Congestion on Risky Routes with Risk Averse Drivers

ANDRE DE PALMA

THEMA, Université de Cergy-Pontoise, Cergy-Pontoise, FRANCE and Ecole Nationale des Ponts et Chaussées, Paris, FRANCE and Senior Member of Institut Universitaire de France.

NATHALIE PICARD

THEMA, Université de Cergy-Pontoise, Cergy-Pontoise, FRANCE

ERSA congress, Amsterdam, 2005

We study the impact of information on risk averse drivers who maximize their von Neumann-Morgenstern expected utility (rather than minimizing expected travel time). For two routes in parallel, equilibrium travel times depend on the distribution of risk aversion as well as on provision of information. Besides the (potentially inconsistent) mean variance model used so far in transportation, we consider three other standard utility functions. Interestingly, we find that optimal route choice may depend on global factors (and not only on local traffic conditions). Finally, we study the benefit of information according to the level of risk aversion.

Keywords: Congestion, expected utility, risk aversion, information, compensating variation

INTRODUCTION

Congestion and associated time loss represent a very severe problem in many cities. Increasing car ownership tends to deteriorate traffic conditions. This fact is very well documented, contrarily to another dimension, which is as important, but has received little attention. This second source of user cost, which results from travel time variability, is often referred to as non recurrent congestion. Non recurrent congestion represents a large fraction of congestion in urban areas. In this case, besides average travel time, drivers incur an additional cost due to the fact that congestion (and therefore travel time) are unpredictable. For example, simulation experiments which were performed with the dynamic traffic simulator, METROPOLIS, suggest that the user cost associated to congested time in Ile-de-France (Paris area and the surroundings) is of the same order of magnitude as the cost associated to the variability of travel time (see de Palma and Picard, 2004a).

Much research has been devoted to the measure of the impact of information on congestion (see, for example, Levinson, 2003 or Zhang and Verhoef, 2004). These results show that ITS systems do decrease the average level of congestion, but that the impacts depend on a large variety of factors, such as the structure of the demand or the type of networks involved. The majority of those results are based either on simulation experiments or on laboratory experiments (see, e.g., Abdel-Aty et al., 1994). We propose here a third avenue of research based on analytical methods, which explicitly introduces a key element for the analysis of information: the individual level of risk aversion.

The idea is simple: the majority of drivers dislike travel time variability because they do not like arrival time variability: a random arrival time at destination makes the planning of a business trip more difficult or the appointment with a doctor more costly (if one wishes to be sure not to miss the appointment). The users are prepared to pay in order to reduce the variability of travel time: this corresponds to the willingness to pay to avoid risk. Noland and Small (1995) also use the term VOR for Value of Reliability (which is an additional component to the user generalized cost, with respect to travel time cost). The same phenomenon takes place on the stock market, since the average return of a stock is larger than the return on money market (see Prigent and Toumi, 2004 and Gollier, 2001). The difference between these two returns is an increasing function of the level of risk aversion. Interestingly, risk aversion not only characterizes human being, but also animals like rats, who are tested when facing safe and risky route choices (see Kagel and Roth, 1995).

The analysis that we present here begins with the choice of an individual selecting between two uncongested routes, one is deterministic and the other is random. The shortest route is also the risky one, so that there is no universally better route: the risk neutral drivers (who ignore travel time variability) will prefer the risky route and the drivers who dislike enough risk will agree to choose the safe route, even though the travel time on this route is larger than the expected travel time on the other route. This is the price to pay for certainty. Note that Arnott, de Palma and Lindsey (1991, 1996) have shown that, theoretically, the provision of information may have a negative impact on welfare. This last issue is not analyzed in this paper.

We then extend the analysis by introducing congestion and by considering that the travel time on each route depends on the number of drivers selecting this route. In other words, travel time is uncertain on the risky route but also depends on road occupancy. The equilibrium of this system is not given by the Wardrop principle because this principle does not incorporate travel time variability. The idea is then to uncover a drivers' preferences structure, which incorporates travel time variability. Noland and Small (1995) and Noland, Small, Koskenoja and Chu (1998) have introduced and estimated a generalized cost function which is the weighted sum of the expected travel time and of the variance of travel time. This is the Mean-Variance model that we consider in this paper. However, we show that this model may be inconsistent, a point which has been disregarded in the literature till now. We also consider three other standard formulations (utility functions). When the travel time is deterministic, the maximization of the utility function is the same as the minimization of the travel time, so that the principle of expected utility maximization, that we use in this paper, generalizes the standard Wardrop principle in a non trivial way.

The main objective of this paper is to study the impact of information, when the capacities of the roads are uncertain. We compare two extreme cases: (1) the drivers have no information (but they know the statistical laws which govern from day to day traffic conditions), and (2) the drivers know, beforehand, the level of capacity on each route (and in this case the equilibrium is given by the standard Wardrop principle). The three main questions we address in this paper are then: (1) how are the travel times on the different routes modified with the provision of information? (2) How do the travel times incurred by different users (which differ by their levels of risk aversion) vary as the result of the provision of information? And (3) how is the benefit from the provision of information distributed across the population, heterogeneous with respect to risk aversion?

The paper is organized as follows: in Section 1, we introduce the main assumptions (concerning the road systems and the drivers' preferences), as well as the notations used in the paper and results in the no congestion case, on which the remaining sections rely. In Section 2, we treat the case when users are fully informed on the road conditions before they make their route choice decisions. In Section 3, we treat the other polar case, when the users have no information about road conditions, and analyze how endogenous congestion modifies the impact of the probability of bad traffic conditions on route choice. In Section 4, we compare the two regimes (no and full information) and study the impact of information on travel time, for the different classes of users. Section 5 concludes the paper. Proofs are partially relegated to the Appendix.

1 CONTEXT AND ASSUMPTIONS

1.1 Definitions and notations

We consider N drivers travelling from a common origin to a common destination. Each user has to choose between a safe route, denoted by S, with a deterministic travel time t_S and a risky route, denoted by R, with a travel time T_R which depends on the state of the nature.

We consider two states of nature: good and bad. For example, the road capacity could be high (good state) or low (bad state). In this latter case, the capacity could be reduced given the occurrence of a significant accident or given a road repair. The probability of the bad state of the nature is denoted by $p \in]0; 1[$. This probability is assumed fixed and exogenous. Moreover, this probability is common knowledge (that is known

by all drivers) when the state of the nature is unknown. The upper script denotes the state of the nature: "$-$" for the good state and "$+$" for the bad state, so that $\mathbb{P}(T_R = t^-) = 1 - p$ and $\mathbb{P}(T_R = t^+) = p$, where t^- (resp. t^+) denotes the travel time on the risky route in the good (resp. bad) state of the nature. Travel time on route j may depend on the (endogenous) number n_j of users who choose route j, $j = R, S$. The total number of users $N = n_R + n_S$ is fixed and exogenous. This assumption of inelastic total demand is rather realistic in the case of out-of-town week-end trips, or commuting in the short run.
We introduce two assumptions concerning travel time functions, which guarantee, as shown below, the existence of interior equilibrium solutions.

Assumption 1 *Travel time $t_S(n_S)$ is continuous and strictly increasing in traffic volume on route S, n_S. On good days, travel time on route R, t^-, is constant. On bad days, travel time $t^+(n_R)$ is continuous and strictly increasing in traffic volume on route R, n_R.*

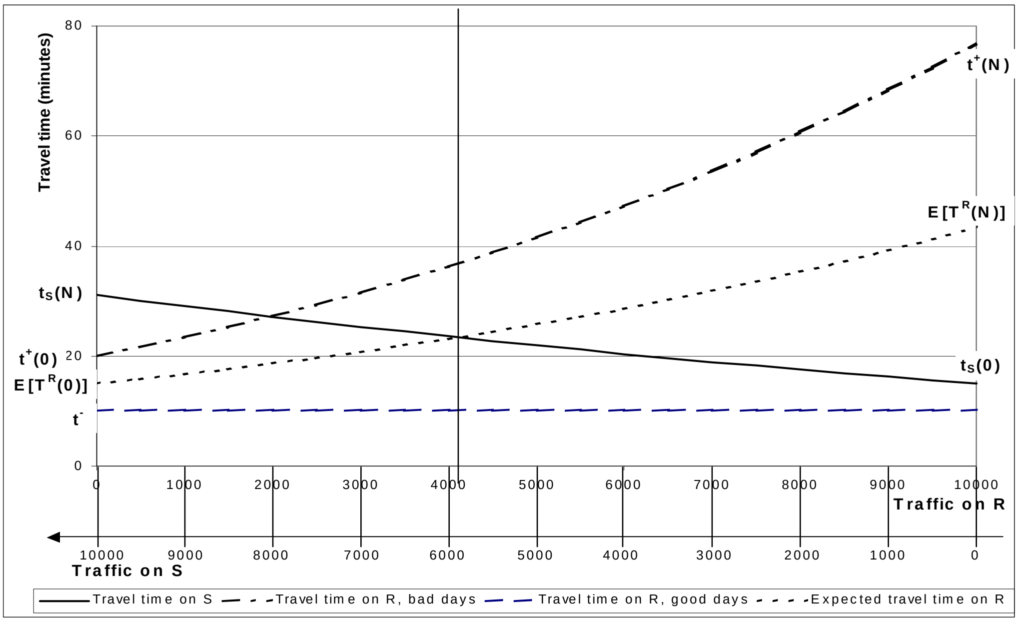

Figure 1: Travel times on the two routes, good days and bad days

The hypothesis that t^- is constant simplifies computations, but it is by no means essential for our analysis. It corresponds to an infinite capacity (or, at least, a capacity sufficiently high that free-flow travel can be maintained). Assumption 1 is illustrated in Figure 1. Traffic on route R corresponds to the usual horizontal axis, whereas traffic on route S corresponds to the reversed axis. Assumption 2 below implies that the decreasing solid curve representing travel time on route S crosses the increasing dash-dotted curve representing travel time on route R bad days, but that these two curves lie above the dotted horizontal line representing travel time on route R good days.

Assumption 2 *The following inequalities hold:*

$$t^+(0) > t^- \tag{2a}$$

$$t_S(0) > t^- \tag{2b}$$

$$t_S(0) < t^+(N) \ \ \textit{and}\ t_S(N) > t^+(0)\,. \tag{2c}$$

Assumption 2a means that, on route R, bad day travel time is larger than good day travel time when route R is unused and therefore (according to Assumption 1) whatever the traffic: $t^+(n_R) > t^-$, $\forall\, n_R,\ 0 \leq n_R \leq N$.

Similarly, Assumption 2b implies that travel time on the safe route is always larger than good day travel time on the risky route, whatever traffic volumes: $t_S(n_S) > t^-,\ \forall\, n_S,\ 0 \leq n_S \leq N$. Finally, Assumption 2c implies that, on bad days, no route dominates the other (that is no route is preferred to the other whatever the distribution of traffic between the two routes).
The first and second moments (expectation $\mathbb{E}(T_R)$ and variance $\sigma^2(T_R)$) of travel time on route R can be computed easily given the properties of the binomial distribution:

$$\mathbb{E}(T_R) = pt^+(n_R) + (1-p)\,t^-$$
$$\sigma^2(T_R) = p(1-p)\left[t^+(n_R) - t^-\right]^2,$$

see de Palma and Picard (2004b) for details. Their results are presented for symmetric deviations of travel time on route R. The correspondence between their symmetric notations (Δ, δ, τ) and the endogenous quantities $(t^-, t^+(n_R), t_S(n_S))$ (conditional on traffic volumes) is given in Appendix 6.1.

Assumption 3 *Expected travel time on route R is not larger than travel time on route S:*

$$pt^+(n_R) + (1-p)\,t^- \leq t_S(N - n_R).$$

Note that Assumptions 1, 2b and 2c imply Assumption 3 when $n_R = 0$. By contrast, Assumption 2c implies that Assumption 3 does not hold when $n_R = N$ and $p = 1$. By continuity, Assumption 3 does not hold when n_R and p are large enough. Assumption 3 therefore means that there is not too much traffic on the risky route (we restrict attention to the part on the left of the vertical line on Figure 1).

1.2 Users' preferences

Users' preferences are represented by a utility functions $U^V(t;\theta^V)$, which is decreasing in travel time t and where $\theta^V > 0$ denotes the risk aversion parameter. Each user chooses the route which maximizes her expected utility $U^V(t_S;\theta^V)$ on route S or $\mathbb{E}U^V(t;\theta^V) = pU^V(t^+;\theta^V) + (1-p)\,U^V(t^-;\theta^V)$ on route R. This corresponds to the von Neumann-Morgenstern expected utility framework (see, e.g., Gollier, 2001).
We envisage four different (expected) utility functions $V = MV, MS, CR, CA$ for Mean-Variance, Mean-Standard deviation, Constant Relative Risk Aversion (CRRA) and Constant Absolute Risk Aversion (CARA), respectively, as specified below:

$$\begin{array}{r} \mathbb{E}U^{MV}(T_R;\theta^{MV}) = -\mathbb{E}(T_R) - \theta^{MV}\sigma^2(T_R), \\ \mathbb{E}U^{MS}(T_R;\theta^{MS}) = -\mathbb{E}(T_R) - \theta^{MS}\sigma(T_R), \\ U^{CR}(t;\theta^{CR}) = -t^{1+\theta^{CR}}/\left(1+\theta^{CR}\right), \\ U^{CA}(t;\theta^{CA}) = \left[1 - \exp\left(\theta^{CA}t\right)\right]/\theta^{CA}. \end{array} \tag{1}$$

The first two lines correspond to expected utility. The Mean-Variance expected utility could be obtained from CARA utility with a normal distribution for travel time. However, the Mean-Standard deviation expected utility corresponds to no explicit utility function. Risk neutrality corresponds to the limiting case $\theta^V \to 0$ for $V = MV, MS, CR, CA$. The risk aversion parameter θ^V is assumed to follow a continuous distribution over an interval I, with $I = \mathbb{R}^+$ (unbounded case) or $I = \left[0;\bar{\theta}^V\right] \subset \mathbb{R}^+$ (bounded case). The bounded case is more appropriate for Mean-Variance and Mean-Standard deviation preferences, which often impose an upper bound on the risk aversion parameter. Indeed, de Palma and Picard (2004b) discuss potential inconsistencies for the Mean-Variance and Mean-Standard deviation preferences ($V = MV, MS$). They show that when the probability p of the bad state of the nature and/or risk when aversion parameter θ^V is too large, the expected utility may be an increasing function of p, which violates rationality. Indeed, a "rational" user would never select a dominated route, that is a route for which the probability p of a high travel time is larger. Consistency requires the following:

Assumption 4 *The risk aversion parameter θ^{MV} and the probability p of the bad state satisfy:*

$$\begin{array}{l} (2p-1)\,\theta^{MV}\left[t^+(n_R) - t^-\right] < 1 \textit{ for } MV \textit{ preferences,} \\ (2p-1)\sqrt{1 + (\theta^{MS})^2} < 1 \textit{ for } MS \textit{ preferences} \end{array}.$$

Assumption 4 guaranties that expected utility is decreased when the probability p of the bad state is increased. A sufficient condition for both inequalities to hold whatever $\theta^V > 0$ is $p \leq 1/2$, i.e. adverse traffic conditions occur with a low enough probability. Moreover, note that the first inequality holds whatever $p \in [0;1]$ when $\theta^{MV} < 1/\left[t^+(n_R) - t^-\right]$. According to Assumption 1, a sufficient condition for Assumption 4 to be true whatever p and whatever traffic on each route is $\theta^{MV} < 1/\left[t^+(N) - t^-\right]$. In contrast, the second inequality always imposes an upper limit on p, which is independent from traffic conditions.

1.3 Probability thresholds and risk aversion thresholds

This section sums up the computation of the threshold values when congestion n_R is exogenous and resulting travel times t^-, $t^+(n_R)$ and $t_S(N-n_R)$ are fixed. The presentation of Sections 1.3 and 1.4 builds on de Palma and Picard (2004b), where the interested reader will find details of the proofs for Theorem 1 and Proposition 1, corresponding to their Propositions 4, 10 and 11. With exogenous congestion and plausible assumptions, we can show (Theorem 1) that the risky route R is preferred to the safe route S ($R \succ S$) if and only if the probability p of the bad state of the nature is less than a threshold $\tilde{p}^V\left(t^-, t^+(n_R), t_S(N-n_R), \theta^V\right) \equiv \tilde{p}^V_\theta(n_R)$ or, equivalently, iff the user's risk aversion θ^V is less than a strictly positive threshold value $\tilde{\theta}^V\left(p, t^-, t^+(n_R), t_S(N-n_R)\right) \equiv \tilde{\theta}^V_p(n_R)$ (the threshold values can be expressed as functions of n_R because the travel times t^+ and t_S depend on traffic n_R). Moreover, the user is indifferent between the two routes ($R \approx S$) at equality. More precisely, we have:

Theorem 1 *Assume Assumptions 3 and 4 hold. For the utility function $V = MV, MS, CR$ or CA, there exists a unique probability threshold $\tilde{p}^V_\theta(n_R) \in [0;1]$ and a unique risk aversion threshold parameter $\tilde{\theta}^V_p(n_R) > 0$, which is strictly decreasing in p, such that:*

$$\begin{aligned} R \succ S \Leftrightarrow p &< \tilde{p}^V\left(t^-, t^+(n_R), t_S(N-n_R), \theta^V\right) = \tilde{p}^V_\theta(n_R) \\ \Leftrightarrow \theta^V &< \tilde{\theta}^V\left(p, t^-, t^+(n_R), t_S(N-n_R)\right) = \tilde{\theta}^V_p(n_R). \\ R \approx S \Leftrightarrow p &= \tilde{p}^V_\theta(n_R) \Leftrightarrow \theta^V = \tilde{\theta}^V_p(n_R). \end{aligned} \tag{2}$$

Equivalently, the probability thresholds $\tilde{p}^V_\theta$ is strictly decreasing in θ^V. For all the preferences envisaged, more risk averse users are ready to incur lower probabilities for the largest travel time than less risk averse users, which is in accord with intuition. The proof is left to the reader. It is similar to (but simpler than) the one of Theorem 2 below. The above thresholds, which are specific to the utility function V, are computed in Appendix 6.2 and the decreasing relationships $\tilde{\theta}^V(p, .)$ are depicted in Figure 2. The parameters are $t^- = 10$ min., $t^+ = 20$ min., and $t_S = 15$ min. According to Theorem 1, the user with preferences V chooses route R below the corresponding curve (i.e. when $\theta^V < \tilde{\theta}^V_p$) and route S above the curve. A different scale was used (from 0 to 14 on the right) for CRRA preferences (upper curve) since, for the chosen parameters (t^-, t^+ and t_S), the relevant values for the risk aversion threshold $\tilde{\theta}^{CR}$ are far larger than for the other preferences. In addition, under Assumptions 1 and 2b, for the four preferences envisaged, the risk aversion threshold goes to infinity when p goes to zero, which means that all users prefer the risky route when the probability of the bad state is low enough. This is also intuitive given Assumptions 1 and 2b. However, Figure 2 shows that, for CARA preferences, a very low probability would be necessary for users with reasonable values of risk aversion (say θ^{CA} around 1.5) to prefer the risky route.
Note that, if Assumption 3 does not hold, then all (risk neutral or risk averse) users prefer route S. With the values considered for t^-, t^+ and t_S, Assumption 3 holds iff $p \leq 0.5$. This is why the four curves cross at $p = 0.5,\ \theta^V = 0$.

Theorem 2 *Assume Assumptions 1, 3 and 4 hold. For the utility function $V = MV, MS, CR$ or CA, the risk aversion threshold parameters $\tilde{\theta}^V_p$ are strictly decreasing in traffic n_R on route R.*

Proof. See Appendix 6.3. ■
Theorem 2 goes along intuition since more traffic on route R makes it less attractive compared to route S. This theorem will be useful to get the equilibrium solution. It corresponds to the demand side, whereas (on the supply side) endogenous traffic on route R is increasing with risk aversion θ^V, as shown in Section 3.

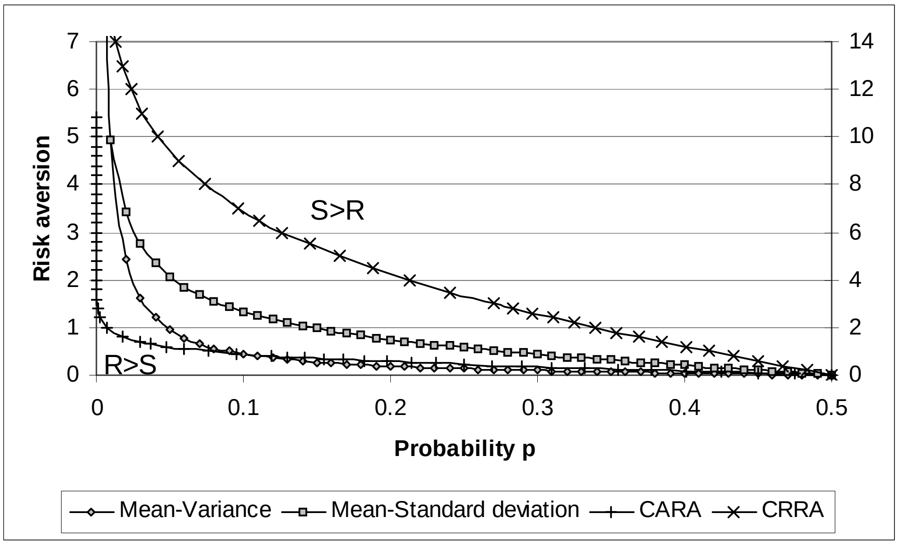

Figure 2: Risk aversion threshold for the different preferences

1.4 Adding a fixed travel time on each route

We now present the effect of adding the same constant to each travel time, focusing on the differences obtained with different utility functions. This point is crucial for road pricing, since (under the assumption of constant value of time) it is equivalent to adding the same fixed cost on both routes. In the standard case (deterministic travel times), this transformation has no impact on route choice in the sense that a driver indifferent between the two routes before the change will remain indifferent between the two routes after the change. As shown below, this result does not necessarily hold with random travel time and risk averse drivers!

Proposition 1 deals with the impact of adding a fixed travel time segment to the network with two routes in parallel. We find that such an additive expansion has no effect on route choice for some preferences, while it tends to favor the risky route for other preferences.

Proposition 1 *For an agent with MV, MS or CA preferences, route choice is unchanged when the same constant travel time is added on each route.*
Consider an agent with CR preferences who is indifferent between routes R and S. When a common travel time $\mu > 0$ is added on each route, she prefers route R.

With CRRA preferences, adding a common constant, μ, to both routes, tends to favor route R. If route R was initially preferred to route S, then it is still preferred with the additional term μ. If route S was initially preferred to route R, then there exists a unique threshold value $\breve{\mu} > 0$ such that the user becomes indifferent between the two routes when $\breve{\mu}$ is added to travel times on both routes. Proposition 1 is rather intuitive, since the *relative* variability of travel time is reduced when adding a constant travel time. Users with CRRA preferences, which are only concerned with the relative variability of travel times, therefore consider that route R becomes less risky when adding a constant travel time.

Proposition 1 implies that the advice provided by traveller information systems may depend on global variables such as the total length of the journey (instead of the travel time on the two routes only) and should then be individual specific and global. It also implies, for example, that if a bottleneck occurs before the decision node (between the safe and the risky route), this may affect route choice. This goes along basic intuition (e.g. drivers fed up refuse to take another chance) but, strangely enough, such behavior is not taken into account by current transport models. Finally, Proposition 1 implies that introducing the same

toll on both routes may affect route choice (if users have CRRA preferences). Note however that the degree of risk aversion for travel time may differ from the degree of risk aversion for money, which is out of the scope of this paper.
In Section 2, we turn to more realistic assumptions, assuming that the travel time on a route depends endogenously on the traffic on that route. However, in order to simplify computations, good day travel time is assumed independent from traffic.

2 FULL INFORMATION (STATE OF THE NATURE KNOWN)

From now on, we assume that travel times on each route depend on the traffic on that route. We assume in this section that, before they begin their journey, all users know the state of the nature. In this case, denoted by full information, the drivers have access to information concerning accidents, or road repairs. In this way, they know the value of road capacity and from that they are able to perfectly anticipate the value of travel time (details of those calculations are provided below). With full information, travel time is deterministic on both routes (but depends of course on the state of the nature). As a consequence, each driver chooses each day the route which minimizes her travel time (this is because utility is a decreasing function of travel time, so that maximizing utility amounts to minimizing travel time). At equilibrium, either both routes are used and they have the same travel time or only the route with the smallest travel time is used.

2.1 Good days

First, we characterize the equilibrium when a good day (high capacity on the risky route) is announced.

Lemma 1 *Let Assumptions 1 and 2b hold. On good days, equilibrium traffic on route R is N.*

Proof. Assumptions 1 and 2b imply that, if good state of the nature is common knowledge, each user prefers route R even though she anticipates that all users will choose it, since $t^- < t_S(0)$. When the good state of the nature is announced, the equilibrium entails $n_R^- = N$ and $n_S^- = 0$. ■
On good days, there exists a unique (corner) equilibrium solution in the good state of the nature, for which all users select route R. In that case, travel time is constant and equal to t^-. Note that, the good days equilibrium traffic (and hence travel time) depends only on the total number of users, N (and does not depend on the congestion functions $t_S(.)$ and $t^+(.)$, nor on users' preferences).

2.2 Bad days

Next, we study the equilibrium split when a bad signal (low capacity on route R) is sent to drivers. The existence and unicity of the equilibrium solution is assured whatever the functional form for congested travel times.

Lemma 2 *Let Assumptions 1 and 2c hold. On bad days, there exists a unique equilibrium traffic on route R, which solves $t_S\left(N - n_R^+\right) = t^+\left(n_R^+\right)$, with $n_R^+ \in]0; N[$.*

Proof. First note that there cannot be corner equilibrium solutions ($n_R = 0$ or $n_R = N$) because of Assumption 2c. Second, at equilibrium (if any), the travel times on the two routes must therefore be the same since they are both used. Finally, Assumptions 1 and 2c imply that there exists a unique number $n_R^+ \in]0; N[$ which equalizes both travel times (and therefore corresponds to an equilibrium). It is given by the solution to the equation $t_S\left(N - n_R^+\right) = t^+\left(n_R^+\right)$. ■
On bad days, there exists a unique (interior) equilibrium solution $\left(n_R^+, n_S^+ = N - n_R^+\right)$. Note that, the bad days equilibrium traffic (and hence travel times on both routes) depends only on the total number of users, N and on the congestion functions $t_S(.)$ and $t^+(.)$, but not on users' preferences.

When a bad state of nature is announced (common knowledge), each user anticipates the equilibrium $\left(n_R = n_R^+, n_S = N - n_R^+\right)$. The equilibrium is stable in the sense that all users would prefer route R if less traffic used R ($n_R < n_R^+$) and all users would prefer route S if more traffic used R ($n_R > n_R^+$). Lemmas 1 and 2 will be useful to study the equilibrium without information.

The optimal strategy can be interpreted as a mixed strategy where each user selects route R with probability $\frac{n_R^+}{N}$ and selects route S with probability $\frac{N-n_R^+}{N}$. An alternative interpretation is that n_R^+ users deterministically choose route R and $\left(N - n_R^+\right)$ users deterministically choose route S.

To illustrate the results, we consider a parametric case:

Assumption 5 *On bad days, when n_R drivers use route R, travel times are given by:*

$$\begin{cases} t^+ (n_R) = (1 + n_R/\lambda_R)^b \, \tau^+ \text{ on the risky route and} \\ t_S (N - n_R) = (1 + (N - n_R)/\lambda_S)^b \, \tau_S \text{ on the safe route,} \end{cases}$$

with $\tau^+, \tau_S, \lambda_R, \lambda_S$ and $b > 0$.

For that specification, τ_S and τ^+ denote respectively the free flow travel times on the safe route and on the risky route (bad days), and λ_j denotes the capacity on route j, $j = R, S$. Recall that, on the risky route on good days, capacity is infinite and travel time is constant (equal to t^-). Note that, with these parameter values, Assumption 1 is trivially satisfied, since Assumption 5 implies Assumption 1. Assumption 2b then corresponds to $\tau_S > t^-$ and Assumption 2c corresponds to:

$$1/ (1 + N/\lambda_R)^b < \tau^+/\tau_S < (1 + N/\lambda_S)^b . \tag{3}$$

Note also that it is always possible to find parameters values τ^+ and τ_S which meet the inequalities 3. With the above specification, we can solve explicitly for the equilibrium solution described in Lemma 2. Under Assumptions 2c and 5, bad day equilibrium traffic on route R is:

$$n_R^+ = \left[1 + N/\lambda_S - \left(\tau^+/\tau_S\right)^{1/b}\right] \Big/ \left[\left(\tau^+/\tau_S\right)^{1/b}/\lambda_R + 1/\lambda_S\right] \in \,]0; N[\, .$$

Indeed, under Assumption 5, the bad day equilibrium traffic n_R^+ solves the travel time equality condition:

$$\begin{aligned} &\left(1 + n_R^+/\lambda_R\right)^b \tau^+ = \left(1 + \left(N - n_R^+\right)/\lambda_S\right)^b \tau_S \\ \Leftrightarrow\, & n_R^+ = \left[1 + N/\lambda_S - (\tau^+/\tau_S)^{1/b}\right] \Big/ \left[(\tau^+/\tau_S)^{1/b}/\lambda_R + 1/\lambda_S\right] . \end{aligned}$$

Assumption 2c (which corresponds to Equation 3 in the example) ensures the existence of an interior solution (i.e. that $0 < n_R^+ < N$).

Note that, under Assumption 5, the solution for bad day equilibrium traffic does not depend separately on free flow travel time on each route (τ^+ and τ_S), but only on the free flow travel time ratio τ^+/τ_S. Bad day equilibrium traffic n_R^+ on the risky route R is increasing in the power index b iff bad day free flow traffic on the risky route is larger than free flow on the safe route ($\tau^+ > \tau_S$). The other comparative statics goes along intuition: equilibrium traffic on route R is increasing in total traffic N and in the capacity λ_R on R and decreasing in the capacity λ_S on the other route. The effect of free flow travel times τ^+ and τ_S go in the opposite direction. Equilibrium travel time, denoted by $t^+\left(n_R^+\right)$, is given by:

$$t^+\left(n_R^+\right) = t_S\left(N - n_R^+\right) = \left\{[\lambda_R + \lambda_S + N] \Big/ \left[\left(\tau^+\right)^{1/b}\lambda_S + (\tau_S)^{1/b}\lambda_R\right]\right\}^b \tau^+\tau_S,$$

which is symmetric in R and S. As an illustration, with the following values of the parameters: $p = 0.5$, $b = 4$, $N = 10,000$, $\tau^+ = 20$ min., $\tau_S = 15$ min., $t^- = 10$ min., $\lambda_R = 25,000$, $\lambda_S = 50,000$, equilibrium traffic is $n_R^+ = 1,991$ (19.9%) when the bad state of the nature is common knowledge. Travel time is then 27.18 min., which represents a 81% increase on route S and a 36% increase on route R (compared to free flow travel time on each route). This corresponds to the intersection of the solid and dash-dotted curves on Figure 1.

3 NO INFORMATION ($p \in]0;1[$ KNOWN)

3.1 Introduction

Assume now that users choose their route without knowing the state of the nature. This corresponds to the case where drivers have no access to a driver information system. However, they know the occurrence of good and bad days. In other words, they all know the probability p of the bad state of the nature (which is common knowledge). In this case, the solution is given by the *unconditional* number of users who select each route. We will show below that the most risk averse users select route S, while the least risk averse users select route R.
The standard solution provided in the literature assumes that all the users are risk neutral. In this case, they all minimize the expected travel time. For an interior solution $\left(n_R^0, n_S^0 = N - n_R^0\right)$, where n_R^0 denotes the no information equilibrium traffic with only risk neutral users, this means that

$$t_S\left(N - n_R^0\right) = pt^+\left(n_R^0\right) + (1-p)\,t^-, \tag{4}$$

which is at the boundary of Assumption 3. Under Assumption 5, the solution n_R^0 solves:

$$\left[1 + \left(N - n_R^0\right)/\lambda_S\right]^b \tau_S = p\left(1 + n_R^0/\lambda_R\right)^b \tau^+ + (1-p)\,t^-.$$

Note that the above solution has no analytical solution in general (except in the case $b = 1$ or $b = 2$). Recall that, under full information, the travel time on the good day is t^- and that, on the bad days, it solves:

$$t_S\left(N - n_R^+\right) = t^+\left(n_R^+\right) > pt^+\left(n_R^+\right) + (1-p)\,t^-. \tag{5}$$

The inequality in 5 is implied by Assumptions 1 and 2a. Expressions 4 and 5 and Assumption 1 imply that $n_R^0 > n_R^+$. A similar argument holds for good day traffic under full information and shows that $n_R^0 < n_R^-$. Therefore, we have shown:

Proposition 2 *Let Assumptions 1 to 2c hold and assume all users are risk neutral. On route R, equilibrium traffic with no information is larger than bad day equilibrium traffic, and lower than good day equilibrium traffic:* $n_R^+ < n_R^0 < n_R^-$.

See Lemmas 1 and 2 for good day and bad day equilibrium traffic, respectively. When travel time is random and users have different levels of risk aversion, their route choice depends on their level of risk aversion. We discuss below the case where travel time is exogenous (which is the case from the user perspective) and later on we endogenize the travel time.

3.2 Individual route choice decision

We first consider individual route choice when the travel times on both routes are given and constant from the individual perspective. Note that if the population size N is large, then the impact of an individual choice on congestion can be neglected, so travel times can be considered fixed and exogenous when solving an *individual* program.
Each user chooses the route which maximizes her expected utility $\mathbb{E}U^V\left(t;\theta^V\right)$ (see Equations 1). Since travel times are given, we can use the results obtained in Section 1.3, with exogenous travel times. In particular, the threshold values for the risk aversion parameter, $\tilde{\theta}_p^V$ of the individual indifferent between the two routes are given in Appendix 6.2. Recall that $\tilde{\theta}_p^V$ is decreasing in n_R (see Theorem 2).
We consider a scaling factor $\bar{\theta}^V$and a continuous distribution $\mathcal{L}$ over an interval I for the risk aversion parameter θ^V in the case of $V = MV$, MS, CR or CA preferences. The distribution $\mathcal{L}$ is characterized by the cumulative distribution function $F_{\mathcal{L}}\left(\theta^V;\bar{\theta}^V\right) > 0, \forall\,\theta^V \in I$, with $I = \mathbb{R}^+$ or $I = \left[0;\bar{\theta}^V\right] \subset \mathbb{R}^+$. We consider two distributions of risk aversion to illustrate our approach: $\mathcal{L} = \mathcal{U}$ for the uniform distribution on $\left[0;\bar{\theta}^V\right]$ and $\mathcal{L} = \mathcal{G}$ for the log-logistic distribution on $\mathbb{R}^+ : F_{\mathcal{G}}\left(\theta;\bar{\theta}^V\right) = 1/\left(1 + \bar{\theta}^V/\theta\right)$.

Assumption 6 *Users' preferences are described by the utility function $V = MV, MS, CR$ or CA; the risk aversion parameter θ^V is distributed according to a continuous distribution $\mathcal{L}$ either over $\mathbb{R}^+$ or over a compact interval $\left[0; \bar{\theta}^V\right]$ included in $\mathbb{R}^+$. In the case of a bounded distribution, $\bar{\theta}^V > \tilde{\theta}_p^V (N)$.*

Assumption 6 is necessary for an interior equilibrium to exist. Otherwise, all the users would select the risky route when p is unknown, as shown below. Indeed, $\tilde{\theta}_p^V (N) \geq \bar{\theta}^V$ would imply that $\tilde{\theta}_p^V (n) \geq \tilde{\theta}_p^V (N) \geq \bar{\theta}^V$ for any $n \leq N$ and all users would select route R if Assumption 6 did not hold. Note that $\tilde{\theta}_p^V (N)$ is a finite number when $p \in]0; 1[$, so no equivalent assumption is necessary in the case of an unbounded distribution. In the remainder of this section, we consider that n_R is endogenous and that preferences are described by the (expected) utility V, with a risk aversion parameter θ^V distributed according to $\mathcal{L}$. We then show that there exists a unique equilibrium interior solution $\left(n_{R,\mathcal{L}}^V, n_{S,\mathcal{L}}^V, \tilde{\theta}_p^V\right)$. The solution is such that, for the equilibrium traffic on both routes $\left(n_{R,\mathcal{L}}^V, n_{S,\mathcal{L}}^V = N - n_{R,\mathcal{L}}^V\right)$, there exists a unique value of risk aversion parameter $\tilde{\theta}_p^V \left(n_{R,\mathcal{L}}^V\right)$ such that, according to conditions 2, the user with $\theta^V = \tilde{\theta}_p^V \left(n_{R,\mathcal{L}}^V\right)$ is indifferent between the two routes, the $n_{R,\mathcal{L}}^V$ users with a lower risk aversion choose route R and the $n_{S,\mathcal{L}}^V$ users with a larger risk aversion choose route S. Note that, with a continuous distribution for users' risk aversion parameter, the probability to find such a user is zero, so no user should be concerned by the strict equality and all users have a strict preference for one route. As a consequence, under Assumption 6, the number of users corresponds to the integral of the density over an interval included in I:

$$n_{R,\mathcal{L}}^V \left(\tilde{\theta}_p^V\right) = N \int_0^{\tilde{\theta}_p^V} f_{\mathcal{L}} \left(\theta; \bar{\theta}^V\right) d\theta = F_{\mathcal{L}} \left(\tilde{\theta}_p^V; \bar{\theta}^V\right).$$

Next we show that risk neutral users necessarily select route R and the most risk averse users necessarily select route S. The route choice behavior of a risk neutral user is given by:

Lemma 3 *Let Assumptions 1, 2b and 2c hold. With no information, risk neutral users choose route R.*

Proof. We proceed by contradiction. Assume that risk neutral users choose route S, then conditions 2 implies that all users choose S, which implies $n_S = N$ and $n_R = 0$. Assumption 2c then implies that route R is preferred to route S in the bad state of the nature. Assumptions 1 and 2b imply that $t^- < t_S (0) < t_S (N)$, so route R is preferred to route S also in the good state of the nature. Route R is therefore preferred to route S whatever the state of the nature and all users should choose R, a contradiction. ■

Lemma 3 means that $\tilde{\theta}_p^V (0) > 0$, so $F_{\mathcal{L}} \left(\tilde{\theta}_p^V (0); \bar{\theta}^V\right) > 0$. Note that, by continuity, this result implies that the least risk averse users select route R.

We have a symmetric result for the most risk averse users:

Lemma 4 *Let Assumptions 1, 2b, 2c and 6 hold. With no information, the most risk averse users choose route S.*

Proof. Once again, we proceed by contradiction. Assume that the most risk averse users choose route R, then Theorem 1 implies that all users choose R, which implies $n_R = N$ and $\theta^V > \tilde{\theta}_p^V (N)$ for all users. This contradicts Assumption 6 when the distribution of θ^V is bounded. Consider now the case when the distribution of θ^V is unbounded and note that, for the four utility functions considered, when $n_R = N$,

$$\mathbb{E}U^V \left(t; \theta^V\right) - U^V \left(t_S; \theta^V\right) \underset{\theta^V \to +\infty}{\longrightarrow} -\infty$$

(or it is proportional to a function which tends to $-\infty$, see the $\psi^V (.)$ functions in Appendix 6.3). This is because, according to Assumptions 2b and 2c, $t^- < t_S (0) < t^+ (N)$. Therefore, $\mathbb{E}U^V \left(t; \theta^V\right) - U^V \left(t_S; \theta^V\right) < 0$ when θ^V is large enough and route S is preferred to route R for some (large) values of risk aversion, a contradiction. ■

Lemma 4 implies that $\tilde{\theta}_p^V (N) < \bar{\theta}^V$ in the case of a bounded distribution and $\tilde{\theta}_p^V (N) < \infty$ otherwise. In both cases, $F_{\mathcal{L}} \left(\tilde{\theta}_p^V (N)\right) < 1$. Lemmas 3 and 4 show that, under the no information regime, there cannot be equilibrium corner solutions (each route is selected by some users at equilibrium). By continuity, and using Theorem 2, we prove in Section 3.3 that there exist an interior equilibrium traffic $n^V_{R,\mathcal{L}} \in]0; N[$ and an associated risk aversion threshold $\tilde{\theta}^V_p\left(n^V_{R,\mathcal{L}}\right)$ such that the $n^V_{R,\mathcal{L}}$ least risk averse users $(\theta^V < \tilde{\theta}^V_p\left(n^V_{R,\mathcal{L}}\right))$ choose route R, the $N - n^V_{R,\mathcal{L}}$ most risk averse users $(\theta^V > \tilde{\theta}^V_p\left(n^V_{R,\mathcal{L}}\right))$ choose route S.

3.3 Equilibrium

We are now ready to study the equilibrium solution without information, which is given by:

Theorem 3 *Let Assumptions 1, 2b, 2c and 6 hold. With no information, when the risk aversion parameter* θ^V *is distributed according to* $\mathcal{L}\left(\bar{\theta}^V\right)$, *there exists a unique equilibrium traffic* $n^V_{R,\mathcal{L}} \in]0; N[$, *which solves* $F^{-1}_{\mathcal{L}}\left(n^V_{R,\mathcal{L}}/N; \bar{\theta}^V\right) = \tilde{\theta}^V_p\left(n^V_{R,\mathcal{L}}\right)$.

Proof. See Appendix 6.4. ■

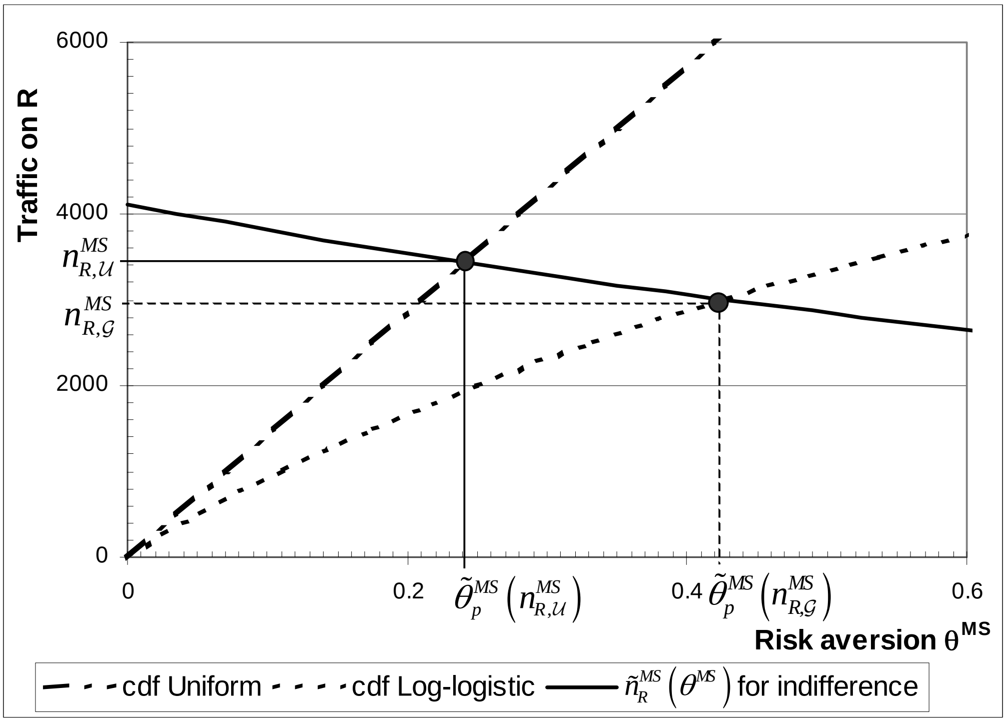

Figure 3: Equilibrium for MS preferences with uniform and log-logistic distributions for risk aversion

The solution for equilibrium traffic $n^V_{R,\mathcal{L}}$ is generally not explicit, but it can easily be solved numerically, as illustrated in Figure 3 for the Mean-Standard deviation case. For the uniform distribution $\mathcal{L} = \mathcal{U}$,

$$F_{\mathcal{U}}\left(\theta; \bar{\theta}^V\right) = \theta/\bar{\theta}^V = n_R/N \text{ and } \theta = F^{-1}_{\mathcal{U}}\left(n_R/N; \bar{\theta}^V\right) = \bar{\theta}^V n_R/N. \tag{6}$$

This corresponds to the dash-dotted increasing straight line on Figure 3. For the log-logistic distribution $\mathcal{L} = \mathcal{G}$,

$$F_{\mathcal{G}}\left(\theta; \bar{\theta}^V\right) = \frac{1}{1 + \bar{\theta}^V/\theta} = n_R/N \Leftrightarrow \theta = F^{-1}_{\mathcal{G}}\left(n_R/N; \bar{\theta}^V\right) = \frac{\bar{\theta}^V}{N/n_R - 1}. \tag{7}$$

This corresponds to the dotted increasing and concave curve on Figure 3. Equilibrium traffic is obtained by inserting these expressions in the relevant line of Equation 14 or 15 in Appendix 6.2, in which t_S and t^+

have to be replaced by their expressions as functions of $n_R = n^V_{R,\mathcal{L}}$. The (inverse) relation between n_R and $\tilde{\theta}^{MS}_p(n_R)$ (for Mean-Standard deviation preferences) corresponds to the decreasing solid curve on Figure 3. For a uniform distribution and Mean-Standard deviation preferences, under Assumption 5 for the travel time functions, Equation 6 inserted in the second line of Equation 14 gives:

$$\begin{aligned}\left(\tfrac{n_R}{N}\right)\bar{\theta}^{MS} &= \left\{\left[1+\tfrac{N-n_R}{\lambda_S}\right]^b \tau_S - t^- - p\left[\left(1+\tfrac{n_R}{\lambda_R}\right)^b \tau^+ - t^-\right]\right\} \Big/ \left\{\sqrt{p\,(1-p)}\left[\tau^+\left(1+\tfrac{n_R}{\lambda_R}\right)^b - t^-\right]\right\}\\ &\Leftrightarrow \left[\bar{\theta}^{MS}\sqrt{p\,(1-p)}n_R/N+p\right]\tau^+\left(1+n_R/\lambda_R\right)^b - \tau_S\left[1+\left(N-n_R\right)/\lambda_S\right]^b\\ &\quad -\bar{\theta}^{MS}\sqrt{p\,(1-p)}t^- n_R/N + (1-p)\,t^- = 0,\end{aligned}$$

which results in a polynomial expression in n_R, of order $1+b$ when b is an integer. As an illustration, Figure 3 is obtained with the values of the parameters given at the end of Section 2. For the uniform distribution $\mathcal{U}$ of risk aversion on the interval $[0; 0.7]$ ($\bar{\theta}^{MS} = 0.7$), the equilibrium without information is obtained for $n^{MS}_{R,\mathcal{U}} = 3,437$ and $\tilde{\theta}^{MS}_p\left(n^{MS}_{R,\mathcal{U}}\right) = 0.241$ ($\theta^{MS} < \tilde{\theta}^{MS}_p\left(n^{MS}_{R,\mathcal{U}}\right)$ for 34% of users). Note that $\theta^V > 1$ for mean-standard deviation expected utility would mean that the user is more sensitive to the variability in travel time than to expected travel time, which would not be very realistic. This represents a congestion of 64% on route S (from 15 min. to 24.57 min.) and of 67% (from 20 to 33.48 min.) on route R in the bad state. The numerical results are slightly different for the log-logistic distribution of risk aversion with $\bar{\theta}^{MS} = 1$. The equilibrium without information is obtained for $n^{MS}_{R,\mathcal{G}} = 3,000$ and $\tilde{\theta}^{MS}_p\left(n^{MS}_{R,\mathcal{G}}\right) = 0.43$ ($\theta^{MS} < \tilde{\theta}^{MS}_p\left(n^{MS}_{R,\mathcal{G}}\right)$ for 30% of users). This represents a congestion of 69% on route S (from 15 min. to 25.3 min.) and of 57% (from 20 min. to 31.5 min.) on route R in the bad state.

3.4 Impact of the probability p on the risk aversion thresholds

The figures with endogenous congestion and travel times are significantly different from those obtained with exogenous constant travel times, especially concerning the impact of the probability p of the bad state of the nature, as illustrated in Figure 4 for Mean-Variance and Mean-Standard deviation preferences and a uniform distribution for risk aversion. The solid curves represent the no information equilibrium risk

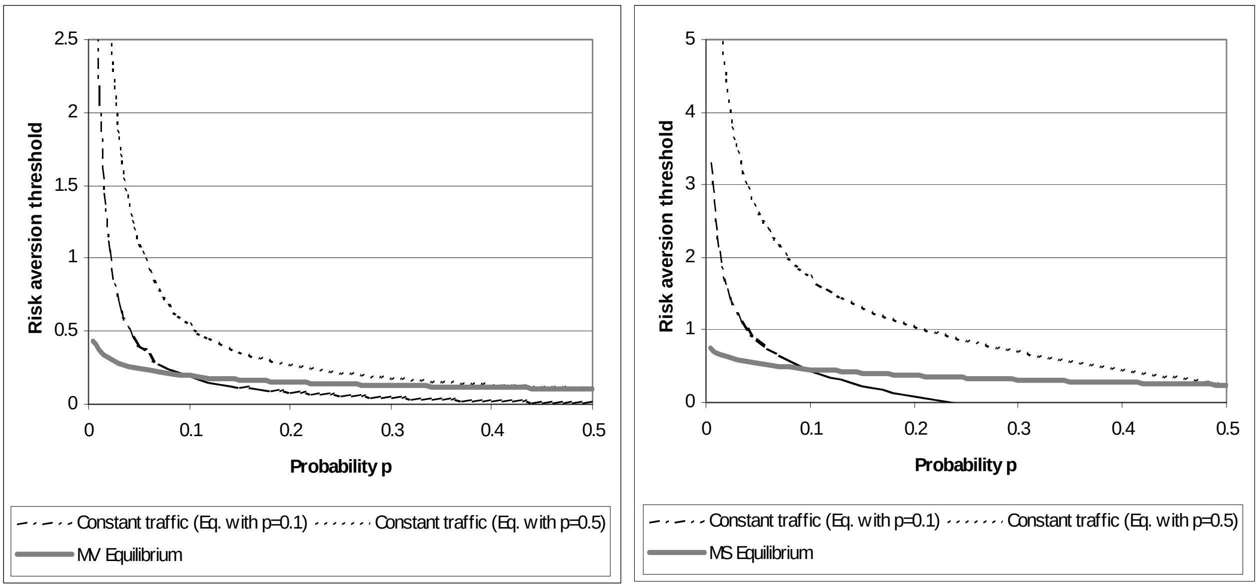

Figure 4: Impact of the probability p on risk aversion threshold with and without endogenous congestion, for MV and MS preferences

aversion threshold with endogenous congestion. The dotted curve represents the risk aversion threshold for a constant traffic corresponding to the equilibrium traffic when $p = 0.5$ (as in Figure 2), that is $n_{R,\mathcal{U}}^{MS} = 3,437$ and $\tilde{\theta}_p^{MS}\left(n_{R,\mathcal{U}}^{MS}\right) = 0.241$ for MS preferences, and $n_{R,\mathcal{U}}^{MV} = 2102$ and $\tilde{\theta}_p^{MV}\left(n_{R,\mathcal{U}}^{MV}\right) = 0.105$ for MV preferences (with $\bar{\theta}^{MV} = 0.5$). The dash-dotted curves are similar, with a probability $p = 0.1$. Note that, without congestion, the risk aversion threshold is highly sensitive to the assumed constant travel times, especially in the MS case. As expected, in both cases, the endogenization of travel time dramatically reduces the impact of the probability p on the risk aversion threshold, and consequently on the distribution of traffic between the two routes, since the bold curves are far flatter than the dotted and dash-dotted curves. With a very low probability of bad state, most drivers would choose route R, which induces much congestion on route R, thus reducing its attractivity.

4 COMPARISON OF THE TWO CASES (STATE OF THE NATURE KNOWN OR NOT)

4.1 Travel time

We first generalize Proposition 2 when users have different levels of risk aversion (we still assume that all users are either risk averse or risk neutral, i.e. there are no risk loving drivers, see Assumption 6).
Traffic depends on the state of nature and on information. When the state of the nature is unknown, the equilibrium traffic $n_{R,\mathcal{L}}^{V}$ on the risky route lies between the equilibrium traffic $n_R^- = N$ on the risky route in the good state of the nature and the equilibrium traffic n_R^+ on the risky route in the bad state of the nature.

Proposition 3 *Assume that Assumptions 1 to 4 hold and that users' preferences are described by the utility function $V = CR$ or CA, with a risk aversion parameter distributed according to $\mathcal{L}$. Then, on route R, equilibrium traffic with no information is larger than bad day equilibrium traffic, and lower than good day equilibrium traffic: $n_R^+ < n_{R,\mathcal{L}}^{V} < n_R^-$.*

Proof. See Appendix 6.5. ■
This result is rather intuitive. Less users are willing to use the risky route when bad state of the nature is common knowledge than when the state of the nature is unknown: the least risk averse users are willing to take their chance in the latter case. However, no similar result holds for MV or MS expected utility. For those preferences, depending on the values of the parameters, equilibrium traffic on route R with no information, $n_{R,\mathcal{L}}^{V}$, may be either larger or lower than bad day equilibrium traffic n_R^+. As a counter-example, consider the numerical values used at the end of Section 3.3, MV preferences and a risk aversion parameter distributed uniformly on the interval $\left[0; \bar{\theta}^{MV} = 1\right]$. Then the no information equilibrium traffic for MV preferences is $n_{R,\mathcal{U}}^{MV} = 1,622$, which is less than bad day equilibrium traffic $n_R^+ = 1,991$. The case $\bar{\theta}^{MV} = 0.5$ developed in Section 3.3 led to the intuitive results $n_{R,\mathcal{U}}^{MV} = 2,102 > n_R^+$.
The proof of Proposition 3 is based on utility computed for a given fixed travel time, not only on expected utility. This proof does not hold for MV or MS expected utility because they do not come from a standard utility function with a binary distribution for travel time. This is another source of inconsistency of MV expected utility (in addition to the one pointed out before Assumption 4). It is not very plausible that providing the information that traffic conditions are bad on one route may increase traffic on that route! We therefore focus our attention from now on to "real" utility functions $V = CR$ or CA. For the sake of comparison, some results are also provided for MV or MS expected utility, but we choose parameters values so that $n_R^+ < n_{R,\mathcal{L}}^{V} < n_R^-$.
The impact of information is usually measured by the difference in travel time before and after the provision of information. We show below that although all users benefit from information, some users benefit from travel time savings more than other users do. Later on, we will also examine the potential utility gains from information provision. The impact of information on route choice and travel time for the least and most risk averse users is summed up in Table 1 and discussed below.

	Route choice and travel time	
State of the nature	$\theta^V < \tilde{\theta}_p^V \left(n_{R,\mathcal{L}}^V\right)$	$\theta^V > \tilde{\theta}_p^V \left(n_{R,\mathcal{L}}^V\right)$
State unknown good	$R,\, t^-$	$S,\, t_S\left(N - n_{R,\mathcal{L}}^V\right)$
State unknown bad	$R,\, t^+\left(n_{R,\mathcal{L}}^V\right)$	
State unknown expected	$pt^+\left(n_{R,\mathcal{L}}^V\right) + (1-p)\, t^-$	
State known good	$R,\, t^-$	$R,\, t^-$
State known bad	$R \approx S,\, t^+\left(n_R^+\right)$	$R \approx S,\, t^+\left(n_R^+\right)$
State known expected	$pt^+\left(n_R^+\right) + (1-p)\, t^-$	$pt^+\left(n_R^+\right) + (1-p)\, t^-$
Expected time gain	$p\left[t^+\left(n_{R,\mathcal{L}}^V\right) - t^+\left(n_R^+\right)\right]$	$t_S\left(N - n_{R,\mathcal{L}}^V\right) - pt^+\left(n_R^+\right) - (1-p)\, t^-$

Table 1: Impact of information on route choice and travel time

When the state of the nature is unknown, the least risk averse users select the risky route and their travel time is t^- in the good state of the nature, $t^+\left(n_{R,\mathcal{L}}^V\right)$ in the bad state of the nature, so their expected travel time is $pt^+\left(n_{R,\mathcal{L}}^V\right) + (1-p)\, t^-$. The most risk averse drivers select route S and their travel time is $t_S\left(N - n_{R,\mathcal{L}}^V\right) = t^+\left(n_{R,\mathcal{L}}^V\right)$ whatever the state of the nature. When the state of the nature is known, both types of users select route R in the good state of the nature, are indifferent between route R and route S in the bad state of the nature and their expected travel time is $t^+\left(n_R^+\right)$. The expected time gain is therefore $p\left[t^+\left(n_{R,\mathcal{L}}^V\right) - t^+\left(n_R^+\right)\right]$ for the least risk averse users and $t_S\left(N - n_{R,\mathcal{L}}^V\right) - pt^+\left(n_R^+\right) - (1-p)\, t^-$ for the most risk averse users.
We will now see that expected time gain is larger for the most risk averse users.

Proposition 4 *Assume users' preferences are described by the utility function $V = CR$ or CA, with a risk aversion parameter distributed according to $\mathcal{L}$ and that Assumptions 1 to 4 hold. Then expected travel time decreases for all the users when information is provided. Moreover, travel time decrease is more significant for the most risk averse users ($\theta^V > \tilde{\theta}_p^V \left(n_{R,\mathcal{L}}^V\right)$ than for the least risk averse users ($\theta^V < \tilde{\theta}_p^V \left(n_{R,\mathcal{L}}^V\right)$).*

Proof. See Appendix 6.6. ■
Most research on ITS focus on congestion and restrict their attention to expected travel time. This important impact of drivers information systems is studied in Proposition 4, which shows that the most risk averse users are (apparently) those who benefit the most from the provision of information (in terms of expected travel time). However, we will see in the next section that this analysis provides a partial and potentially misleading view, since it does not consider the cost of uncertainty (which can only be neglected when drivers are risk neutral). Indeed, the benefit of information perceived by the least risk averse users is larger than suggested by expected travel time since information also reduces travel time uncertainty for those users. We show in next section that the impact of information is more ambiguous for the most risk averse users.
The aggregate impact of the provision of information is characterized by:

Corollary *When users have CRRA or CARA preferences, information reduces aggregate expected travel time.*
Proof. This result is obvious since expected travel time is reduced for all users (see Proposition 4). ■
Aggregate expected time gain AET^V is the weighted sum of travel time gained by the $n_{R,\mathcal{L}}^V$ least risk averse users and by the $\left(N - n_{R,\mathcal{L}}^V\right)$ most risk averse users:

$$AET^V = n_{R,\mathcal{L}}^V p\left[t^+\left(n_{R,\mathcal{L}}^V\right) - t^+\left(n_R^+\right)\right] + \left(N - n_{R,\mathcal{L}}^V\right)\left[t_S\left(N - n_{R,\mathcal{L}}^V\right) - pt^+\left(n_R^+\right) - (1-p)\, t^-\right]$$

With MS preferences and with the parameter values used in the previous examples, expected travel time with information is reduced to 18.59 min. (27.18 min. in the bad state and 10 min. in the good state). Without information, expected travel time was 24.57 min. (for sure) for the most risk averse users and 21.74 min. for the least risk averse users (33.48 min. in the bad state and 10 min. in the good state).
Expected travel time reduction is $24.57 - 18.59 = 5.98$ min. for the $6,563$ most risk averse users and $21.74 - 18.59 = 3.15$ for the $3,437$ least risk averse users. This represents a gain of $6,563 * 5.98 + 3,437 * 3.15 = 50,073$ min for the $10,000$ users, that is 5.01 min by user.

4.2 Compensating variation

We now study how much time each user would be eager to pay for information. However, we have to be more explicit in two dimensions: whether information is private or public (common knowledge), and what is the reference point.

Lemma 5 *Private information has no value when all other users are informed.*

Proof. Consider the reference situation in which all users have access to information. Equilibrium travel time is then t^- on good days and $t^+\left(n_R^+\right) = t_S\left(N - n_R^+\right)$ on bad days. If a driver were to deviate from the equilibrium with costly information, she would choose route R. This is because, at the equilibrium with traffic n_R^+, $t^+\left(n_R^+\right) = t_S\left(N - n_R^+\right) > t^-$, so $\mathbb{E}U^V\left(T_R\left(n_R^+\right);\theta^V\right) > U^V\left(t_S\left(N - n_R^+\right);\theta^V\right)$ for any uninformed user, who would therefore choose route R. The value of private information in then zero because it does not modify choices. ■

This result is rather intuitive since, when all users are informed, equilibrium is such that they are all indifferent between the two routes in the bad state of the nature and they all strictly prefer the risky route in the good state. Any user would then prefer the risky route when she does not know the state of the nature whereas the others have access to this information.

We do not explore the value of private information when all other users are uninformed because equilibrium would make no sense in that case. The case in which an endogenous fraction of users is informed is left for future research. Here, we rather focus on the value of public information, and we consider:

Definition *The compensating variation $CV^V\left(\theta^V\right)$ corresponds to the time a user with preferences V and risk aversion θ^V is ready to pay for public information.*

With costly public information, equilibrium travel time is $t^+\left(n_R^+\right) = t_S\left(N - n_R^+\right)$ on bad days and t^- on good days, so expected utility is:

$$\mathbb{E}U^V\left(T_R\left(n_R^+\right) + CV^V\left(\theta^V\right);\theta^V\right) = pU^V\left(t^+\left(n_R^+\right) + CV^V\left(\theta^V\right);\theta^V\right) + (1-p)\,U^V\left(t^- + CV^V\left(\theta^V\right);\theta^V\right). \tag{8}$$

Without costly public information, equilibrium traffic is $n_{R,\mathcal{L}}^V$ and expected utility depends on individual choice. Recall that, without information, the least risk averse drivers ($\theta^V < \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$) choose route R and get the expected utility

$$\mathbb{E}U^V\left(T_R\left(n_{R,\mathcal{L}}^V\right);\theta^V\right) = pU^V\left(t^+\left(n_{R,\mathcal{L}}^V\right);\theta^V\right) + (1-p)\,U^V\left(t^-;\theta^V\right), \tag{9}$$

while the most risk averse ones ($\theta^V > \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$) choose route S and get the deterministic utility

$$U^V\left(t_S\left(N - n_{R,\mathcal{L}}^V\right);\theta^V\right) = U^V\left(t_S\left(N - n_{R,\mathcal{L}}^V\right);\theta^V\right). \tag{10}$$

Proposition 5 *The compensating variation $CV^V\left(\theta^V\right)$ is given by:*

$$\begin{cases} \mathbb{E}U^V\left(T_R\left(n_R^+\right) + CV^V\left(\theta^V\right);\theta^V\right) = \mathbb{E}U^V\left(T_R\left(n_{R,\mathcal{L}}^V\right);\theta^V\right) \text{ if } \theta^V < \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right) \\ \mathbb{E}U^V\left(T_R\left(n_R^+\right) + CV^V\left(\theta^V\right);\theta^V\right) = U^V\left(t_S\left(N - n_{R,\mathcal{L}}^V\right);\theta^V\right) \text{ if } \theta^V > \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right) \end{cases}. \tag{11}$$

Assume that Assumptions 1 to 4 hold and that users' preferences are described by the utility function $V = CR$ or CA. Then $CV^V\left(\theta^V\right)$ is guaranteed to be positive and increasing in θ^V for the least risk averse users ($\theta^V < \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$), and decreasing in θ^V for the most risk averse users ($\theta^V > \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$).

Proof. Equation 11 comes from the definition of $CV^V\left(\theta^V\right)$ and from Equations 8 to 10.

Proposition 3 means that $n_{R,\mathcal{L}}^V > n_R^+$, at least for CR and CA preferences. This implies that, for the least risk averse users (who would choose route R without information), travel time is reduced in the bad state of the nature and unchanged in the good state of the nature. Therefore, both the expectation and the variability of travel time are reduced by information for the least averse drivers. This implies that the benefit of information, and therefore the compensating variation, are strictly positive and increasing in θ^V

for all the least risk averse users ($\theta^V < \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$). By continuity, the compensating variation is strictly positive for some of the most risk averse users.
The most risk averse users would switch from the safe to the risky route when public information is provided, so information would increase the variability of their travel time (from 0 to a positive value) at the same time as it decreases their expected travel time. This implies that the benefit of information, and therefore the compensating variation, are decreasing in θ^V when $\theta^V > \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$, and it may become negative for the very risk averse users. ■
Note that Equation 11 holds for any preferences, although the remainder of Proposition is guaranteed only for CR and CA preferences. Note also that $\mathbb{E}U^V\left(T_R\left(n_{R,\mathcal{L}}^V\right);\theta^V\right) = U^V\left(t_S\left(N - n_{R,\mathcal{L}}^V\right);\theta^V\right)$ when $\theta^V = \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$, so the two expressions of $CV^V\left(\theta^V\right)$ converge at $\theta^V = \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$. The compensating variation for the different cases is computed in Appendix 6.7 and represented in Figure 5 (solid curves), together with the expected travel time reduction (stair-shaped dash-dotted curves). Whatever the preferences, the compensating variation exactly corresponds to the expected travel time reduction at $\theta^V = 0$ (risk neutral users).
Note that $CV^V\left(\theta^V\right)$ is a piecewise linear function of θ^V for $V = MV$ and MS. With MV or MS preferences, $CV^V\left(\theta^V\right)$ may be negative for any user for certain values of the parameters (this is not the case in Figure 5). Indeed, with MV preferences, the sign of $CV^V\left(\theta^V\right)$ is given by the sign of $\left[t^+\left(n_{R,\mathcal{L}}^{MV}\right) - t^+\left(n_R^+\right)\right]$, which is not guaranteed to be positive (see the example following Proposition 3).
Those users are ready to pay for public information because it helps a better distribution of users between the two routes in the bad state of the nature. With the numerical values of the parameters considered in the

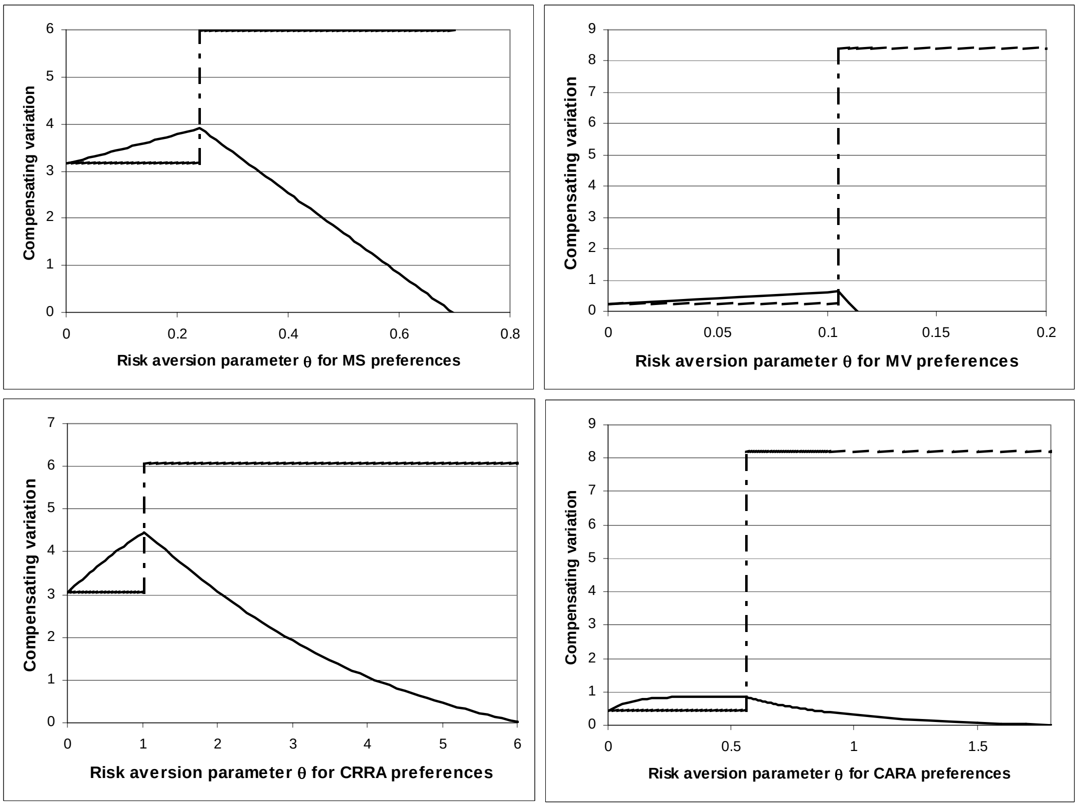

Figure 5: Compensating variation and expected travel time reduction for the different preferences

example, for MS and CR preferences, the limit above which the compensating variation becomes negative is far away from the risk aversion threshold and very few users are concerned (1.4% for MS preferences and

25% for CRRA preferences). On the opposite, utility would be reduced by public information for a large fraction of users with the two other functional for preferences (77% with MV expected utility and 53% with CARA utility).

5 CONCLUSION

We have proposed in this paper a framework to analyze the rational behavior of drivers facing uncertain travel time conditions. We have shown the importance of the risk aversion parameter to explain route choice behavior and the impact of information. The users with high enough risk aversion modify their choice when they are informed of good conditions, while risk neutral individuals stay on the risky route whether or not they receive information. However, we show that both types of users benefit from information given that the expected travel time is reduced for all users. Interestingly, we have shown that individual benefit depends on their level of risk aversion. Moreover, this dependency is not monotonic (see Figure 3).
The validation of this model with empirical data will require to imbed our model in a discrete choice framework (see McFadden, 2001). This would add unobserved heterogeneity, as in de Palma and Picard (2004a), to explain the route choice behavior of users. We also refer the reader to Hartog et al. (2000) for another measurement of risk aversion when individuals face simple lotteries. In our view, the main hypotheses to check are the following (1) the individuals are able to estimate the probability of each state of the nature: de Palma and Picard (2004c) consider a situation in which users discover from day to day the distribution of travel time and incorporate the information acquired daily, by resorting to a Bayesian update. In this paper, we consider a situation in which learning has already taken place. (2) The analysis of route choice with uncertain travel time could allow to identify the specification of the utility function and the risk aversion parameters. However, it is likely that stated as well as revealed information will be needed to estimate such models. (3) Finally, the above analysis relies on the expected utility theory. Different authors since Kahneman and Tversky (1979) have argued that individuals may overestimate small probabilities and underestimate large probabilities. In the transportation context, it would be interesting to see how such theory (referred to as Cumulative Prospect Theory) applies. See Camerer (1989), Machina (1982), as well as the visionary paper by Allais (1953). This amounts to analyze how drivers may change their route when there is a small probability that an important event occurs. For example, in France, the department of transportation has started to publish the number of accidents which occurred on each major interurban highway and roadway (which could be used to compute the probability that a driver has an accident). It remains to analyze how this information would affect route choice for a fraction of drivers. In addition, according to Ellsberg's paradox (see Kagel and Roth, 1995), individuals may be averse not only to risk, but also to ambiguity. This would represent an additional value for acquiring information from day to day, thus increasing the advantage of selecting the risky route (at least at the beginning of the learning process). Much research, both at a theoretical and at an empirical point of view, needs to be devoted in order to better understand all the unexplored facets of the Drivers Information System.

ACKNOWLEDGMENTS

The authors thank Simon Anderson, Frédéric Koessler and Robin Lindsey for their helpful comments and suggestions on earlier versions of this paper. They also benefitted from suggestions from participants at the 2005 TRB meeting.

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6 APPENDIX

6.1 Correspondence between (Δ, δ, τ) and $(t^-, t^+(n_R), t_S(n_S))$

The correspondence between the symmetric notations (Δ, δ, τ) used in de Palma and Picard (2004b) and the endogenous travel times is given by:

$$\left\{\begin{array}{l}(1-\Delta)\tau = t^- \\ (1+\Delta)\tau = t^+(n_R) \\ (1+\delta)\tau = t_S(n_S)\end{array}\right. \Leftrightarrow \left\{\begin{array}{l}\Delta = [t^+(n_R) - t^-] / [t^+(n_R) + t^-] \\ \delta = 2t_S(n_S) / [t^+(n_R) + t^-] - 1 \\ \tau = [t^+(n_R) + t^-]/2\end{array}\right. . \tag{12}$$

6.2 Thresholds

The probability thresholds $\tilde{p}_\theta^V$ (such that $\mathbb{E}U^V(T_R;\theta^V) = U^V(t_S;\theta^V)$) are given by analytic formulas for the four specifications of preferences considered:

$$\left\{\begin{array}{l}\tilde{p}_\theta^{MV} = \left\{1 + (t^+ - t^-)\theta^{MV} - \sqrt{1 + \left[(t^+ - t^-)\theta^{MV} - \frac{2t_s}{t^+ + t^-} - 1\right](t^+ - t^-)\theta^{MV}}\right\} \Big/ \left\{2(t^+ - t^-)\theta^{MV}\right\}, \\ \tilde{p}_\theta^{MS} = \left\{(\theta^{MS})^2 + 1 + 2\frac{t_S - t^-}{t^+ - t^-} - \theta^{MS}\sqrt{(\theta^{MS})^2 + 1 - \left(2\frac{t_S - t^-}{t^+ - t^-}\right)^2}\right\} \Big/ \left\{2\left[1 + (\theta^{MS})^2\right]\right\}, \\ \tilde{p}_\theta^{CR} = \left[(t_S)^{1+\theta^{CR}} - (t^-)^{1+\theta^{CR}}\right] \Big/ \left[(t^+)^{1+\theta^{CR}} - (t^-)^{1+\theta^{CR}}\right] \\ \text{and} \\ \tilde{p}_\theta^{CA} = \left[\exp\left(t_s\theta^{CA}\right) - \exp\left(t^-\theta^{CA}\right)\right] / \left[\exp\left(t^+\theta^{CA}\right) - \exp\left(t^-\theta^{CA}\right)\right].\end{array}\right. \tag{13}$$

In addition, formulas are far more simple for the risk aversion thresholds $\tilde{\theta}_p^{MV}$ and $\tilde{\theta}_p^{MS}$:

$$\left\{\begin{array}{l}\tilde{\theta}_p^{MV} = [t_S - t^- - p(t^+ - t^-)] / \left[p(1-p)(t^+ - t^-)^2\right] \\ \text{and} \\ \tilde{\theta}_p^{MS} = [t_S - t^- - p(t^+ - t^-)] / \left[\sqrt{p(1-p)}(t^+ - t^-)\right].\end{array}\right. \tag{14}$$

However, no analytic formula can be obtained for he risk aversion thresholds $\tilde{\theta}_p^{CR}$ and $\tilde{\theta}_p^{CA}$, which solve, respectively:

$$\left\{\begin{array}{l}p(t^+)^{1+\tilde{\theta}_p^{CR}} + (1-p)(t^-)^{1+\tilde{\theta}_p^{CR}} = (t_S)^{1+\tilde{\theta}_p^{CR}} \\ \text{and} \\ p\exp\left(t^+\tilde{\theta}_p^{CA}\right) + (1-p)\exp\left(t^-\tilde{\theta}_p^{CA}\right) = \exp\left(t_S\tilde{\theta}_p^{CA}\right).\end{array}\right. \tag{15}$$

6.3 Proof of Theorem 2

Equations 14 clearly show that $\tilde{\theta}_p^{MV}$ and $\tilde{\theta}_p^{MS}$ are increasing in t_S and decreasing in t^+. Assumption 1 then implies that $\tilde{\theta}_p^{MV}$ and $\tilde{\theta}_p^{MS}$ are decreasing in n_R.
Let

$$\begin{aligned}\psi^{CR}\left(p, t^-, t^+, t_S; \theta^{CR}\right) &= (t_S)^{1+\theta^{CR}} - \left[p(t^+)^{1+\theta^{CR}} + (1-p)(t^-)^{1+\theta^{CR}}\right] \\ &= \left(1+\theta^{CR}\right)\left[\mathbb{E}U^{CR}\left(T_R;\theta^{CR}\right) - U^{CR}\left(t_S;\theta^{CR}\right)\right] \text{ and} \\ \psi^{CA}\left(p, t^-, t^+, t_S; \theta^{CA}\right) &= \exp\left(t_S\theta^{CA}\right) - \left[p\exp\left(t^+\theta^{CA}\right) + (1-p)\exp\left(t^-\theta^{CA}\right)\right] \\ &= \theta^{CA}\left[\mathbb{E}U^{CA}\left(T_R;\theta^{CA}\right) - U^{CA}\left(t_S;\theta^{CA}\right)\right].\end{aligned}$$

The user with preferences $U^V\left(t;\theta^V\right)$, for $V = CR, CA$ is then indifferent between the two routes when $\psi^V\left(p, t^-, t^+, t_S; \theta^V\right) = 0$. She prefers route R when $\psi^V(.) > 0$ and route S when $\psi^V(.) < 0$. The function

$\psi^V(.)\,,\ V=CR,CA$ is increasing in t_S and decreasing in t^+, so it is decreasing in n_R. We will now prove that $\psi^V(.)\,,\ V=CR,CA$ is locally decreasing in θ^V, at the indifference point $\tilde{\theta}_p^V$ which solves $\psi^V\left(.;\tilde{\theta}_p^V\right)=0$. Consider first CRRA preferences. $\psi^V\left(.;\tilde{\theta}^{CR}\right)=0$ corresponds to:

$$\begin{aligned} (t_S)^{1+\tilde{\theta}^{CR}} &= p\,(t^+)^{1+\tilde{\theta}_p^{CR}} + (1-p)\,(t^-)^{1+\tilde{\theta}_p^{CR}} \\ \Leftrightarrow \left(1+\tilde{\theta}_p^{CR}\right)\ln(t_S) &= \ln\left[p\,(t^+)^{1+\tilde{\theta}_p^{CR}} + (1-p)\,(t^-)^{1+\tilde{\theta}_p^{CR}}\right] \end{aligned}$$

which can be used to eliminate t_S in the derivative:

$$\begin{aligned} \frac{\partial\psi^{CR}}{\partial\theta^{CR}}\left(.;\tilde{\theta}_p^{CR}\right) &= \left(1+\tilde{\theta}_p^{CR}\right)\ln(t_S)\,(t_S)^{1+\tilde{\theta}_p^{CR}} - \left(1+\tilde{\theta}_p^{CR}\right)\left[p\ln\left(t^+\right)\left(t^+\right)^{1+\tilde{\theta}_p^{CR}} + (1-p)\ln\left(t^-\right)\left(t^-\right)^{1+\tilde{\theta}_p^{CR}}\right] \\ &= \ln\left[p\left(t^+\right)^{1+\tilde{\theta}_p^{CR}} + (1-p)\left(t^-\right)^{1+\tilde{\theta}_p^{CR}}\right]\left[p\left(t^+\right)^{1+\tilde{\theta}_p^{CR}} + (1-p)\left(t^-\right)^{1+\tilde{\theta}_p^{CR}}\right] \\ &\quad - \left[p\left(1+\tilde{\theta}_p^{CR}\right)\ln\left(t^+\right)\left(t^+\right)^{1+\tilde{\theta}_p^{CR}} + (1-p)\left(1+\tilde{\theta}_p^{CR}\right)\ln\left(t^-\right)\left(t^-\right)^{1+\tilde{\theta}_p^{CR}}\right], \end{aligned}$$

which is in the form: $z\ln(z) - [px\ln(x) + (1-p)\,y\ln(y)]$, with $z = px + (1-p)\,y$. The convexity of the function $x\ln(x)$ then implies that $\frac{\partial\psi^{CR}}{\partial\theta^{CR}}\left(\tilde{\theta}_p^{CR}\right) < 0$, so $\frac{\partial\tilde{\theta}_p^{CR}}{\partial n_R} = -\frac{\partial\psi^{CR}}{\partial n_R}\left(.;\tilde{\theta}_p^{CR}\right)/\frac{\partial\psi^{CR}}{\partial\theta^{CR}}\left(.;\tilde{\theta}_p^{CR}\right) < 0$. We have therefore proved that $\tilde{\theta}_p^{CR}$ is decreasing in n_R.
The proof goes a similar way for CARA preferences. In that case, $x=\exp\left(t^+\theta^{CA}\right)$, $y=\exp\left(t^-\theta^{CA}\right)$ and $\tilde{\theta}_p^{CA}\frac{\partial\psi^{CA}}{\partial\theta^{CA}}\left(.;\tilde{\theta}_p^{CA}\right) = z\ln(z) - [px\ln(x) + (1-p)\,y\ln(y)]$. The details of the proof for CARA preferences are left to the reader.

6.4 Proof of Theorem 3

The equilibrium number of users on route R is the number $n_{R,\mathcal{L}}^V$ of users which are less risk averse than indifferent user $\tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$: $n_{R,\mathcal{L}}^V = NF_{\mathcal{L}}\left(\tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right);\bar{\theta}^V\right)$, or $n_{R,\mathcal{L}}^V/N = F_{\mathcal{L}}\left(\tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right);\bar{\theta}^V\right)$. Let $\phi_{\mathcal{L}}(n_R) = F_{\mathcal{L}}\left(\tilde{\theta}_p^V(n_R);\bar{\theta}^V\right)$. Since $F_{\mathcal{L}}(.)$ is strictly increasing on its support I and $\tilde{\theta}_p^V(.)$ is strictly decreasing in n_R (see Theorem 2), $\phi_{\mathcal{L}}(n_R)$ is strictly decreasing in n_R. According to Lemma 3, we have $0 < \phi_{\mathcal{L}}(0)$, and according to Lemma 4, we have $1 > \phi_{\mathcal{L}}(N)$. Since in addition n_R/N is strictly increasing in n_R there exists a unique solution $n_{R,\mathcal{L}}^V$, with $0 < n_{R,\mathcal{L}}^V < N$ such that $n_{R,\mathcal{L}}^V/N = \phi_{\mathcal{L}}\left(n_{R,\mathcal{L}}^V\right)$.

6.5 Proof of Proposition 3

Consider the individual with $\theta^V = \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$, who is indifferent between the two routes when the state of the nature is unknown (and traffic on the risky route is $n_{R,\mathcal{L}}^V$). Her expected utility satisfies:

$$\begin{aligned} pU^V\left(t^+\left(n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right) + (1-p)\,U^V\left(t^-;\tilde{\theta}_p^V\right) &= U^V\left(t_S\left(N-n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right) \\ \Leftrightarrow p\left[U^V\left(t^+\left(n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right) - U^V\left(t_S\left(N-n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right)\right]& \\ + (1-p)\left[U^V\left(t^-;\tilde{\theta}_p^V\right) - U^V\left(t_S\left(N-n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right)\right] &= 0 \end{aligned}$$

Since $t^- < t_S\left(N-n_{R,\mathcal{L}}^V\right)$, one gets

$$U^V\left(t^-;\tilde{\theta}_p^V\right) - U^V\left(t_S\left(N-n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right) > 0,$$

which implies, from the last equation, that

$$U^V\left(t^+\left(n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right) - U^V\left(t_S\left(N-n_{R,\mathcal{L}}^V\right);\tilde{\theta}_p^V\right) < 0,$$

and $t^+\left(n_{R,\mathcal{L}}^V\right) > t_S\left(N - n_{R,\mathcal{L}}^V\right)$.
Note that the function

$$\Omega\left(n_R\right) \equiv U^V\left(t^+\left(n_R\right); \tilde{\theta}_p^V\right) - U^V\left(t_S\left(N - n_R\right); \tilde{\theta}_p^V\right)$$

is decreasing in n_R. Therefore, $\Omega\left(n_R^+\right) = 0$ (see Proposition 2) and $\Omega\left(n_{R,\mathcal{L}}^V\right) < 0$ implies that $n_{R,\mathcal{L}}^V > n_R^+$.

6.6 Proof of Proposition 4

Equilibrium traffic $n_{R,\mathcal{L}}^V > n_R^+$, so $t_S\left(N - n_{R,\mathcal{L}}^V\right) < t_S\left(N - n_R^+\right)$. Without information on the state of the nature, the least risk averse users choose route R, which is not subject to congestion in the good state of the nature. Consequently, the least risk averse users incur the same travel time with and without information in the good state of the nature. In the bad state of the nature, $n_R^+ < n_{R,\mathcal{L}}^V$, so travel time on the risky route is reduced by information. Expected travel time is therefore reduced by information for users who choose R without information, that is users with $\theta^V < \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)$.
When $n_R = n_{R,\mathcal{L}}^V$, the risky route is chosen by some risk averse users, those with $\theta^V \in \left]0; \tilde{\theta}_p^V\left(n_{R,\mathcal{L}}^V\right)\right[$, so the expected travel time is less on route R than on route S when $n_R = n_{R,\mathcal{L}}^V$: $pt^+\left(n_{R,\mathcal{L}}^V\right) + (1-p)\,t^- < t_S\left(N - n_{R,\mathcal{L}}^V\right)$. In addition, $n_R^+ < n_{R,\mathcal{L}}^V$, so $t^+\left(n_R^+\right) < t^+\left(n_{R,\mathcal{L}}^V\right)$ and $pt^+\left(n_R^+\right) + (1-p)\,t^- < pt^+\left(n_{R,\mathcal{L}}^V\right) + (1-p)\,t^- < t_S\left(N - n_{R,\mathcal{L}}^V\right)$, which implies that information reduces expected travel time also for the most risk averse users.
Expected travel time variation (due to the provision of information), which corresponds to the difference between expected travel times without and with information, is therefore positive both for the most and the least risk averse users.
Note that expected travel time is the same for the most and the least risk averse users when the state of the nature is known (see Table 1), so the difference between the expected travel time gain by the most and the least risk averse users is equal to the difference between expected travel time for the most and the least risk averse users without information. That is:

$$t_S\left(N - n_{R,\mathcal{L}}^V\right) - pt^+\left(n_R^+\right) - (1-p)\,t^- - p\left[t^+\left(n_{R,\mathcal{L}}^V\right) - t^+\left(n_R^+\right)\right] = t_S\left(N - n_{R,\mathcal{L}}^V\right) - \left[pt^+\left(n_{R,\mathcal{L}}^V\right) + (1-p)\,t^-\right] > 0.$$

The positive sign corresponds to the fact that risk neutral users select route R when the state of the nature is unknown, which implies that expected travel time is lower on route R than on route S.

6.7 Computation of $CV\left(\theta^V\right)$

Recall that $\sigma^2\left(T_R\left(n_R\right)\right) = p\,(1-p)\left(t^+\left(n_R\right) - t^-\right)^2$.
In the Mean-Variance case, we obtain:

$$\begin{aligned} CV^{MV}\left(\theta^{MV}\right) &= \mathbb{E}\left(T_R\left(n_{R,\mathcal{L}}^{MV}\right)\right) - \mathbb{E}\left(T_R\left(n_R^+\right)\right) + \theta^{MV}\left[\sigma^2\left(T_R\left(n_{R,\mathcal{L}}^{MV}\right)\right) - \sigma^2\left(T_R\left(n_R^+\right)\right)\right] \\ &= p\left[t^+\left(n_{R,\mathcal{L}}^{MV}\right) - t^+\left(n_R^+\right)\right] + \theta^{MV} p\,(1-p)\left[t^+\left(n_{R,\mathcal{L}}^{MV}\right) - t^+\left(n_R^+\right)\right]\left[t^+\left(n_{R,\mathcal{L}}^{MV}\right) + t^+\left(n_R^+\right) - 2t^-\right] \end{aligned}$$

when $\theta^{MV} < \tilde{\theta}_p^{MV}\left(n_{R,\mathcal{L}}^{MV}\right)$ and

$$\begin{aligned} CV^{MV}\left(\theta^{MV}\right) &= t_S\left(N - n_{R,\mathcal{L}}^{MV}\right) - \mathbb{E}\left(T_R\left(n_R^+\right)\right) - \theta^{MV}\sigma^2\left(T_R\left(n_R^+\right)\right) \\ &= t_S\left(N - n_{R,\mathcal{L}}^{MV}\right) - pt^+\left(n_R^+\right) - (1-p)\,t^- - \theta^{MV} p\,(1-p)\left[\left(t^+\left(n_R^+\right) - t^-\right)^2\right] \end{aligned}$$

when $\theta^{MV} > \tilde{\theta}_p^{MV}\left(n_{R,\mathcal{L}}^{MV}\right)$.
In the Mean-Standard deviation case, we obtain

$$\begin{aligned} CV^{MS}\left(\theta^{MS}\right) &= \mathbb{E}\left(T_R\left(n_{R,\mathcal{L}}^{MS}\right)\right) - \mathbb{E}\left(T_R\left(n_R^+\right)\right) + \theta^{MS}\left[\sigma\left(T_R\left(n_{R,\mathcal{L}}^{MS}\right)\right) - \sigma\left(T_R\left(n_R^+\right)\right)\right] \\ &= p\left[t^+\left(n_{R,\mathcal{L}}^{MS}\right) - t^+\left(n_R^+\right)\right] + \theta^{MS}\sqrt{p\,(1-p)}\left[t^+\left(n_{R,\mathcal{L}}^{MS}\right) - t^+\left(n_R^+\right)\right] \end{aligned}$$

when $\theta^{MS} < \tilde{\theta}_p^{MS}\left(n_{R,\mathcal{L}}^{MS}\right)$ and

$$\begin{aligned} CV^{MS}\left(\theta^{MS}\right) &= t_S\left(N - n_{R,\mathcal{L}}^{MS}\right) - \mathbb{E}\left(T_R\left(n_R^+\right)\right) - \theta^{MS}\sigma\left(T_R\left(n_R^+\right)\right) \\ &= t_S\left(N - n_{R,\mathcal{L}}^{MS}\right) - pt^+\left(n_R^+\right) - (1-p)\,t^- - \theta^{MS}\sqrt{p\,(1-p)}\left[t^+\left(n_R^+\right) - t^-\right] \end{aligned}$$

when $\theta^{MS} > \tilde{\theta}_p^{MS}\left(n_{R,\mathcal{L}}^{MS}\right)$.
In the CARA case, we obtain:

$$CV^{CA}\left(\theta^{CA}\right) = \frac{1}{\theta^{CA}}\ln\frac{p\exp\left(t^+\left(n_{R,\mathcal{L}}^{CA}\right)\theta^{CA}\right) + (1-p)\exp\left(t^-\theta^{CA}\right)}{p\exp\left(t^+\left(n_R^+\right)\theta^{CA}\right) + (1-p)\exp\left(t^-\theta^{CA}\right)}$$

when $\theta^{CA} < \tilde{\theta}_p^{CA}\left(n_{R,\mathcal{L}}^{CA}\right)$ and

$$CV^{CA}\left(\theta^{CA}\right) = \frac{1}{\theta^{CA}}\ln\frac{\exp\left(t_S\left(N - n_{R,\mathcal{L}}^{CA}\right)\theta^{CA}\right)}{p\exp\left(t^+\left(n_R^+\right)\theta^{CA}\right) + (1-p)\exp\left(t^-\theta^{CA}\right)}$$

when $\theta^{CA} > \tilde{\theta}_p^{CA}\left(n_{R,\mathcal{L}}^{CA}\right)$.
No analytical formula is available for $CV^{CR}\left(\theta^{CR}\right)$, which solves:

$$p\left[t^+\left(n_R^+\right) + CV^{CR}\left(\theta^{CR}\right)\right]^{1+\theta^{CR}} + (1-p)\left[t^- + CV^{CR}\left(\theta^{CR}\right)\right]^{1+\theta^{CR}} = p\left[t^+\left(n_{R,\mathcal{L}}^{CR}\right)\right]^{1+\theta^{CR}} + (1-p)\left[t^-\right]^{1+\theta^{CR}}$$

when $\theta^{CR} < \tilde{\theta}_p^{CR}\left(n_{R,\mathcal{L}}^{CR}\right)$ and which solves:

$$p\left[t^+\left(n_R^+\right) + CV^{CR}\left(\theta^{CR}\right)\right]^{1+\theta^{CR}} + (1-p)\left[t^- + CV^{CR}\left(\theta^{CR}\right)\right]^{1+\theta^{CR}} = \left[t_S\left(N - n_{R,\mathcal{L}}^{CR}\right)\right]^{1+\theta^{CR}}$$

when $\theta^{CR} > \tilde{\theta}_p^{CR}\left(n_{R,\mathcal{L}}^{CR}\right)$.