Analyzing structural change: two new biproportional tools.
Application to the input-output table of France (1980-1996)

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ABSTRACT. The biproportional filter was created to analyze structural change between two input-output matrices by removing the effect of differential growth of sectors without predetermining if the model is demand or supply-driven. It has the disadvantage that projecting a first matrix on a second is not the same thing than projecting the second matrix on the first. Here two alternative methods are proposed which has not this last drawback, with the additional advantage for the biproportional bimarkovian filter that effects of sector size are also removed. The methods are compared with an application for France, 1980 and 1996.

I. Introduction

To evaluate change in the structure of exchanges, you can compare two matrices of technical coefficients to remove the differences in the column margins. A technical coefficient is the ratio z_{ij} of the flow from a sector *i* to a sector *j*, over the output x_j of sector *j*: it indicates how much a sector has to buy of commodity *i* to produce one unit of commodity *j*. For Leontief, these coefficients are assumed to be stable, what implies that the economy is demand driven, so two technical coefficient matrices **A** and $\mathbf{A}^*$ are compared, an initial matrix **A** and a final matrix $\mathbf{A}^*$, belonging of two different periods. The same thing can be done with allocation coefficients: in the Ghosh perspective, the allocation coefficients are assumed to be stable, what imply that the economy is supply driven. An allocation coefficient is a coefficient that divides the flow z_{ij} by the output x_i of sector *i*: it indicates how the output of a sector, *i*, will be allocated to other sectors, *j*. One compares two allocation coefficient matrices **B** and $\mathbf{B}^*$. Both hypotheses are incompatible and the results in the first case are not comparable with the results in the second case: if technical coefficients are assumed to be stable, allocation coefficients cannot be stable, except in the case of absolute join stability (Chen and Rose, 1986 and 1991), and reciprocally. So, there is a large literature about what model can be considered as the more attractive (Oosterhaven, 1988 and 1989), (Miller, 1989), (Gruver, 1989), (Rose and Allison, 1989), (Dietzenbacher, 1997). Generally, the demand driven model is considered so. However, there is doubt about the real stability over time of one type of coefficient or the other. Clearly, if technical coefficients are not stable, the model can be declared as not demand driven but the reciprocal of this proposition is false: if technical coefficients are stable, the model is not necessarily demand driven; if allocation coefficients are not stable, the model cannot be declared as supply driven but the reciprocal is false again. So, one may want to compare technical coefficients over time by assuming the demand driven hypothesis and the normal stability of technical coefficients, but one will have no information about allocation coefficients. Or one may want to compare allocation coefficients over time by assuming the supply driven hypothesis by assuming the normal stability of allocation coefficients, but one will have no information about technical coefficients.

This is a dilemma that could be removed without assuming *ex ante* that technical and allocation coefficients are stable (their stability will be measured eventually *ex post*, what could help to dismiss one of the alternative hypothesis or both eventually): in (Mesnard, 1990a, 1990b, 1996, 1977), a biproportional filter was proposed to analyze structural change. It is a generalization of the comparisonof technical coefficients and of allocation coefficients: the basic idea inside biproportional methods to measure structural change is to become free from the orientation of the economy -- demand-driven or supply-driven hypothesis -- . Note that comparing two technical coefficients is the same thing than comparing two absolute values $z_{ij}\,\frac{x_j^*}{x_j}$ and z_{ij}^*: $a_{ij} \leftrightarrow a_{ij}^* \Leftrightarrow \frac{z_{ij}}{x_j} \leftrightarrow \frac{z_{ij}^*}{x_j^*} \Leftrightarrow z_{ij}\,\frac{x_j^*}{x_j} \leftrightarrow z_{ij}^*$, where the symbol "↔" signifies "compared to". So, starting from an initial flow matrix $\mathbf{Z}$ and a final flow matrix $\mathbf{Z}^*$, the principle consists into compute a matrix the closer as possible to $\mathbf{Z}$ but with the row and column margins of $\mathbf{Z}^*$ (a margin is the column sum or a row sum); see figure 1.

Figure 1 here

The tool to perform this is biproportion: i.e. $K(\mathbf{Z},\ \mathbf{Z}^*)$; and then to compare the result to $\mathbf{Z}^*$ by calculating the difference matrix $\mathbf{Z}^* - K(\mathbf{Z},\ \mathbf{Z}^*)$. This method is called the *ordinary biproportional filter*. Variation indicators are then computed: the absolute variation between $\mathbf{Z}$ and $\mathbf{Z}^*$ (with the *Frobenius norm of this difference matrix),*

$$\Sigma = \sqrt{\sum_i \sum_j \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}$$

or the absolute variation of vectors of this matrix, for demanding sectors (column vectors, Σ_j) or supplying sectors (row vectors, Σ_i):

$$\Sigma_j = \sqrt{\sum_i \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2} \text{ and } \Sigma_i = \sqrt{\sum_j \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}$$

In the last step, to remove the effect of the size differences of sectors, the relative variation is calculated by dividing the absolute variation by, e.g., the total of the row or the column of final flow matrix $\mathbf{Z}^*$ (Mesnard, 199a, 1990b, 1977):

$$\sigma = \frac{\sqrt{\sum_i \sum_j \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_i \sum_j z^*_{ij}}$$

$$\sigma_j = \frac{\sqrt{\sum_i \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_i z^*_{ij}} \text{ and } \sigma_i = \frac{\sqrt{\sum_j \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_j z^*_{ij}}$$

A first difficulty is that there are other manners to compute these relative variabilities. A second difficulty is that it is also possible to project $\mathbf{Z}^*$ on the margins of **Z** to compare the result to **Z**: one obtain two different results without a criterion to declare the superiority of one over the other.

The aim of this paper is to allow to remove the technical problems encountered with the ordinary biproportional filter. A first part of this paper will expose these problems and a second part will introduce two new biproportional methods that avoid them and a third part will present two applications, structural change in France.

II. The difficulties of the ordinary biproportional filter

A. Computing the relative variabilities

At least, two other possibilities could have been chosen to compute the relative variabilities. The first possibility is to divide the absolute variation by the margins of the initial flow matrix **Z**, for example:

$$\sigma = \frac{\sqrt{\sum_i \sum_j \left[z_{ij}^* - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_i \sum_j z_{ij}}$$

The second possibility is to divide the absolute variation by the margins of the initial and the final flow matrices **Z** and $\mathbf{Z}^*$, for example:

$$\sigma = \frac{\sqrt{\sum_i \sum_j \left[z_{ij}^* - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_i \sum_j \left(\frac{z_{ij} + z_{ij}^*}{2} \right)}$$

So, we have three possibilities to the total: this is an axiomatic difficulty, that we must try to remove. Nevertheless, the method removes the change effect of both margins of the tables **Z** and $\mathbf{Z}^*$, i.e. the effect of differential growth of demanding and supplying sectors, without predetermining if the model is supply or demand driven.

<u>Remark</u>. Also, a χ^2 is able to be computed, for example:

$$\sqrt{\sum_i \frac{\left[z_{ij}^* - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}{z_{ij}^*}} \quad \text{or} \quad \sqrt{\sum_i \frac{\left[z_{ij}^* - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}{z_{ij}}} \ \text{x}$$

B. Computing in reverse order

Instead of projecting $\mathbf{Z}$ on the margins of $\mathbf{Z}^*$, what is called the *direct* calculation, different results were also obtained when $\mathbf{Z}^*$ was projected on the margins of $\mathbf{Z}$ what is called the *reverse* calculation. Are calculated, $K(\mathbf{Z}^*,\ \mathbf{Z})$, then $\mathbf{Z} - K(\mathbf{Z}^*,\ \mathbf{Z})$ and, for example:

$$\Sigma = \sqrt{\sum_i \sum_j \left[z_{ij} - K\left(\mathbf{Z}^*, \mathbf{Z}\right)_{ij} \right]^2} \text{ or } s = \frac{\sqrt{\sum_i \sum_j \left[z_{ij} - K\left(\mathbf{Z}^*, \mathbf{Z}\right)_{ij} \right]^2}}{\sum_i \sum_j z_{ij}}$$

As these reverse results are not the same than direct results, it was necessary to conduct both computations. This is a disadvantage, not only because it forces to have two computations, but also because results can diverge strongly: the ordering of sectors -- from the most changing to the less changing -- could be difficult as the application below will prove.

Remark. When one evaluates the *cumulative* change with the ordinary biproportional filter, what consists into comparing, for example, 1981-1980, then 1982-1980, ..., up to 1996-1980, by opposition to a "slipping" evaluation of change, for example 1981-1980, then 1982-1981, ..., up to 1996-1995, one have to compute a lot:

- for the direct calculation: $(\mathbf{Z}_t) - K(\mathbf{Z}_0,\ \mathbf{Z}_t)$ with year 0 as reference year,

  or: $(\mathbf{Z}_T) - K(\mathbf{Z}_t,\ \mathbf{Z}_T)$ with T as reference year,

- and for the reverse calculation: $(\mathbf{Z}_0) - K(\mathbf{Z}_t,\ \mathbf{Z}_0)$ with year 0 as reference year,

  or: $(\mathbf{Z}_t) - K(\mathbf{Z}_T,\ \mathbf{Z}_t)$ with T as reference year,

that is, for 16 years, 4×15 biproportional projections. Avoiding the reverse computation divides this large amount of computation by 2. x

C. *Synthesis*

To summarize, two axiomatic difficulties come across with the ordinary biproportional filter, a method that succeeds to compare two flow matrices without posing the demand-driven hypothesis or the supply-driven hypothesis, by removing the effect of differential growth of sectors. The main difficulty is that there are two ways to do the calculation, the direct calculation from $\mathbf{Z}$ to $\mathbf{Z}^*$ and the reverse calculation from $\mathbf{Z}^*$ to $\mathbf{Z}$. The second difficulty is caused by the fact the this method fails to remove naturally the differences in size of sectors, what obliges to calculate the relative variabilities along with the absolute variabilities and there are three ways to remove the differences of size of sectors.

Understanding that the basic idea of biproportional filtering consists into giving to the flow matrices $\mathbf{Z}$ and $\mathbf{Z}^*$ the same margins, to remove these difficulties, one can try to find a third matrix $\mathbf{Z}^B$ to provide these margins. If $\mathbf{Z}^B$ has the same margins than $\mathbf{Z}$, or is equal to $\mathbf{Z}$, then $K(\mathbf{Z},\ \mathbf{Z}^B) = \mathbf{Z}$ and $K(\mathbf{Z}^*,\ \mathbf{Z}^B) = K(\mathbf{Z}^*,\ \mathbf{Z})$, so one have the reverse projection of the ordinary biproportional projector; if $\mathbf{Z}^B$ has the same margins than $\mathbf{Z}^*$, then $K(\mathbf{Z}^*,\ \mathbf{Z}^B) = \mathbf{Z}^*$ and $K(\mathbf{Z},\ \mathbf{Z}^B) = K(\mathbf{Z},\ \mathbf{Z}^*)$, so one have the direct projection of the ordinary biproportional projector. For all positions between these two "polar" matrices, one can obtain a wide range of results.

A good idea could consist into choosing $\mathbf{Z}^B$ such a manner that the variance would be minimized, either by measuring the variance in absolute terms as the square of the Frobenius norm of the difference matrix:

$$\| K(\mathbf{Z},\ \mathbf{Z}^B) - K(\mathbf{Z}^*,\ \mathbf{Z}^B) \|_F^2$$

either by measuring it in relative terms:

$$\frac{\| K(\mathbf{Z},\ \mathbf{Z}^B) - K(\mathbf{Z}^*,\ \mathbf{Z}^B) \|_F^2}{\sum_i \sum_j z_{ij}^B}$$

Unfortunately, these expressions are not linear regarding to the terms of $\mathbf{Z}$, $\mathbf{Z}^*$ and to the margins of $\mathbf{Z}^B$ and they have no analytical solution because biproportion is a transcendent operator. So, such a problem can be solved only by a succession of computations, what is too much heavy even for small matrices.

However, there are two particular matrices that are good candidates to play the role of $\mathbf{Z}^B$:

- a matrix, function of $\mathbf{Z}$ and $\mathbf{Z}^*$, for example the mean of $\mathbf{Z}$ and $\mathbf{Z}^*$, denoted $\bar{\mathbf{Z}}$ with $\bar{\mathbf{Z}} = \frac{1}{2}(\mathbf{Z} + \mathbf{Z}^*)$. I call this the *biproportional mean filter*.

- the bimarkovian matrix $\mathbf{1}^M$, a matrix of which all column margins are equal and all row margins are equal: this is the *biproportional bimarkovian filter*.

  Remark. $\mathbf{Z}^B$ could be, for example, a third matrix of an intermediary year, 1988, if $\mathbf{Z}$ is 1980 and $\mathbf{Z}^*$ is 1996, but remember that only the margins of this matrix are important.

III. The new methods

A. The biproportional mean filter

1. First step

Each matrix $\mathbf{Z}$ and $\mathbf{Z}^*$ is projected to the margins of another intermediary matrix $\mathbf{Z}^B$ that provide a fixed base, for example the mean $\bar{\mathbf{Z}}$ of $\mathbf{Z}$ and $\mathbf{Z}^*$, to give $K(\mathbf{Z}, \bar{\mathbf{Z}})$ and $K(\mathbf{Z}^*, \bar{\mathbf{Z}})$. This operation allows to remove the effects of differential growth of sectors.

There are many tools to perform this operation of projection of a matrix and the problem is to choose one of these tools, or, in other words, there are an infinite number of matrices that can have the same margins and the problem is to choose one of these matrices. The resulting matrix may vary depending on the tool chosen to perform the projection, and consequently the results of the methods may vary (see annex). Here, I choose biproportion. In particular, some methods as the orthogonal projection or the minimization of the least squares between a matrix, $\mathbf{Z}$ or $\mathbf{Z}^*$, and the matrix $\bar{\mathbf{Z}}$, that is $\min \sum_i \sum_j (z_{ij} - \bar{z}_{ij})^2$, may create negative terms in the projection $K(\mathbf{Z}, \bar{\mathbf{Z}})$ because the form becomes additive: $K(\mathbf{Z}, \bar{\mathbf{Z}}) = \mathbf{U} + \mathbf{Z} + \mathbf{V}$, where $\mathbf{U}$ and $\mathbf{V}$ are diagonal matrices: non negativity of terms of the projected matrix is not guaranteed (Mesnard, 1990a). This is why the tool that we choose to perform the projection is biproportion. With biproportion, one have $K(\mathbf{Z}, \bar{\mathbf{Z}}) = \mathbf{P\,Z\,Q}$, and,

$$p_i = \frac{\bar{z}_{i\bullet}}{\sum_{j=1}^{m} q_j\, z_{ij}} \text{, for all } i \text{, and } q_j = \frac{\bar{z}_{\bullet j}}{\sum_{i=1}^{n} p_i\, z_{ij}} \text{, for all } j$$

so, if all terms $p_i^{(k)}$ are positive, all terms $q_j^{(k)}$ will be also positive, as soon as all terms of $\mathbf{Z}$ are positive (Mesnard, 1994, 1997). The same reasoning holds for $\mathbf{Z}^*$. The operation of projection by a biproportional method signifies that the projected matrix is the nearest to the initial matrix, in the sense of information theory, among other theories (Mesnard, 1990a), but with the guarantee that there will be no negative terms inside the projected matrix if $\mathbf{Z}$ or $\mathbf{Z}^*$ have not.

2. Second step

$K(\mathbf{Z},\ \bar{\mathbf{Z}})$ is compared to $K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})$ by calculating the Frobenius norm of the difference matrix $K(\mathbf{Z}^*,\ \bar{\mathbf{Z}}) - K(\mathbf{Z},\ \bar{\mathbf{Z}})$. This is done in absolute values,

- for one single coefficient: $\Sigma_{ij} = \left| K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij} \right|$.

- for demanding sectors (i.e. for column vectors): $\Sigma_j = \sqrt{\sum_i \left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij} \right)^2}$.

- for supplying sectors ((i.e. for row vectors): $\Sigma_i = \sqrt{\sum_j \left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij} \right)^2}$.

- or for the whole economy: $\Sigma = \sqrt{\sum_i \sum_j \left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij} \right)^2}$.

This is done also in relative terms, by dividing absolute variabilities Σ by the value of the margins of $\bar{\mathbf{Z}}$:

- for one single coefficient: $\sigma_{ij} = \dfrac{\left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij} \right)^2}{\bar{z}_{ij}}$.

- for demanding sectors (i.e. for column vectors): $\sigma_j = \dfrac{\sqrt{\sum_i \left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij} \right)^2}}{\bar{z}_{\bullet j}}$.

- for supplying sectors (i.e. for row vectors): $\sigma_i = \dfrac{\sqrt{\sum_j \left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij}\right)^2}}{\bar{z}_{i\bullet}}$.

- or for the whole economy: $\sigma = \dfrac{\sqrt{\sum_i \sum_j \left(K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})_{ij} - K(\mathbf{Z},\ \bar{\mathbf{Z}})_{ij}\right)^2}}{\bar{z}_{\bullet\bullet}}$.

Note that there is a unique way to calculate the relative variabilities: the two problems caused by the ordinary biproportional filter are removed at the same time. However, the effect of size differences of sectors is not removed and one is obliged to continue to calculate the relative variabilities along with the absolute variabilities.

Remarks.

1) Because of their non linear nature, relative variabilities have not the property of aggregation: $\Sigma_i\, \sigma_i \neq \sigma$. So, one can compare relative variabilities for sectors between them, or relative variabilities for individual cells between them, or even relative variabilities for the whole economy at different dates, but one must not compare the relative variability of the whole economy to the relative variabilities of sectors, or relative variabilities of sectors to relative variabilities of individual cells.

2) It is ineffective to project $K(\mathbf{Z},\ \bar{\mathbf{Z}})$ to $K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})$ by a biproportion because both matrices have the same margins (Mesnard, 1994):

$$K\left[K(\mathbf{Z},\ \bar{\mathbf{Z}}),\quad K(\mathbf{Z}^*,\ \bar{\mathbf{Z}})\right] \ = \ K(\mathbf{Z},\ \bar{\mathbf{Z}})$$

This is why both matrices can be compared directly.

3) Unlike for the ordinary biproportional filter, where a direct computation, $K(\mathbf{Z},\ \mathbf{Z}^*)$, and a reverse computation, $K(\mathbf{Z}^*,\ \mathbf{Z})$, can be done with a different result in both cases, here the

same results are found when $K(\mathbf{Z}, \bar{\mathbf{Z}})$ is compared to $K(\mathbf{Z}^*, \bar{\mathbf{Z}})$ or when $K(\mathbf{Z}^*, \bar{\mathbf{Z}})$ is compared to $K(\mathbf{Z}, \bar{\mathbf{Z}})$. This is a real advantage: it is no more necessary to have a complicated and approximately empirical procedure to interpret the results by comparing two series of results. For this reason, the new method is more satisfying. x

3. More about the justification of the method

There is a clear difference between these two methods: all column vectors have the same margins and all row vectors have the same margins in the biproportional bimarkovian filter, where it is not the case with the biproportional mean filter. A figure based on an Edgeworth box will illustrate it. Consider the matrices:

$$\mathbf{Z} = \begin{bmatrix} 5 & 5 \\ 4 & 1 \end{bmatrix} \begin{matrix} 10 \\ 5 \end{matrix} \quad \text{and} \quad \mathbf{Z}^* = \begin{bmatrix} 3 & 1 \\ 6 & 5 \end{bmatrix} \begin{matrix} 4 \\ 11 \end{matrix}$$
$$\begin{matrix} 9 & 6 \end{matrix} \qquad\qquad\qquad \begin{matrix} 9 & 6 \end{matrix}$$

so, $$K(\mathbf{Z}, \mathbf{Z}^*) = \begin{bmatrix} 1.42 & 2.58 \\ 7.58 & 3.42 \end{bmatrix} \text{ and } K(\mathbf{Z}^*, \mathbf{Z}) = \begin{bmatrix} 6.74 & 3.26 \\ 2.26 & 2.74 \end{bmatrix}$$

This matrix is represented by the following Edgeworth box (see figure 2), where the sides of the box correspond to the column constraints of matrix **Z**, the line *AB* corresponds to the row constraints of **Z** and where the point *z* corresponds to **Z**. For matrix $\mathbf{Z}^*$, column constraints are the same and row constraints become the line *CD*, when $\mathbf{Z}^*$ is represented by point z^*. The length of segment $\{K(z, z^*), z^*\}$, which corresponds to the variation by the direct projection, is closed to the length of segment $\{K(z^*, z), z\}$, which corresponds to the variation by the reverse projection. Consider another matrix $\mathbf{Z}_1$ with the same margins than **Z**:

$$\boldsymbol{Z}_1 = \begin{bmatrix} 8 & 2 \\ 1 & 4 \end{bmatrix} \begin{matrix} 10 \\ 5 \end{matrix}$$
$$\begin{matrix} 9 & 6 \end{matrix}$$

so, $$K(\boldsymbol{Z}_1,\ \boldsymbol{Z}^*) = \begin{bmatrix} 3.74 & 0.26 \\ 5.26 & 5.74 \end{bmatrix}$$

As $\mathbf{Z}$ and $\boldsymbol{Z}_1$ have the same margins, $K(z^*,\ z_1)$ is confused with $K(z^*,\ z)$. The segment $\{K(z_1,\ z^*),\ z^*\}$ is clearly shorter than the segment $\{K(z^*,\ z_1),\ z_1\}$. This is because the projection of z_1 is near the limit of the box: the orthogonal projection of z_1, found by an additive method, is even outside the limit of the box (it corresponds to negative terms in the projected matrix) and the ordinary biproportional projection corrects it.

Figure 2 here

Consider the matrix $\bar{\mathbf{Z}}$ represented by the segment *EF* in figure 3:

$$\bar{\boldsymbol{Z}} = \frac{1}{2}(\boldsymbol{Z} + \boldsymbol{Z}^*) = \begin{bmatrix} 4 & 3 \\ 5 & 3 \end{bmatrix} \begin{matrix} 7 \\ 8 \end{matrix}$$
$$\begin{matrix} 9 & 6 \end{matrix}$$

Then,

$$K(\boldsymbol{Z},\ \bar{\boldsymbol{Z}}) = \begin{bmatrix} 3.00 & 4.00 \\ 6.00 & 2.00 \end{bmatrix},\quad K(\boldsymbol{Z}^*,\ \bar{\boldsymbol{Z}}) = \begin{bmatrix} 5.00 & 2.00 \\ 4.00 & 4.00 \end{bmatrix},\quad K(\boldsymbol{Z}_1,\ \bar{\boldsymbol{Z}}) = \begin{bmatrix} 6.25 & 0.75 \\ 2.75 & 5.25 \end{bmatrix}$$

Figure 3 here

B. The biproportional bimarkovian filter

The biproportional bimarkovian filter could appear as a particular case of the biproportional mean filter because the matrix $\bar{\mathbf{Z}}$ is replaced by the bimarkovian matrix $\mathbf{1}^M$. Not only the effect of the differential growth of sectors will be removed without predetermining if the economy is demand or supply - driven as in the ordinary biproportional filter, not only the problem of the double result will be removed as in the biproportional mean filter, but the effect of differential size of sectors will be removed: after projection all sectors in column will have the same margin, i.e. the same size, and all sectors in column will have the same margin.

1. First step

Each matrix $\mathbf{Z}$ and $\mathbf{Z}^*$ is transformed into a bimarkovian matrix, $\mathbf{Z}^M$ and $\mathbf{Z}^{*M}$. A bimarkovian or binormalized matrix is a matrix of which all margins of both sides, column and row, are equal to 1: this is exactly possible only for square matrices. Any other number can be chosen, the important thing is that all margins of the same side would be equal. For rectangular matrices or dimension $(n,\ m)$ matrices, the margins of one side -- say the side of dimension n -- are equal to μ, and the margins of the other side are equal to λ: $\mathbf{1}^M\,\mathbf{s} = \mu\,\mathbf{s}$ and $\mathbf{s}'\,\mathbf{1}^M = \lambda\,\mathbf{s}'$. For example, one can take:

(1) $$\mathbf{1}^M = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{matrix} 1 \\ \cdots \\ 1 \end{matrix}$$
$$\frac{n}{m} \quad \cdots \quad \frac{n}{m}$$

or,

$$(2) \qquad \mathbf{1}^M = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{matrix} m \\ \dots \\ m \end{matrix}$$
$$n \;\dots\; n$$

As said before, there are many tools to project a matrix and the results may vary depending on the tool chosen to perform it; here biproportion is chosen: $\mathbf{Z}^M = K(\mathbf{Z},\ \mathbf{1}^M)$ and $\mathbf{Z}^{*M} = K(\mathbf{Z}^*,\ \mathbf{1}^M)$. In particular, with some methods like the minimization of the least squares between a matrix, $\mathbf{Z}$ or $\mathbf{Z}^*$, and the matrix $\mathbf{1}^M$, this may again create negative terms in $K(\mathbf{Z},\ \mathbf{1}^M)$. This is why the tool that we choose to perform binormalization is biproportion. With biproportion, one have $K(\mathbf{Z},\ \mathbf{1}^M) = \mathbf{P}\ \mathbf{Z}\ \mathbf{Q}$, and,

w with matrix $\mathbf{1}^M$ defined in (1):

$$(3) \qquad p_i = \frac{1}{\sum_{j=1}^{m} q_j\, z_{ij}}\text{, for all } i\text{, and } q_j = \frac{n}{m}\,\frac{1}{\sum_{i=1}^{n} p_i\, z_{ij}}\text{, for all } j$$

w or with the matrix $\mathbf{1}^M$ defined in (2):

$$(4) \qquad p_i = \frac{m}{\sum_{j=1}^{m} q_j\, z_{ij}}\text{, for all } i\text{, and } q_j = \frac{n}{\sum_{i=1}^{n} p_i\, z_{ij}}\text{, for all } j$$

The same reasoning holds for $\mathbf{Z}^*$. Note that the above expressions (3) and (4) are equivalent because a separable modification of $\mathbf{Z}$ (or $\mathbf{Z}^*$) is ineffective (Mesnard, 1994, 1997): if $\mathbf{Z}$ is replaced by $\tilde{\mathbf{Z}} = \quad \mathbf{Z}$, then $K(\tilde{\mathbf{Z}},\ \mathbf{1}^M) = K(\mathbf{Z},\ \mathbf{1}^M)$, and the exact form of the bimarkovian matrix has no importance.

2. Second step

$\mathbf{Z}^M$ is compared to $\mathbf{Z}^{*M}$, i.e. the Frobenius norm of the difference matrix $\mathbf{Z}^{*M} - \mathbf{Z}^M$ is calculated. The rest of the method is similar to the biproportional mean filter, except that the absolute variations and relative variations are homothetical: with a bimarkovian matrix like $\mathbf{1}^M$ in (2), one divide by m for columns and by n for rows the absolute variations when calculating relative variations. So it is sufficient to construct only relative variations.

w For one single coefficient: $\sigma_{ij} = \left| z_{ij}^{*M} - z_{ij}^{M} \right|$.

Remark. In this case, the real value of the term $\{i, j\}$ in the matrix $\mathbf{1}^M$ is completely arbitrary and does not play a role in the calculation (even if matrices are square). But, for simplicity, according to Bernoulli-Laplace, it seems logical to consider that all terms inside are equal:

$$\mathbf{1}^M = \begin{bmatrix} 1 & \dots & 1 \\ .. & & .. \\ 1 & \dots & 1 \end{bmatrix} \begin{matrix} m \\ \dots \\ m \end{matrix}$$
$$\begin{matrix} n & \dots & n \end{matrix}$$

so, the absolute variability of an individual cell is divided by 1. x

w For demanding sectors (i.e. for column vectors): $\sigma_j = \frac{1}{n} \sqrt{\sum_i \left(z_{ij}^{*M} - z_{ij}^{M} \right)^2}$.

w For supplying sectors (i.e. for row vectors): $\sigma_i = \frac{1}{m} \sqrt{\sum_j \left(z_{ij}^{*M} - z_{ij}^{M} \right)^2}$.

w For the whole economy: $\sigma = \frac{1}{n\,m} \sqrt{\sum_i \sum_j \left(z_{ij}^{*M} - z_{ij}^{M} \right)^2}$.

Remark. The influence of a multiplicative parameter λ is neutral; for example:

$$\sigma(\lambda) = \frac{1}{n\,m\,\lambda}\sqrt{\sum_i \sum_j \left(\lambda\, z_{ij}^{*M} - \lambda\, z_{ij}^{M}\right)^2} = \sigma$$

so, the choice between forms (1) or (2) for matrices $\mathbf{1}^M$ is itself neutral. x

One can note that, even if in both methods two biproportional projections are required and so the amount of computations remains similar, there is only one calculation of relative variabilities (with simpler computations) and no computations of absolute variabilities in the biproportional bimarkovian filter.

C. Example

$$\mathbf{Z} = \begin{bmatrix} 5 & 5 & 6 \\ 4 & 1 & 3 \\ 3 & 4 & 5 \end{bmatrix} \begin{matrix} 16 \\ 8 \\ 12 \end{matrix} \quad \text{and} \quad \mathbf{Z}^* = \begin{bmatrix} 2 & 3 & 8 \\ 6 & 1 & 4 \\ 1 & 2 & 6 \end{bmatrix} \begin{matrix} 13 \\ 11 \\ 9 \end{matrix}$$
$$\begin{matrix} 12 & 10 & 14 \end{matrix} \qquad\qquad \begin{matrix} 9 & 6 & 18 \end{matrix}$$

1) Ordinary biproportional filter.

$$K(\mathbf{Z},\ \mathbf{Z}^*) = \begin{bmatrix} 3.124 & 2.920 & 6.956 \\ 4.190 & 0.979 & 5.831 \\ 1.686 & 2.100 & 5.213 \end{bmatrix} \text{and } K(\mathbf{Z}^*,\ \mathbf{Z}) = \begin{bmatrix} 4.056 & 5.228 & 6.716 \\ 5.637 & 0.807 & 1.555 \\ 2.307 & 3.964 & 5.729 \end{bmatrix}$$

w For the direct projection, the results between $K(\mathbf{Z},\ \mathbf{Z}^*)$ and $\mathbf{Z}^*$ are respectively the following with relative variabilities. Overall: 9.63%; for rows: 11.82 %, 23.41 %, 11.65 %; for columns: 24.87 %, 2.17 %, 12.50 % .

w For the reverse projection, the relative variabilities between $K(\mathbf{Z}^*, \mathbf{Z})$ and $\mathbf{Z}$ are the following. Overall: 7.49 %; for rows: 7.54 %, 27.39 %, 8.39 %; for columns: 16.77 %, 3.01 %, 12.64 % .

2) Biproportional mean filter.

$$\bar{\mathbf{Z}} = \frac{1}{2}\,(\mathbf{Z}+\mathbf{Z}^{*}) = \begin{bmatrix} 3.5 & 4 & 7 \\ 5 & 1 & 3.5 \\ 2 & 3 & 5.5 \end{bmatrix} \begin{matrix} 14.5 \\ 9.5 \\ 10.5 \end{matrix}$$
$$\begin{matrix} 10.5 & 8 & 16 \end{matrix}$$

then $K(\mathbf{Z},\ \bar{\mathbf{Z}}) = \begin{bmatrix} 4.011 & 3.953 & 6.536 \\ 4.195 & 1.033 & 4.272 \\ 2.294 & 3.014 & 5.192 \end{bmatrix}$ and $K(\mathbf{Z}^{*},\ \bar{\mathbf{Z}}) = \begin{bmatrix} 2.925 & 4.119 & 7.456 \\ 6.008 & 0.940 & 2.552 \\ 1.567 & 2.942 & 5.991 \end{bmatrix}$

and the relative variations between $K(\mathbf{Z},\ \bar{\mathbf{Z}})$ and $K(\mathbf{Z}^{*},\ \bar{\mathbf{Z}})$ are the following. Overall: 8.92 %; for rows: 9.88 %, 26.32 %, 10.32 %; for columns: 21.29 %, 2.55 %, 13.17 % .

3) Biproportional bimarkovian filter.

$$\mathbf{Z}^{M} = \begin{bmatrix} 0.853 & 1.204 & 0.943 \\ 1.468 & 0.518 & 1.014 \\ 0.679 & 1.278 & 1.043 \end{bmatrix} \begin{matrix} 3 \\ 3 \\ 3 \end{matrix} \quad \text{and} \quad \mathbf{Z}^{*M} = \begin{bmatrix} 0.604 & 1.271 & 1.126 \\ 1.942 & 0.454 & 0.604 \\ 0.454 & 1.275 & 1.271 \end{bmatrix} \begin{matrix} 3 \\ 3 \\ 3 \end{matrix}$$
$$\begin{matrix} 3 & 3 & 3 \end{matrix} \qquad\qquad \begin{matrix} 3 & 3 & 3 \end{matrix}$$

The results between $\mathbf{Z}^{M}$ and $\mathbf{Z}^{*M}$ are the following with relative variabilities. Overall: 8.61 %; for rows: 10.54 %, 21.02 %, 10.67 %.; for columns: 19.37 %, 3.08 %, 16.80 % . As it can be seen, the bimarkovian filter may distort strongly the data. For example, z_{13} is twice z_{23} when z_{13}^{M} is approximately equal to z_{23}^{M}. This could seem to violate the good sense, but one must keep in mind that both $\mathbf{Z}$ and $\mathbf{Z}^{*}$ are simultaneously projected, and eventually distorted. Remember also that the more the point -- $\mathbf{Z}$ or $\mathbf{Z}^{*}$ -- is near the limits of the Edgeworth box, the more the data are distorted: this distortion occurs only to avoid to have negative points.

IV. Applications: Input-output table for France

The two new filters will be compared to the ordinary biproportional filter by an application based on data for France for the period 1980-1996. These two tables, (INSEE, various years), used in their original form, not aggregated, are given in the base of 1980; the deflation used is those made by the INSEE itself: all tables are at the prices of 1980, so price effects are removed. Only the intermediate block of tables is used.

Biproportions are calculated after only 30 iterations because convergence is fast. Remember that, if the percentages of variation obtained with all methods can be compared (both provide relative variations), there are two ways of projection in the ordinary biproportional filter and only one the two other filters: one is obliged to synthesize these results, direct and reverse, in a completely empirical way, by computing the average of these two. The overall change is indicated in table 1. The biproportional bimarkovian filter has the lower overall relative variability, so it appears to be the more satisfactory because this indicates that the measure distorts the less the data globally (but not always locally: see the example): the measure tries to capture how the structure has changed, so, among many indicators, the better is the one that exhibits the lower overall variability.

Table 1 here

In the two following tables, the results for the biproportional bimarkovian filter will be presented in a first column, a second column will contain the results of the biproportional mean filter, a third and fourth column will present the results for the ordinary biproportional filter for direct and reverse computations and a last column will indicate the average of column two and three. Table 2 presents results for column sectors and table 3 for row sectors. Just for a help in reading the tables, higher values for relative variations (more than 10%) are indicated in bold for the biproportional bimarkovian filter, the biproportional mean filter and for the average of the ordinary biproportional filter.

With all methods, the main result is the overwhelming role of sector T37 (*Financial Services*), for both column and row vectors. This is caused by the strong development of exchanges between financial institutions, which can appear partially artificial because only balances are really exchanged each month. This is why in the future reform of the French national accounting system, only these balances will be taken into account. However a discussion remains concerning these phenomenons. For other sectors, the results are the following with the biproportional bimarkovian filter:

- For columns: T32 (*Telecommunications and Mail*) and T06 (*Electricity, Gas and Water*), two monopolistic or oligopolistic, sectors are largely changing; also T36 (*Insurance*), T38 (*Non Marketable Services*), T22 (*Printing and Publishing*), T34 (*Marketable Services to Private Individuals*), T17 (*Shipping, Aircrafts And Arms*) are significantly changing (by more than 10%). Remember that change in columns reflects change in the production function.

- For rows: T32 (*Telecommunications and Mail*), T15B (*Household Appliances*), T17 (*Shipping, Aircrafts And Arms*), T24 (*Building Trade, Civil and Agricultural Engineering*), T08 (*Ores and non Ferrous Metals*), T29 (*Automobile Trade and Repair Services*), T35 (*Housing Rental and Leasing*), T22 (*Printing and Publishing*). Remember that change in rows reflects change in the distribution function.

The biproportional mean filter and the average of the ordinary biproportional filter provide very similar results. When one switches from the biproportional bimarkovian filter to the biproportional mean filter or to the average of the ordinary biproportional filter,

- One must remove to the list of highly variable column vectors: T38 (*Non Marketable Services*) and T22 (*Printing and Publishing*), but one must add T15B (*Household Appliances*), T24 (*Building Trade, Civil and Agricultural Engineering*), T25 (*Trade*), T29 (*Automobile Trade and Repair Services*) and T35 (*Housing Rental and Leasing*).

- One must add to the list of highly variable row vectors: T12 (*Miscellaneous Chemicals, Pharmaceuticals*), T21 (*Paper and Cardboard*), T22 (*Printing and Publishing*), T33 (*Business Services*), T34 (*Marketable Services to Private Individuals*).

Table 2 here

Table 3 here

Remark. Some large differences between direct and reverse projections can be noted with the ordinary biproportional filter (these cases are indicated in italics in the tables 2 and 3),

- For columns: T10 (*Glass*), T13 (*Smelting Works, Metal Works*), T14 (*Mechanical Engineering*), T15A (*Electric Industrial Equipment*), T18 (*Textile Industry, Clothing Industry*), T21 (*Paper and Cardboard*), T24 (*Building Trade, Civil and Agricultural Engineering*), T25 (*Trade*), T29 (*Automobile Trade and Repair Services*), T34 (*Marketable Services to Private Individuals*).

- For rows: T21 (*Paper and Cardboard*), T22 (*Printing and Publishing*), T24 (*Building Trade, Civil and Agricultural Engineering*), T29 (*Automobile Trade and Repair Services*), T32 (*Telecommunications and Mail*), T33 (*Business Services*), T34 (*Marketable Services to Private Individuals*), T35 (*Housing Rental and Leasing*). x

All this can be summarized in two series of figures, one for columns and one for rows: in each figure, the results of one of the three methods -- bimarkovian filter, mean filter and the average between direct and reverse ordinary filter -- are compared to the results of one of the two remaining methods. When points are along the first diagonal, both methods provide the same result. When points are to the right of the diagonal, the variation found with the filter in the X-axis is higher than the variation found with the filter in Y-axis, and conversely when points are to the left of the diagonal; for example, with T37 in figure 5, the biproportional bimarkovian filter gives a higher result than the biproportional mean filter when it is the inverse for T06 or T32 in the same figure.

As it can be seen in figures 5, 6, 8 and 9, more than half the number of sectors is above the diagonal when comparing the bimarkovian filter to the other methods: often the biproportional bimarkovian filter provides lower estimation for relative variabilities than the ordinary biproportional filter.

In figure 4, when the biproportional mean filter is compared to the ordinary biproportional filter for columns, points seem to be correctly aligned along the first diagonal: ordinary biproportional filter and biproportional mean filter provide similar results.

Figure 4 here

In figure 5, when comparing the biproportional bimarkovian filter to the biproportional mean filter for columns, except for point T37 (*Financial Services*) which is to the right of the first diagonal, most points are to the left.

Figure 5 here

Figure 6, that compares the biproportional bimarkovian filter to the ordinary biproportional filter for columns, is very similar to figure 4: this is normal because points are along the diagonal in figure 3. The conclusion is that, for rows, the bimarkovian filter provides results that differ from those of the ordinary filter and the mean filter.

Figure 6 here

In figure 7, when comparing the biproportional mean filter to the ordinary biproportional filter for rows, the points are along the diagonal but less than perfectly than for figure 4: some points are to the right of the first diagonal, for example T24 (*Building Trade, Civil and Agricultural Engineering*), T29 (*Automobile Trade and Repair Services*) and T32 (*Telecommunications and Mail*), T35 (*Housing Rental and Leasing*), but T37 (*Financial Services*) is far to the left. The mean filter and the average of direct and reverse filters for rows do not give results so similar than for columns, even if they remain globally similar as indicated by table 3.

Figure 7 here

In figure 8, when the biproportional bimarkovian filter is compared to the biproportional mean filter for rows, almost all points are to the left of the first diagonal for high values as T32 (*Telecommunications and Mail*), T24 (*Building Trade, Civil and Agricultural Engineering*), T29 (*Automobile Trade and Repair Services*), T35 (*Housing Rental and Leasing*), etc., except for T37 (*Financial Services*).

Figure 8 here

Figure 9, that compares the biproportional bimarkovian filter to the ordinary biproportional filter for rows, is similar to figure 8, but in a less clear way than for figures 6 and 5. Again, this signifies that the bimarkovian filter differs the more to the two other methods, even if, in this application, it is less clear for rows than for columns.

Figure 9 here

V. Conclusion

In this paper, two methods heve been proposed to perform the analysis of structural change by avoiding the above difficulties. Only one set of results is obtained by choosing to project on only one matrix $\bar{\mathbf{Z}}$, the mean of flow matrices $\mathbf{Z}$ and $\mathbf{Z}^*$: the method has been called the *biproportional mean filter*. As for the ordinary biproportional filter, the advantage is that the orientation of the economy is not predetermined but the additional advantage is that one have not of two different sets of results; however the problem of the size differences of sectors has not been removed. So a second method has been introduced as a derivation of the first, but it appears also as a **direct** generalization of the simple comparison of two coefficient matrices $\mathbf{A}$

and $\mathbf{A}^*$ (or two Markovian matrices). This second method has been called the *biproportional bimarkovian filter* because it is based on the transformation of both matrices **Z** and $\mathbf{Z}^*$ into bimarkovian matrices by a biproportion.

In the *ordinary biproportional filter,* one matrix **Z** (respectively $\mathbf{Z}^*$) is projected on the margins of the another matrix $\mathbf{Z}^*$ (respectively **Z**) by the mean of biproportion and then the projected matrix is compared to $\mathbf{Z}^*$ (respectively **Z**). This method avoids to remove the differential effects of sectors without predetermining if the model is demand-driven or supply-driven, but the two different results, **Z** on $\mathbf{Z}^*$ or $\mathbf{Z}^*$ on **Z**, are able to be strongly diverging.

To avoid this difficulty, a first generalization is the *biproportional mean filter* that provides results that are close to those of the ordinary biproportional filter. However, as in the previous method, the differences of sizes of sectors are not removed naturally, but they are removed only acrobatically by a calculation of relative variabilities along with absolute variabilities.

In the *biproportional bimarkovian filter*, both matrices **Z** and $\mathbf{Z}^*$ are transformed into bimarkovian matrices by biproportion and then these two transformed matrices are compared. This method also avoids all difficulties: it does not predetermine if the model is demand or supply driven by removing the effect of differential growth of sectors -- as all biproportional filtering method -- and it does it with only a single result as the biproportional mean filter. However, it adds a considerable advantage: the differences of sizes of sectors are removed what is more satisfying.

Results for France for 1980-1996 depend, naturally, on the method chosen but, along with its theoretical advantages, as the biproportional bimarkovian filter has the lower overall variation, it seems to be the best.

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	Bimarkovian	Biproportional mean	Biprop. direct	Biprop. reverse	Average direct + inverse
Overall change	2.20	5.30	6.61	1.23	3.92

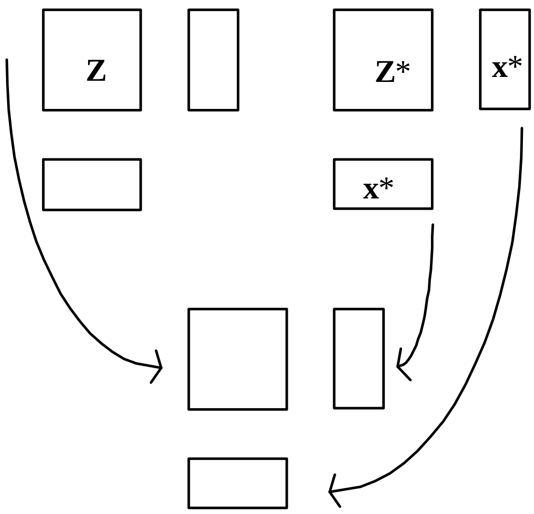
Table 1. Comparison of methods for France, overall change, in %

Sectors	Bimark.	Biprop. mean	Biprop. direct	Biprop. reverse	Average direct + reverse
T01 Farming, Forestry, Fishing	7,75	5,22	5,55	4,67	5,11
T02 Meat and Milk Products	5,73	3,94	4,53	3,60	4,07
T03 Other Agricultural and Food Products	4,99	7,05	8,57	5,34	6,96
T04 Solid Fuels	3,92	5,46	7,03	3,79	5,41
T05 Oil Products, Natural Gas	4,83	3,45	4,86	1,90	3,38
T06 Electricity, Gas and Water	**18,55**	**29,60**	28,45	30,97	**29,71**
T07 Ores and Ferrous Metals	4,23	5,74	7,49	4,24	5,87
T08 Ores and non Ferrous Metals	3,69	4,20	5,08	2,41	3,75
T09 Building Materials, Misc. Minerals	5,32	8,60	11,10	6,15	8,63
T10 Glass	3,60	9,05	*12,22*	*2,69*	7,46
T11 Basic Chemicals, Synthesized Fibers	6,48	7,78	9,42	5,05	7,23
T12 Miscellaneous Chemicals, Pharmaceuticals	3,95	6,92	9,05	3,91	6,48
T13 Smelting Works, Metal Works	3,76	7,48	*10,07*	*2,78*	6,43
T14 Mechanical Engineering	3,04	7,41	*10,23*	*1,51*	5,87
T15A Electric Industrial Equipment	5,60	8,95	*11,24*	*3,95*	7,60
T15B Domestic Equipment Goods for Households	8,48	**11,14**	11,42	11,43	**11,43**
T16 Motor Cars for Land Transport	3,08	7,70	9,52	4,44	6,98
T17 Shipping, Aircrafts And Arms	**10,38**	**13,71**	14,53	11,56	**13,05**
T18 Textile Industry, Clothing Industry	3,24	7,27	*9,88*	*2,22*	6,05
T19 Leather and Shoe Industries	5,15	8,15	8,83	7,80	8,32
T20 Wood, Furnitures, Varied Industries	2,63	4,31	5,65	2,34	4,00
T21 Paper and Cardboard	2,49	8,69	*11,05*	*2,97*	7,01
T22 Printing and Publishing	**13,21**	6,67	8,72	3,24	5,98
T23 Rubber, Transformation of Plastics	5,97	5,72	6,83	4,48	5,66
T24 Building Trade, Civil and Agric. Eng.	6,52	**16,53**	*21,51*	*2,92*	**12,22**
T25 Trade	3,48	**11,55**	*15,28*	*3,12*	9,20
T29 Automobile Trade and Repair Services	6,41	**20,19**	*25,91*	*4,74*	**15,33**
T30 Hotels, Cafés, Restaurant	3,11	4,65	5,94	2,97	4,46
T31 Transports	5,23	7,76	9,68	5,03	7,36
T32 Telecommunications and Mail	**19,18**	**28,71**	31,49	21,73	**26,61**
T33 Business Services	2,77	4,34	5,63	2,80	4,22
T34 Marketable Services to Private Individuals	**13,18**	**15,44**	*20,07*	*4,28*	**12,18**
T35 Housing Rental and Leasing	3,95	**14,41**	14,32	11,91	**13,12**
T36 Insurance	**14,21**	**14,55**	19,19	9,91	**14,55**
T37 Financial Services	**63,72**	**36,38**	29,97	49,56	**39,77**
T38 Non Marketable Services	**13,92**	7,38	7,84	6,00	6,92

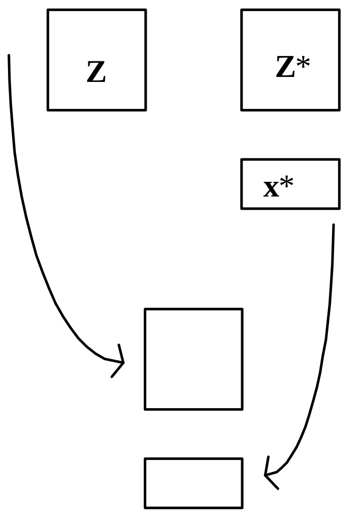
Table 2. Comparison of methods for France, column vectors, in %

Sectors	Bimark.	Biprop. mean	Biprop. direct	Biprop. reverse	Average direct + reverse
T01 Farming, Forestry, Fishing	5,75	3,18	3,35	3,01	3,18
T02 Meat and Milk Products	5,75	5,61	5,61	5,69	5,65
T03 Other Agricultural and Food Products	5,86	7,40	7,01	7,94	7,48
T04 Solid Fuels	9,20	**14,50**	18,11	10,00	**14,06**
T05 Oil Products, Natural Gas	8,20	5,35	6,17	4,37	5,27
T06 Electricity, Gas and Water	6,54	6,21	7,46	5,00	6,23
T07 Ores and Ferrous Metals	3,48	3,00	3,38	2,94	3,16
T08 Ores and non Ferrous Metals	**11,26**	**12,15**	13,05	11,13	**12,09**
T09 Building Materials, Misc. Minerals	6,10	5,21	6,42	3,06	4,74
T10 Glass	2,82	2,73	3,08	1,78	2,43
T11 Basic Chemicals, Synthesized Fibers	2,23	2,65	2,69	2,41	2,55
T12 Miscellaneous Chemicals, Pharmaceuticals	6,74	9,49	9,17	11,22	**10,20**
T13 Smelting Works, Metal Works	4,05	2,53	2,86	1,83	2,35
T14 Mechanical Engineering	3,71	5,44	5,94	3,93	4,94
T15A Electric Industrial Equipment	7,94	5,14	5,83	3,65	4,74
T15B Domestic Equipment Goods for Households	**17,33**	**19,28**	19,43	20,48	**19,96**
T16 Motor Cars for Land Transport	3,23	2,95	3,29	1,99	2,64
T17 Shipping, Aircrafts And Arms	**14,78**	**20,82**	21,21	18,51	**19,86**
T18 Textile Industry, Clothing Industry	3,62	2,00	2,40	1,62	2,01
T19 Leather and Shoe Industries	3,44	4,47	4,02	5,27	4,65
T20 Wood, Furnitures, Varied Industries	4,88	5,51	7,45	3,11	5,28
T21 Paper and Cardboard	3,53	**14,29**	*17,62*	*3,05*	**10,34**
T22 Printing and Publishing	9,24	**12,66**	*15,32*	*3,00*	9,16
T23 Rubber, Transformation of Plastics	4,93	3,96	4,06	3,88	3,97
T24 Building Trade, Civil and Agric. Eng.	**14,62**	**37,23**	*42,26*	*8,10*	**25,18**
T29 Automobile Trade and Repair Services	**10,69**	**32,14**	*38,45*	*6,96*	**22,71**
T30 Hotels, Cafés, Restaurant	2,60	3,12	4,16	2,15	3,16
T31 Transports	3,43	4,13	5,24	2,30	3,77
T32 Telecommunications and Mail	**20,25**	**32,21**	*37,44*	*8,35*	**22,90**
T33 Business Services	6,86	**13,64**	*17,14*	*3,37*	**10,26**
T34 Marketable Services to Private Individuals	4,13	**15,79**	*19,12*	*5,80*	**12,46**
T35 Housing Rental and Leasing	**10,30**	**23,37**	*27,61*	*8,70*	**18,16**
T36 Insurance	6,00	6,96	6,95	7,19	7,07
T37 Financial Services	**57,65**	**35,04**	*28,22*	*68,79*	**48,51**

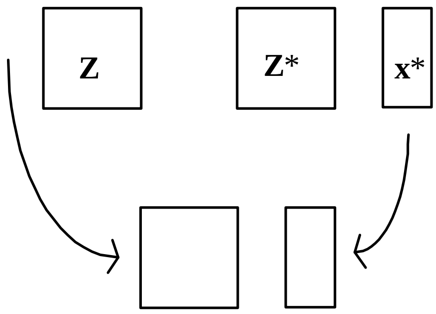
Table 3. Comparison of methods for France, row vectors for France, in %



Comparison of both coefficients
by a biproportion



Comparison of technical
coefficients by a proportion



Comparison of allocation
coefficients by a proportion

Figure 1. Principle of matrix comparisons over time.

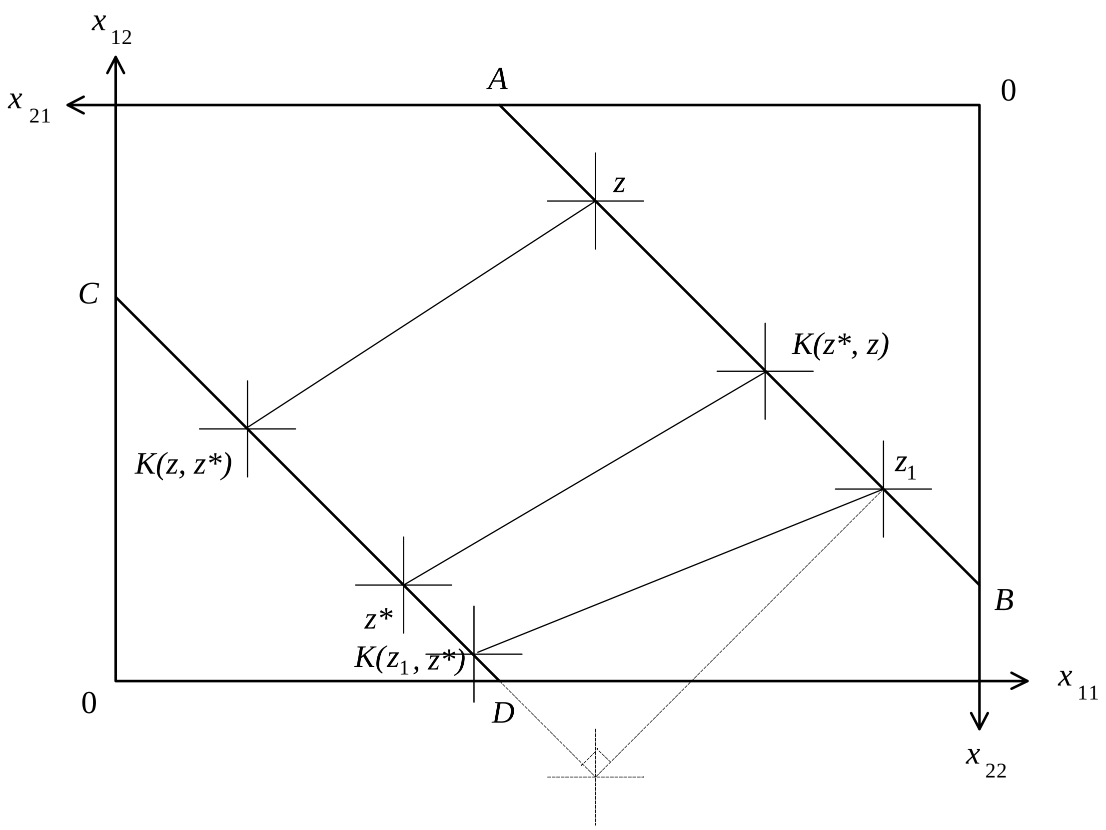


Figure 1. Edgeworth box for the ordinary biproportional projector

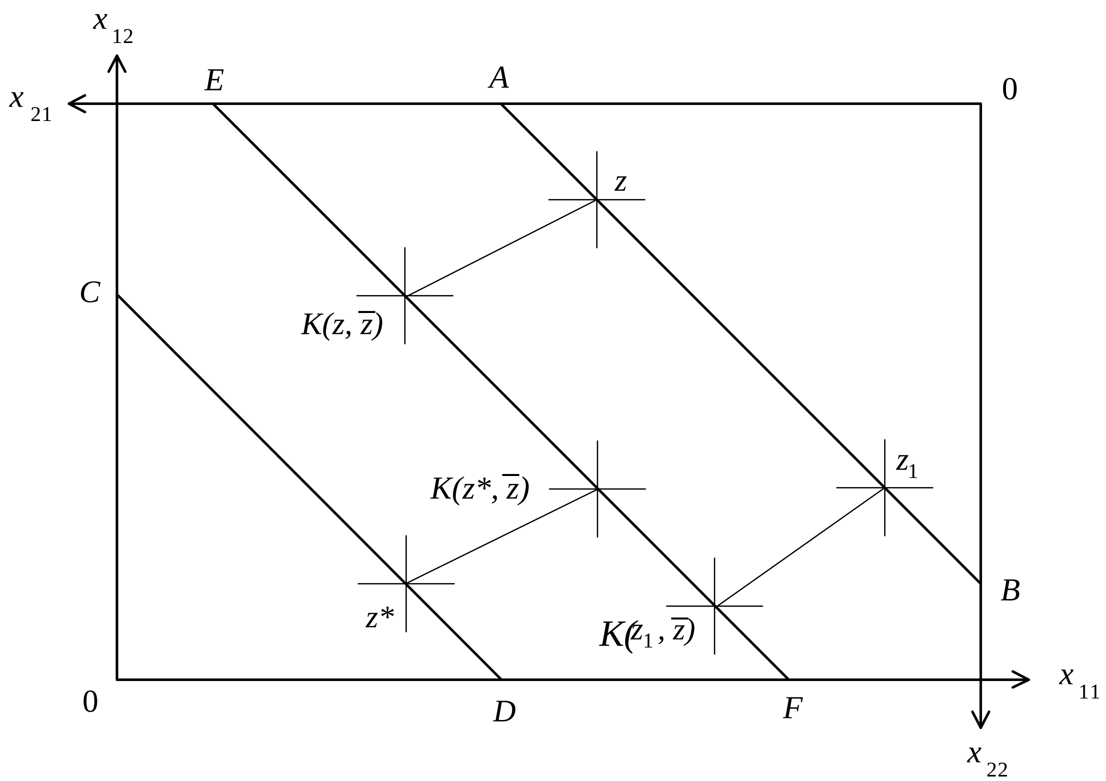


Figure 2. Edgeworth box for the single-base biproportional projector

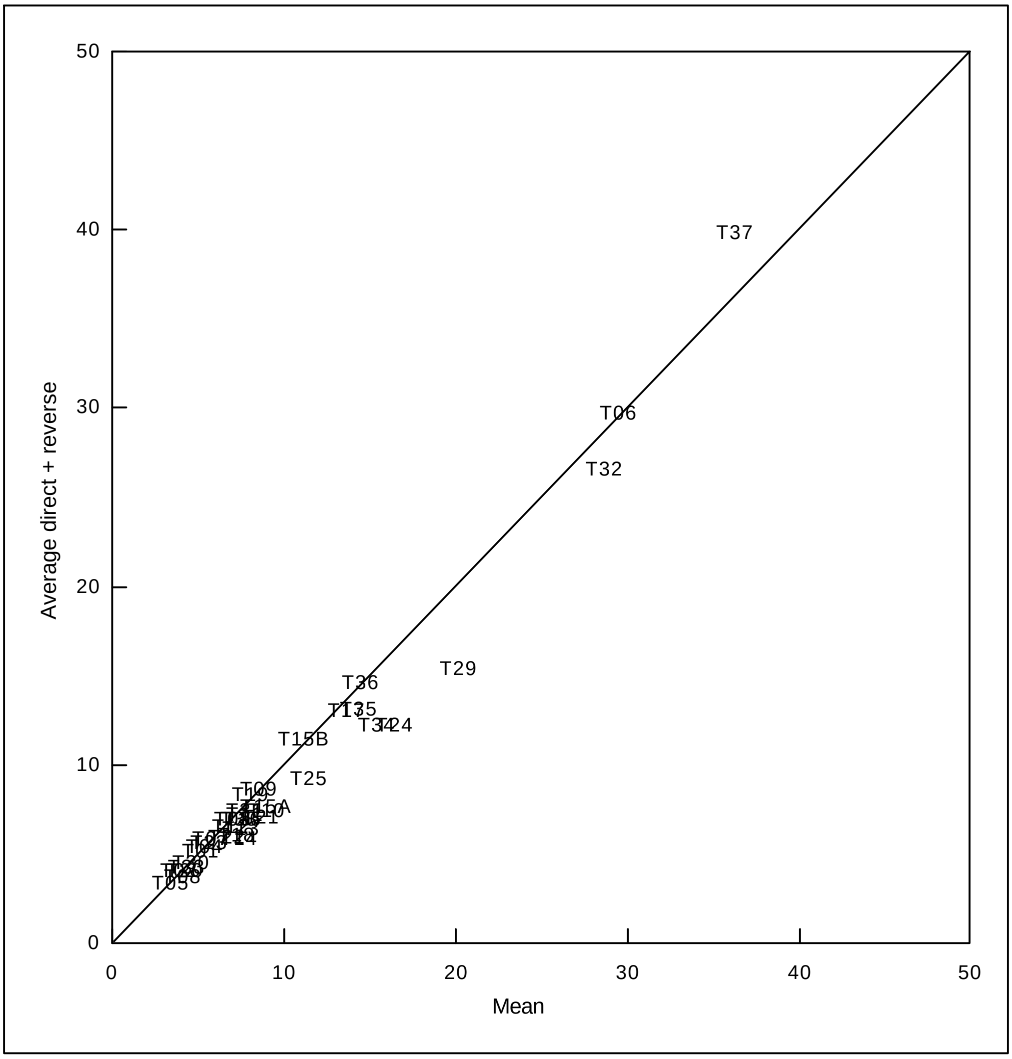


Figure 3. Comparison of methods: mean Vs average direct + reverse, for columns

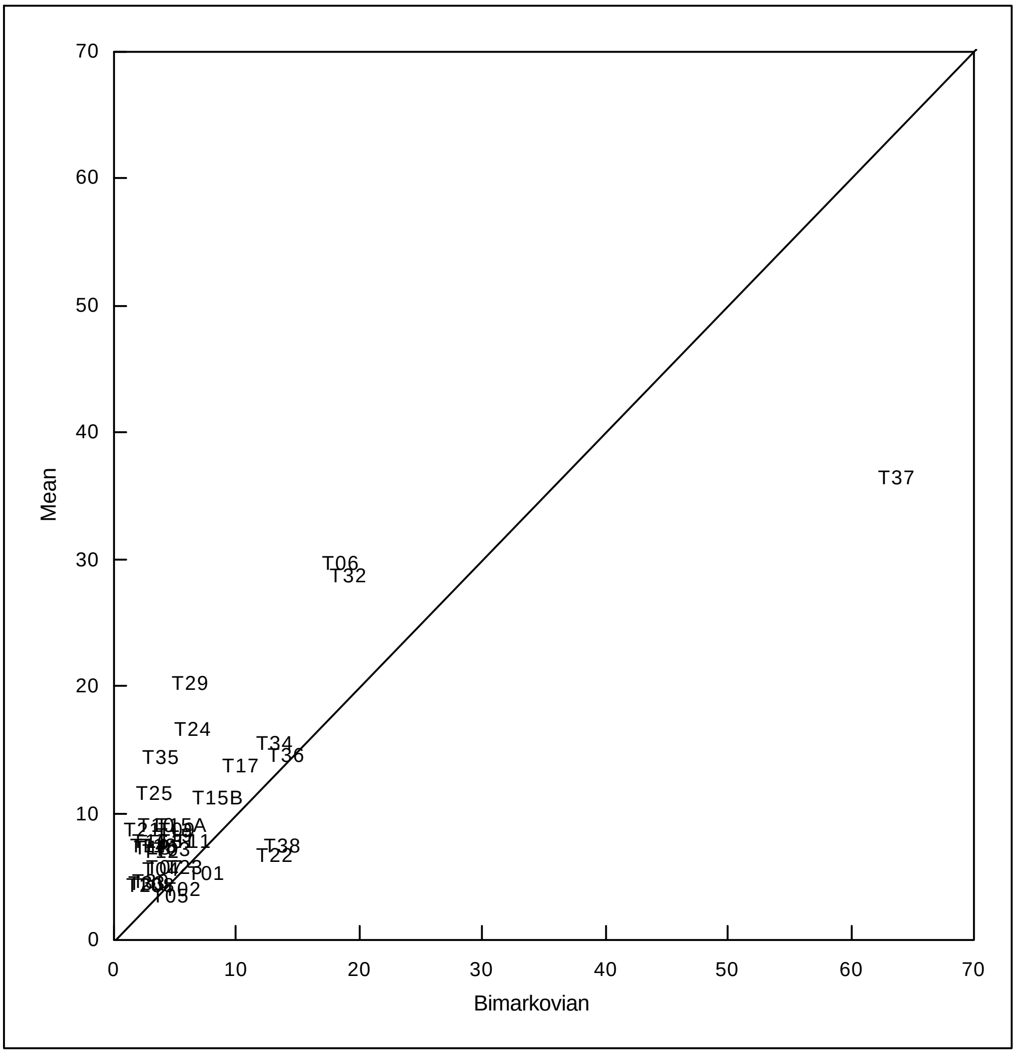


Figure 4. Comparison of methods: bimarkovian Vs mean, for columns

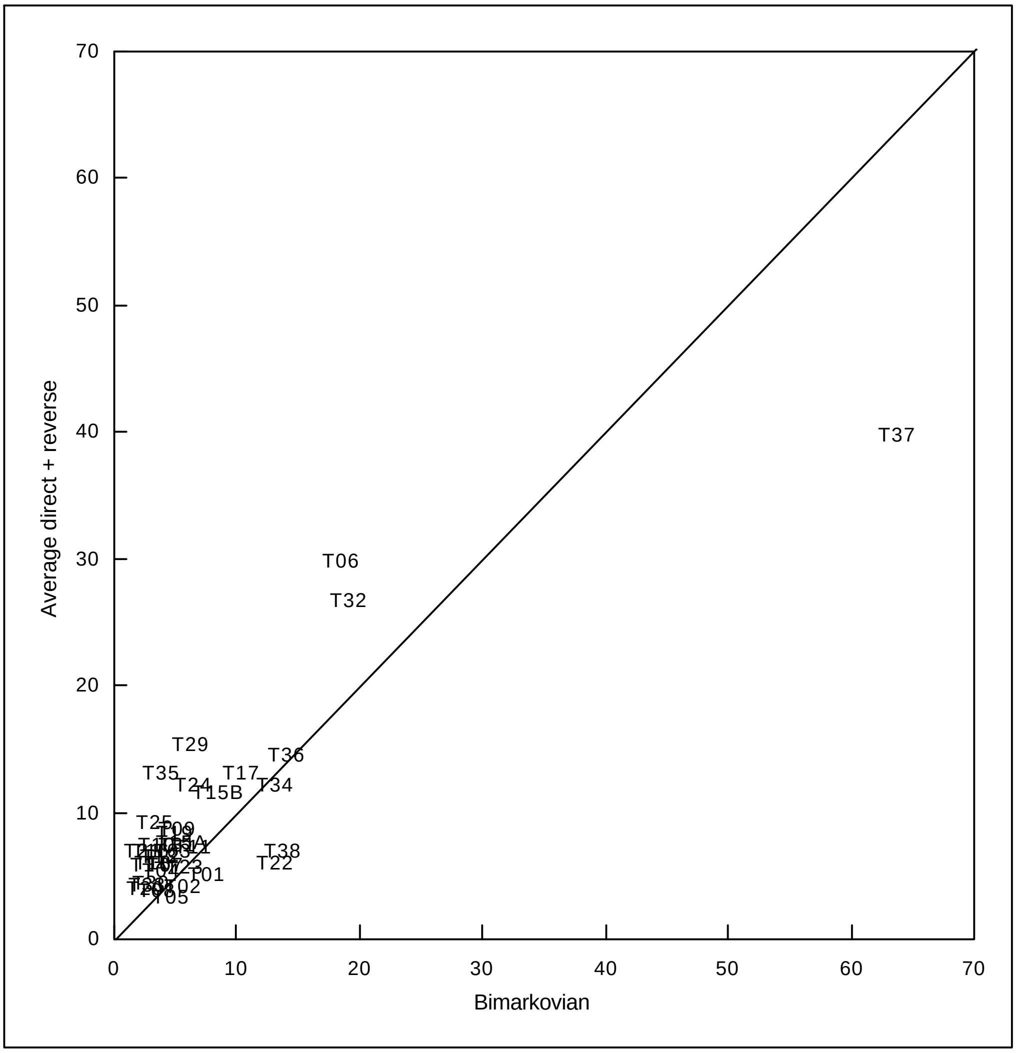


Figure 5. Comparison of methods: Bimarkovian Vs average direct + reverse, for columns

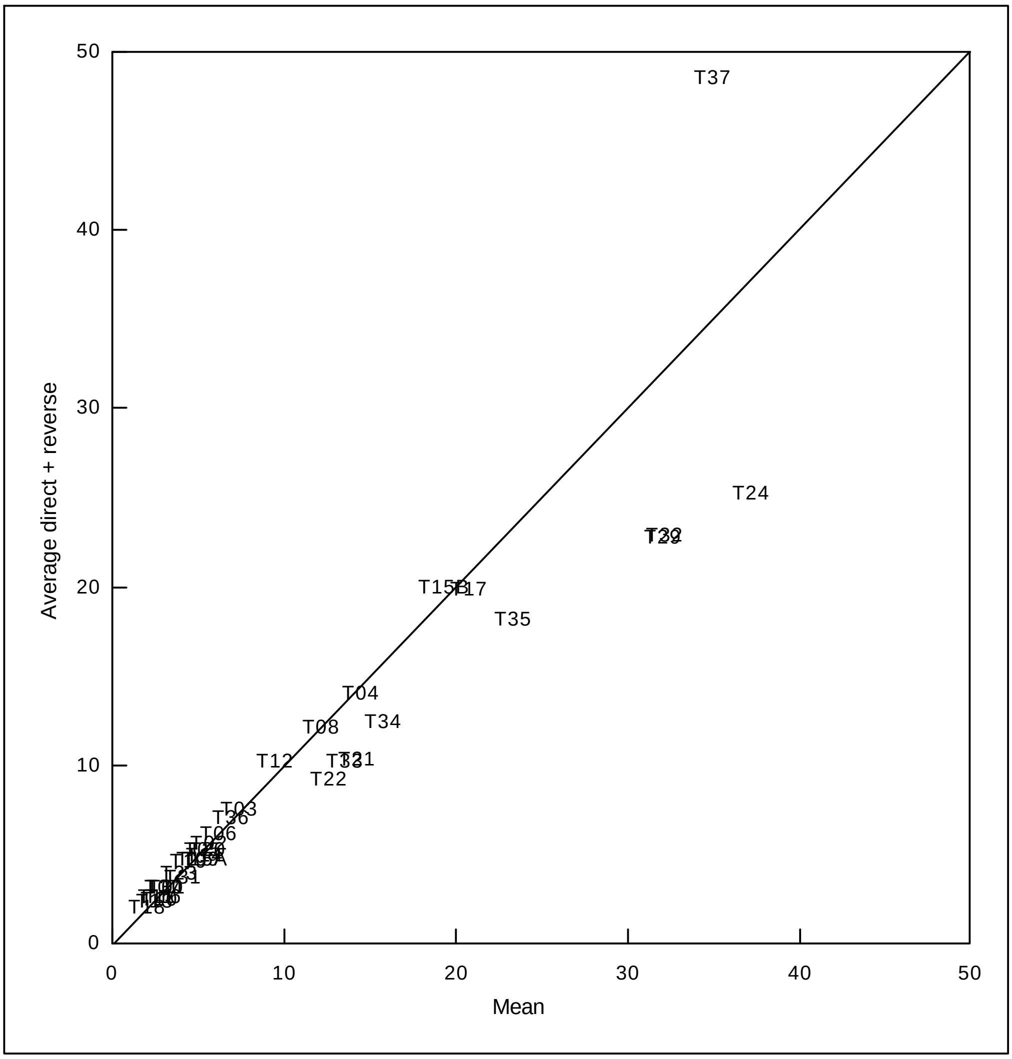


Figure 6. Comparison of methods: mean Vs average direct + reverse, for rows

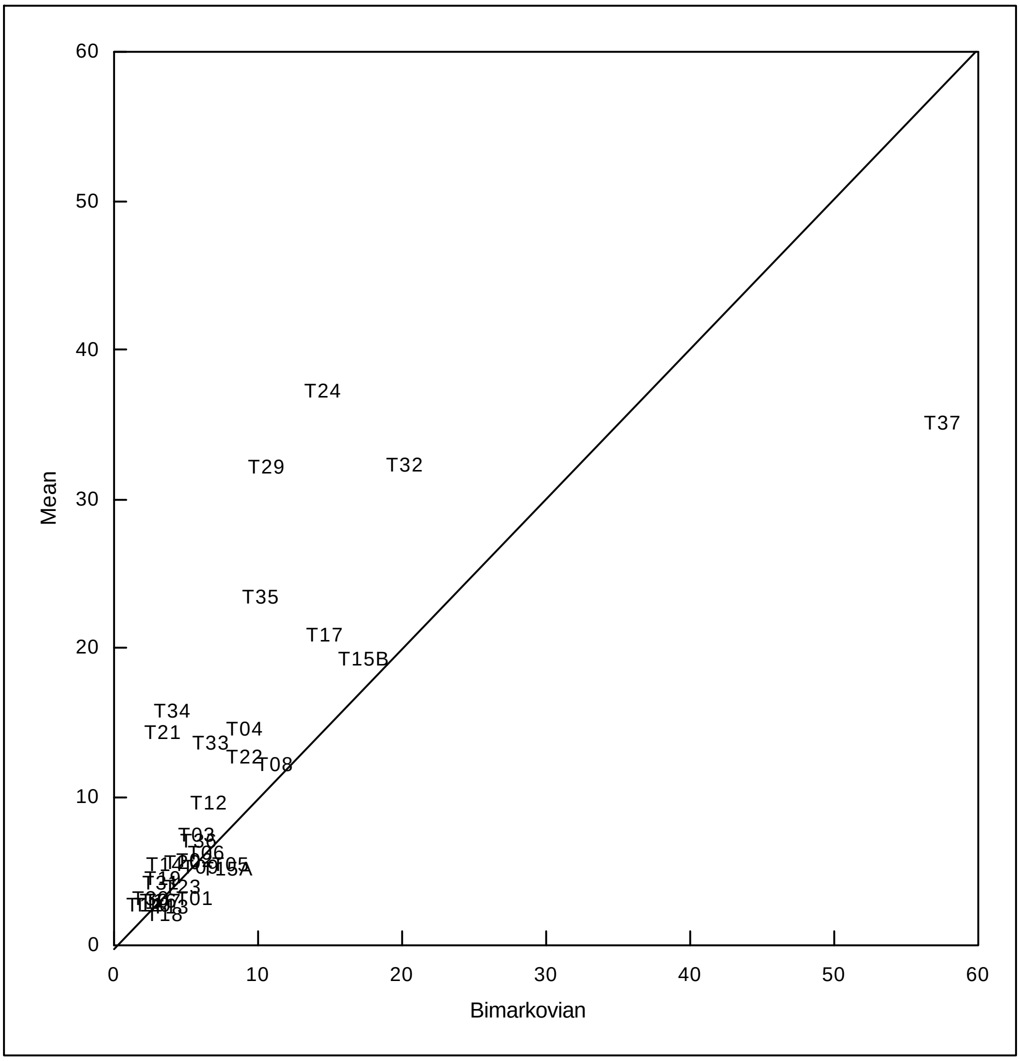


Figure 7. Comparison of methods: bimarkovian Vs mean, for rows

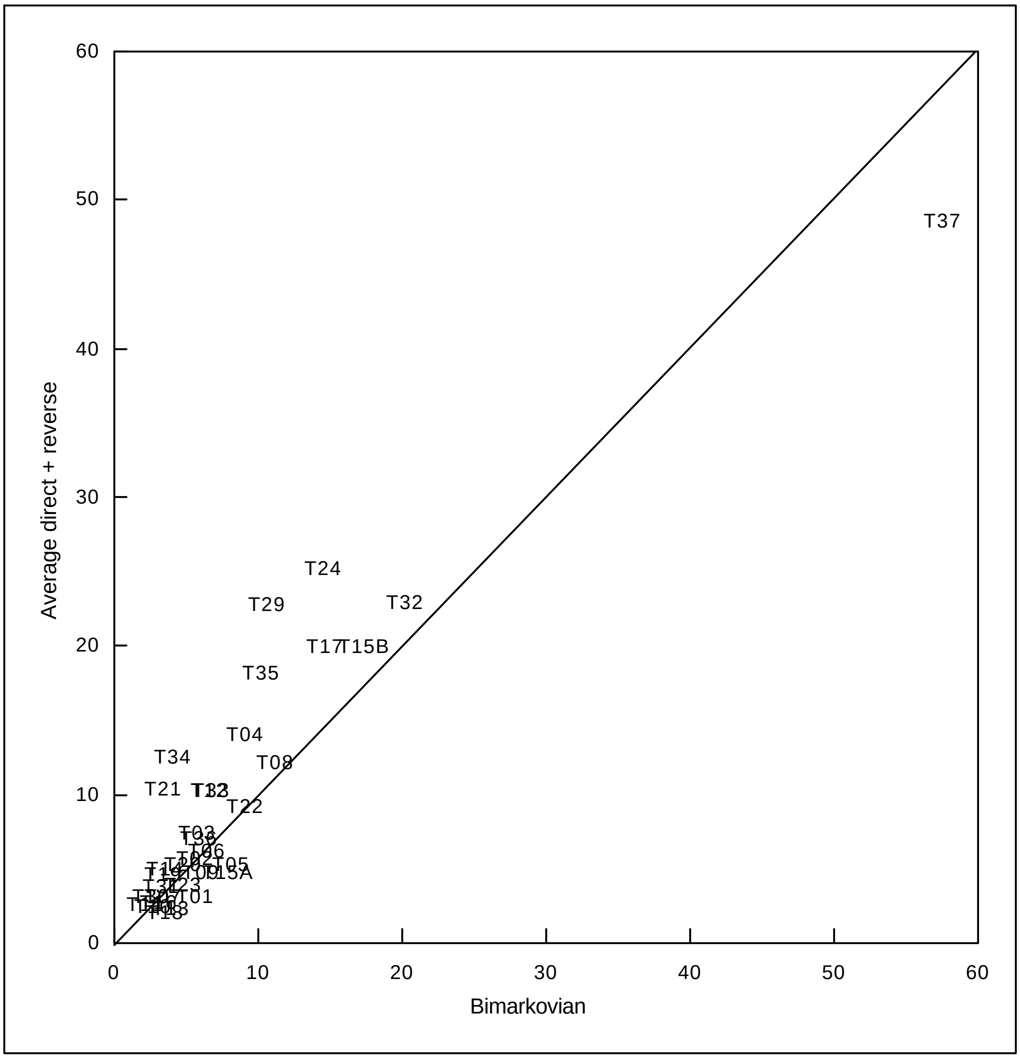


Figure 8. Comparison of methods: Bimarkovian Vs average direct + reverse, for rows