

# OPTIMAL ENDOWMENTS OF PUBLIC INVESTMENT: AN EMPIRICAL ANALYSIS FOR THE SPANISH REGIONS\*

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## Abstract

In this paper we try to find some evidence on the optimality in the provision of government capital in the Spanish regions, during the period 1965-1995. To this end, an optimality condition is derived from an optimization growth model, generalising the condition previously derived by Karras (1997) to the case of any kind of returns to scale over all inputs in the production function. Then, a simple production function is estimated, from which the marginal products of both private and government capital are computed. By comparing the estimates of those marginal products, we are able to infer whether the public capital stock in the Spanish regions is underprovided or not, relative to the private capital stock.

Key words: Economic growth, government capital, regions

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## 1. Introduction

Following Aschauer's (1989) influential contribution, the role of public investment has been stressed as a crucial factor leading to higher private capital productivity, which would lead in turn to higher growth rates. According to this author, the decline in productivity growth experienced by the US economy during the seventies would have been explained to a great extent by the decrease in the provision of public infrastructures during that period. In this way, the following years witnessed the appearance of a great amount of empirical literature that analysed the impact of public investment on economic growth; a comprehensive survey of that literature can be found in Sturm, Kuper and de Haan (1998).

Although the first empirical studies made use of aggregate time series for countries, this approach has been also extended to a regional framework using panel data, obtaining results that were quantitatively lower than those found with aggregate data [see, e.g., Holtz-Eakin (1994)]. The reason would be the spillover effects related to the regional endowments of public capital, whose effect would extend not only to the own region, but also to the neighbouring regions. In any case, public infrastructures seem to play an important role in the growth process of regions that should not be neglected (Button, 1998).

On the other hand, the issue of the optimal endowments of public infrastructure has not been the subject of extensive research. In principle, the impact of public investment on economic performance should depend on the stage of development of the economy. For instance, in an empirical analysis for Sweden, Berndt and Hansson (1992) pointed that, since, according to their estimates, public infrastructure capital would have been above its optimal level, this could help to explain the relatively weak effect found for the latter on productivity growth. On the contrary, in a less advanced country such as Spain, a higher investment effort by the public sector should be more effective.

A recent contribution by Karras (1997) develops a simple condition to assess whether public capital is optimally provided, namely, whether the marginal productivities of both private and public capital are equal or not. By estimating a simple growth equation for fifteen European countries during the period 1960-1992, he was unable to reject the null hypothesis that the

marginal productivities of private and public capital were equal, so that government investment would be neither underprovided nor overprovided in the fifteen countries of his sample.

In this paper we try to address this issue (i.e., whether the endowments of public investment are optimal or not) in a regional framework, using Spanish data for the period 1965-95. Unlike Karras (1997), who assumes that the production function exhibits constant returns to scale in all factors, we will generalise his result without needing this constraint. On the other hand, the Spanish economy can provide an interesting case of study, since it has experienced a sustained period of growth in the last forty years, which has been accompanied by a strong process of structural change. In particular, the establishment of new regional governments after the restoration of democracy in 1977, coupled with the strong increase experienced by public investment since then, are reasons that can justify the interest of the Spanish case for the objectives of this paper.

The paper is organised as follows. In section 2, the theoretical condition under which public capital would be optimally provided is derived from an optimization growth model. In section 3, we provide an empirical application of the model, for the case of the Spanish regions during the period 1965-1995. Finally, the main conclusions are summarised in section 4.

## 2. Theoretical framework

In this section we will derive the condition that will allow us to assess whether public capital is optimally provided or not, generalising the approach of Karras (1997). The theoretical framework is based on Ramsey's optimization growth model [see Blanchard and Fischer (1989) or Romer (1996) for an overview], extended to incorporate the role of government capital into the production function.

We begin by assuming an aggregate production function such as:

$$Y_t = A_t F(K_t, KG_t, L_t) \quad (1)$$

where  $Y$  denotes real output, which depends on the amounts utilized of private capital,  $K$ , government capital,  $KG$ , and labour,  $L$ ;  $A$  is an index of the level of technology. The function  $F$  is assumed twice continuously differentiable, with  $F_x = \frac{\partial F}{\partial X_t} > 0$  and  $F_{xx} = \frac{\partial^2 F}{\partial X_t^2} < 0$  (for  $X = K, KG, L$ ); however, unlike Karras (1997), we will assume that the function  $F$  is homogeneous of degree  $z$  ( $z \geq 1$ ) in all production factors. In other words, we make no particular presumption on the kind of returns to scale over all three inputs, which might be increasing, constant or decreasing, according if  $z$  is greater than, equal to, or lower than one, respectively.

Next, we write the production function in *per capita* terms:

$$y_t = L_t^{z-1} A_t f(k_t, kg_t) \quad (2)$$

where  $x = X/L$  denote a variable in *per capita* terms (for  $X = Y, K, KG$ ), with  $f_x = \frac{\partial f}{\partial x_t} > 0$  and

$$f_{xx} = \frac{\partial^2 f}{\partial x_t^2} < 0 \quad (\text{for } x = k, kg).$$

The output is either consumed or invested, so that, in *per capita* terms:

$$\dot{k} = L_t^{z-1} A_t f(k_t, kg_t) - c_t - (\delta + n)k_t - \tau_t \quad (3)$$

where  $\dot{k} = \frac{dk_t}{dt}$ ,  $c$  is *per capita* consumption,  $\delta$  is the rate of depreciation of private capital,  $n$  is

the rate of population growth, and  $\tau$  denotes taxes *per capita*. The latter are used to finance government capital's accumulation following the government budget constraint, also in *per capita* terms:

$$\dot{k}g = \tau_t - (\delta + n)kg_t \quad (4)$$

where  $\dot{k}g = \frac{dkg_t}{dt}$ , and government capital is assumed to depreciate at the same rate than private capital.

On the other hand, the representative individual is assumed to maximize utility, which depends on *per capita* consumption, over an infinite planning horizon:

$$U = \int_0^{\infty} u(c_t) e^{-\rho t} dt \quad (5)$$

where  $\rho$  is the rate of time preference and  $u_c = \frac{du}{dc_t} > 0$ , subject to (3), (4), and  $k_0, kg_0 > 0$ . This

optimization problem is solved by setting the Hamiltonian:

$$H_t = u(c_t) e^{-\rho t} + \lambda_1 (L_t^{z-1} A_t f(k_t, kg_t) - c_t - (\delta + n)k_t - \tau_t) + \lambda_2 (\tau_t - (\delta + n)kg_t)$$

from which the first-order conditions would be:

$$\frac{\partial H_t}{\partial c_t} = e^{-\rho t} u_c - \lambda_1 = 0$$

$$\frac{\partial H_t}{\partial \tau_t} = -\lambda_1 + \lambda_2 = 0$$

$$\frac{\partial H_t}{\partial k_t} = \lambda_1 L_t^{z-1} A_t f_k - \lambda_1 (\delta + n) = -\dot{\lambda}_1$$

$$\frac{\partial H_t}{\partial kg_t} = \lambda_1 L_t^{z-1} A_t f_{kg} - \lambda_2 (\delta + n) = -\dot{\lambda}_2$$

In this way, from the first three conditions we get:

$$-\frac{\dot{u}_c}{u_c} = [L_t^{z-1} A_t f_k - (\delta + n)] - \rho \quad (6)$$

and, from the last three:

$$L_t^{z-1} A_t f_k = L_t^{z-1} A_t f_{kg} \quad (7)$$

where  $L_t^{z-1} A_t f_k$  and  $L_t^{z-1} A_t f_{kg}$  are the marginal products of private and government capital, respectively. Equation (6) is the Euler condition, which implies that, the higher the marginal product of private capital (net of depreciation and population growth) relative to the rate of time preference, the more it pays to depress the current level of consumption in order to enjoy higher consumption later. In turn, equation (7) states that optimal accumulation of private and government capital requires that their marginal products be equal, and coincides with the condition already derived by Karras (1997), but generalised to the case of any kind of returns to scale over all inputs in the production function .

In this way, the latter condition would imply that, given the marginal product of private capital, if the marginal product of government capital would be higher than that of private capital, it would be profitable for the government to raise public investment; in other words, and assuming that private capital is optimally provided, a marginal product of government capital above (below) the marginal product of private capital would mean that government capital is underprovided (overprovided), relative to private capital. In the next section we will provide an empirical test of equation (7), using Spanish regional data.

### 3. Empirical model and results

In order to test empirically equation (7), we start from the production function above, equation (1), written for simplicity in a Cobb-Douglas form:

$$Y_t = A_t K_t^\alpha KG_t^\beta L_t^\gamma \quad (8)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are the output elasticities of the production factors private capital, government capital, and labour, respectively. In *per capita* terms, the production function (8) becomes:

$$y_t = L_t^{\alpha+\beta+\gamma-1} A_t k_t^\alpha kg_t^\beta \quad (9)$$

so that  $\alpha + \beta + \gamma$  would amount to  $z$  in the preceding section (see equation (2)), indicating the degree of returns to scale for all production factors. Finally, assuming that  $A_t = A_0 e^{\mu t}$ , where  $A_0$  is the initial level of technology and  $\mu$  the rate of technical progress, we can write equation (9) in logarithms as follows:

$$\log y_t = \log A_0 + \mu t + (\alpha + \beta + \gamma - 1) \log L_t + \alpha \log k_t + \beta \log kg_t \quad (10)$$

We have estimated equation (10) for the 17 regions (“comunidades autónomas”) established after the approval of the current Spanish Constitution in 1978, along the period 1965-1995. The data are taken from Fundación BBVA (various years) for GDP; from Mas, Pérez and Uriel (various years) for the private and public capital stock; and from Mas, Pérez, Uriel and Serrano (various years) for employment. Notice that our public capital variable embodies only the directly productive items included into the whole government capital stock (i.e., roads, water infrastructures, urban structures, ports, railroads, and airports), hence excluding the non directly productive items (i.e., education and health); see Mas, Pérez and Uriel (various years) for details. On the other hand, *per capita* variables ( $y$ ,  $k$ , and  $kg$ ) are defined in terms of employment ( $L$ ), and valued in real terms (at 1986 prices).

There is some available evidence on the favourable effect of the public capital stock on the productivity of private capital for the Spanish case, both with aggregate data, as in Bajo-Rubio and Sosvilla-Rivero (1993); and with regional data, as in Mas, Maudos, Pérez and Uriel (1996) or Gil, Pascual and Rapún (1998). We can also quote some other studies specifically addressed to the analysis of economic growth, such as Bajo-Rubio and Sosvilla-

Rivero (1998), who found a positive effect on growth for public investment as a percentage of GDP using aggregate data for the whole Spanish economy; and Bajo-Rubio, Díaz-Roldán and Montávez-Garcés (1999), where the same result was obtained when estimating a convergence regression using regional data over the period 1967-91.

Finally, we refer now to other two recent studies, more directly related to the objective of this paper. Boscá, Escribá and Murgui (2001) find, using the dual approach based on cost functions for the period 1980-93, that there would still remain a substantial gap between observed and optimal public capital, which would justify a further increase in public investment. However, when computing real and observed relative profitabilities for public capital, de la Fuente (2001) concludes that public investment would have been too redistributive, in the sense that too much public capital would have been located in poorer regions. Therefore, we will try to address this somewhat conflicting evidence by using the more traditional approach of estimating a production function, as explained above.

Some descriptive evidence is provided in Figures 1 and 2. These figures show, for the first and last year of our sample period and for the 17 Spanish regions, the levels of real GDP per employee, and the GDP share of the government capital stock, respectively. As can be seen in Figure 1, GDP per employee would have experienced a significant increase between both dates, reaching twice its initial level in most regions; the growth of this variable would have been somewhat stronger in the case of poorer regions, supporting previous findings on convergence [see, e.g., Cuadrado-Roura, García-Greciano and Raymond (1999) or de la Fuente (2002)]. In turn, the evolution of the GDP share of the government capital stock would have been also impressive, being its increase especially remarkable after the first eighties, when the first Socialist government took office.

The results of the econometric estimation of equation (10) are shown in Table 1. The estimated equations include individual effects for each region, which would proxy the initial level of technology  $A_0$ . On the other hand, due to the potential endogeneity of some of the explanatory variables, the above equation has been estimated using the Generalized Method of Moments, which might be thought as a generalization of the Instrumental Variables estimator.



This method derives linear transformations of the original disturbances and instruments that are orthogonal, using these orthogonality conditions to estimate the parameters optimally; useful summaries can be found in Pagan and Wickens (1989) or Greene (2000).

The results for the whole set of regions appear in column (1). Notice that the coefficient on employment would be negative and significantly different from zero, so that the hypothesis of decreasing returns to scale over all inputs would not be rejected. Both capital stocks, private and public, would have a positive and significant effect on the evolution of output per employee, with estimated elasticities of 0.49 and 0.04, respectively.

We have also divided the whole set of regions into two groups, i.e., those enjoying a GDP per employee above and below the Spanish level, on average over the whole period of analysis. This procedure allows us to classify regions into “richer” and “poorer” or, more precisely, into “more productive” or “less productive”, according whether GDP per employee (that is, average labour productivity) is above or below the Spanish average level. The results for both groups of regions appear, respectively, in columns (2) and (3) for the more productive regions (Madrid, Baleares, Cataluña, País Vasco, Navarra, Rioja, and Canarias), and in columns (4) and (5) for the less productive regions (Comunidad Valenciana, Aragón, Cantabria, Asturias, Murcia, Andalucía, Castilla y León, Castilla-La Mancha, Extremadura, and Galicia). As can be seen in columns (2) and (4), the coefficient on employment would not be significantly different from zero, so that the hypothesis of constant returns to scale over all inputs would not be rejected when both groups of regions are taken separately. Accordingly, in columns (3) and (5) this variable was dropped, so that positive and significant elasticities for the stocks of both private and public capital were obtained: 0.45 and 0.05 for the more productive regions, and 0.58 and 0.05 for the less productive regions (even though the latter coefficient is significant only at the 10% level).

The next step would be computing the marginal products of private and government capital (denoted by *MPK* and *MPKG*, respectively) from their estimated elasticities,  $\alpha$  and  $\beta$ , as follows:

$$MPK = \alpha \frac{Y}{K}$$

and

$$MPKG = \beta \frac{Y}{KG}$$

where  $\frac{Y}{K}$  and  $\frac{Y}{KG}$  are the average products of private and government capital, respectively, taken as their mean values over the sample period. As can be seen in Table 2, for the whole set of regions the marginal product of private capital would be higher than that of government capital, which would mean that government capital would be overprovided, relative to the available endowment of private capital. A more complete picture, however, can be obtained when the same computation is performed for the two groups of regions differentiated above. Although the previous result keeps (even more strongly) for the less productive regions, it turns that the marginal product of private capital would be lower than that of government capital for the more productive regions, so that in this case government capital would be underprovided, relative again to the available endowment of private capital.

## 4. Conclusions

In this paper we have tried to find some evidence on the optimality in the provision of government capital in the Spanish regions. To this end, we have derived a theoretical condition allowing us to assess whether public capital would be under or overprovided, which generalises the condition previously derived by Karras (1997) to the case of any kind of returns to scale over all inputs in the production function. This theoretical condition makes use of the result that the marginal products of private and government capital should be equal in an optimum, according to a Ramsey-type optimization growth model.

This condition has been tested empirically using Spanish regional data over the period 1965-1995, by estimating a simple production function, and then computing the marginal products of private and government capital from their estimated elasticities and their mean average products over the sample period. Our results showed a positive effect of government capital on the evolution of GDP per employee, although its marginal product would be below that of private capital. However, when dividing the whole set of regions between those with a GDP per employee above and below the Spanish average, the previous result would only kept for the less productive regions, whereas the opposite result (i.e., a higher marginal product for government capital than for private capital) was obtained in the case of the more productive regions. Therefore, assuming that private capital was optimally provided, these results would suggest that government capital would be underprovided in the more productive regions, unlike the less productive regions, where the opposite result would hold. The policy implications of the results in this paper would be in line with those recently derived in de la Fuente (2001): although government capital would have been a relevant factor behind the growth process experienced by Spanish regions in last years, on strictly efficiency grounds public investment efforts should be more intensively addressed to the more productive regions at the expense of the less productive ones, assuming that private capital was optimally provided.

This last qualification is essential, and leads us to take with some caution the above results. So, leaving aside equity considerations, which could justify by themselves the use of a regional infrastructures policy as a redistributive device (de la Fuente, 2001), the need of an even increased investment in both private and public capital in order to enhance real

convergence with the more advanced EU countries (Martín and Velázquez, 2001) could be an important reason to provide a higher investment in government infrastructures, even in poorer regions. So, a regional infrastructures policy directed to poorer regions could be justified if it would serve as a means of attraction of private investment to those regions. In this sense, the positive effect of public productive investment on enhancing private investment at the regional level, found in a recent study using the same time period than in this paper (Martínez-López, 2002), could validate this conclusion.

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**Table 1: Estimation of a production function for the Spanish regions, 1965-1995**

(Dependent variable:  $\log y$ )

	(1) All regions	(2) More productive regions	(3) More productive regions	(4) Less productive regions	(5) Less productive regions
$t$	0.0070 (13.8427)	0.0074 (10.0734)	0.0069 (11.6834)	0.0057 (6.8684)	0.0060 (7.6316)
$\log L$	-0.1557 (-2.0515)	-0.1911 (-1.2990)	-	0.0581 (0.3421)	-
$\log k$	0.4889 (18.4356)	0.4524 (13.2606)	0.4507 (12.4587)	0.5987 (9.1069)	0.5770 (21.4252)
$\log kg$	0.0402 (2.2125)	0.0392 (1.3864)	0.0536 (2.1672)	0.0498 (1.7217)	0.0480 (1.6053)
$R^2$	0.9885	0.9835	0.9815	0.9893	0.9897

Note:  $t$ -statistics in parentheses.

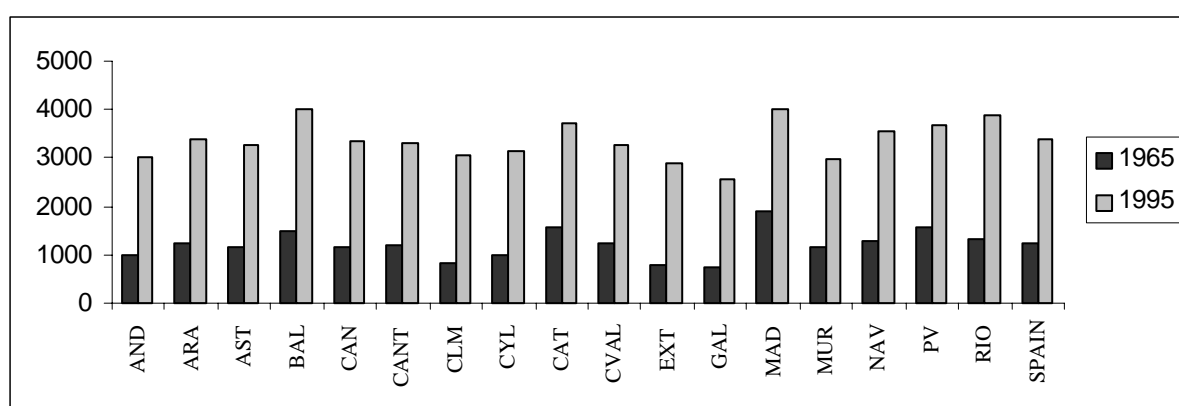


**Table 2: Marginal products of private and government capital in the Spanish regions, 1965-1995**

	(1) All regions	(2) More productive regions	(3) Less productive regions
Private capital	0.1340	0.1252	0.1518
Government capital	0.1016	0.1705	0.0944

### Figure 1: GDP per employee in the Spanish regions, 1965 and 1995

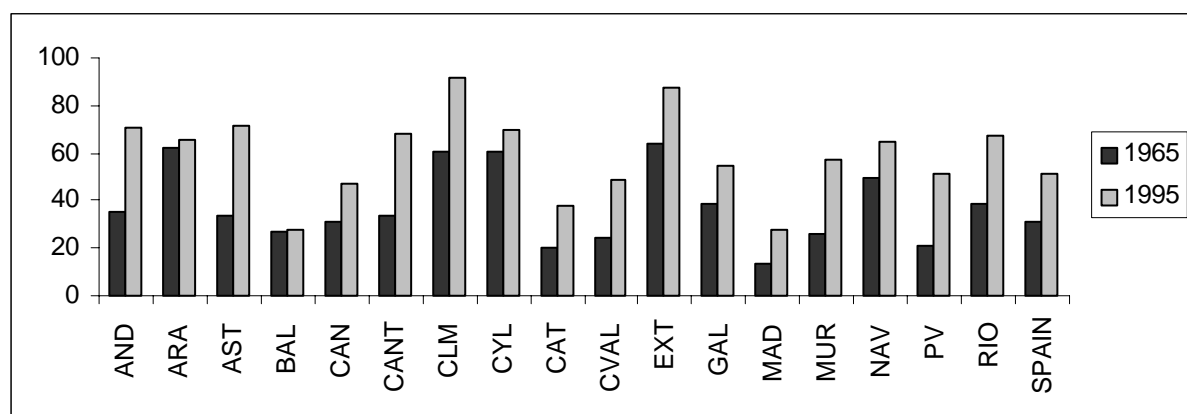
(Thousand Pesetas per occupied person, at 1986 prices)



Note: The regions shown in the figure are, from left to right, Andalucía, Aragón, Asturias, Baleares, Canarias, Cantabria, Castilla-La Mancha, Castilla y León, Cataluña, Comunidad Valenciana, Extremadura, Galicia, Madrid, Murcia, Navarra, País Vasco, and Rioja.

Source: Fundación BBVA (various years) and Mas, Pérez, Uriel and Serrano (various years).

**Figure 2: Government capital stock as a percentage of GDP in the Spanish regions, 1965 and 1995**



Note: See Figure 1.

Source: Fundación BBVA (various years) and Mas, Pérez and Uriel (various years).