Some Aspects on the Interregional Spatial Distribution of Local Sector Activities (preliminary version)

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Abstract

This paper focuses on the spatial distribution of economic activities that serve intraregional demand. The level of local sector activities is measured by employment per inhabitant. The basic hypothesis relates to how this proportion varies systematically over space, from high values in a central business district, through low values in suburban areas, and asymptotically approaching the average regional level as the distance from the center increases. This hypothesis is examined both analytically and through simulation experiments where the location decision of firms is assumed to reflect the net effect of agglomeration economies, economies of scale and transportation costs. We further discuss to what degree the relevant hypothesis is consistent with specific assumptions on the distribution of wages (transport cost) between consumers, and the spatial distribution of the prices on goods and services. The model formulation we propose is useful as a part of economic base modeling for predicting regional development.

1. Introduction

In the literature one can find extensive results on the modeling of locational decisions of firms and households. Complementary to this, much research is directed towards the modeling and prediction of traffic flows. Surprisingly little attention, however, is paid to the important issue of treating location decisions and traffic flows simultaneously within the same modeling framework.

One area of research where this issue has been addressed, is the formulation of large-scale models for urban structure and development. In almost a decade after Lee's "Requiem for large-scale models" (Lee 1973), the activity within this area of research was very low. After this period there was a renaissance of large-scale models, see for example Boyce (1988). This renaissance was a result of progress within mathematical methods, solution algorithms and computers, estimation procedures and data, and the theoretical framework. Concerning the theoretical framework it is particularly important that standard methods of spatial interaction analysis has been demonstrated to be consistent with utility theory, see for example Ben Akiva and Lerman (1985). During the last decade a multitude of large-scale models has been constructed and applied for specific metropolitan areas, for reviews see Wegener (1994, 1998), or Batty (1994).

Many of the operational large-scale models are based on ideas from the Lowry model (Lowry 1964). Anas (1987) offers a more recent presentation and evaluation of this modeling tradition. The Lowry model is pivoted on the central idea of economic base theory, where production is split between local and basic production sectors. The activity level in local production sectors is determined by demand that originates within the study area, while production in basic sectors is exogenously given, independent of the relevant intraregional components of demand. Increased activity in basic sectors attracts workers to the region. This further increases the demand for locally produced goods, and a

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multiplier process is initiated that converges towards a higher level of regional production and employment. For a textbook presentation of economic base theory, see for instance Treyz (1993).

Any operational comprehensive model of urban and regional development has to take into account the interdependency between location decisions of firms and households. Most of the existing large scale model formulations are according to the basic idea in the Lowry model, where the residential location pattern responds to changes in the spatial configuration of basic sector activity, initiating the well-known multiplier process. Contrary to this the IMREL model (Integrated Model of Residential and Employment Location) defines residential location decisions to be the driving force of the development within an urban area, see Anderstig and Mattsson (1991). As a result of improved mobility, households are argued to be less dependent on work location, while location decisions of firms are influenced by the accessibility relative to labor supply, or, in other words, the recruiting potential to labor.

In this paper, however, we will not enter into a discussion of what is a reasonable sequence in the process towards a new location pattern in the study area. One important component of such a process is nevertheless how the spatial configuration of local sector firms relates to the residential location pattern and household shopping behavior. This is the subject of our paper. We neither focus on location decisions of basic sector firms, nor on how the residential location decisions of households relate to job market accessibility or retailing facilities.

Unlike the set of large-scale models in the literature, our approach is not restricted to urban, or metropolitan, areas. Rather, we take on a more macroscopical view of the geography, as we focus on a regional perspective, potentially including several urban areas. In this respect our approach is corresponding to the modified version of the Lowry model that is presented in Thorsen (1998). In Thorsen (1998) this part of the model is, however, introduced on an ad hoc basis, ignoring a set of relevant aspects. The main ambition of this paper is to offer a refined and theoretically more satisfying specification of the spatial configuration of local sector production, based on the shopping behavior of households.

To be more precise, a crucial part of our construction is to model the spatial dispersion of the fraction between employment and labor in a region with a central business district (CBD). Letting E denote employment and L labor, we claim that the fraction E/L, viewed as a function of the traveling distance d from the CBD can be expected to trace a graph similar to the one shown in Figure 1.

E ê L
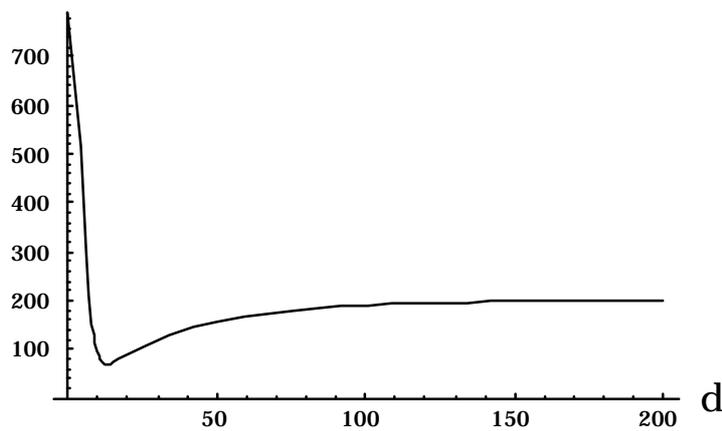
In the following sections we will demonstrate that a function of this kind can be argued to be a result of two competitive forces. First, a concentration of local sector activities in a city, or a central business district, results from a combination of technological scale economies and specific kinds of external economies of scale that generate co-location of local sector firms (see for example Quigley (1998)). Contrary to the Weberian tradition in location theory, however, we take into account that economic activities are space-consuming. Hence, the center is not concentrated to one single point in the geography, but the various kinds of scale and agglomeration economies can be expected to fall off rapidly at a certain distance from the city center. This explains the first part of the curve in Figure 1. The exact form of this part of the curve of course depends on the size and dispersion of a specific city. At some distance from the city center, however, a second principle can be expected to start dominating the picture. Like for example in de Palma et al. (1994), it can be argued that centrally located stores set lower prices and offer a greater variety of consumer goods than more peripherally located stores. In this paper we focus on the price aspect. Price reductions might be due to scale economies, externalities and competition effects. The price reductions, however, are counteracted by an increased traveling cost as the distance to the CBD increases. The price reductions are not constant, but can be expected to exhibit a widely distributed probability distribution over a large class of different firms and services.

The paper will be organized as follows. In Section 2 we consider the situation where the CBD is reduced to a single point, and emphasis is put exclusively on the balance between (consumer) savings versus generalized traveling costs. We model these effects starting from a simplified case and then gradually adding on more structure to the model. In Section 3 we suggest how one can incorporate the spatial component of the agglomeration part. In Section 4, we generalize the construction to the case where there is more than one CBD. In Section 5 we use empirical findings from a region on the western coast of Norway and try to calibrate our model with respect to these observations. Finally in Section 6, we offer some concluding remarks.

2. Consumer savings versus traveling costs

Traditionally, urban economic models are based on the assumption of a monocentric city center, see for example Fujita (1986) for a survey. To a certain degree this will also be the case in this paper. We do no attempts to explain the spatial configuration of central places, and we consider the location of a city center to be predetermined. As mentioned in the introduction, agglomerations of local sector activities in a city center is typically argued to be a result of specific increasing returns and external scale economies. External scale economies, or agglomeration economies, are traditionally divided into urbanization and localization economies (Isard1956). Urbanization economies relates to the overall economic activity in an area, while localization economies reflect interdependencies between firms that supply/produce similar goods or services. Through the emergence of the New Economic Geography there has been an increased focus on scale economies to explain the spatial

structure of an economy. Krugman (1998) also discusses the formation of urban areas in a dynamic context. By drawing on economic base theory and potential analysis the concentration of production is argued to be self-reinforcing.

In the model to be presented the only known characteristic of the geography is the location of the city center (CBD). The spatial distribution of population is not specified, and we do not model the level of local sector activities in specific locations. What we model is the propensity that households do their shopping locally rather than in the city center. This propensity is represented by the fraction of local sector employment relative to population in specific locations; E/L. Hence, this fraction reflects the spatial shopping behavior of households. The spatial distribution of E/L can be explained through the same kind of mechanisms that are relevant when the center structure in an area is to be explained. For this purpose Krugman (1995) distinguishes between two general sorts of interdependence of business activities. First, the "centrifugal" forces reflect the competition for customers, workers and land. This competition promotes a spatial dispersion of business. Second, the "centripetal" forces reflect positive external scale effects of a cluster of stores that offer a variety of goods and services. Such forces attract customers to an area, and promote agglomerations of business activities.

In this paper we consider E/L as the net result of centrifugal and centripetal forces. The centripetal forces explain why a city center in general offers a larger variety of consumer goods, and different kind of scale economies explain why a positive relationship can be expected between city size and productivity, see Quigley (1998). For the same reason it can be argued that stores in the city center in general will offer goods and services at lower prices than more peripherally located stores. Such a tendency can also result from a game theoretical approach to spatial price competition. In de Palma et al. (1994) findings suggest that prices tend to be lower at centrally located stores, which face the most competition.

In this paper the centripetal forces will be represented by price reductions, while transportation costs of potential customers represent the centrifugal forces. We start out with a situation where there is only one type of good. Hence, we first ignore the possibility of multipurpose shopping in a setting with a diversity of goods. We also ignore the possibility of congestion. Providing the consumers with this service, a number of E_1 employees is needed pr 1000 customers. This in turn defines the ratio $E/L=E_1/1000$ (which we conveniently measure in terms of employees pr 1000 workers). As long as the traveling cost remains below the price reduction, nobody wants to do their shopping in local firms. Hence $E/L=0$ in this case. At some particular distance, however, the traveling cost will begin to exceed the price reduction. At this point we assume that local stores will take over the whole market, and correspondingly $E/L=E_1/1000$ from this point on. This situation is illustrated in Figure 2.
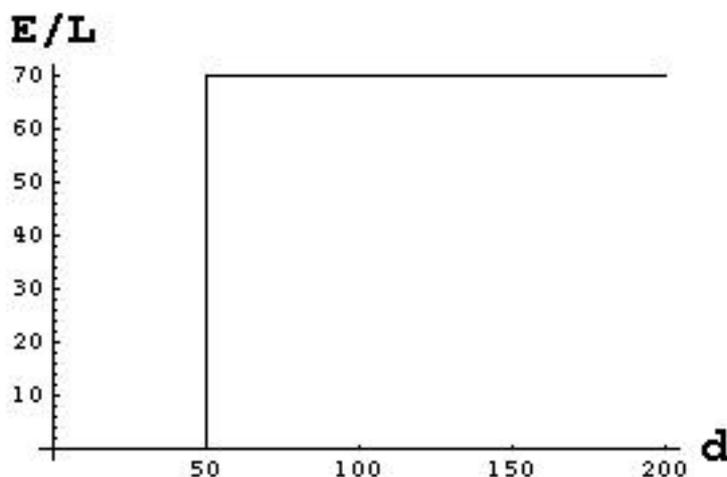
Figure 2
Fractions of Employment/Labor as a function of traveling distance d to CBD

In Figure 2, $E_1=70$, and we have assumed a price reduction of 10 in the CBD, counteracted by a constant traveling cost $TC= 0.1\$/km$. To take the model one step further, we consider a collection of N different types of stores and goods, $F_1,F_2,...,F_N$, each of which requiring $E_1,E_2,...,E_N$ employees to serve 1000 customers. Each different type of store F_i can offer a price reduction PR_i in the CBD, $i=1,2,...,N$. Without loss of generality, we can assume that the different types of stores have been sorted in such a way that the price reductions increase with i. We keep the basic assumption that the customers do their shopping locally by F_i if $TC \cdot distance = PR_i$, $i=1,2,...,N$, and at the CBD otherwise. Note, however, that if the customers benefit from more than one good at the same time the total savings from traveling to the CBD might exceed the traveling cost even when each saving is separately exceeded by TC. We will deal with this case of multipurpose shopping later. An example with 3 different types of firms is shown in Figure 3.
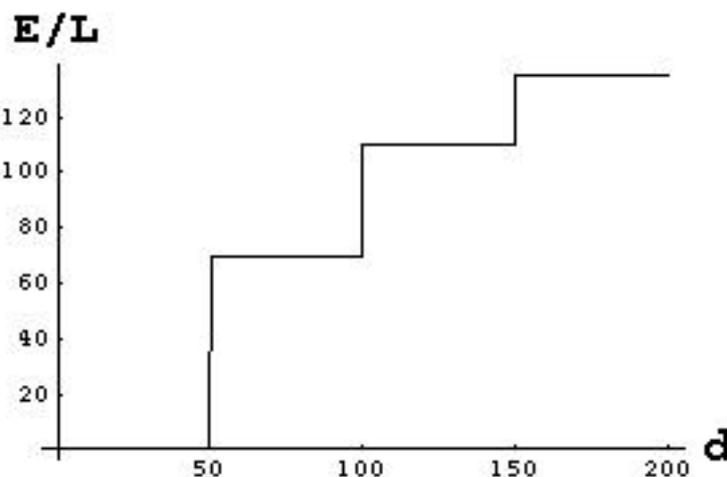
Figure 3
Fractions of Employment/Labor as a function of traveling distance d to CBD

In Figure 3 we have put $E_1=70$, $E_2=40$, $E_3=25$, $PR_1=\$10$, $PR_2=\$20$, $PR_3= \$30$ and $TC=0.1\$/km$. As a next step we replace the constant TC by a random variable. Traveling to the CBD has an important time component, and the value of time savings relates to the wage level. Hence, traveling costs can be expected to vary considerably over the population. In the following we will assume that wages are distributed according to a probability distribution carrying a (usually continuous) density $Y=Y[w]$, where w denotes the wage level and $Y=0$ on $(-8,0)$. The relationship between wage and the valuation of time naturally depends on trip purpose. Norwegian authorities recommend that an hour spent on journey-to-work should be evaluated by NOK 46, see Håndbok-140 (1995). This estimate is based on information of average hourly earnings in manufacturing in 1995. To be more precise NOK 46 represents roughly 42% of the hourly earnings. The estimate of NOK 46 corresponds reasonably well to empirically based estimates in Tretvik (1995). Tretvik (1995) in addition estimates the value of time for different income groups. The numerical experiments to be carried out in this paper are based on an assumption that the value of time savings represents a constant fraction of the wage level. This is assumed to apply for any trip purpose. The recommendations of Norwegian transportation

authorities are not, however, specific about shopping trips. For this purpose we follow the procedure in Forslund and Johansson (1995), where reduced transportation time for shopping trips are valued as equivalent to 84% of the money values of reduced transportation time for journeys-to-work. Summarized, this means that the value of time savings is assumed to be about 35% of the relevant wage level. Our numerical calculations are based on an assumption that Ψ is defined in terms of a lognormal distribution with mean 15, and standard deviation of 3. This mean value is according to the average hourly wage level in manufacturing, see (NHO) ...

In addition to the value of time savings, transportation costs include gasoline consumption and specific service and capital costs. In Håndbok-140 (1995) such vehicle costs are estimated to be represented by an average of NOK 0.86 per km for light vehicles.

Without loss of generality, we may effectively assume that the speed is constant (any effect of differences in traveling speed may be incorporated into Y). Phrased in terms of the distribution, a certain fraction of the population will prefer travel to the CBD, while the rest will prefer to take advantage of local firms. To be precise, we let $W=W[PR,d]$ denote the wage level corresponding to a generalized traveling cost equal to PR in the distance d. We denote the ratio E/L by R and get

(2.1)

$$R[d] = \sum_{i=1}^{N} E_i \int_{W[PR_i,d]}^{\infty} \Psi[w]\,dw$$

As a result of this, we obtain that the jump discontinuities in Figure 2 and Figure 3, will be smeared out continuously over a large part of the interval. An example of this sort is presented in Figure 4 below.
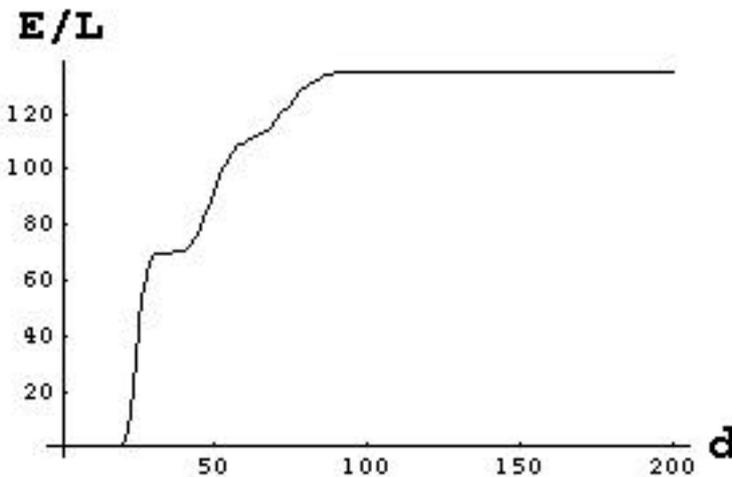
Figure 4
Fractions of Employment/Labor as a function of traveling distance d to CBD

In Figure 4 we have assumed that Y is defined in terms of a lognormal distribution with mean $15 and with a standard deviation of $3. Otherwise the scenario is the same as in Figure 3. Note the increase in values due to the increase in traveling cost when time is included.

In reality a community will be equipped with a multitude of different goods and services, each with a potentially different price reduction at the CBD. This puts our attention in the direction of continuous distributions, in particular when we take into account such services with a flexible price, e.g., paid according to the actual time spent on the service. Again we assume that all services have been sorted in the direction of increased savings at the CBD. The density $F=F[pr]$ is defined in such terms that the integral

$$\int_a^b \Phi[\, pr]\,dpr$$

denotes the number of employees needed to serve 1000 customers in all services offering price reductions in the interval *[a,b]* . In this case the ratio *E/L* is calculated according to the formula (2.2)

$$R[d] = \int_0^\infty \Phi[\, pr] \int_{W[\, pr,d]}^\infty \Psi[w]\,dw\,dpr$$

A numerical simulation making use of the result in (2.2) is shown in Figure 5.
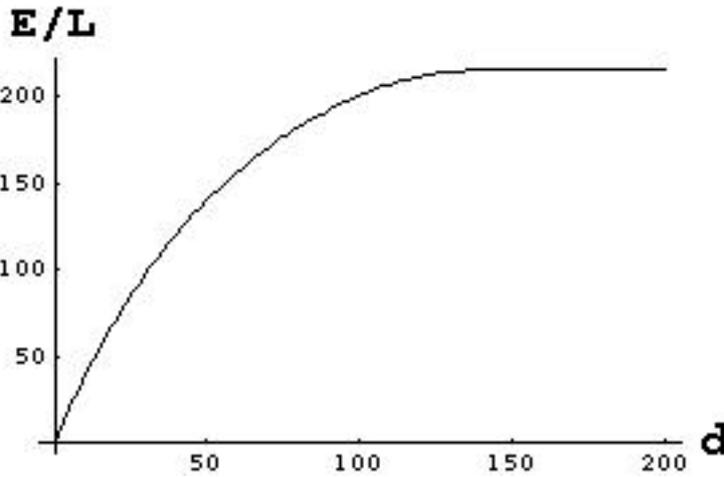
Figure 5

Fractions of Employment/Labor as a function of traveling distance *d* to CBD

The construction above ignores the possibility of multipurpose shopping. Conveniently rephrasing the problem, however, we are able to include multipurpose shopping within the same setup. Consider the situation shown in Table 1.

	F1	F2	F3
#Employees/1000customers	10	30	20
Savings	$10,00	$50,00	$60,00
Together with F1 only	40 %	10 %	30 %
Together with F2 only	20 %	60 %	20 %
Together with F3 only	30 %	10 %	40 %
F1, F2 and F3	10 %	20 %	10 %

Table 1: Savings/shopping frequencies

As shown in Table 1, the customers will sometimes take advantage of more than one service at the time. The idea is now simply to view a shopping combination as a new service. So we introduce new combinations $F_4=(F_1,F_2)$, $F_5=(F_1,F_3)$, $F_6=(F_2,F_3)$ and $F_7=(F_1,F_2,F_3)$. Using the conditional frequencies from Table 1, we get the accumulated savings shown in Table 2. In Table 2 we really consider, e.g., the service F_1 as the service provided when the customers take advantage of service 1 only. Hence only 4, i.e., 40% of the 10 employees are needed for this particular service. All the other employees are split according to this pattern.

	F1	F2	F3	F4	F5	F6	F7
#E/1000 C	4	18	8	5	9	7	9
Savings	10	50	60	60	70	110	120

Table 2: Accumulated savings

Clearly the construction above can be extended to the general case. Given a density function $F=F(pr)$ and a collection of conditional probabilities for all combined patterns, it is possible to construct an agglomerated density $F_a=F_a(pr)$ reproducing local sector activities through the formula

(2.3)

$$R[d] = \int_0^\infty \Phi_a[pr] \int_{W[pr,d]}^\infty \Psi[w]\,dw\,dpr$$

E/L
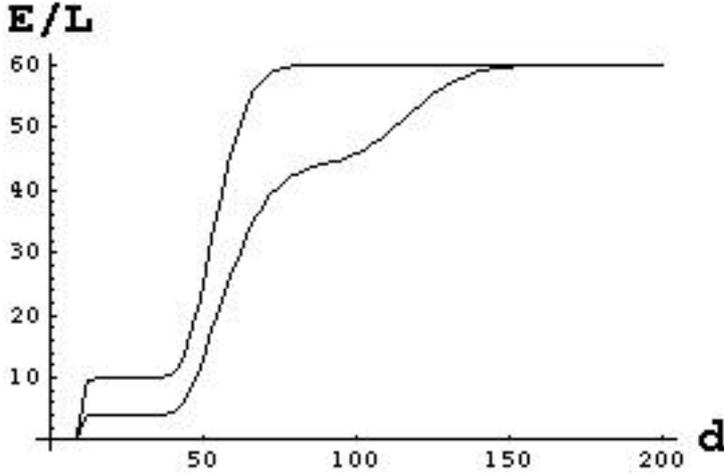
Figure 6
Fractions of Employment/Labor as a function of traveling distance d to CBD

The possibility of multipurpose shopping represents a centripetal force that attracts customers to the city center. Figure 6 illustrates the effect of this force. The upper curve refers to a situation where only one type of good is considered. The lower curve indicates how the propensity to do their shopping locally is reduced for customers with residence outside the city center.

Using the set of ideas above, we are also able to include various other effects in our model. One such aspect is an increased willingness to travel to the CBD during weekends, which may partly be explained through a tendency to put less weight on the time consumption part in this case. Any particular service may be split into 7 daily services, the time component part can be allowed to vary on a daily basis. The local sector activities are then reproduced using a weighted average of the expression (2.3).

If we interchange the order of integration in (2.3), we get
(2.4)

$$R[d] = \int_0^\infty \int_0^{TC[w,d]} \Phi(pr)\,dpr\,\Psi(w)\,dw$$

where TC=TC[w,d] is the generalized traveling cost for traveling a distance d when the wage level is w. Differentiating this expression, we see that
(2.5)

$$R'[d] = \int_0^\infty \Phi_a(TC[w,d])\frac{\P TC[w,d]}{\P d}\,dpr\,\Psi(w)\,dw$$

If we assume that the mapping $d \circledR TC[w,d]$ is linear or more generally concave, we get
(2.6)

$$R''[d] \le \int_0^\infty \Phi_a'(TC[w,d])\left(\frac{\P TC[w,d]}{\P d}\right)^2\,dpr\,\Psi(w)\,dw$$

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Hence, if it is largely the case that services offering large discounts also require fewer employees, i.e., $F'_a<0$, then we can expect to find $R''[d]<0$. Even when this fails in general, we will still expect to see such kind of effect at the tail of the distribution. Note that in the presence of increasing congestion effects on approach to the CBD, there will be a proportionally smaller time penalty when the customers travel from more remote locations. The generalized traveling cost will be concave in this case.

If we continue the reasoning above, we can proceed indefinitely to include additional effects into the model. Once a new effect is introduced, however, we observe a clear tendency that local anomalies will be further dispersed in space. The large-scale picture we end up with, is a concave function asymptotically increasing towards a limit where all services are covered by the local sector. In this large-scale picture the local effects are wiped out, and on the basis of this we suggest to model the function through an expression of the form
(2.7)

$$R_{\text{local sector}}[d] = R_\infty(1 - \exp[-\boldsymbol{b}\,d])$$

Here R_8 denotes the value corresponding to the case where all activity remains in the local sector, and $\boldsymbol{b}$ is a parameter measuring the speed of which the limiting value is obtained. In what follows the function in (2.7) will be used to model the local sector part of the employment ratio.

3 Spatial dispersion of the CBD

We will now turn to the modeling of the centripetal forces, which promote concentration of business activities in the city center. As mentioned in section 2, those forces are due to increasing returns and external economies of scale. External economies of scale, or agglomeration economies, make stores to clump together in clusters rather than being more evenly spread-out across an area. Based on such arguments we assumed the existence of a central city, where goods and services are provided at lower prices (and larger diversity) than in stores at more peripheral locations in the market. We will, however, take into account that business activities are space-consuming; the city center is not restricted to one single point in the geography. The dispersion of an urban area can be given numerous specifications. For example, a trend that has been observed in many metropolitan areas is the rise of urban subcenters, or edge cities (see Krugman 1995). We will not deal with this kind of spatial configurations. Our model formulation is based on a monocentric urban area with a traditional downtown which represents the highest level of agglomeration economies in the geography. Though such a city center might be dense, it is in general dispersed over a certain distance. In addition, the relevant centripetal forces are in general effective also in short distances from the city center. Hence, the concentration of local sector activities might be high also in short distances from the city center, though they can be expected to fall off rapidly with small increases in distance from the CBD.

We let D represent the dispersion of the city. This means that the population and the business activities in the city is distributed within the interval $[-D,D]$. We let E_0 and L_0 denote the observed employment/labor within the city, and we wish to model the ratio $E/L[x]=R_{agglomeration}[x]=R_{a[x]}$ subject to the balancing condition
(3.1)

$$\int_{-D}^{D} R_a[x]l[x]dx = E_0$$

where $l[x]$ denotes the population density within the city. It is reasonable to assume that R_a is symmetric and decreasing with $|x|$, and we will denote the fraction

$$\frac{R_a(D)}{R_a(0)} = a$$

as the marginal fraction, which we may think of as a number approximately equal to zero w.r.t. the problem in question. As a simple device of this sort we will consider
(3.2)

$$R_a[x] = K \cdot \frac{E_0}{L_0} e^{-\left(\frac{gx}{D}\right)^2}$$

After a change of variables, we see that (3.1) implies
(3.3)

$$K = \frac{g\, L_0 / D}{\displaystyle\int_{-g}^{g} e^{-u^2} l(Du/g)\, du}$$

where

$$g = \sqrt{-\ln[a]}$$

If we consider the case where the population is uniformly distributed within a 2-dimensional disc, $l[x] = C|x|$ and we get
(3.4)

$$K = C_g^{(1)} = \frac{g^2}{1 - e^{-g^2}}$$

If on the other hand the population is uniformly distributed along a truly 1-dimensional geography, $l[x] = C$ and this gives
(3.5)

$$K = C_g^{(2)} = \frac{g}{\displaystyle\int_{0}^{g} 2e^{-u^2}\, du}$$

The point to be made here is that $C_g^{(1)}$ and $C_g^{(2)}$ are never very much different. If the marginal level $a \in [0.01, 0.50]$, then $2 = C_g^{(1)}/C_g^{(2)} = 4$. Hence $C_g^{(1)}$ and $C_g^{(2)}$ always have the same order of magnitude. Moreover, it follows from (3.3) that any population density satisfying a condition of the form
(3.6)

$$C_1|x| \le l|x| \le C_2 D \quad -D \le x \le D$$

will satisfy
(3.7)

$$C_g^{(2)} \frac{C_1}{C_2} \leq K \leq 2 C_g^{(1)} \frac{C_2}{C_1}$$

Thus all reasonable population densities will give $K = \dfrac{g^2}{1 - e^{-g^2}}$ to within the same order of magnitude. This is the value we will use in the sequel.

This modeling of the centripetal forces has implicitly been based on a set of simplifying assumptions. One such assumption is that distance to the city center is the same, no matter where in the CBD the relevant shopping destination is located. Hence, distance represents an estimate of average distance to retailing facilities in the city center, and this can be stated as an assumption that internal distances downtown are so short that they can be ignored when shopping from more peripheral locations is considered.

Another simplifying assumption is that the agglomeration tendencies are continuously reduced as the distance from the city center increases. This continuous reduction is not explained by our modeling framework. One possible explanation is spatial variation in land prices and economic rents within the city center. This might explain a high density of stores in the most central parts of the CBD. Still, the analysis in this paper has been based on the assumption of uniform prices of consumption goods within the urban center; there is no spatial variation in price reductions. Internal distances can be thought to be too short to allow for spatial price variations in a market equilibrium. We will not, however, enter into a discussion of spatial price competition in this paper, as we don't consider this to be of basic importance for the problem that is focused.

As a last step we find the relative level of local sector activities as the net result of centripetal and centrifugal forces:
(3.8)

$$R[d] = R_{\text{agglomeration}}[d] + R_{\text{local sector}}[d]$$

This, too, is a simplification. Internally in the CBD, the two parts are acting together and the model does not take the effect of this into account. When D is reasonably small, however, the local sector effect is small anyway so this do not significantly change the model. The curve in Figure 1 shows a numerical simulation of the function in (3.8). Here we used the parameter values D=10 (km), *a=5%* , E_0/L_0=250 (pr 1000 customers), R_8=200 (pr 1000 customers), and *b=0.03* . The values inside the CBD may seem surprisingly high. Note, however, that the scaling constant K is calculated from a hypothesis where the population is uniformly distributed within a two dimensional disc. Only a very small proportion of the population will then be situated close to the center.

4. Extension to several CBDs

The construction in Section 2 and 3 applies to the situation where there is only one CBD. In this paragraph we wish to extend this construction to the case where there is more than one CBD. To this end we will base our construction on a convex combination of functions constructed from the

case with one CBD. These convex combinations we construct via distance deterrence functions. Such functions were introduced by the authors in Thorsen et al. (1999), see this paper for a motivation and discussion of the concept. As an example of this sort, we will make use of a logistic function
(4.1)

$$D(x) = \frac{1}{1 + e^{-k(x-x_0)}} \quad \text{with} \quad x_0 = \frac{1}{2}(d_0 + d_\infty), k = \frac{2\ln(1/a - 1)}{(d_\infty - d_0)}$$

Here a is defined as the marginal level of interaction. If x is very small, $D(x)\tilde{} 0$ (i.e., no deterrence), and if x is large, then $D(x)\tilde{} 1$ (i.e., full deterrence). The parameter d_0 signifies the distance at which D is marginally close to no deterrence, and d_8 signifies the distance at which D is marginally close to full deterrence.

Now consider the situation where there are two CBDs, CBD_1 and CBD_2. Consider a point in space at distances d_1 to CBD_1 and d_2 to CBD_2. We let $R_1[d]$ and $R_2[d]$ be the functions found from (3.9) using the construction in Section 2 and 3. $R_{12}[d]$ is the corresponding expression when we merge CBD_1 and CBD_2 together. Now we propose to model E/L using the expression
(4.2)

$$R[d_1, d_2] = D(d_1)D(d_2)R_\infty + (1 - D(d_1))D(d_2)R_1[d_1]$$
$$+ (1 - D(d_2))D(d_1)R_2[d_2] + (1 - D(d_1))(1 - D(d_2))R_{12}[(d_1 + d_2)/2]$$

As the reader may wish to verify, the four numbers $D(d_1)D(d_2)$, $(1-D(d_1))D(d_2)$, $(1-D(d_2))D(d_1)$, and $(1-D(d_1))(1-D(d_2))$ always add to 1. Hence E/L is modeled as a convex combination of the four states R_8, R_1, R_2, and R_{12}. From (4.2) we obtain the following:

• If d_1, d_2 are both large, the model suggest the expression R_8.
• If d_1 is small and d_2 is large, the model suggest the expression $R_1[d_1]$. (All other terms are small in this case).
• If d_2 is small and d_1 is large, the model suggest the expression $R_2[d_2]$.
• If d_1 and d_2 are both very small, the model suggests that CBD_1 and CBD_2 are acting as a single unit.

At first sight the inclusion of a merged state R_{12} may seem somewhat superficial. One application of this kind of model is, however, to provide predictions for a change in the spatial pattern of retailing facilities when the internal distances within the system are subject to change. Consider the network shown in Figure 7. If a new road connection is introduced between the nodes B and E, one would expect a stronger kind of response if B and E are both CBDs than if any other pair of nodes has this property.
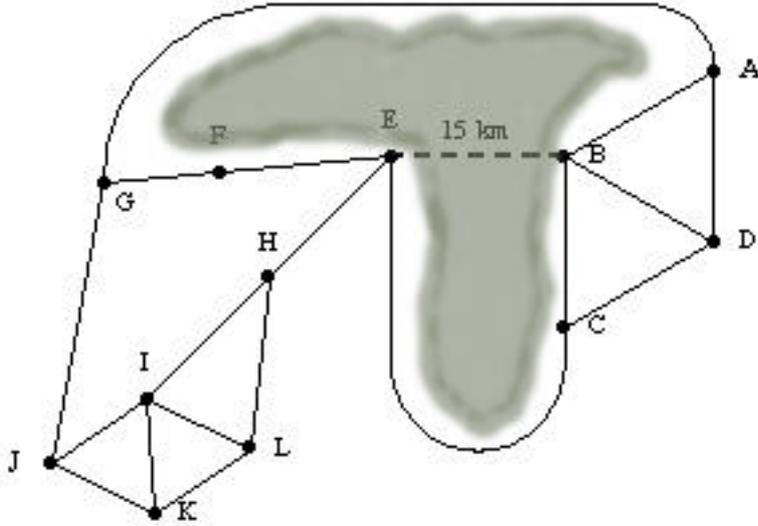
FIGURE 7: A system subject to change

Figure? shows a numerical simulation of the case above. For the purpose of illustration, we show a case with a 1-dimensional geography, i.e., we assume that all the population is located along the same road. The position is measured in terms of the distance to CBD_1, while CBD_2 is located at a distance d_{CBD} from CBD_1.

It is straightforward to extend (4.2) to the general case. If there are 3 CBDs in the system, we put (4.3)

$$
\begin{aligned}
R[d_1, d_2, d_3] = {}& D(d_1)D(d_2)D(d_2)R_\infty \\
& + (1 - D(d_1))D(d_2)D(d_3)R_1[d_1] \\
& + (1 - D(d_2))D(d_1)D(d_3)R_2[d_1] \\
& + (1 - D(d_3))D(d_1)D(d_2)R_3[d_1] \\
& + (1 - D(d_1))(1 - D(d_2))D(d_3)R_{12}[(d_1 + d_2)/2] \\
& + (1 - D(d_1))(1 - D(d_3))D(d_2)R_{13}[(d_1 + d_3)/2] \\
& + (1 - D(d_2))(1 - D(d_3))D(d_1)R_{23}[(d_2 + d_3)/2] \\
& + (1 - D(d_1))(1 - D(d_2))(1 - D(d_3))R_{123}[(d_1 + d_2 + d_3)/3]
\end{aligned}
$$

Expression (4.3) then defines *E/L* in terms of a convex combination of the 8 states $R_8, R_1, R_2, R_3, R_{12}, R_{13}, R_{23}$, and R_{123}. The extension to the general case with *N* CBDs follows similarly. Note that the number of states in the model will increase exponentially with *N*. This, however, is not a problem since we will always expect that the total number of CBDs is quite small.

5. Empirical results from the model

In preparation.

6. Concluding remarks

In preparation.

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