

⁵ See, for example: Fetter (1924), Hyson and Hyson (1950), and, more recently, Parr (1995).

former. This paper combines these two streams of research, obtaining overlapping but limited market areas.

The remainder of this paper is organized as follows. In Section 2, the model is presented. In Section 2.1, we characterize the solution of the model and we find the conditions under which a firm will limit its "reach," under perfect price discrimination. In Section 2.2, previous analysis is presented, under the assumption of uniform prices. In Section 3, we discuss whether or not the equilibrium of the model is "constrained" optimum from a social point of view. Finally, Section 4 concludes the paper.

2 The Model

The market is composed of consumers with identical preferences for all brands, and with unitary mass.

Let M be the number of firms that are on the market. Each firm can decide to sell their product on the entire market, or to focus on a portion of the market.

We assume that firms set their market areas independently and that they all compete with each other for the same customers. More precisely, we assume that each couple of firms, even when they limit their market areas, continue to compete with each other for a particular subgroup of consumers.⁶ In other words, a firm has a few competitors for each consumer, but many competitors in its market area.⁷ The

⁶ This formulation is similar to Hart (1985), although he focuses on consumers and not on firms. In fact, he assumes that consumers like only a finite subset of brands which are potentially available, and that this subset is generally different for different consumers. Moreover, he assumes that the valuations the consumer puts on the brands he likes are independent of which particular brands these are.

⁷ The independence of market areas is not necessary for deriving the model, but it helps us to set the problem into a framework of monopolistic competition, ensuring that the number of competitors is sufficiently large. In footnote 9, we provide an explanation of the independence of market areas in

following example clarifies this point.

Example 1: *There are 12 consumers labeled $\{a,b,c,d,e,f,g,h,i,j,k,l\}$ and 4 firms $\{1,2,3,4\}$. Firm i serves only consumers in its market area, A_i . For example:*

$$A_1 = \{a,b,c,d,e,f,g,h,i\}, \quad A_2 = \{d,e,f,g,h,i,j,k,l\},$$

$$A_3 = \{a,b,c,d,e,f,j,k,l\}, \quad A_4 = \{a,b,c,g,h,i,j,k,l\}.$$

The size of the market area is 3/4 of the entire market; yet, despite this fact, each firm competes with each other.

Consumer demand. Let M be the number of firms and let us call $\eta_i \in [0,1]$ the size of the market area that firm $i \in M$ wants to serve. Let $\psi_i \in [0,1]$ be the quality offered by firm i and $p_i > 0$ be the price paid by the consumer. Preferences are described in equation (1) by a constant elasticity of substitution utility function:

$$u_l = \left(\sum_{i=1}^M I_{\{l \in i\}} (x_i t_i \psi_i)^\rho \right)^{\frac{1}{\rho}} = \left(\sum_{j=1}^N (x_j t_j \psi_j)^\rho \right)^{\frac{1}{\rho}} \quad I_{\{l \in i\}} = \begin{cases} 1 & \text{if } l \in i \\ 0 & \text{if } l \notin i \end{cases} \quad (1)$$

where $\rho \in (0,1)$ and $1/(1-\rho)$ is the elasticity of substitution between any two products, N is the number of varieties that consumer l can buy, $j \in N$ refers to a firm whose market area consumer l belongs to, and x_j and ψ_j are, respectively, the quantity and the quality provided by firm j to consumer l . Transport costs will be assumed to be of the "iceberg" type; that is, only a fraction $t_j \leq 1$ of the goods shipped by firm j arrives to consumer l . Note that a consumer has a probability η_i of belonging to the market area of firm i , hence $N = \sum_{i=1}^M \eta_i$.⁸ The consumer budget constraint is:

terms of firms' costs related to the "reach."

⁸ For simplicity, the integer number problem is omitted.

$$\sum_{j=1}^N p_j x_j = I \quad (2)$$

where I is the expenditure for the goods. Following Dixit and Stiglitz (1977), the quantity purchased by consumer l from firm j , namely x_j , is (see the Appendix for the computation):

$$x_j(\psi_j, p_j, t_j) = (\psi_j t_j)^{\frac{\rho}{1-\rho}} \cdot y \cdot \left(\frac{q}{p_j} \right)^{\frac{1}{1-\rho}} \quad (3)$$

where q is a price index and y is a quantity index, as defined in the Appendix.

Consumer demand is the result of two factors: $\hat{x}_j(\psi_j, p_j)$, that is, the quantity demanded net of the effects of transport costs, and $\tau_j := t_j^{\frac{\rho}{1-\rho}} \leq 1$ which represents the negative impact of transport costs.

Thus, equation (3) can be written in the following way:

$$x_j(\psi_j, p_j, t_j) = (\psi_j)^{\frac{\rho}{1-\rho}} \cdot y \cdot \left(\frac{q}{p_j} \right)^{\frac{1}{1-\rho}} \cdot \tau_j = \hat{x}_j(\psi_j, p_j) \cdot \tau_j. \quad (4)$$

Costs. A firm faces two different types of costs: variable costs and fixed costs.⁹ Variable costs, denoted by $c(\eta_i, \psi_i)x_i$, are linear in output and are affected by the "reach" and quality. Fixed costs, denoted by $h(\eta_i, \psi_i)$, do not depend on output, but only upon the "reach" and quality.

We assume that h is strictly positive, twice continuously differentiable, and weakly increasing and

⁹ The independence of market areas relies upon the capability of firms to select consumers they like and on the firm's ranking for consumers. If the ranking of one firm differs from that of the others in such a way that the most favoured consumers for one are the worst for the others, then market areas are independent. Differences in ranking can be explained by the logistic system of the firm, such as the location of malls or the stores, by differences in managers' knowledge of markets, by the choice of different distribution channels, by different costs in customising products to be exported, etc.

convex in both arguments. In order to simplify the analysis, we assume that $c(\eta_i, \psi_i) = \psi^\theta c_1(\eta_i)$, with $\theta \in (0,1)$ and c_1 strictly positive, twice continuously differentiable, and weakly increasing and convex. The specification of h and c is sufficient to describe the many different cases we have depicted in the introduction.¹⁰

Note that the model accommodates two different situations: the first one, where transport costs are sustained by consumers (i.e., the costs a consumer encounters when he/she wants to buy a product, or the costs a consumer encounters, if the product is not precisely as he/she likes). And, the second situation, where transport costs are sustained by firms (i.e., the costs for delivering the goods or to customize the product. They are included in the unit costs, which depend on the “reach”).¹¹

Now, we introduce the concept of economies of size, with respect to the shape of $h(\eta_i, \psi_i)/\eta_i$. We call the previous expression, fixed costs averaged by the size of the market area, or, simply, average fixed costs.

Definition 1: Economies of size occur when, by expanding the size of the market area, the average fixed costs decrease. This can be written as:

$$\frac{h(\eta_i, \psi_i)}{\eta_i} - \frac{\partial h(\eta_i, \psi_i)}{\partial \eta_i} > 0. \quad (5)$$

Similarly, there are diseconomies of size when equation (5) holds with the opposite sign.

Figure 1 depicts two different occurrences: in panel (a), there are diseconomies of size in the range $[0, \eta^m)$ and economies of size in the range $(\eta^m, 1]$; in panel (b), there are only economies of size. Since

¹⁰ An exception is the cream-skimming model, as firms present a similar ranking for customers. Hence, market areas are not independent.

¹¹ The main reason to introduce two different types of transport costs is that when firms cannot price discriminate (see Section 2.2), there are different results in the two situations.

$h > 0$ and continuous, there are no situations where there are diseconomies of size for all η . We call η^m the minimum size of the market area, that is, $\eta^m = \arg \min_{\tilde{\eta}} h(\tilde{\eta}, \psi) / \tilde{\eta}$. Notice that η^m may depend on ψ .

Insert figure 1 about here

2.1 Firm supply under perfect price discrimination

After a brief analysis of the cost structure, we move to the solution of the model. In what follows, we solve the model in two steps. In the first step, we solve the firm optimization problem, deriving firm supply in a monopolistic competition framework. In the second step, we find the market equilibrium, allowing firms to enter until profits go to zero. This section analyzes the case when firms can perfect price discriminate. Section 2.2 examines the case when they cannot.

The profit of firm i is given by the following expression:

$$\pi_i(\eta_i, \psi_i, p_i) = \int_0^{\eta_i} (p_i(\tilde{\eta}) - c(\tilde{\eta}, \psi_i)) \cdot \hat{x}_i(\psi_i, p_i(\tilde{\eta})) \cdot \tau(\tilde{\eta}) d\tilde{\eta} - h(\eta_i, \psi_i). \quad (6)$$

Firm i maximizes (6) by simultaneously choosing three strategic variables: price, quality and size of the market area. First order conditions imply that:

$$p_i(\eta_i) = \frac{c(\eta_i, \psi_i)}{\rho}, \text{ for each } \eta_i \quad (7)$$

$$(p_i(\eta_i) - c(\eta_i, \psi_i)) \cdot \hat{x}_i(\psi_i, p_i(\eta_i)) \cdot \tau(\eta_i) = \frac{\partial h(\eta_i, \psi_i)}{\partial \eta_i}, \text{ if } \eta_i \in (0, 1) \quad (8)$$

and

$$\int_0^{\eta_i} \left(\frac{p_i(\tilde{\eta}) - c(\tilde{\eta}, \psi_i)}{\beta \psi_i} - \frac{\theta c(\tilde{\eta}, \psi_i)}{\psi_i} \right) \cdot \hat{x}_i(\psi_i, p_i(\tilde{\eta})) \cdot \tau(\tilde{\eta}) d\tilde{\eta} = \frac{\partial h(\eta_i, \psi_i)}{\partial \psi_i}, \text{ if } \psi_i \in (0, 1) \quad (9)$$

where $\beta := (1 - \rho) / \rho$. Equation (7) is the pricing rule, equation (8) determines the "reach," and equation (9) concerns quality.¹² Constant elasticity of the utility function implies that the price does not depend on the number of firms or varieties faced by consumers. But, it is charged on a markup basis. Equation (8) has a clear interpretation. The potential demand of a consumer located just beyond the boundary of the market area is $\hat{x}_i(\psi_i, p_i(\eta_i)) \cdot \tau(\eta_i)$. Thus, the marginal benefit for a small expansion in η_i is $(p_i(\eta_i) - c(\eta_i, \psi_i)) \cdot \hat{x}_i(\psi_i, p_i(\eta_i)) \cdot \tau(\eta_i)$, while the marginal cost of such an expansion is $h_{\eta_i}(\eta_i, \psi_i)$. Condition (8) says that firm i chooses η_i in such a way that the marginal benefit equates the marginal cost. The interpretation of equation (9) is more complex. In this framework, quality impacts not only on the demand of the marginal consumer, but on the demand of all consumers. From equation (4), we know that the impact of a small quality change on the quantity demanded of a generic consumer $\tilde{\eta}$ is $\hat{x}_i(\psi_i, p_i(\tilde{\eta})) \cdot \tau(\tilde{\eta}) / \beta \psi_i$. Thus, a firm's benefits from a small change of quality are $(p_i(\tilde{\eta}) - c(\tilde{\eta}, \psi_i)) \cdot \hat{x}_i(\psi_i, p_i(\tilde{\eta})) \cdot \tau(\tilde{\eta}) / \beta \psi_i$ for each consumer. However, a quality change also increases variable costs of an amount equal to $(\theta \cdot c(\tilde{\eta}, \psi_i) / \psi_i) \cdot \hat{x}_i(\psi_i, p_i(\tilde{\eta})) \cdot \tau(\tilde{\eta})$ for each consumer, and fixed costs for an amount equal to $h_{\psi_i}(\eta_i, \psi_i)$. Summing up previous considerations, condition (9) states that firm i chooses ψ_i in such a way that the overall benefits arising from a small increase of quality just equate the overall costs.

¹² To be consistent with the assumptions of monopolistic competition, we have to check that the impact of a change of price, quality and size of the market area on the demand is negligible. The assumption that firms will compete with each other for all consumers (even if not contemporaneously) allows us to evaluate the elasticity $\partial \log q / \partial \log p_i$ and $\partial \log q / \partial \log \psi_i$ that are of the order $1/M$, and the elasticity of $\partial \log q / \partial \log \eta_i$ that is of the order β/M . Hence, assuming M is reasonably large we can neglect the effect of p_i , ψ_i and η_i on q .

Now, substituting equation (7) into equations (8) and (9), we obtain:

$$\beta \cdot c(\eta_i, \psi_i) \cdot \hat{x}_i(\psi_i, c(\eta_i, \psi_i)/\rho) \cdot \tau(\eta_i) = \frac{\partial h(\eta_i, \psi_i)}{\partial \eta_i}, \text{ if } \eta_i \in (0,1) \quad (10)$$

$$\eta_i \underline{c\hat{x}\tau}(\eta_i, \psi_i) = \psi_i^+ \frac{\partial h(\eta_i, \psi_i)}{\partial \psi_i}, \text{ if } \psi_i \in (0,1) \quad (11)$$

where $\underline{c\hat{x}\tau}(\eta_i, \psi_i) := \frac{1}{\eta_i} \int_0^{\eta_i} c(\tilde{\eta}, \psi_i) \cdot \hat{x}_i(\psi_i, c(\tilde{\eta}, \psi_i)/\rho) \cdot \tau(\tilde{\eta}) d\tilde{\eta}$ and $\psi_i^+ := \psi_i / (1-\theta) > \psi_i$. From now on, we assume that $\tau(\tilde{\eta})$ is decreasing in $\tilde{\eta}$, i.e., customers who are more costly for the firm are also those with higher transport costs. Equations (10) and (11) suggest that the solution is interior only if $h_{\eta_i}(\eta_i, \psi_i) > 0$ and $h_{\psi_i}(\eta_i, \psi_i) > 0$.¹³ Previous results can be summarized in the following proposition:

Proposition 1. *Under perfect price discrimination, when $h_{\eta_i}(\eta_i, \psi_i) = 0$, a firm serves the entire market; when $h_{\eta_i}(\eta_i, \psi_i) > 0$, it can be profitable for the firm to limit the size of the market area. When the solution is internal, a firm's optimal choice is described by equations (7), (10) and (11).*

Market equilibrium. Monopolistic competition implies that firms enter the market until profits go to zero. Zero-profit condition assures that variable profits equal fixed costs. Hence:

$$\beta \eta_i \underline{c\hat{x}\tau}(\eta_i, \psi_i) = h(\eta_i, \psi_i). \quad (12)$$

We look for an internal, unique and symmetric solution. Symmetry implies that $x_i = x$, $p_i = p$, $\eta_i = \eta$, $\psi_i = \psi$. Combining (10), (11) and (12), we obtain equation (13):

$$h(\eta, \psi) = \eta^+ h_{\eta}(\eta, \psi) = \beta \psi^+ h_{\psi}(\eta, \psi) \quad (13)$$

¹³ We assume that second order conditions are satisfied. Second order conditions imply that:

$$\pi_{\eta_i \eta_i} = (\beta \tau_{\eta_i} c - \tau c_{\eta_i}) \cdot \hat{x}_i - h_{\eta_i \eta_i} \leq 0, \quad \pi_{\psi_i \psi_i} = (1-\theta)(1-\beta-\theta) / \beta \cdot \eta_i \underline{c\hat{x}\tau} - h_{\psi_i \psi_i} \leq 0 \quad \text{and} \quad \pi_{\eta_i \eta_i} \pi_{\psi_i \psi_i} - \left((1-\theta) \eta_i \underline{c\hat{x}\tau} - h_{\eta_i \psi_i} \right)^2 \geq 0.$$

Note that the first condition is always satisfied and the second condition is certainly satisfied if $\beta > 1-\theta$, i.e., the consumer appreciates varieties or unit costs are sufficiently increasing in "reach."

where $\psi^+ := \psi / (1 - \theta) > \psi$ and $\eta^+ := \eta \underline{c\hat{x}\tau} / c\hat{x}\tau \geq \eta$. Note that equation (13) implies that $h(\eta, \psi) / \eta^+ = h_\eta(\eta, \psi)$, meaning that the optimal size of the market area corresponds to a situation where the average fixed costs, modified to include transport and unit costs, must equal the marginal increase of fixed costs due to the "reach." It is simple to prove that since $\eta^+ \geq \eta$, it follows that, in equilibrium, the optimal size of the market area, η^* , is smaller than, or equal to the minimum size, η^m , i.e. $\eta^* \leq \eta^m$. Figure 1 depicts two different situations. Panel (a) shows the optimal "reach" when there are first diseconomies of size and then economies of size. The existence of increasing transport costs and/or of increasing unit costs distorts the equilibrium, thereby reducing the optimal market area. Panel (b) shows the optimal "reach" when there are diseconomies of scale. In that case, the minimum size is 1 and the optimal solution is smaller than 1, provided that transport cost and unit costs are sufficiently high. In both situations, firms do not exploit all the economies of size, and the equilibrium is in the portion of the averaged fixed cost curve where there are economies of size.¹⁴

Insert figure 2 about here

In a similar way, equation (13) implies that $h(\eta, \psi) / \psi^+ = \beta h_\psi(\eta, \psi)$. Optimal quality depends on two factors: costs and preferences. When $\beta = 1$, the result mimics the choice of the optimal size. The optimal quality is such that fixed costs averaged by quality and corrected by unit costs of quality equals the marginal increase of fixed costs due to the improvement in quality.

Figure 2 provides additional intuition to the result when $\beta \neq 1$. We define $\psi^m := \arg \min_{\tilde{\psi}} h(\eta, \tilde{\psi}) / \tilde{\psi}$. Panel (a) depicts the case in which fixed costs averaged by quality are first decreasing and then

¹⁴ This implies that diseconomies of size are a sufficient condition for having internal solution.

increasing. The picture shows that when $\beta = 1 - \theta$, the optimal quality, ψ^* , coincides with the quality which minimizes the averaged fixed costs, i.e. $\psi^* = \psi^m$, when $\beta > 1 - \theta$ (i.e. consumers strongly appreciate varieties) we have $\psi^* < \psi^m$ and when $\beta < 1 - \theta$ (consumers weakly appreciate varieties) we have $\psi^* > \psi^m$. Panel (b) shows the firm's choice when fixed costs averaged by quality are decreasing. In this occurrence a firm will provide a quality less than 1 only if $\beta > 1 - \theta$.

The choice of quality is driven by considerations of costs and of consumers' preferences. This fact produces a trade-off between quality and varieties and it is the reason why, in equilibrium, the optimal quality, ψ^* , can be different from ψ^m (even if τ and c are constant). In this model, *quality* plays the same role as *quantities* in the Dixit-Stiglitz model. When consumers strongly appreciate varieties, firms are induced to produce a lower quality/quantity. On the other hand, when consumers weakly appreciate varieties, then they push firms to expand quality/quantity.

Previous results are summarized in the following proposition.

Proposition 2. *Let η^* and ψ^* be internal, symmetric and unique solution, then the equilibrium condition is provided by equation (13). In equilibrium, $\eta^* \leq \eta^m := \arg \min_{\tilde{\eta}} h(\tilde{\eta}, \psi) / \tilde{\eta}$ and $\psi^* \leq \psi^m := \arg \min_{\tilde{\psi}} h(\eta, \tilde{\psi}) / \tilde{\psi}$ when $\beta \geq 1 - \theta$ and $\psi^* \geq \psi^m$ when $\beta < 1 - \theta$.*

This corollary completes Propositions 1 and 2.

Corollary 1. *Assuming unit costs and transport costs are constant, when there are diseconomies of size, it is optimal to limit the market area; and, the optimal size of the market area coincides with the minimum size.¹⁵*

In order to identify the number of firms and varieties that are found on the market, starting from

¹⁵ Notice that most of the literature on logistics has as its objective the minimisation of logistic costs (see, for example: Burns et al. 1985, Campbell, 1993, McCann, 1998, and Hsu and Tsai, 1999).

equation (2) we integrate over consumers. With a little abuse of notation, we can write:

$$\int_0^1 \sum_{j=1}^N p_j x_j dl = M \int_0^\eta p(\tilde{\eta}) \cdot \hat{x}(\psi, p(\tilde{\eta})) \cdot \tau(\tilde{\eta}) d\tilde{\eta} = I. \quad (14)$$

Noting that the zero-profit condition can be written as $(1-\rho) \int_0^\eta p(\tilde{\eta}) \cdot \hat{x}(\psi, p(\tilde{\eta})) \cdot \tau(\tilde{\eta}) d\tilde{\eta} = h$, and exploiting symmetry, we have:

$$M^* = \frac{(1-\rho)I}{h(\eta^*, \psi^*)}, \quad N^* = \eta^* M^* = \frac{(1-\rho)I}{h(\eta^*, \psi^*)/\eta^*}. \quad (15)$$

As expected, the number of firms is increasing in the consumer income I and is decreasing in the elasticity of substitution $1/(1-\rho)$ and fixed costs h . Note that the maximum number of varieties is obtained when η minimizes the average fixed costs. When transport cost and unit variable costs are not constant, $\eta^m > \eta^*$ and the number of varieties is, in general, smaller.¹⁶ When transport costs and unit variable costs are constant, then the optimal solution maximizes the number of varieties and firms operate at the minimal size.

2.2 Firm supply under uniform price

In this section, now we repeat the previous analysis for the case when firms cannot price discriminate and choose a uniform price for every market. The profit function is the same of equation (6). The pricing rule is now:

$$\bar{p}_i = \frac{1}{\rho} \frac{c \underline{\tau}(\psi_i, \eta_i)}{\underline{\tau}(\eta_i)} \quad (16)$$

where $\underline{\tau}(\eta_i, \psi_i) = \frac{1}{\eta_i} \int_0^{\eta_i} \tau(\tilde{\eta}) d\tilde{\eta}$ and $c \underline{\tau}(\eta) = \frac{1}{\eta} \int_0^\eta c(\tilde{\eta}, \psi_i) \tau(\tilde{\eta}) d\tilde{\eta}$, and we use \bar{p}_i to indicate that prices

¹⁶ In Section 3, we show that choosing $\eta^* < \eta^m$ is optimal in terms of second best analysis, since extending the reach to η^m reduces consumption too much and, hence, it lowers the welfare.

are uniform. First order conditions are:

$$(\bar{p}_i - c(\eta_i, \psi_i)) \cdot \hat{x}_i(\psi_i, \bar{p}_i) \cdot \tau(\eta_i) = \frac{\partial h(\eta_i, \psi_i)}{\partial \eta_i}$$

$$\frac{\eta_i}{\beta \psi} (\bar{p}_i \tau(\eta_i) - c \tau(\psi_i, \eta_i) \cdot (1 - \beta \theta)) \cdot \hat{x}_i(\psi_i, \bar{p}_i) = \frac{\partial h(\eta_i, \psi_i)}{\partial \psi_i}.$$

And zero-profit condition is:

$$\eta_i \cdot (\bar{p}_i \tau(\eta_i) - c \tau(\psi_i, \eta_i)) \cdot \hat{x}_i(\psi_i, \bar{p}_i) = h(\eta_i, \psi_i).$$

Now, substituting equation (16) into previous equations, and after some simplifications, we obtain that:

$$\left(\frac{1}{\rho} \frac{c \tau(\psi_i, \eta_i)}{\tau(\eta_i)} - c(\eta_i, \psi_i) \right) \cdot \hat{x}_i(\psi_i, \bar{p}_i) \cdot \tau(\eta_i) = \frac{\partial h(\eta_i, \psi_i)}{\partial \eta_i}, \text{ if } \eta_i \in (0, 1) \quad (17)$$

$$\eta_i (1 - \theta) c \tau(\eta_i, \psi_i) \hat{x}_i(\psi_i, \bar{p}_i) = \psi_i \frac{\partial h(\eta_i, \psi_i)}{\partial \psi_i}, \text{ if } \psi_i \in (0, 1) \quad (18)$$

$$\eta \beta c \tau(\eta_i, \psi_i) \hat{x}_i(\psi_i, \bar{p}_i) = h(\eta_i, \psi_i).$$

In perfect price discrimination, $h_{\eta_i}(\eta_i, \psi_i) > 0$ is a necessary condition for firms to limit their “reach.”

This is no longer true when firms cannot price discriminate. Assume that $h_{\eta_i}(\eta_i, \psi_i) = 0$. Thus, first order condition (17) is satisfied when $c \tau(\eta_i, \psi_i) = \rho c(\eta_i, \psi_i) \tau(\eta_i)$. Under uniform prices, firms may decide to limit their coverage when their unit costs are increasing in the “reach.” Notice that if $h_{\eta_i}(\eta_i, \psi_i) = 0$, and unit variable costs are constant, increasing consumer’s transport costs does not induce a firm to limit the coverage. Previous results can be summarized in the following proposition:

Proposition 3. *Under uniform prices, it can be profitable for the firm to limit the size of the market area when $h_{\eta_i}(\eta_i, \psi_i) > 0$ or when $h_{\eta_i}(\eta_i, \psi_i) = 0$ and unit costs, $c(\eta_i, \psi_i)$, are increasing in the “reach.” When the solution is internal, the firm’s optimal choice is described by equations (16), (17) and (18).*

The firm’s optimal choice, zero-profit condition and symmetry yield equation (19) which describes the

market equilibrium:

$$h = \eta \cdot \frac{\underline{\tau}}{\tau} \frac{\partial h(\eta, \psi)}{\partial \eta} + \eta \cdot \left(c(\eta, \psi) - \frac{c \underline{\tau}(\psi, \eta)}{\underline{\tau}(\eta)} \right) \cdot \tau(\eta) \cdot \hat{x}(\psi, \bar{p}) = \psi^+ \beta \frac{\partial h(\eta, \psi)}{\partial \psi}. \quad (19)$$

The quality condition is exactly the same as in previous section. In order to interpret the decision to limit the "reach," we write the first part of equation (19) in the following way:

$$\frac{h}{\eta} = \frac{\underline{\tau}}{\tau} \frac{\partial h(\eta, \psi)}{\partial \eta} + \left(c(\eta, \psi) - \frac{c \underline{\tau}(\psi, \eta)}{\underline{\tau}(\eta)} \right) \cdot \tau(\eta) \cdot \hat{x}(\psi, \bar{p}).$$

This equation presents two sources of distortion with respect to the case when transport costs and unit costs are constant. The first factor, $\underline{\tau}/\tau > 1$, captures the role of increasing transport costs and the second factor, $c - \underline{c}\underline{\tau}/\underline{\tau} > 0$, captures the role of increasing unit costs. Both effects are responsible for a reduction in the size of the market area. The first factor is effective only if $h_\eta(\eta, \psi) > 0$, while the second one plays a role even if fixed costs are not affected by the "reach."

These results are summarized in the following proposition, which is analogous to Proposition 2.

Proposition 4. *Let η^* and ψ^* be internal, symmetric and unique solution, then the equilibrium condition is provided by equation (19). In equilibrium, $\eta^* \leq \eta^m := \arg \min_{\tilde{\eta}} h(\tilde{\eta}, \psi) / \tilde{\eta}$ and $\psi^* \leq \psi^m := \arg \min_{\tilde{\psi}} h(\eta, \tilde{\psi}) / \tilde{\psi}$ when $\beta \geq 1 - \theta$ and $\psi^* \geq \psi^m$ when $\beta < 1 - \theta$.*

Finally, equation (15) provides the number of varieties and the number of firms that are in equilibrium.

3 Social Optimum, Market Area and Quality

This section questions whether the outcome of the market produces too many or too few products, with respect to the constrained social optimum.¹⁷ We propose the same analysis as in Dixit and Stiglitz

¹⁷ The constrained social optimum assumes that the planner cannot use lump-sum amounts to transfer

(1977) regarding the social welfare. Their main result (p. 301) states that “we have a rather surprising case where the monopolistic competition equilibrium is identical to the optimum constrained by the lack of lump sum subsidies.” In the following proposition, we show that this result continues to hold, if we extend the framework.

We consider the case in which c does not depend on η . In this occurrence, price discrimination and uniform price coincide¹⁸. We assume symmetry among consumers, i.e., $\sum_{j=1}^N \tau_j$ is constant among consumers.

Proposition 5. *Under symmetry, when c does not depend on η , the market equilibrium coming from the monopolistic competition corresponds to the constrained social optimum.*

Proof. The planner chooses the price p , the reach η , the quantity x function of the “reach,” the quality ψ , and the number of firms on the market M as to maximize utility, satisfying the consumer budget constraint (2) and the zero-profit condition (12). Aggregation over consumers implies that the budget constraint is $\int_0^1 \sum_{j=1}^N p_j x_j dl = Mp \int_0^\eta x(\tilde{\eta}) d\tilde{\eta} = I$, and the zero-profit condition requires that: $(p - c) \int_0^\eta x(\tilde{\eta}) d\tilde{\eta} = h$. Combining previous equations we have:

$$\int_0^\eta x(\tilde{\eta}) d\tilde{\eta} = \frac{I - Mh}{cM} . \quad (20)$$

The welfare maximization can be computed in two steps. First, we find the optimal quantity as a function of transport costs, then we replace equation (20) into the welfare function in order to find η^S , ψ^S and M^S , which are the solutions of the constraint optimum.

To begin, the planner maximizes the utility of the consumer l , subject to the budget constraint. Since

resources from consumers to firms.

¹⁸ The two pricing strategies coincide because consumer’s transport costs are of iceberg type. See: Hsu (1979) and Fujita and Thisse (2002, chap. 9).

there is no price discrimination, the maximum utility is obtained when quantities are chosen in such a way as to maximize $\psi \left(\sum_{j=1}^N (t_j x_j)^\rho \right)^{\frac{1}{\rho}}$ subject to $\sum_{j=1}^N x_j = I / p$. This implies that $x_j / \tau_j = x_g / \tau_g =: k$.

Replacing it in the utility function, after some calculations, it emerges that $u = k \psi \left(\sum_{j=1}^N \tau_j \right)^{\frac{1}{\rho}}$.

Integrating over different consumers, we obtain:

$$W = k \psi \int_0^1 \left(\sum_{j=1}^N \tau_j \right)^{\frac{1}{\rho}} dl = k \psi \left(M \int_0^\eta \tau(\tilde{\eta}) d\tilde{\eta} \right)^{\frac{1}{\rho}} \quad (21)$$

where the last equality holds as $\sum_{j=1}^N \tau_j$ is constant. Noting that $k = x_j / \tau_j$ is constant for every j and replacing equation (20) into (21), we obtain:

$$W = \psi \frac{\int_0^\eta x(\tilde{\eta}) d\tilde{\eta}}{\int_0^\eta \tau(\tilde{\eta}) d\tilde{\eta}} \left(M \int_0^\eta \tau(\tilde{\eta}) d\tilde{\eta} \right)^{\frac{1}{\rho}} = \frac{\psi}{c} (I - Mh) \left(M \int_0^\eta \tau(\tilde{\eta}) d\tilde{\eta} \right)^\beta.$$

Finally, taking the first derivative, with respect to η , M and ψ , we have the same equations as in Section 2:

$$M^s = \frac{(1-\rho)I}{h(\eta^s, \psi^s)}, \quad \frac{h}{\eta^s} = \frac{\tau}{\tau} \frac{\partial h(\eta^s, \psi^s)}{\partial \eta}, \quad \frac{h}{\psi^s} = \beta \frac{\partial h(\eta^s, \psi^s)}{\partial \psi}.$$

4 Concluding Remarks

In this paper, we have derived the conditions under which a firm prefers to limit its "reach" and we have considered the case of two different pricing rules. In the case of perfect price discrimination, we find that a firm limits the size of its market area only when fixed costs are increasing in the "reach." Alternatively, in the case of uniform prices, it emerges that increasing variable costs may be sufficient for firms to limit their "reach," even if fixed costs are not affected by the spatial coverage.

Literature on international trade holds that fixed costs and unit variable costs are affected by the "reach." Hence, our main assumptions are grounded on strong empirical evidence. Our theoretical

findings can be useful to formally introduce trade barriers into international trade models. In particular, our framework can provide an additional explanation to the overestimation of the trade flows obtained by gravity models, as in McCallum (1995). These models usually estimate a demand function similar to equation (3), which relates the quantity demanded as a decreasing function of transport costs. From our point of view, lack of trade is not only due to increasing trade costs induced by borders, but, also, to increasing fixed costs, which induce firms not to offer their products on international markets. Finally, we have showed that the monopolistic competitive equilibrium is constrained socially optimal.

5 Appendix

This Appendix is devoted to computing the consumer demand, following the approach presented by Dixit and Stiglitz (1977).

A consumer maximizes equation (1) subject to equation (2). Let y and q be, respectively, the quantity index and price index. They are defined as follows:

$$y := \left(\sum_{j=1}^N (x_j t_j \psi_j)^\rho \right)^{\frac{1}{\rho}} \quad q := \left(\sum_{j=1}^N \left(\frac{p_j}{t_j \psi_j} \right)^{-\frac{1}{\beta}} \right)^{-\beta} \quad (22)$$

where $\beta := (1 - \rho) / \rho$. The first order condition with respect to x_j is:

$$\psi_j (x_j t_j \psi_j)^{\rho-1} \left(\sum_{g=1}^N (x_g t_g \psi_g)^\rho \right)^{\frac{1}{\rho}-1} = \lambda p_j \quad (23)$$

or:

$$\left(\frac{\psi_g t_g x_g}{\psi_j t_j x_j} \right)^{\rho-1} = \frac{p_g / \psi_g t_g}{p_j / \psi_j t_j}. \quad (24)$$

From (24) we have: $(\psi_g t_g x_g)^\rho = \left(\frac{p_g / \psi_g t_g}{p_j / \psi_j t_j} \right)^{\rho-1} (\psi_j t_j x_j)^\rho$. Summarily, it follows:

$$\left(\sum_{g=1}^N (\psi_g t_g x_g)^\rho \right)^{\frac{1}{\rho}} = \left(\sum_{g=1}^N \left(\frac{p_g / \psi_g t_g}{p_j / \psi_j t_j} \right)^{\frac{\rho}{\rho-1}} (\psi_j t_j x_j)^\rho \right)^{\frac{1}{\rho}}.$$

Simplifying and solving for x_j , we have (3).

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Figures

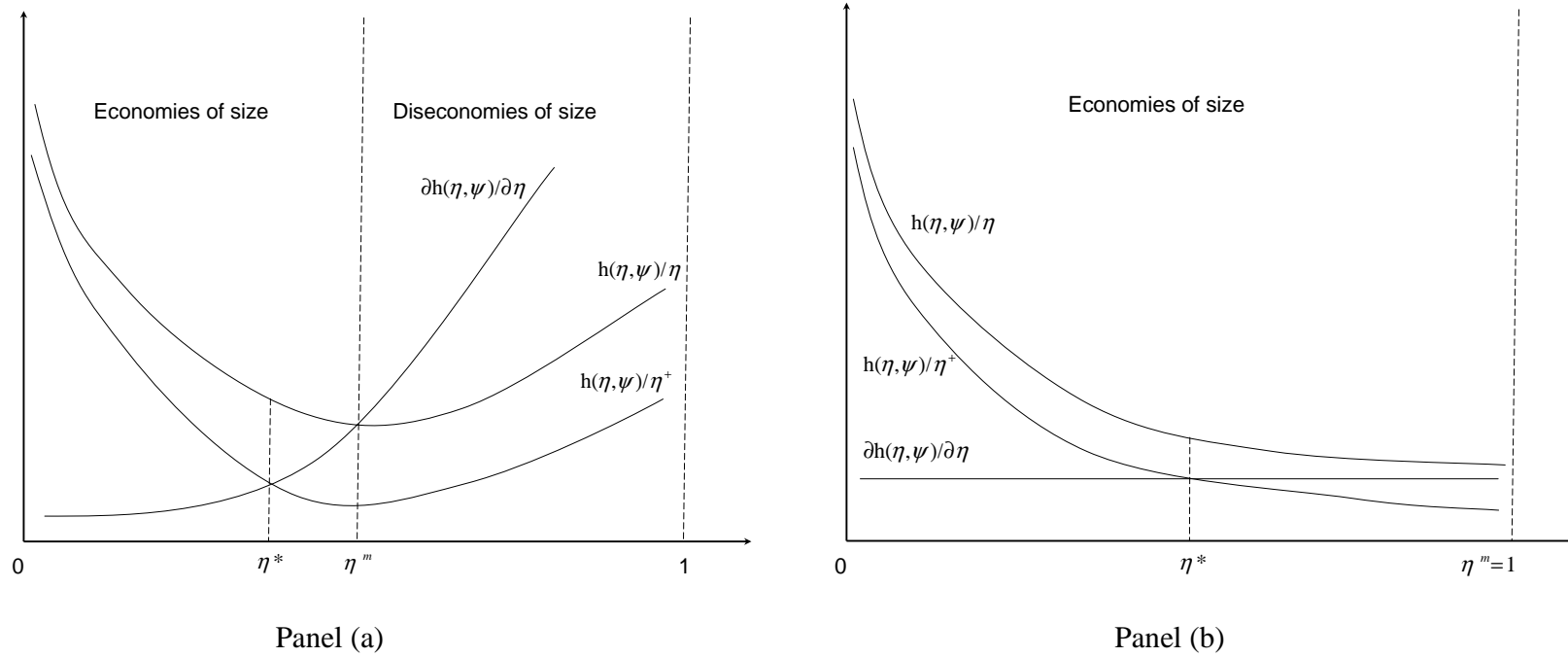


Figure 1 depicts two different situations. In Panel (a), there are diseconomies of size in the range $[0, \eta^m)$ and economies of size in the range, $(\eta^m, 1]$. In Panel (b), there are only economies of size. In equilibrium, $\eta^* \leq \eta^m$.

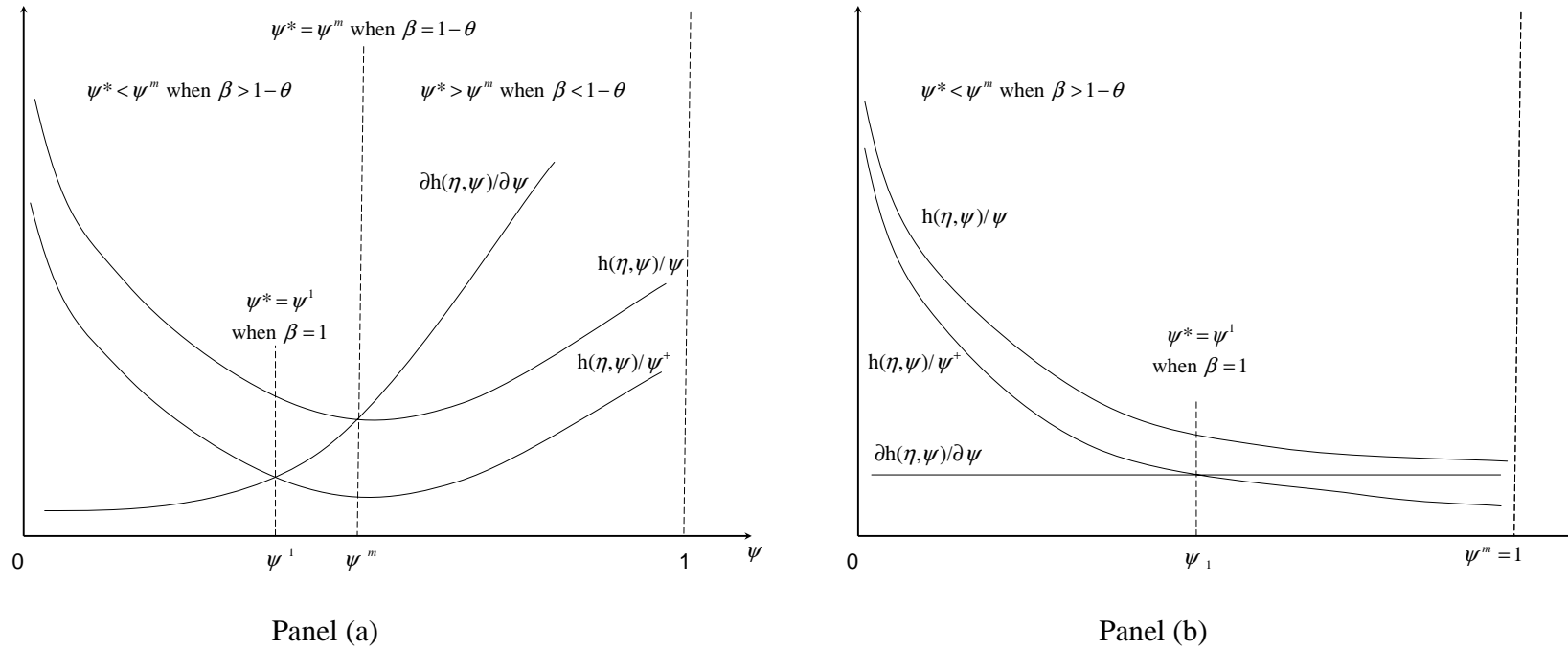


Figure 2 depicts two different situations. Panel (a) shows the case in which fixed costs averaged by quality are first decreasing and then increasing. In equilibrium, if $\beta > 1 - \theta$ then $\psi^* < \psi^m$ and if $\beta < 1 - \theta$ then $\psi^* > \psi^m$. Panel (b) shows the firm's choice when fixed costs averaged by quality are decreasing. In equilibrium, if $\beta > 1 - \theta$ then $\psi^* \leq \psi^m$ and if $\beta \leq 1 - \theta$ then $\psi^* = \psi^m = 1$.