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Consumer Price Index formulas at the elementary aggregate. A new proposal from the economic point of view

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Abstract
This paper proposes a new formula to calculate the elementary indices of a Consumer Price Index. Firstly, we demonstrate that formulas habitually used by Statistic Agencies do not reflect the expected consumer's behaviour. Secondly, we show that the elementary index defined as the harmonic average inversely weighted by prices in the base period is, from the economic and axiomatic point of view, superior to any other habitually used formula when only information for prices are available and are referred to homogenous goods.

Key words: Consumer Theory, Cost of living, Elementary Aggregate Price Index, outlet substitution bias

1. Introduction

The price level in the aggregate economy and, more concretely, controlling its changes, has become one of the high-priority objectives within the framework of the regional macroeconomic analysis. Its different evolution could modify the interregional capital and commercial flows, being able to cause strong shocks, and of asymmetric nature, in each economy.

The first step to reach this objective is obtaining a trustworthy and comparable measurement of the inflation in the different regions to be compared. The Index Number Theory is then used to calculate Consumer Price Indexes (CPI) the regional level.

The calculation of CPI is made, at least, in two phases. In the first one, Elementary Price Index is considered (EPI). In the second and later phases, these EPI are combined, along with weighting information based on household's expenditure, to obtain CPI for different aggregation levels to the country level.

As previous step to the calculation of the IPE and CPI, the set of goods and services has to be defined based on households' consumption behaviour. These sets are grouped in layers, named elementary aggregates, based on their homogeneity of satisfying consumer's necessities. The COICOP (Classification Of Individual Consumption by Purpose) has important implications at the time of analyzing the behaviour of the consumer within each elementary aggregate, because of a high possibility of substitution between products. Nevertheless, this possibility diminishes and can get to be null when the goods and services satisfy necessities with very different nature. Whether what is wanted it is to calculate an EPI that correctly reflects the consumer behaviour, the described homogenous character cannot be forgotten, especially if, in addition, we take into account that National Statistics Agencies have no expenditure information available for weighting purposes, only data of prices to calculate EPI. This paper is focussed on analysis of the formula used to obtain the IPE, with the limitations of available information just commented.

The election of the formula for the IPE has not been widely studied in the economic literature, being the proposal by Carli in 1764 and Dutot in 1738 [extracted Reference of OIT (2003), chapter 20, pages 12-13] the most often used for practical purposes. Nevertheless, Fisher (1922) had already recommended not using the Carli's formula because of the bias to the rise that it introduces [Fisher (1922), pages 29-30]. Throughout the 20th century different authors has continued looking for the ideal formula extending possible approaches to the subject: the approach of Divisia, the stochastic approach, the economic approach and the axiomatic approach. The final summary of these studies can be synthesized in "Toward to Dwells Accurate Measure of The Cost of Living" by the Advisory Commission To The Study The Consumer Price Index presented in 1996. This report, also known as Boskin's Report, suggests the use of geometric mean price indices at the elementary aggregate for the EPI, this formula is attributed to Jevons in 1983 [OIT (2003), chapter 20, pages 12-13].

In the present paper, we demonstrate that all usually formulas for the calculation of the IPE are incoherent with the theory of consumer behaviour, in an aggregate characterized by the high level of substitution caused by homogeneity in the consumption purpose. In addition, the formula proposed by Rodriguez, González and Rodriguez (2005), is not only superior from the axiomatic point of view, but also from the economic approach, is the only one that is able to reflect the expected consumer behaviour. In the following point, the most outstanding aspects for the EAI elaboration are presented. In the third section, consumer's behaviour and its consequences are studied using three habitually used utility functions. In section 4, the price index for each utility function are derived, and related to the proposed formulas by Dutot, Carli, Jevons and Rodriguez, Gonzalez and Rodriguez (2005). In section 5 the main conclusions are shown.

2. Some important aspects on EAI calculation

The Elementary Aggregate Index is the price index of an elementary aggregate comprising only price data, because an elementary aggregate is, as defined by the Commission Regulation (EC) nº 1749/96, referring to the expenditure or consumption covered by the most detailed level of stratification of the Harmonized Index of Consumer Prices and within which reliable expenditure information is not available for weighting purposes.

Theoretically, prices for all products in both time periods should be known. Nevertheless, EAI are actually estimated using data from samples in the outlets of products, not to the consumer, at both time periods.

Therefore, sampling must define for each elementary aggregate, what are the points of sale and products in the sample. Criteria to select establishments are several, but all of them are oriented to reach the higher degree of representativeness of consumption habits of the population. Criteria to select products are basically two: fixed products are selected for each aggregate (this is the case of the Spanish EAI), or a simple random sample of products within the aggregate. Each alternative has important consequences. The fixed article criterion is based on its representation capacity, i.e., chosen products are the most frequently consumed. This alternative assumes that all the products in an elementary aggregate have, in prices, a common tendency.

This assumption is not made when doing a simple random sample within the products of the CPI basic level, but the problem is that the selected products can not be representative of consumption habits.

The fundamental characteristic of a CPI is that it measures changes in prices of a set of goods and services that are representative of the consumption habits of a population. Simple random sample alternative can obtain no very realistic results. For example, following the definition of elementary aggregate, a simple random sample could lead us to use prices of the meat of camel, horse and wild boar to calculate the EAI of the elementary aggregate meat. It is evident that this index would not represent the consumption habits of many countries.

Another important characteristic related to product selection is the homogeneous/heterogeneous nature of the final prices set. Using fixed product, prices will correspond with very homogenous products. However, when using simple random sample, the degree of the heterogeneity of products can be high and the space comparability of prices very low. The higher the heterogeneity of products, the more difficult EAI interpretation will be.

If goods or services are homogenous, it would have no sense to consider, within an elementary aggregate, the product level and we could interpret that each price corresponds with an establishment. In this case, the approached problem is related to outlet substitution bias. Since the prices are measured in the shops and not at homes, the calculated consumer price index does not reflect changes in consumer's purchasing habits due to shops substitution. Nevertheless, the prices of the same or similar goods and services are often very different in shops and it favours substitution between shops . "Not accounting for the shopping substitution in the CPI equals to the assumption that all the price difference between stores is because their service level is different" [Guonason (2003), p. 22]. But, pure price difference can exist and a change in market condition make is possible for some households to switch from purchasing at higher to lower prices.

3. Study of the main consumer's utility functions in the CPI and Their suitability at the elementary aggregate level

The economic approach to Index Number Theory starts with the Consumer Theory and the paper by Konus [Konus (1939)]. In a market with k products, the consumer must decide the amount of each good he wishes to consume. Consumer will reach a certain level of satisfaction with that set. It implies that consumer has preferences at the time of consuming, that are sorted by the utility that each product combination produces. We assume that consumer has well defined preferences over different combinations of k consumer commodities or items. Moreover, denoting by q_i the amount of item i={1,2,…,k}, consumer's utility function gives, for any consumption vector $Q=\{q_1,q_2,\ldots,q_k\}$, the utility value that the consumer reaches when consuming Q.

The objective of the consumer is to maximize his utility level, under the budget constraint. Denoting by $U=U(q_1, q_2, \ldots, q_k)$ the utility function and by Y the budget constraint, and solving the maximization problem, the ordinary demand functions (Marshallian demands) are obtained, for a given vector of prices $P=\{p_1, p_2, \ldots, p_k\}$, These functions measure the amounts of each item to be consumed to reach the higher utility level given Y. Indirect utility function can be obtained by replacing the ordinary demand functions in the utility function. It measures the higher utility level that consumer can reach with a certain budget and vector of prices. The cost function C(P,U) is obtained as the inverse of indirect utility function, and it gives the minimum expenditure to reach a certain utility level. Given a vector prices at the base period, P_0, and a vector of prices at the present moment, P_t, the Konus family of true cost of living indices [Konus (1939)] is defined as the quotient of costs functions associated to each one of the price vectors and to a same utility level, this is

$$I(P_1,P_0,U)=\frac{C(P_1,U)}{C(P_0,U)} \tag{1}$$

The main utility functions that have been used in the CPI context are the Leontief and Cobb-Douglas utility function, both from the Production Theory. In addition, in this paper we also use a Bergson utility function, because of its properties that will be shown throughout the paper. The common characteristic to them is that they present income elasticity equal to 1. This means that an increase of 1% in the income modifies the consumed amount of each product in the same percentage.

In order to simplify expressions, in which it follows, we will consider only two products. Therefore, the vectors of prices and quantities are reduced to $P=\{p_1, p_2\}$ and $Q=\{q_1, q_2\}$ and the budget constraint is $Y=p_1q_1+p_2q_2$.

3.1. Study of the Leontief utility function in the CPI and its suitability at the elementary aggregate level

The Leontief utility function is defined as the U function in (2), being a_1 and a_2 two positive constants, whose relation measures the preferences between products and outlets.

$$U = \min\{a_1 q_1, a_2 q_2\} \tag{2}$$

From (2) can be deduced that any additional product unit does not imply any improvement in the utility level. It is necessary that both amounts are increased. Consequently, the use Leontief preferences supposes that products are complementary. What is more, it establishes a fixed relation between the quantities consumed of both products that it is determined by the constants a_i, i.e., $a_1 q_1 = a_2 q_2$ is fulfilled. From the graphical point of view, with quantities of both products in a Cartesian axis, the utility functions have a L shape with the vertex towards the origin. Consequently, to reach a higher utility level, it would be necessary to increase the consumed quantity of both goods, and in the proportion established by the a_i.

Considering that optimization of (2) is equivalent to the fulfilment of $a_1 q_1 = a_2 q_2$, and adding the constrain budget, a system of equations is obtained, from which is immediate to obtain the ordinary demand functions in (3).

$$q_1 = \frac{Y}{p_1 + p_2 \frac{a_1}{a_2}}; q_2 = \frac{Y}{p_2 + p_1 \frac{a_2}{a_1}} \tag{3}$$

Price elasticity and cross-price elasticity are calculated, using the ordinary demand functions, to analyze the substitution capacity of this type of preferences. Thus, for example, for product 1 the price elasticity and cross-price elasticity are in (4).

$$\varepsilon_{q_1}^{p_1} = -\frac{p_1}{p_1 + p_2 \frac{a_1}{a_2}}; \varepsilon_{q_1}^{p_2} = -\frac{p_2 \frac{a_1}{a_2}}{p_1 + p_2 \frac{a_1}{a_2}} \tag{4}$$

It can be observed that cross-price elasticity is negative because the complementary character of the Leontief type preferences. If the price of product 1 is increased there will be a reduction of the consumed quantities of both products.

This result is hardly compatible with the consumer's behaviour within an elementary aggregate, because, the later is formed by a quite homogenous set of goods, so they have a high degree of substitution.

The expected consumer's behaviour within an elementary aggregate should be analyzed on the bases that are defined in section 2. This is, each price can be identified like coming from a different establishment for an only product or service. Actually, if price in an outlet is increased, it would cause the displacement of the consumer's demand toward another point of sale.

To sum up, the use of Leontief consumer's preferences function as theoretical base of EAI calculation is not really adapted because it corresponds with complementary goods and it does not allow any degree of substitution, neither between products nor between establishments.

3.2. Study of the Cobb-Douglas utility function in the CPI and its suitability at the elementary aggregate level

The Cobb-Douglas consumer's preferences function for two products can be expressed as in (5), being $a_1 + a_2 = 1$ two constants that measures preferences by products and/or outlets.

$$U = q_1^{a_1} q_2^{a_2} \qquad (5)$$

The maximization of this function subject to the constraint budget results in the ordinary demand functions (6).

$$q_i = \frac{Ya_i}{p_i}, \ i = \{\ 1,\ 2\ \} \qquad (6)$$

The first element to emphasize in (6) is that cross-price effect does not exist. Changing the price of product 1 does not affect the demanded amount of product 2, since the cross-price elasticity is equal to zero, and the price elasticity is constant and equal to -1. This means that expenditure is fixed for each product and the Cobb-Douglas preferences doesn't keep in mind the relation of prices between products. A consumer would spend $a_i /(a_1 + a_2)$ of his total available rent in product i= {1, 2}.

Therefore, the Cobb-Douglas consumer's preference function, in strict sense, does not allow substituting between products either. It allows substituting between the amount and price of the same product. Nevertheless, this substitution is very limited, since an increase of a 1% in price also produces a reduction of 1% in its demanded amount, not affecting the demanded quantity of the rest of products.

Transferring from these results to the EAI calculation scene implies that the consumer would spend the same amount in each outlet, because of the constants values are not known and they are assumed to be equal. In addition, when a price is increased, consumer would reduce his demand only in that establishment but it would not be increased in a cheaper one.. This entails

a lack of optimization in the consumer's behaviour. Deepening in the limitations of the price elasticity with this preference function, Tellis (1988) developed a meta-analysis of the price elasticity for some specific products. He estimated that the average elasticity was -1.76, clearly superior to the Cobb-Douglas consumer's preferences function price elasticity.
Therefore, the Cobb-Douglas consumer's utility function is not either adapted to represent consumer's behaviour within a set of goods in an elementary aggregate.

3.3. Study of the Bergson utility function in the CPI and its suitability at the elementary aggregate level

The utility function considered in this section belongs to the set of preference functions known as Bergson family. In particular, we will work with (7), being a_i two constants with a similar interpretation to the previous cases.

$$U = \left[a_1 \sqrt{q_1} + a_2 \sqrt{q_2}\right]^2 \quad (7)$$

The ordinary demand functions (8) are obtained maximizing expression (7) subject to the constraint budget $Y = p_1 q_1 + p_2 q_2$.

$$q_1 = \frac{a_1^2 p_2 Y}{p_1\left[p_2 a_1^2 + p_1 a_2^2\right]};\ q_2 = \frac{a_2^2 p_1 Y}{p_2\left[p_2 a_1^2 + p_1 a_2^2\right]} \quad (8)$$

Direct observation of (8) shows that the utility function defined in (7) allows substitution between products. Analytically, the demonstration can be done by calculating the price elasticity and cross-price elasticity. Their expressions are shown in (9).

$$\varepsilon_{q_1}^{p_1} = -1 - \frac{p_1 a_2^2}{p_2 a_1^2 + p_1 a_2^2};\ \varepsilon_{q_1}^{p_2} = \frac{p_1 a_2^2}{p_2 a_1^2 + p_1 a_2^2} \quad (9)$$

As it could be observed, for positive values of a_i, the price elasticity is between -1 and -2, depending, among others, on the price relation, being compatible with the work of Tellis (1988). In addition, cross-price elasticity has the expected sign by the Consumer's Theory for substitute goods, being its value between zero and one. Moreover, the substitution form is (10) and shows that the consumed amount in each outlet and for each price is proportional to the price relation between both goods or services. Consequently consumers will spend more in the lowest price establishment, ceteris paribus.

$$p_1 q_1 = \frac{a_1^2 p_2}{a_2^2 p_1} p_2 q_2 \quad (10)$$

For example, supposing that p_1 and p_2 are the prices of a same product in two establishments and only one consumer. If p_1 is much higher than p_2, it would be expected that the amount spent in the later would be very small. On the other hand, if the prices are equal, it would be expected the same expenditure in both outlets, conditioned to the values of a_1 and a_2. These constants could measure preferences for a certain establishment (in the previous example), or preferences for a certain product mark in an elementary aggregate.

Consequently, the preference function defined in (7) can be representative of consumer's behaviour within an elementary aggregate and can be used to define which is the theoretical price index that must be used for EAI calculation.

After studying the three utility functions, it can be affirmed that the Leontief and Cobb-Douglas preference functions are not adapted to the assumptions in the EAI calculation at an elementary aggregate formed by a quite homogenous set of goods with a high sustituibility degree. Leontief preferences type is for complementary goods and it allows sustituibility neither between products nor between establishments. The Cobb-Douglas preferences function either does not allow changes between products in strict sense, since, although it allows substituting between the amount and the price of the same product, this one is very limited, and so it does not affect to the rest of products. On the other hand, Bergson preference function presented in this paper, not only can measure preferences between establishments but, it can measure preference for a certain mark in an elementary aggregate. Therefore the conclusion is that, in the scope of the elementary indices, only the Bergson utility function is appropriate Consumer's Theory and with the empirical evidence approaches to price elasticity, that indicates it is clearly superior, in absolute terms, to 1.

4. Price index expressions based on utility function. EAI applications

Once the three utility and ordinary demands functions are presented, associated cost functions are obtained by their indirect utility functions using expression (1). Table 1 shows Theoretical Price Index for each studied functions when applying (1), denoting by p_i^0 and p_i^1 to the prices of the product i={1,2} at the base period (0), and present time (t).

Table 1. Theoretical Price Index

Leontief	Cobb-Douglas	Bergson
$\frac{p_1^t a_2 + p_2^t a_1}{p_1^0 a_2 + p_2^0 a_1}$	$\left[\frac{p_1^t}{p_1^0}\right]^{a_1}\left[\frac{p_2^t}{p_2^0}\right]^{a_2}$	$I_B = \frac{\frac{(p_2^t a_1^2 + p_1^t a_2^2)\, p_1^t p_2^t}{(p_1^t + p_2^t)^2}}{\frac{(p_2^0 a_1^2 + p_1^0 a_2^2)\, p_1^0 p_2^0}{(p_1^0 + p_2^0)^2}}$

The Price Index formulas for the Leontief and Cobb-Douglas utility functions are widely known and, consequently, it is not abounded with its demonstration. To obtain the Price Index expression for the Bergson utility function we substitute the ordinary demand functions (8) in the function (7), so the indirect utility function U^I (11) is obtained.

$$U^I = \left[a_1 \sqrt{\frac{a_1^2 p_2 Y}{p_1 \left[p_2 a_1^2 + p_1 a_2^2 \right]}} + a_2 \sqrt{\frac{a_2^2 p_1 Y}{p_2 \left[p_2 a_1^2 + p_1 a_2^2 \right]}} \right]^2 \tag{11}$$

From (11) we can write (12).

$$U^I = \frac{a_1^2 a_2^2 (p_1 + p_2)}{p_2 a_1^2 + p_1 a_2^2} Y \frac{p_1 + p_2}{p_1 p_2} \tag{12}$$

Solving the equation (12) for Y, the cost function (13) is obtained.

$$C(P,U) = \frac{U p_1 p_2 (p_2 a_1^2 + p_1 a_2^2)}{a_1^2 a_2^2 (p_1 + p_2)} \tag{13}$$

Finally, considering (1), the Price Index formula for the Bergson utility function can be written like (14), same expression as in Table 1.

$$I_B = \frac{\dfrac{(p_2^t a_1^2 + p_1^t a_2^2) p_1^t p_2^t}{(p_1^t + p_2^t)^2}}{\dfrac{(p_2^0 a_1^2 + p_1^0 a_2^2) p_1^0 p_2^0}{(p_1^0 + p_2^0)^2}} \tag{14}$$

Once the theoretical expressions for the Price Index of the studied utility functions are obtained, the following points are dedicated to relate them with the EAI usual formulas. To obtain these relations we will consider that, as it is already commented in the introduction, in the scope of the EAI only information on prices are available, and constants (a_i) values are unknown.

4.1.- Leontief utility function and its relation with Carli and Dutot expressions

First of all, taking a_1=a_2=0.5, the Leontief Price Index (I_L), is the quotient of the arithmetic mean of prices in t and 0. This formula is also known as Dutot formula and we demonstrate that it is not suitable for the EAI calculation, because it can be derived from the Leontief utility function. Secondly, starting off Leontief utility price index derived in (15),

$$\frac{p_1^t a_2 + p_2^t a_1}{p_1^0 a_2 + p_2^0 a_1} \tag{15}$$

and dividing numerator and denominator by $p_1^0 p_2^0$, defining I$_i$=$\frac{p_i^t}{p_i^0}$ and $w_i^t = \frac{p_i^t q_i^t}{Y}$, it can be obtained that the Leontief Price Index is (16), i.e., the average of price indices.

$$I_L = \sum_{i=1}^{2} w_i^0 I_i \tag{16}$$

This last expression is the weighted Carli Price Index. Under the restriction on the constants value are equal to 0.5, un-weighted Carli Price Index is obtained, i.e., the arithmetic average of Price Indices. Again the results of section 2 would indicate the inconvenience to use this formula for EAI calculation, since it is also derived from a preferences function that does not suitably represent consumer's behaviour within an elementary aggregate.

Thirdly, returning to the general formula in the Leontief Price Index and considering $a_1 q_1^0 = a_2 q_2^0$ is fulfilled, (17) can be written.

$$a_1 = a_2 \frac{p_1^0}{p_2^0} \frac{w_2^0}{w_1^0} \tag{17}$$

Replacing (17) in the general formula of the Leontief Price Index (15), this can be written as (18).

$$I_L = \frac{\sum_{i=1}^{2} w_i^0 p_i^t}{\sum_{i=1}^{2} w_i^0 p_i^0} \tag{18}$$

(18) is the weighted Dutot Price Index. This is, the quotient between the arithmetic means of prices at the base and t period, using weights from the base period. When these weights are equal to 0.5, the unweighted Dutot Price Index is obtained. It has to be noticed that, the assumption of equal weights implies that $p_1^0 q_1^0 = p_2^0 q_2^0$ and since $a_1 q_1^0 = a_2 q_2^0$ is fulfilled, the assumption under using weighted Dutot Price Index in the EAI calculation is that constants (a_1, a_2) are equal to prices (p_1, p_2). Therefore, the use of Leontief preferences implies that outlets with higher prices at the base period are preferred by consumers.

We can conclude that Leontief preference type do not represent consumer's behaviour within an elementary layer. Since Carli and Dutot expressions are derived from this type of preferences, both formulas are not adapted to resume the information that comes from product prices within an elementary aggregate.

4.2.- Cobb-Douglas utility functions and its relation with Jevons expression

In sections 3 we have shown that Cobb-Douglas preference type are not adapted for EAI calculation. With the information in table 1, it could be clearly deduced that it agrees with the weighted price index formula of Jevons, when all the constants are positive and adding to 1. Moreover, when no information is available and these constants are taken equal to 0.5, the un-weighted Jevons formula (19) is obtained.

$$I_J = \sqrt{q_1 q_2} \qquad (19)$$

This means that the un-weighted Jevons formula is not adapted for EAI calculation, since it supposes the same expenditure in all the establishments or in each price, and a change in price in an establishment would not modify the demand in another one.

4.3- The Bergson utility function and its relation with Rodriguez, Gonzalez y Rodriguez (2005) proposal

From Consumer's Theory, expenditure is expected to be proportional to the price relation. The Bergson preference fulfils this property. EAI expression for this utility family is shown in (14), where the constant values are un-known. Taken them all equal, expression (14) becomes (20).

$$I_B = \frac{\dfrac{a^2(p_2^t + p_1^t)p_1^t p_2^t}{(p_1^t + p_2^t)^2}}{\dfrac{a^2(p_2^0 + p_1^0)p_1^0 p_2^0}{(p_1^0 + p_2^0)^2}} = \frac{\dfrac{p_1^t p_2^t}{(p_1^t + p_2^t)}}{\dfrac{p_1^0 p_2^0}{(p_1^0 + p_2^0)}} \qquad (20)$$

Rearranging (20) as in (21), the theoretical price index formula of the Bergson utility function is obtained.

$$I_B = \frac{\dfrac{p_1^t p_2^t}{(p_1^t + p_2^t)}}{\dfrac{p_1^0 p_2^0}{(p_1^0 + p_2^0)}} = \frac{p_1^t p_2^t (p_1^0 + p_2^0)}{p_1^0 p_2^0 (p_1^t + p_2^t)} = \frac{\dfrac{(p_1^0 + p_2^0)}{p_1^0 p_2^0}}{\dfrac{(p_1^t + p_2^t)}{p_1^t p_2^t}} = \frac{\sum_{i=1}^{2} \dfrac{1}{p_i^0}}{\sum_{i=1}^{2} \dfrac{1}{p_i^t}} \qquad (21)$$

Expression (21) agrees with the proposal by Rodriguez, Gonzalez and Rodriguez (2005) for the EAI calculation in an elementary aggregate j for 2 prices. Generalizing expression (21) for K_j prices does not suppose any difficulty. The authors demonstrate that I_B is equal to the harmonic average of the simple Price Indices weighted by the inverse of prices in the base period, resulting in expression (22).

$$I_B = I_{A(t/0)} = \left[\sum_{i=1}^{K_j} {}^j w_i^0 \times \left(\frac{{}^j p_i^t}{{}^j p_i^0} \right)^{-1} \right]^{-1}, \text{con } {}^j w_i^0 = \frac{1}{{}^j p_i^0 \times \sum_{i=1}^{K_j} \frac{1}{{}^j p_i^0}} \tag{22}$$

Also, Rodriguez, Gonzalez and Rodriguez (2005) have demonstrated that, from the axiomatic point of view, (22) is superior to the Jevons expressions when products in an elementary aggregate are homogenous, since the former fulfils more desirable properties than the latter.

What is more, working again with two products and no information on the constants, it is easily to demonstrate that the proposed index, derived from a Bergson preference function, is the only of the studied ones in which expenditure in each product is proportional to the price relation.

The demonstration can be made by studying the implicit weights in expenditure that the different formulas derived from the studied utility functions have. Denoting by G_i the expenditure made in product i, its weight is G_i/Y.

Implicit weight for Carli expression is 0.5. Therefore, (23) is fulfilled.

$$\frac{G_i}{Y} = 0,5 \Rightarrow \frac{p_i q_i}{p_1 q_1 + p_2 q_2} = 0,5 \tag{23}$$

It is clear, from this last expression that $G_1=G_2$. Dutot expression can be written as in (24) [Rodriguez, Gonzalez and Rodriguez (2005)].

$$I_D = \sum_{i=1}^{2} \frac{p_i}{\sum_{i=1}^{2} p_i} I_i \tag{24}$$

Repeating the process made with Carli in (23), (25) is obtained.

$$\frac{G_i}{Y} = \frac{p_i}{\sum_{i=1}^{2} p_i} \Rightarrow \frac{p_i q_i}{p_1 q_1 + p_2 q_2} = \frac{p_i}{\sum_{i=1}^{2} p_i} \tag{25}$$

Clearing q_1 in (25) we can conclude that the use of the weight $\frac{p_i}{\sum_{i=1}^{2} p_i}$ in terms of expenditure participation implies that $q_1=q_2$.

For the index proposed by Rodriguez, Gonzalez and Rodriguez (2005), the starting point is (26).

$$\frac{G_i}{Y} = \frac{1}{p_i \times \sum_{i=1}^{2} \frac{1}{p_i}} \tag{26}$$

(26) can be re-written as (27).

$$q_1 = [p_1 q_1 + p_2 q_2] \frac{1}{p_1^2 \left[\frac{1}{p_1} + \frac{1}{p_2} \right]} \tag{27}$$

Clearing q_1 in (27), (28) can be obtained.

$$q_1 = \frac{\frac{p_2^2 q_2}{p_1[p_1 + p_2]}}{1 - \frac{p_2}{p_1 + p_2}}, \tag{28}$$

From (28) is immediate to demonstrate that the weight of the index proposed by Rodriguez, Gonzalez and Rodriguez (2005) implies that expenditure in product 1 is proportional to the expenditure in product 2, being the constant the price relation, this is (29).

$$p_1 q_1 = \frac{p_2}{p_1} p_2 q_2 \tag{29}$$

The application of this procedure to the formula of Jevons is more complex because of its multiplicative character. Nevertheless, for the two goods case the following approach can be made. The un-weighted Jevons index for two products is $I_J = I_1^{0.5} I_2^{0.5}$. This it is equivalent to a generic additive index of the form $I_J = S_1 I_1 + S_2 I_2$, being $S_1 + S_2 = 1$. Therefore (30) is fulfilled.

$$\frac{S_1 I_1 + S_2 I_2}{I_1^{0,5} I_2^{0,5}} = 1 \tag{30}$$

(30) can be re-written as (31), where $\alpha = \sqrt{\frac{I_1}{I_2}}$.

$$S_1 \alpha + S_2 \frac{1}{\alpha} = 1 \tag{31}$$

Keeping in mind that $S_1 + S_2 = 1$, (32) can be developed.

$$S_1 = \frac{1}{1 + \sqrt{\frac{I_1}{I_2}}} ; S_2 = \frac{\sqrt{\frac{I_1}{I_2}}}{1 + \sqrt{\frac{I_1}{I_2}}} \tag{32}$$

Now, the general procedure can be applied the same way as it was applied to Carli, Dutot, Rodriguez, Gonzalez and Rodriguez (2005) indices. This is, equalling the expenditure weight to the implicit weight, as in (33).

$$\frac{p_1 q_1}{p_1 q_1 + p_2 q_2} = \frac{1}{1 + \sqrt{\frac{I_1}{I_2}}} \qquad (33)$$

Working with (33), it can be shown that the expenditure is proportional to the square root of the quotient of simple indices (34).

$$p_1 q_1 = \sqrt{\frac{I_2}{I_1}} p_2 q_2 \qquad (34)$$

This result is important, since it implies that the relation between the amounts spent in two products does not depend on the price relation, but on their changes in price relation. Therefore, as Rodriguez, Gonzalez and Rodriguez (2004) have demonstrated, this result is compatible with an objective of price stability but not with price convergence. Anyway, this result is not more than a particular case defined by the restriction imposed on (30) and is not completely comparable with the rest of expressions.

5. Conclusions

In the present paper we have demonstrated the inconsistency, from the economic point of view, of using Carli, Dutot, and Jevons expressions for EAI calculation in homogeneous elementary aggregates, because these formulas are derived from preference functions that are not suitable to represent consumer's behaviour.

A more coherent solution with Consumer Theory is derived from Bergson utility functions. Concretely, using the preference function defined in (7). This function has a theoretical price index equal to the quotient between the inversed prices average at the base period divided by the same prices average, but at the present moment.

This index agrees with the one proposed by Rodriguez, Gonzalez and Rodriguez (2005) defined as the harmonic average of the price indices weighted by the inverse of prices at the base moment.

The new formula for the EAI is compatible with the Consumer Theory and fulfils all the desirable properties for an index number within an homogenous elementary aggregate. In addition, it is compatible with the estimations of price elasticity for substitutive products.

Finally, the proposed formula implies that expenditure is proportional to the price relation, being, consequently, superior to the rest of formulas which imply equality between quantities, equality between expenditures or proportional to the price indices relation.

The main conclusion is that the substitution of the formula of the elementary aggregate by the proposed formula in this paper must be studied by Statistic Agencies, especially when its implementation does not suppose any additional cost.

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