

THE EVOLUTION OF CITY SIZE DISTRIBUTION IN PORTUGAL: 1864-2001

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Abstract

The rank-size model - which states that the size distribution of cities in a country follows a Pareto distribution - has been recognized as one of those stylised facts or amazing empirical regularities, in spatial economics.

A common problem in city size distribution studies concerns the definition of “cities”, namely the consistency of those definitions over time. In this paper we use a city-proper data base which uses a consistent definition of cities from 1864 to 1991. Portugal is a country with long established national borders and whose mainland urban system shows a constant number of cities over that period.

In Portugal, empirical evidence on city size distribution based on census data shows that two large cities dominate the urban system, associated with a large number of very small cities and a clear deficit of medium-size cities. In this paper we analyse the evolution of the rank size exponent and examine the effect of varying city size cut-offs on the estimated value of that exponent. Then, we study the deviations of the rank-size distribution from linearity. Finally, we explore the dynamics underlying the evolution of the urban system by examining the relationship between city growth rates and city size.

. **Keywords:** city size distribution, Zipf’s law, rank-size, urban hierarchy, urban primacy

1. Introduction

Zipf's law - which states that the size distribution of cities in a country follows a Pareto distribution - has been recognized as one of those stylised facts or amazing empirical regularities, in spatial economics.

The first geographical analysis of city size distribution goes back to the beginning of the last century, with the pioneering works of Auerbach (1913), Lotka (1924) and Goodrich (1926), further developed by Singer (1936) and Zipf (1949). They stated that the city size distribution follows a Pareto distribution:

$$(1) R(S) = AS^{-a}$$

where $R(S)$ denotes the number of cities with at least S inhabitants, A is a constant, S represents the population of the city and a is the Pareto exponent. The work of Zipf led him to conclude that the city size distribution took a special form where $a = 1$ and A is the size of the largest city in the urban system. In this case, equation (1) can be written as follows:

$$(2) R(S) = P_1 S^{-1}$$

where $R(S)$ indicates the rank of the city with size S and P_1 the size of the largest city. Equation (2) is known as the rank-size rule or Zipf's Law.

Empirical work on city size distribution evolved in the fifties with the work of Allen (1954), who provided evidence based on a cross section analysis of 58 countries and a long-term evolution for 9 of those countries. Another seminal contribution is that of Rosen and Resnick (1980) who examined city size distribution for a sample of 44 developed and developing countries. In the early nineties, empirical studies led to a general acceptance of the rank-size model as a synthetic description of the hierarchical organisation of urban systems. The debate focused on methodological questions related first to the effect of city definition and minimum size threshold, for including a city in the sample, on the estimates of Pareto exponent, and second to the deviations from rank-size regularity. More recent advances such as Guérin-Pace (1995), Eaton and Eckstein (1997), Gabaix (1999), Soo (2002), Black and Henderson (2003) and Ioannides and Overman (2003) addressed both empirical evidence and econometric issues as well as the theoretical foundations of city size distribution, in the line of recent spatial theories of economic growth.

The present paper analyses the evolution of city size distribution in the mainland Portugal using data from Population Census from 1864 up to 2001. First, we present the rank-size model and results from empirical work for multi and single country studies. Then we apply this model to Portugal, beginning by a description of the long term evolution of urban hierarchy using a graph in which we draw the relationship between the rank of each urban centre and its size. We use ordinary least squares (OLS) to estimate the rank-size parameters and examine the sensibility of these estimates to the lower size threshold for including a city in the sample. Then, we study the deviations of the rank-size distribution from linearity. Finally, we explore the dynamics underlying the evolution of the urban system by examining the relationship between city growth rates and city size. This is a

preliminary approach, aiming to isolate some of the relationships that might contribute to understand the underlying forces of rank-size distribution shape and its time evolution.

2. City size distribution

2.1. The Rank-size Model

The estimation of rank-size model requires the ordering of cities from the largest down to the smallest and it relates the rank of a city with its size, measured by its population, as follows:

$$(3) R_{it} = AP_{it}^{-a} \text{ or, in logarithmic form, } (3') \log R_{it} = \log A - a \log P_{it}$$

where R_{it} is the rank of the i^{th} city in time period t , P_{it} is the size (population) of the i^{th} city in time period t , A is a constant and a is the Pareto/Zipf's exponent. This formulation is known as the Pareto equation.

Alternatively, empirical studies use the Lotka's formulation which is given by the following equation:

$$(4) P_{it} = BR_{it}^{-\beta} \text{ or, in logarithmic form, } (4') \log P_{it} = \log B - \beta \log R_{it}$$

where B is a constant and β is the inverse of Pareto exponent. The two formulations can further be related to as $B = A^{\beta}$.

Accordingly to this model, city size distribution is characterised by the number of cities and two parameters: the exponent (a or β) and the constant term (A or B).

Using Lotka's formulation, when $0 < \beta < 1$ the rank-size curve is flatter and city sizes are more evenly distributed than that predicted by Zipf's law ($\beta=1$). In particular, considering the limiting value of $\beta \rightarrow 0$ all cities would have the same size. On the other hand, when $\beta > 1$, the rank-size curve becomes steeper. In this case, urban hierarchy is more contrasted than in Zipf's case and cities in the top of the hierarchy are larger. Here we obtain a more heterogeneous distribution of city sizes. In the limiting case of $\beta \rightarrow \infty$, there would be just one city in the urban system. Thus, β is a measure of city sizes inequality in a given urban system and time period.

Concerning the constant term B , it provides a measure of the size of the largest city in the system. As a rule, we would expect a growing value in result of the process of urbanisation and concentration in the largest city. However, in the last decades, some urban systems exhibit a decreasing value for the constant, in result of spatial rearrangement of urban system, as congestion becomes evident in larger cities and medium and small size cities grow at faster rates.

2.2. Review of empirical studies

Existing empirical studies usually take two different approaches: 1) comparative studies, which consider a sample of different countries and try to provide evidence about how well a power law describes the city size distribution; 2) single country studies which examine city size distribution in a given country and its time evolution. In both perspectives some authors use Lotka's formulation while others recur to Pareto's equation.

In order to compare the OLS estimates for the slope (a or β) we use the transformation in equation (5), which results from OLS estimators and coefficient of determination formula's.

$$(5) \mathbf{b} = \frac{R^2}{\mathbf{a}},$$

where R^2 is the coefficient of determination, whose value is identical in both formulations (Lotka and Pareto).

Table 1 presents the main distinctive aspects of some cross country studies. Generally, these studies employ Pareto's formulation. Sample characteristics vary from study to study, rendering difficult results generalisations.

Table 1 – Cross country studies

Author (date)	Number of countries, minimum city size	Pareto exponent (<i>absolute value</i>)	City definition
Singer (1936)	7 countries, with at least 2000 inh. Time evolution for USA, Germany and France from XIX th century to early XX th century	Min: 0.93 (Canada, 1935) Max: 1.59 (Japan, 1920)	City Proper
Rosen and Resnick (1980)	44 countries, 1970 census data, 50 largest cities for most countries	Min: 0.81 Max: 1.96	City Proper
	6 countries	Min: 0.82 Max: 1.125	Metropolitan Area
Moriconi-Ébrard (1993) (<i>Lotka equation</i>)	78 countries with at least 30 cities with a minimum threshold of 10 000 inhabitants in 1980's	Min: 0.73 Max: 1.38 (<i>Pareto equivalent estimates</i>)	Urban Agglomeration
Soo (2002)	75 countries, from 1972 to 2001 data, usually with a minimum threshold of 15 000 inhabitants	Min: 0.729 Max: 1.719	City Proper
	26 countries	Min: 0.586 Max: 1.23	Urban Agglomeration

Singer's (1936) study, is considered as one of the first systematic empirical analysis of city size distributions. This author demonstrates that the city size distribution could be described by the same relationship used by Pareto to study income distribution (Parr, 1985,

199). He uses the Pareto exponent to provide a measure of interurban concentration degree.

Another key empirical study is that of Rosen and Resnick (1980), who use rank-size and primacy measures to characterise the size distribution of cities in a sample of 44 countries, in 1970. Country samples consider the fifty largest cities. For those cases where over 50 cities had at least 100000 inhabitants, all the cities surpassing this threshold were included in their sample. They found that almost three fourths of the countries had Pareto exponents above the unity,¹ thus concluding that in most countries city size distribution is more even than predicted by rank-size rule (Rosen and Resnick, 1980, 166). They also examine the sensitivity of Pareto exponent estimates to the city definition and to sample size, finding that the value of the exponent is quite sensible to both choices. To test for non linearity in city size distribution, they add quadratic and cubic terms to their starting equation. For 30 of the 44 countries the curvature of the rank-size line showed an upward concavity, suggesting that growth rates of cities are positively correlated to size. This result conflicts with those of previous studies, which concluded that the rank-size data, when graphed on double logarithmic scales, showed a downward concavity (Rosen and Resnick, 1980, 173).

A less well-known contribution is that of Moriconi-Ébrard (1993)². In his work, the author examines city size distributions for 78 countries, with at least 30 urban agglomerations, considering a minimum threshold of 10000 inhabitants in 1980. He uses both a cross country study and a time series analyses for each country, from 1950 up to the eighties. For the cross country study and the last date, he finds an average Lotka exponent of 1,05 with a weak standard deviation (0,138), which is remarkable if we consider the dimension of his sample and the fact that it has very different countries. Based on a world wide database, covering countries with different levels of development and political systems, his results allow him to draw conclusions regarding the influence of political regime and economic system on the shape of city size distribution (Moriconi-Ébrard, op. cit., 194-196).

Regarding Zipf's law, Soo (2002) addresses two key issues: the appropriateness of Pareto distribution to describe city size distribution and the dependence of results on the estimation methods. He concludes that acceptance of Zipf's law depends on the estimation method. Using OLS he rejects the hypothesis of Pareto exponent equal to one for 73% of the countries in his sample, corroborating Rosen and Resnick's (1980) results which reject the same hypotheses in 82% of the countries. As for the Hill estimator, Zipf's law is rejected only for 40% of the countries. His results for the values of the quadratic term are similar to those of Rosen and Resnick's (1980), but less stronger.

From the above mentioned empirical work some conclusions might be drawn:

- There is a large dispersion of the exponent estimates, but on average, these estimates are within the range [0.85, 1.15] - a power law describes quite well the city size distribution (Gabaix and Ioannides, 2003, 13);
- Pareto exponent estimates tend to be smaller for urban agglomerations than for city-proper data;
- Pareto exponent estimates seem to depend on sample thresholds; studies considering only the upper tail distribution of city size tend to show a larger exponent;

¹ Here therein exponent values are taken in absolute values.

² The fact that it has been written in French and not translated, as far as we know, might justify its not so wide diffusion.

- Pareto exponent estimates seem also to depend on the estimation method.

Another line of empirical work considers the characteristics of the rank-size distribution in a single country and its evolution through time. In table 2 we summarise some results of recent work using this approach.

Dobkins and Ioannides (2000) and Black and Henderson (2002) use data from population census, from 1900 up to 1990, to study the US urban system. While the first two authors define cities as metropolitan areas, according to contemporaneous census definitions, Black and Henderson (2002) use consistent over time definitions for the same period. Generally, consistent definition of cities produces smaller estimates. The long term evolution of the US city size distribution exhibits a declining trend, with full sample Pareto exponent estimates smaller than those for the upper tail of the city size distribution. However, Black and Henderson get a contrasting result for the upper tail distribution, obtaining a larger value of the Pareto exponent for 1990 than in the beginning of the century. As for the variation of Pareto exponent estimates with estimation method, Dobkins and Ioannides (2000) obtain higher values when using OLS than with maximum likelihood estimators.

Table 2 – Single country studies

Author	Country / Period	City-definition	Number of cities		Exponent (<i>absolute value</i>)		Notes	
Fujita, Krugman and Venables (1999)	U.S.A. 1991	Metropolitan Areas	130		1.004		Upper tail	
Gabaix (1999b)	U.S.A 1991	Metropolitan areas	135		1.005		Upper tail	
Dobkins and Ioannides (2000)	U.S.A. 1900-1990	Metropolitan areas (contemporaneous definition)	1900	112	1900	1.044	OLS	
			1950	162	1950	0.999		
			1990	334	1990	0.949		
			1900	112	1900	0.953	Maximum likelihood	
			1990	334	1990	0.553		
			1900	56	1900	1.212	Upper tail	
1990	167	1990	0.993					
Black and Henderson (2002)	U.S.A. 1900-1990	Metropolitan areas (consistent over time definition)	1900	194	1900	0.861	Full sample	
			1950	247	1950	0.870		
			1990	282	1990	0.842		
			1900	64	1900	1.01	Upper tail	
			1990	93	1990	1.18		
Moriconi-Ébrard (1993) (<i>Lotka equation</i>)	Time series estimates for 50 up to 78 countries, in the 1950-1980 period	Urban agglomerations with at least 10000 inhs.	1950	252	1950	0.99	0.996*	France
			1990	435	1990	1.06	0.934*	
			1950	737	1950	1.13	0.883*	US
			1990	1163	1990	1.22	0.818*	
Guérin-Pace (1995) (<i>Lotka equation</i>)	France 1831-1990	Urban agglomerations from a 2000 inhs. threshold to a 100000 inhs. threshold	1831	675	1831	0.72	1.375*	Full sample (2000 inhs.)
			1982	1782	1982	1.05	0.949*	
			1831	5	1831	1.13	0.761*	Upper tail (100000 inhs.)
			1982	56	1982	0.85	1.118*	

* Equivalent estimates of Pareto exponent, calculated as in equation (5)

The results for the USA obtained by Moriconi-Ébrard enhance the sensitivity of Pareto's estimates relatively to the definition of cities. His estimates, considering urban agglomeration with at least 10000 inhabitants, are lower than those of previous authors. However, they exhibit the same temporal trend.

Guérin-Pace (1995) uses data referring to urban agglomerations with 2000 inhabitants or more, to study the French urban system, from 1831 to 1990. For each date, he concludes that the Lotka exponent is fairly stable for city size distribution using a threshold between 2000 and 20 000 inhabitants. However, when considering different years, there is a significant variation in the slope estimates with sample size. On the other hand, the time evolution of the exponent shows increasing values except for city populations of 50000 or more inhabitants. He also, for all census dates, finds that the best adjustment quality occurs for thresholds between 2000 and 10 000 inhabitants.

In conclusion, Pareto exponent estimates are sensible to city definition, sample threshold and the estimation methods. Consequently, when comparing results from different studies we must account for these differences. Nevertheless, there is a consensus in the literature about the good performance of the rank-size model to describe, in a synthetic way, the hierarchical organization of an urban system.

3. Application of the rank-size model to Portugal

A common problem in city size distribution studies concerns the definition of "cities", namely the consistency of those definitions over time. In this paper we use two city-proper data bases for mainland Portugal. The first one was developed by Albergaria (1999) and uses a consistent definition of cities from 1864 to 1991. In this data base, cities are defined taking into account the 1998 administrative classification of Portuguese cities. Population was calculated for each city and each census, using the 1998's city definition. In order to analyse the recent evolution of the urban system, we use another city proper data base built by Ferreira, Cardoso and Silva (2003) based on INE (2002) data, for 1991 and 2001. This data base uses the 2001 administrative classification of places and so the number of cities grows from 111 to 123, from one data base to another. As we observed inconsistency between the two data bases we consider them separately.

The above data bases correspond to urban places which have, in the referred dates, the administrative status of "city" regardless their absolute dimension. As these data bases used the 1998 or 2001's definition of cities, we had another problem: some places, in early dates, had zero population or were too small to be considered urban unities. In order to define whether a place qualifies as a city, in our study, we use an absolute cut-off of 2000 inhabitants, in each census date.

Our sample obeys to two criteria: 1) urban places which have in 1998's or 2001's data base the administrative status of "city"; and 2) have at least 2000 inhabitants, in each census date. With the application of these criteria the number of cities in our sample grows from 85 in 1864 up to 110 in 1991, using data from Albergaria; for the 2001 data base, the total number of cities is 122, in both dates.

The problem with a sample based on administrative city definition is that city boundaries may not coincide with functional and economical boundaries of urban places. On the other hand, our sample is not based on contemporaneous administrative definitions. Applying city definitions to prior decades in a single country study, minimizes the problem of city definition and that of building consistent definitions over time. We must note that in Portugal, as in many other countries, data constraints do not allow alternative approaches to city definition over time.

3.1. A brief characterisation of the Portuguese urban system

Portugal is a country with long established national borders whose mainland urban system dates back to some centuries ago: many of the cities have several hundred years and some of them are even older than the nation³. In Table 3 and Table 4 we present some basic data about the Portuguese urban system.

In 2001, the Portuguese urban system is characterised by small cities, with an average city size of 30895 inhabitants and a median city size of 15382 inhabitants, a low urbanisation rate⁴ (38.2%) and a primacy index⁵ of 15% for the top city and 22% when the two largest cities are considered. Another characteristic, not shown in the table, is the concentration of urban population in the two existing metropolitan areas⁶: 56,9% of total urban population lives in cities belonging to Lisboa and Porto metropolitan areas.

Table 3 - Basic data about the Portuguese Urban System (Full sample), 1864-2001

Data Source	Census date	Number of Cities	Average size	Median Size	Minimum Size	Maximum Size	Urbanization rate (%)	Top Two Primacy Index (%)
Albergaria data base	1864	85	8829	4563	2013	190311	18,83	37,26
	1890	91	11791	5469	2172	300964	23,02	41,70
	1900	97	12397	5815	2044	351210	24,05	43,07
	1920	101	14688	6851	2054	484664	26,17	46,31
	1940	105	19502	9277	2075	694389	28,37	46,54
	1950	108	21571	9755	2009	783226	29,41	45,70
	1960	109	23278	10206	2092	802230	30,60	43,58
	1970	108	25057	10520	2141	769044	33,31	39,73
	1981	110	29637	12457	2189	807937	34,92	34,82
	1991	110	29087	13248	2789	663394	34,14	30,19
Atlas Data base	1991	122	29546	13638	2487	661966	38,46	26,65
	2001	122	30895	15382	2578	564657	38,19	21,96

The long term evolution of the urban system shows a slow increase in the number of cities, between 1864 and 1991, while city population more than quadruplicates in the same period. As a consequence average city size increases from 8829 inhabitants, in 1864, to

³ Portugal is an independent nation-state since 1140, whose mainland borders, despite some adjustments in subsequent centuries, date back to the 13th century.

⁴ The urbanisation rate is defined as the relative importance of urban population in total population, expressed in percentage. In this study, urban population is defined as total resident population in cities, for a given year.

⁵ The primacy index is defined as the ratio of resident population in top one or top two cities to total urban population, expressed in percentage.

⁶ Lisboa Metropolitan Area has 16 cities and it represents 36,7% of total urban population; Porto Metropolitan Area contains 12 cities and its share in total urban population is 20,2%.

29087, in 1991. As a rule, urban population grows faster than total population and the urbanisation rate increases from around 19% in 1864 to 34% in 1991.

Growth rates for urban population and average city size are smaller in the fifties and sixties than in previous inter census periods. The maximum growth rate occurs in the 1970-1981 period, but the following decade registers a negative evolution both in terms of total urban population and in average city size and urbanisation rate. This is due to the loss of population in the two largest cities (Lisboa and Porto) and it is associated to a decrease in the level of primacy of the urban system as well as to an increase in the size of middle and small cities.

For the 1991-2001 period, urban population and average city size increase. There is however a slight decrease in the urbanisation rate as well as a decline in primacy indexes, in a context of continued heavy population losses in the cities of Lisboa and Porto and population gains in middle-small cities, as the growth of median city size indicates.

Table 4 – Basic facts about growth, Portugal 1864-2001

Data Source	Inter Census date	Annual average growth rate (%)			
		Number of cities	Total Urban Population	Average city size	Total Population
Albergaria data base	1864-1890	0,26	1,38	1,12	0,60
	1890-1900	0,64	1,15	0,50	0,70
	1900-1920	0,20	1,06	0,85	0,63
	1920-1940	0,19	1,62	1,43	1,22
	1940-1950	0,28	1,30	1,01	0,93
	1950-1960	0,09	0,86	0,76	0,46
	1960-1970	-0,09	0,65	0,74	-0,21
	1970-1981	0,17	1,71	1,54	1,27
	1981-1991	0,00	-0,19	-0,19	0,04
	1864-1991	0,20	1,15	0,94	0,68
Atlas Data base	1991-2001	0,00	0,45	0,45	0,52

3.2. Application of the rank-size model

The long term evolution of urban hierarchy in Portugal can be visualised using a graph in which we draw the relationship between the rank of each urban centre and its size. Figure 1 presents the rank-size graph for each census date from 1864 up to 1991 and Figure 2, presents the same data for the 1991-2001 period.

On the whole, the shape of the rank-size distribution has remained however stable until the eighties, shifting up in the course of time, as a result of urban growth. We can not infer, from these results that individual city ranking has remained unchanged. In fact, excluding Lisboa and Porto, cities relative position in the urban system has changed. In the course of time, the rank-size graph shows a slight enlargement in the bottom and a significant increase in its height. This result points to an urban growth process characterised by a slow increase in the number of cities, with a considerable growth in the size of the largest city.

Generally, the rank-size line shows an upward concavity between the 3rd and the 20th city, as a consequence of the under-dimension of middle size cities. It presents also a downward concavity in the lower tail of the distribution, translating the proliferation of small cities.

From 1991 up to 2001, we denote a downward counter clockwise movement of the rank-size line, due to the decline in the size of the two largest cities. We observe a more even distribution of city sizes, as the two top cities have lost population, whereas middle size cities have experienced population gains and the dimension of the smallest cities in our sample remained stable.

Figure 1 - Rank-size distributions of Portuguese cities, 1864-1991: Full sample

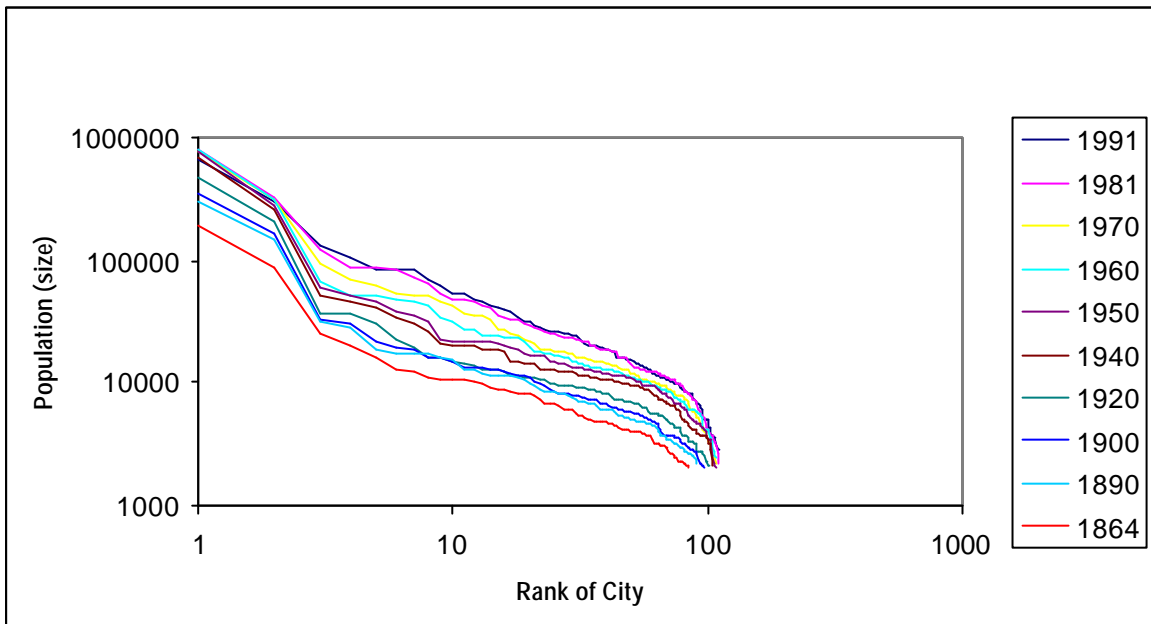
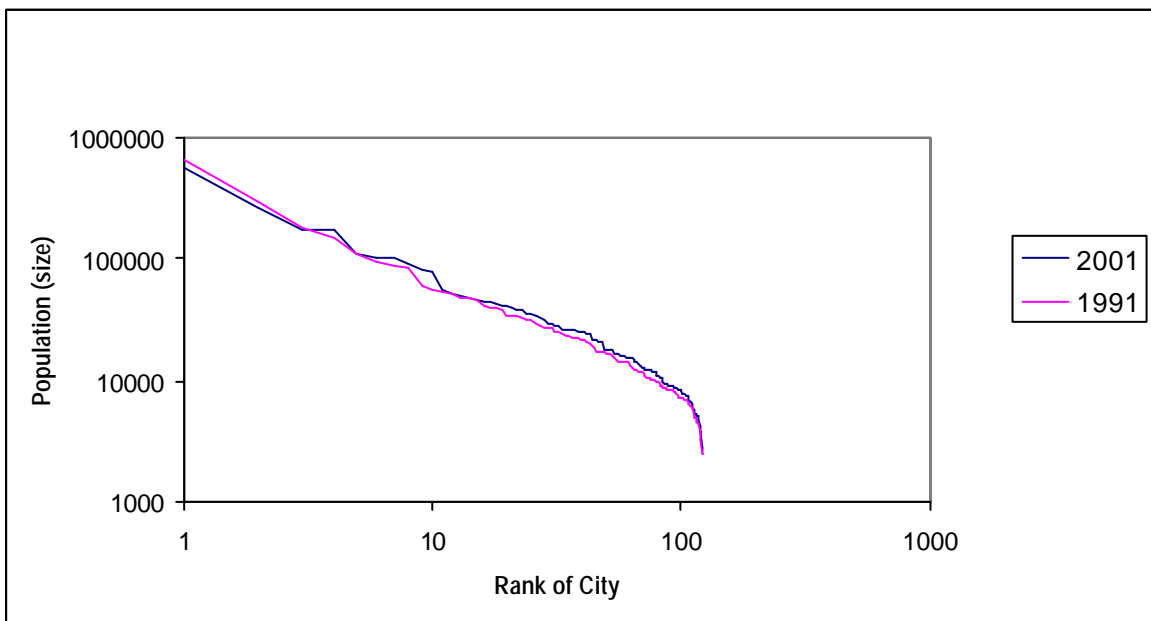


Figure 2 - Rank-size distributions of Portuguese cities, 1991-2001: Full sample (Atlas)



With the purpose of characterising the evolution of the Portuguese urban system we applied rank-size model (cf. equation 4') to our data. We defined two samples of different population thresholds. The first sample (full sample), considers all cities with at least 2000 inhabitants; the second one, uses a minimum threshold of 10000 inhabitants. Comparing the number of cities in the two samples, we conclude that, when the upper tail is used, there is a substantial reduction in the number of cities and the relative importance of very small cities decreases over time. The results of ordinary least squares estimation are shown in Table 5. The estimates of rank-size parameters are all statistically significant at 5% significance level. The quality of the adjustment is quite good, since R^2 are high and close to unity.

Table 5 - Results of OLS estimation of the rank-size parameters⁷

Data base	Census Date	Cities with 2000 inhs. or more				Cities with 10000 inhs. or more			
		Number of cities	Slope	Intercept	R^2	Number of cities	Slope	Intercept	R^2
Albergaria	1864	85	0,796	4,915	0,946	12	1,208	5,160	0,919
	1890	91	0,845	5,095	0,947	19	1,042	5,230	0,875
	1900	97	0,853	5,136	0,936	21	1,024	5,251	0,862
	1920	101	0,839	5,179	0,907	23	1,093	5,360	0,881
	1940	105	0,855	5,332	0,908	45	0,869	5,328	0,874
	1950	108	0,880	5,422	0,899	53	0,822	5,345	0,879
	1960	109	0,898	5,496	0,921	57	0,842	5,425	0,917
	1970	108	0,962	5,630	0,927	59	0,872	5,519	0,955
	1981	110	0,993	5,775	0,931	75	0,864	5,620	0,978
Atlas	1991	110	0,994	5,790	0,947	73	0,875	5,648	0,989
	1991	122	0,991	5,839	0,961	77	0,886	5,711	0,993
	2001	122	0,972	5,855	0,950	85	0,860	5,716	0,991

When the entire system is used we can see that the slope parameter tends to increase through time and approaches the reference value of the rank-size rule, in 1981 and 1991. However, for the last decade and considering Atlas data base, we witness a slight decline in the slope estimate. The estimates of β , ranging from 0,796 to 0,994, indicate that the rank-size curve is flatter than predicted by Zipf's law and city sizes are more evenly distributed.

This result must be interpreted with caution as we have an urban system with primatial characteristics. For instance, if we take the 1991 city size distribution in the Albergaria's database and compare the observed sizes with the expected size of equivalent rank for a top city of 663394 inhabitants and $\beta=1$, all the cities from the 2nd to the 25th rank are under-dimensioned. In particular, population deficit is more notorious for cities ranking from the third to tenth position. The opposite situation occurs from the 26th until the 87th position, where cities are bigger than expected. Finally, for all the remaining positions in the bottom of the distribution, cities are smaller than predicted by rank-size rule – some of them have less than 50% of their expected population.

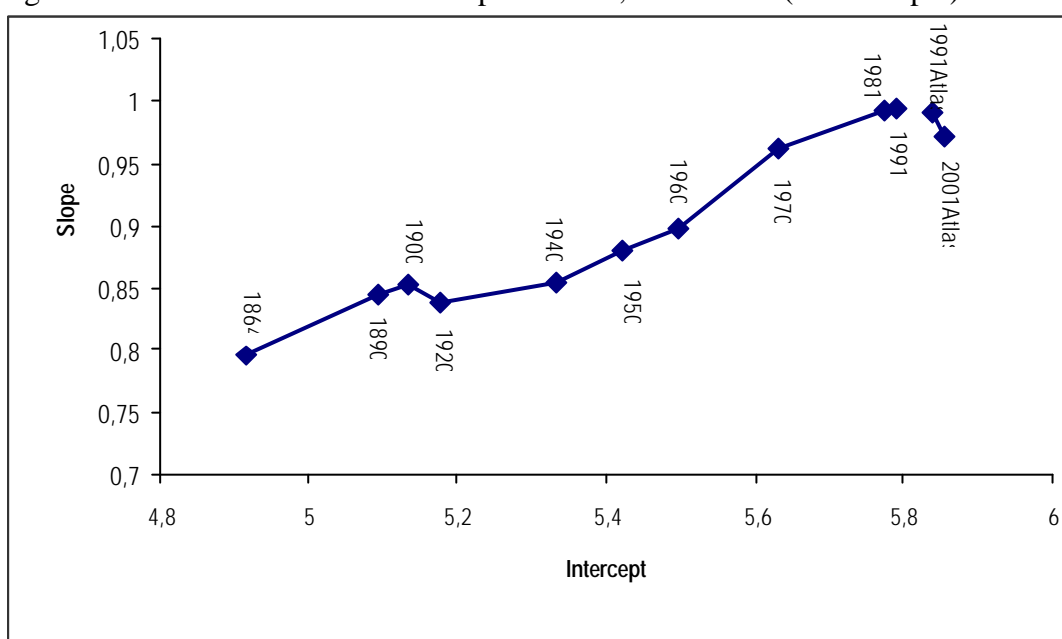
As for the intercept, we observe a continuous increase, reflecting the concentration of urban population in the largest city, as well as the growing urbanisation of the country. However, in the last two decades, the two largest cities have been losing population in pro of contiguous cities, belonging to the same metropolitan area, while medium size cities have experienced significant population gains, especially in the 1991-2001 period. For the

⁷ Complete details available from the authors upon request.

first time, in this last period, the theoretical population of the top city is above its real value, which can be interpreted as a reflex of a process of declining primacy, noticeable in the second half of the 20th century. We must note that positive and significant deviations between the intercept (expected population in the largest city) and the observed size of this city tend to occur when urban systems have more primate city size distributions.

Figure 3 presents the evolution of slope and intercept for the full sample. Until 1991, excluding the 1900-1920 period,⁸ there is a co-evolution of these two parameters, expressing a process of urban concentration in the largest cities, accompanied by the development of the whole urban system. There is, however, in the nineties, a change in this tendency, reflecting the process of redistribution of urban population between central and suburban cities in metropolitan areas.

Figure 3 – Evolution of the rank-size parameters, 1864-2001 (Full Sample)



Finally, for the sample of cities with at least 10000 inhabitants, when we take a long term evolution, the slope decreases with time, remaining fairly stable in the last three decades. This reflects a process of decreasing inequality of the city size distribution when very small cities are excluded. This distinct evolution between the two samples mirrors the changes in growth behaviour of middle-sized cities vis-à-vis the first city. At the beginning of our study period, intermediate cities in the class size of 30000-100000 inhabitants developed more slowly than Lisboa, growing at a faster rate after the fifties.

3.2.1. Sensibility of the slope estimates to sample thresholds

The results obtained in section 2 show that the rank size parameters are sensible to sample threshold definition. In order to make a deep study of this issue we consider different sample cut-offs and compare the OLS results for the slope and R^2 . Sample cut-offs were chosen taking into account the dimension of the Portuguese urban system and current cut-offs for urban definition in the Portuguese statistical system. We did not consider, as most

⁸ Which corresponds to a period of serious political and social instability. In this period Portugal became a Republic. Another relevant event is the participation of the country in the first World War.

of country studies did, sample thresholds of at least 50000 inhabitants because in 2001 we had only 13 cities satisfying this criterion and in 1864 only 2.

Whichever the census date, the estimate of β remains reasonably stable for sample thresholds of 2000 and 5000 inhabitants (Figure 4). For sample sizes of 10 000 or 20000 inhabitants, the slope decreases over time, reflecting a narrowing of inequality among city sizes, in the upper tail of the distribution. For the last decades, slope estimates tend to be stable but decreasing as the threshold increases. In fact, the dynamics of the urban system show a more even distribution as the two top cities loose population. The sensibility of the slope estimates to sample threshold is higher in the beginning of the observation period.

Figure 4 – Sensibility of slope estimates to sample threshold

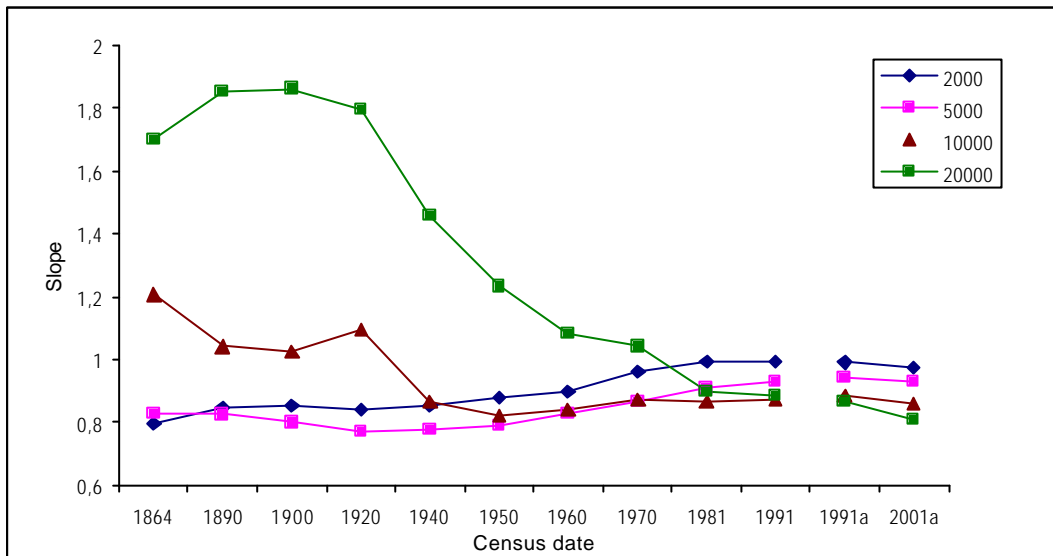
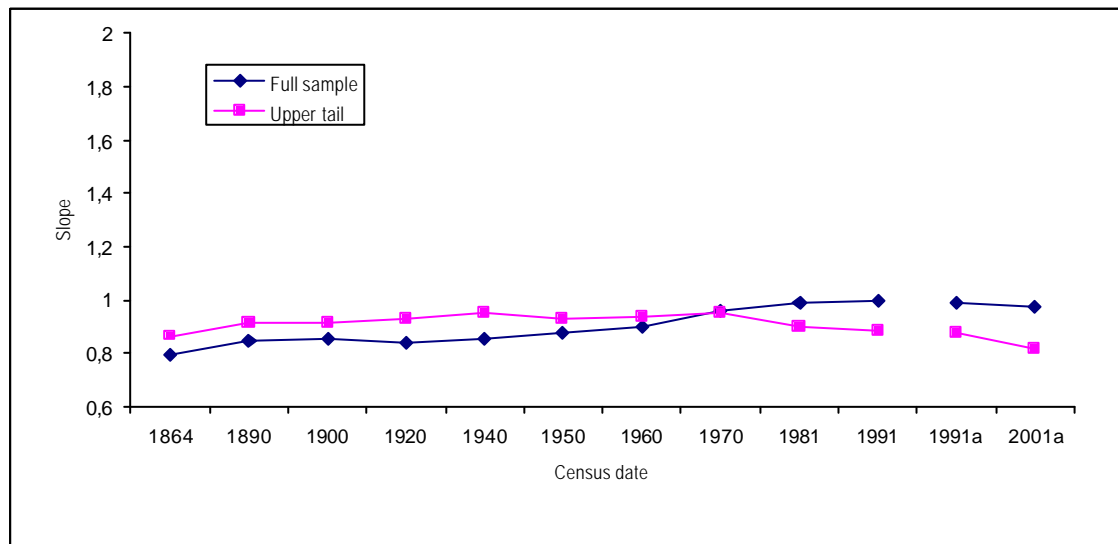


Figure 5 – Sensibility of slope estimates: full sample versus upper 1/3 of cities

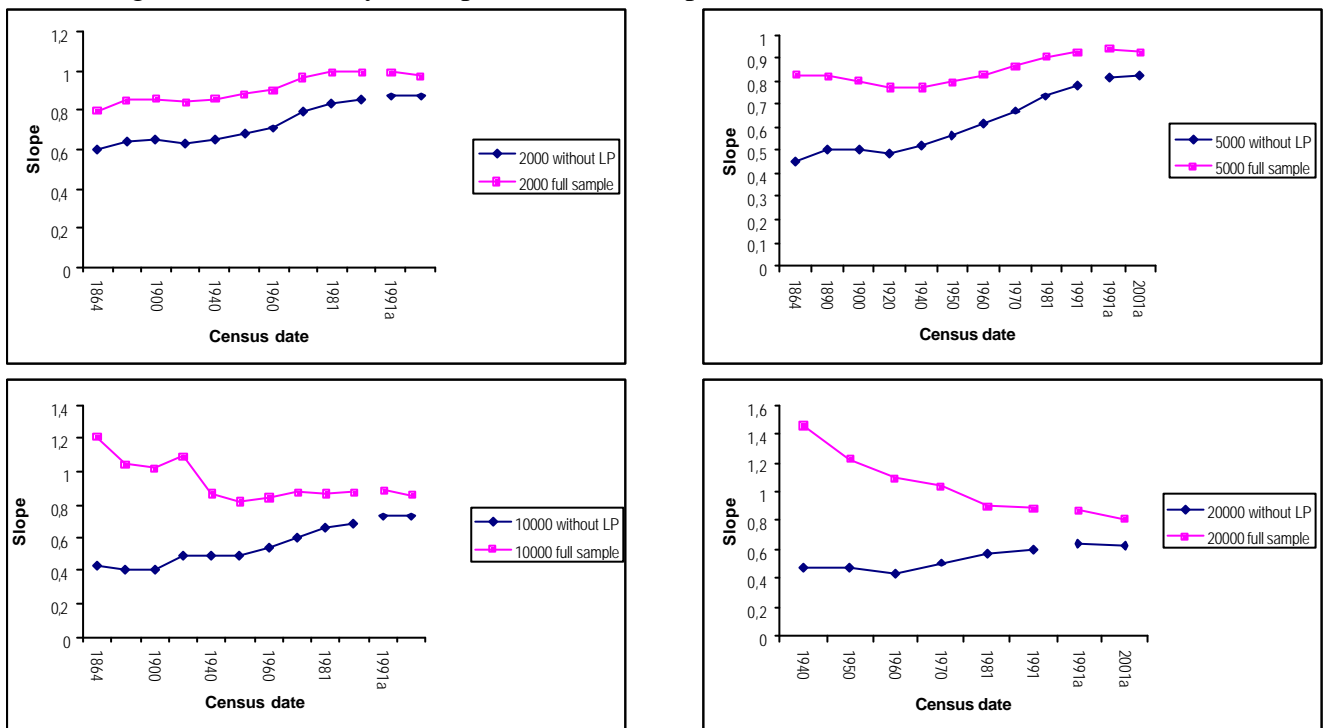


We re-estimated the Zipf's model for the top one-third cities in each census date. Figure 5 compares slope results for the full sample and the upper tail distribution. For the upper tail distribution, slope estimates increase until 1940 and decrease afterwards. Comparing both

samples, until 1960, the estimates are bigger, in absolute terms, when the upper tail is considered. Since then, they are smaller than those obtained for the full sample and differences got more important. This trend reflects a reduction in city size inequality in the upper tail distribution.

As the Portuguese urban system is usually described as a macro-cephalic system, we examine the sensibility of the slope estimate to the exclusion of the two top cities (Figure 6). Regardless the sample threshold, the slope is bigger when all the cities are considered. The exclusion of the two top cities decreases the slope estimate, in all census dates, but there is a clear trend towards convergence.

Figure 6 – Sensibility of slope estimates to top two cities exclusion



There is no clear relationship between the adjustment quality and the threshold. Until 1940 the adjustment quality is better for the 2000 and 20000 population cut offs, whereas in the following two decades the best adjustment occurs in the 5000 and 2000 population thresholds, and henceforth in the 10000 and 5000 cut offs. In Atlas data base the best adjustment occurs in the 10000 and 20000 thresholds. When we exclude the two top cities from the sample, the full sample exhibits always the worst adjustment quality.

For 1864, 1920 and 1940 the best adjustment quality is obtained for the threshold of 20000 inhabitants or more; in 1981 and 1991, the best adjustment quality occurs in the 10000 inhabitants cut-off; for 1950-1970 periods, in the 5000 population threshold and for 1890 and 1900 census date, when the full sample is considered. If we take Atlas Data base, there is no significant difference in the quality of the adjustment for different sample cut-offs but the best adjustment quality is obtained for the population of 10000 or more.

3.2.2. Deviations from rank-size regularity

The fact that slope estimates are sensitive to sample size requires a further analysis of the rank size distribution. In the literature this conflicting results are taken as evidence of non-paretian behaviour of the distribution (the distribution is not log-linear). The Pareto's equation supposes a linear relationship between the logarithm of the size and the logarithm of the rank. The deviations from linearity are also detected when we look at the rank-size graphs. Therefore, we examine the deviations of the rank-size distribution from linearity by adding a quadratic term to equation 3', following the standard approach in literature. Thus, we estimate the following equation:

$$(3'') \log R_{it} = a + b \log P_{it} + c(\log P_{it})^2$$

for the full sample.

The value of the parameter c characterises the curvature: when $c > 0$, the rank-size curve is strictly convex (upward concavity) and when $c < 0$ it is strictly concave (downward concavity). An upward concavity is obtained when the city size distribution has a smaller number of middle-sized cities than predicted by Zipf's Law. In this case, there is a deficit of intermediate cities in favour of largest cities dimension or the number of small cities. A downward concavity means that there is a larger number of middle-sized cities than expected. In this case, there is an excess of intermediate cities relatively to the dimension of the largest cities or to the number of small cities. In rank-size distributions with an upward concavity, the largest city will be larger and smaller cities will be more numerous than expected in a linear relationship between the logarithm of city size and the logarithm of its order. On the other hand, in rank-size distributions with a downward concavity, middle-sized cities are larger than expected in a linear relationship between the logarithms of size and order.⁹

The long term evolution of parameter c is depicted in Figure 7.¹⁰ Until the middle of the 20th century the value of c is positive and decreasing, showing that urban growth was accompanied by concentration in the largest cities and proliferation of small cities. In 1950 and 1960, the value of c is not significantly different from zero meaning that the rank-size distribution tends to conform to linearity.¹¹ In other census dates, the value of the quadratic parameter is negative reflecting the growth of middle-sized cities, reinforced in the last decades.

Our results differ from those of Moriconi-Ébrard (1993), for 1981, and Soo (2002), for 2001, who obtained for Portugal a positive value for the quadratic term of the equation. This discrepancy suggests that estimates of c are sensitive to city and threshold definition. In fact, Moriconi-Ébrard (1993) uses urban agglomerations with at least 10000 inhabitants, while Soo (2002) uses Brinkhoff's data base,¹² with a threshold of 15000 inhabitants.

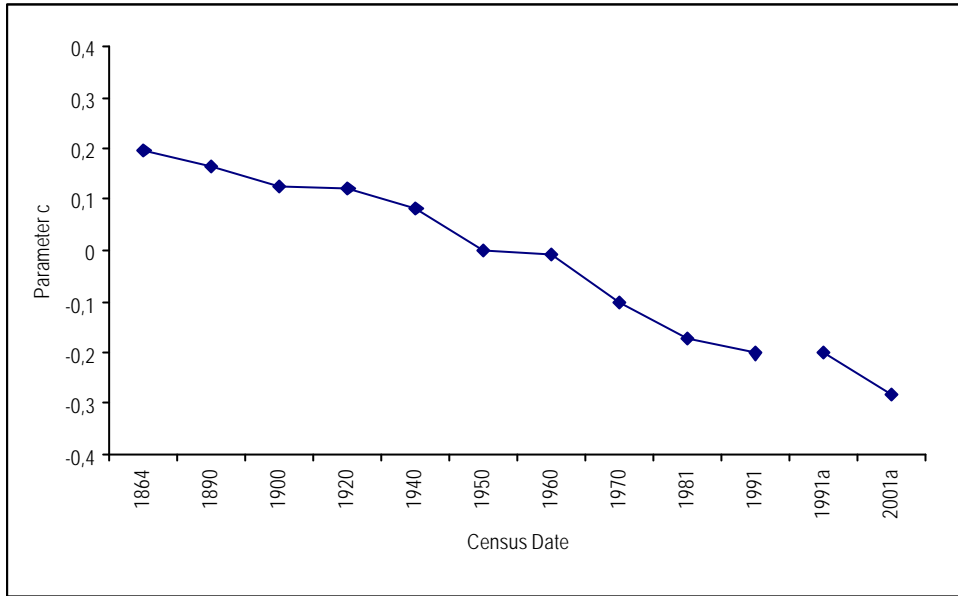
⁹ This interpretation refers to Pareto's formulation and to the corresponding shape of the rank-size curve.

¹⁰ The estimates of c parameter are all statistically significant at 5% significance level, except in 1950 and 1960.

¹¹ We must note that, for these years, using the Pareto's formulation (equation 3') we obtained an estimate of the slope close to one.

¹² Comparing the cities in this data base with INE's list of legal cities, we conclude that Brinkhoff's definition includes places that are not classified as cities.

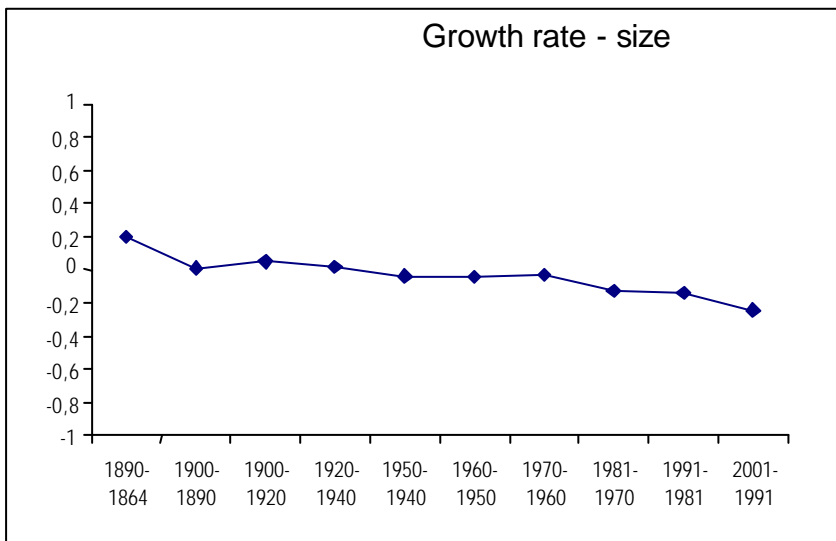
Figure 7 – Evolution of parameter c in equation 3''



On the other hand our results for the long term evolution of c are similar to those of Guérin-Pace's for France in 1831-1990 period. We should note that Guérin-Pace uses, as in our full sample, the 2000 inhabitant's threshold.

The presence of a curvature in the rank-size distribution is seen as a violation of Gibrat's Law. In order to generate a log-normal distribution, city growth rates must be independent of city size and also independent from period to period (Parr, 1976: 286-287; Moriconi-Ébrard, 1993: 245). To analyse this aspect we compute correlation coefficients between annual average growth rates and city size, in the beginning of each inter-census periods (Figure 8), and between successive annual average growth rates (Figure 9).

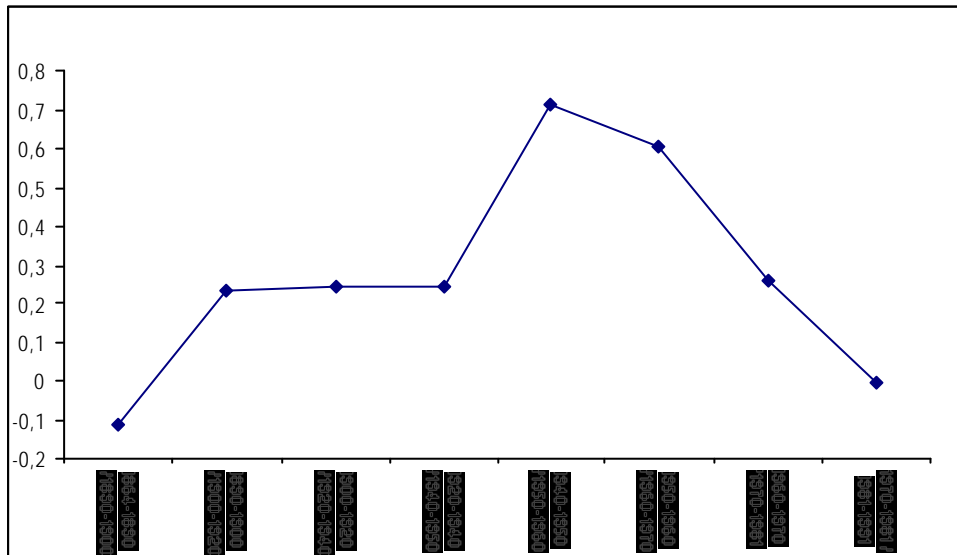
Figure 8 – Pearson's coefficient of correlation between annual average growth rate and city size



From Figure 8 we conclude that there is a weak correlation between the annual average growth rates and the initial size of each city, meaning that there is not a linear relationship between these two variables. Correlation coefficients vary from 0,20 in 1864-1890 to -0,14 in 1981-1991, becoming negative in the 1940-1950 period. The change in coefficient sign points to different spatial patterns in the urban growth process, with growth favouring the largest cities, for positive values of the coefficient, and a tendency towards urban growth decentralisation, for negative values of the correlation coefficient.

The long-term evolution of the correlation coefficient is similar to the evolution of the quadratic term in equation 3''. A positive correlation coefficient is associated with an upward concavity in the rank-size distribution, whereas negative coefficients are associated with a downward concavity. We calculate the correlation coefficient between c and the correlation coefficient of growth rates and city size. We find a positive and high value for that coefficient (0,88).

Figure 9 – Pearson's coefficient of correlation between successive values of annual average growth rates



Non-linearity can also result from the existence of autocorrelation of growth rates over time. In the beginning of the period we have a slight negative correlation between successive growth rates. During the first half of the 20th century, this correlation becomes positive and roughly constant (0,23 - 0,25). After the II World War, correlation coefficients are high (0,71 – 0,61) decreasing afterwards. From this behaviour, we conclude that, from 1940 up to 1970, urban growth follows a cumulative growth process. In fact, when analysing the spatial distribution of cities exhibiting the largest growth rates in the inter-census periods from 1940 up to 1970,¹³ we find that those cities are mainly those belonging to Lisboa Metropolitan Area periphery.

In conclusion, the size distribution of cities in Portugal results from a process of urban growth characterised by concentration of the population in the largest cities, in the early phases of the period considered, followed by a selective growth process beneficial to the

¹³ This is a period of growth and important structural changes in the Portuguese economy.

same cities, in particular those cities that are closer to Lisboa. More than the relationship between size and growth rates, deviation from Pareto distribution seems to arise from autocorrelation in successive growth rates. In this vein, a better understanding of the rank size distribution needs to associate its characteristics and evolving pattern with the spatial pattern of urban growth, which is not accounted by a rank-size study.

4. Conclusions

This paper presents evidence about urban evolution in Portugal over more than a century, focusing on the characteristics and shape of city size distribution. One limitation of our study relates to the nature of our sample. The use of legal cities has important drawbacks since it corresponds to the inclusion of very small places and the exclusion of urban places, with considerable population but lacking the administrative status of city. As a consequence our sample is biased against new urban places.

In our study, the following aspects of the evolution of the urban system emerge:

1. The Portuguese urban system is characterised by the proliferation of very small cities and two dominant cities which are the central cities of Portugal's two metropolitan areas. The urbanisation rate is low. In 2001, despite the heavy population losses in the central cities of metropolitan areas, 57% of urban population was concentrated in the 28 cities belonging to those metropolitan areas. The growth of urban population is faster than that of the number of cities, so urban growth is mainly the result of concentration in existing cities than of the emergence of new cities. In the last two decades, there is a trend towards spatial reorganization of the urban system, since the two top cities have experienced heavy population losses whereas intermediate cities, specially those in the periphery of Lisboa and Porto, have registered significant population gains.
2. The rank size line shifts up in the course of time as a result of urban growth; it exhibits a slight enlargement at the bottom and significant increase in its height; urban growth process was characterised by a slow increase in the number of cities and a considerable enlargement of the dimension of the top city. The line becomes smoother in the course of time, expressing the development of the urban system as a whole, accompanied by a reduction of inequality between city sizes in the upper tail of the distribution.
3. The evolving pattern of the rank-size line is in accordance to the contrasting evolution of the slope estimates, when we consider the full sample versus sample thresholds excluding smaller cities.
4. Deviations from rank-size regularity enhance two different processes in the evolution of the urban system: until the middle of the twenty century urban growth was accompanied by population concentration in the largest cities and proliferation of small cities; afterwards growth benefits middle size cities, reinforced in the last decades by heavy population losses in the two largest cities.

In conclusion, while the size distribution of cities is fairly stable, there is a tendency towards increasing urban concentration in the early phases of the urbanisation process followed by a change in the growth behaviour of the two top cities *vis à vis* the middle size cities. This last tendency may correspond to a process of selective growth since it favours

mainly cities located closer to the central cities in the metropolitan areas of Lisboa and Porto.

The evolution of the Portuguese urban system reflect structural changes in the Portuguese economy, that took place mainly in the second half of the 20th century: modern industrialisation, occurring since the fifties, export orientated growth in the sixties, economic restructuring in the seventies and the eighties, following severe political changes and, finally, integration in the European Union. It reflects also the evolution from a centralised political regime, administrating vast colonial territories to a democratic regime, with a more decentralised administrative organisation and confined to its European borders. Nevertheless the interplay of these factors with the changing pattern of the urban system can not be addressed in the context of rank size models. In order to elucidate these aspects we need to explore the relationship between the measure of city size inequality and relevant economic and political variables and to attempt to model changes in city size distribution and its underlying dynamics.

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