Optimal land use and the allocation of endogenous amenities

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Abstract

This paper explores the implications, from a public sector economics point of view, of combining welfare assessments concerning land use in urban and environmental economics respectively. Urban economics has a long tradition in determining the optimal allocation of land (or space) as a consumption good, while land use issues in environmental economics are predominantly rooted in hedonic pricing as a valuation method for optimising the allocation of public goods. Recently, hedonic pricing methods have been extended by adopting location choice models for the valuation of non-marginal changes in levels of local amenities. Following a possible revision of the location choices by the population, endogenous prices are introduced and compensated for in a willingness to pay. Some of the new methods also allow for social interactions by means of endogenous amenities. While endogenous prices are the

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main contribution of these so-called sorting models to the valuation literature, until now little attention has been paid to the efficiency of the market equilibrium assumed, in terms of the consumption of space. This is surprising, because social interactions as endogenous amenities might alternatively be interpreted as positive external effects. As such, they are likely to result in an oversupply of land in a competitive market. The dominant characterisation of the equilibrium on the land (or housing) market in sorting models is market clearing, given a fixed supply. In this paper, the total amount of land used in the market clearing equilibrium will be compared with the competitive market equilibrium and the allocation by a benevolent social planner maximising social welfare. It is shown that under relatively general conditions and allowing for endogenous amenities, locational sorting models with a fixed supply make strong assumptions regarding the optimal total amount of land used and that in a competitive market this amount is larger than in the case of land use planning. This result suggests that in public policy recommendations, sorting models could benefit from complementing the valuation methodology with the internalisation of external effects for optimising land use.

Keywords: Environmental Issues, Theory, Land Use, Locational Sorting, Welfare Economics

JEL Classification: Q58, R52, D61

1 Introduction

When taking into account the price for land or housing, a formalisation of the land or housing market is needed. Traditionally the land market receives attention from various economic subdisciplines. Land in spatial economics is considered a consumption good or production factor, with a corresponding market, sometimes identified as the housing market for simplicity, when referred to as a rental market for location space. In environmental economics land plays a role in property valuation, where the value of land is assessed in revealed preference methods, especially hedonic pricing; again often projected on housing. And in finance land is considered an asset—with extensions to real estate in general—, competing with other investments in diversified risk portfolios or as a basis for a credit loan (mortgage). These different approaches highlight different aspects of land in economics.

Leaving aside for the moment the asset quality of land, primarily because of the complication of introducing time as yet another dimension, besides space, the question arises which combination of elements from spatial and environmental economics would serve policy interests concerning land markets from a social welfare perspective. Both perspectives seem to accommodate different definition of *efficiency*. On one side there is the spatial economics and capitalisation literature stressing the optimal allocation of land through markets, while on the other side the environmental economics and valuation literature put forward the public good character of local (environmental) quality. From a public policy point of view, an ideal welfare measure would address both aspects simultaneously. If, in a first assessment, the quality of a location would indeed be considered a local public good, exogenous to both consumer and producer (developer), two goods would be traded simultaneously on a land market:

1. *land*, as a consumer good or production factor, as in the spatial economics

land use tradition of von Thünen (1826),

2. *quality*, as a local public good (amenity, environmental quality) in the tradition of Mäler (1974).

In this stylised case, public policy is confronted with two aspects of a socially optimal allocation of land:

1. securing optimal allocation of land by markets,
2. supplying local public goods.

Recently, hedonic pricing methods have been extended by adopting location choice models for the valuation of non-marginal changes in levels of local amenities Timmins (2003); Smith et al. (2004); Bayer et al. (2005). These so-called locational sorting models provide a good starting point. However, since the location choice model is usually limited to deriving demand functions with a fixed supply, the aspect of optimal land use is not addressed. This paper shows how location choice models could in principle be extended with an endogenous total amount of residential land in order to cover both aspects simultaneously.

The structure of the paper is as follows. Section 2 discusses different perspectives on land use in economic theory. In section 3 a model will be developed that addresses the main issues stated above. A welfare analysis with this model is presented in section 4. Conclusions are stated in section 5.

2 Land use in economics

In the next two subsections an overview will be given on the traditional focus on land use in environmental and urban economics respectively. In subsection 2.3, the two aspects of land use mentioned above are highlighted and the relation with recently introduced location choice models in environmental economics is addressed.

2.1 Land use in environmental economics

Housing prices have been used extensively for a long time already in environmental economics. In that respect, the perspective on land use in environmental economics seems to have been dominated by the theoretic underpinnings of hedonic pricing by Rosen (1974). Rosen proposed a perfectly competitive market for the characteristics of consumer goods, making use of a bid rent concept, thereby referring to Alonso (Rosen, 1974, p. 38). In the context of methods applied in environmental economics in general (not restricted to land use), hedonic pricing has a special position. It is one of the few valuation methods with an explicit reference to market prices. Many other methods rather deal exclusively with the valuation of pure public goods. Starting with Mäler (1974), environmental economics has developed a theoretical basis for incorporating public goods and external effects—more generally, non-market goods—in an essentially neoclassical framework.

In broad terms, non-market goods can be thought of as all goods that affect well-being, but which are not traded on a market. Environmental quality for example is assumed to be consumed, but there is no market for it. This allows for a clear separation in the analysis of the maximisation of welfare from the allocation mechanism. If people would consume only goods that are traded on markets, the price mechanism under perfect competition would secure an optimal allocation and thereby yield a maximum level of social well-being. If markets were to supply environmental quality, it would typically lead to an under-supply, by familiar arguments that apply for public goods. From a welfare economics point of view, the state therefore has to intervene in the allocation, either by regulation or by taking care of supply itself.

The most rigorous implementation concerning the valuation of pure public goods is the contingent valuation method (CVM). Because public goods lack a market for achieving an efficient allocation, different criteria had to be developed. Mäler (1974) proposed a concept of shadow prices, or virtual prices, for public goods that is consistent with regular definitions of expenditure minimisation, compensated (Hicksian) demand and compensating or equivalent variation. The problem of expenditure minimisation is the dual problem to utility maximisation. The focus on expenditure minimisation in environmental economics can be explained by the goal of finding a monetary measure for welfare. For keeping the same level of welfare, while changing the supply of non-market goods, a minimal amount of virtual money can be derived. For market goods a change in price results in a change of the Hicksian consumer's surplus that equals the difference in the expenditure needed to maintain the same level of utility. With a similar definition for changes in the supply of non-market goods, a willingness to pay (WTP) can be derived, that serves as a monetary measure for the value of the change in (environmental) quality. The willingness to pay corresponds to a marginal WTP integrated over the amenity improvement. This yields something similar to a Hicksian consumer's surplus for price changes, with the public good treated as a quantity. The general result maintains that the allocation of public goods which yields the highest WTP would also yield the highest increase in utility, and is therefore optimal.

2.2 Land use in urban economics

In spatial economics the concept of the bid rent also plays a central role in land use models. The best-documented tradition of land use models in economics employing a bid rent dates back to von Thünen (1826) for agricultural land use. Von Thn̈en's method was extended by Alonso (1964) for location choices of consumers. These prototype models of land use in economics have always been interpreted as being part of the neoclassical economics tradition, because they conform to the conditions of competitive markets. They have also often been criticised, because of their unrealistic assumptions. However, it seems fair to say that the assumptions are the price of maintaining a reference to the neoclassical framework, identifying market allocation with welfare maximisation. In the land use models of economics, or more particularly urban economics, the market equilibrium price for land is assumed to be identical to the maximum bid rent that represents the price a consumer is willing to pay as the rental price for space, after travel or transport costs are subtracted from her income. Travel or transport costs are the only connotation with geography, as they are calculated based on the distance to a single, exogenously given Central Business District (CBD), or market place. The existence and location of the CBD itself is not explained in traditional urban economics. With location simplified to 'distance to the CBD' accounted for in transportation costs, elements of geography are exclusively introduced as location dependent net income; i.e., net of transportation costs.

2.3 Land use and social welfare

The question arises, which combination of elements from spatial and environmental economics would serve policy interests concerning land markets from a social welfare perspective best. On one hand the environmental economics tradition is concerned with the supply of non-market goods, while urban economics focuses on the market allocation of land. However, if in Rosen (1974)'s theoretical justification for the use of hedonic pricing one or more characteristics of the market good is labelled quality, certain quantity and quality combinations can only be consumed together. From the perspective of the consumer, pure public goods are interpreted as exogenously given non-market goods and are entirely beyond the control of the consumer or producer. In this sense, the value of the local environmental quality of a location could still be derived as in hedonic pricing from the market price, even though it concerns a pure public good. The phenomenon is known as *capitalisation*. The value of non-market good is said to be capitalised in the market price for land or housing, because the demand for land (quantity) is affected by the demand for quality. In this stylised case, public policy is confronted with two aspects of a socially optimal allocation of land: the efficiency of allocation of land by markets and the supply of local public goods.

Most analyses of the relation between the supply of local public goods and the impact on social welfare in a spatial context can be found in the public finance literature that takes Tiebout (1956) as a starting point. Tiebout proposed to interpret a special kind of spatial equilibrium, where a population is distributed over a given number of municipalities—as equivalent to a market equilibrium. The size of the population in a municipality would represent the demand for the locally supplied public good, determining in equilibrium its aggregate price as the municipal expenditure. Tiebout however, does not refer to capitalisation of the value of the public good in property prices directly.

In the next section, it will be shown that an alternative location choice model, in which the local quality characteristic is interpreted analogously to the distance to the CBD itself, is suitable for addressing simultaneously capitalisation and total land consumption. The main inspiration for this approach is supplied by a relatively new valuation method, which is based on so-called locational sorting models (Bayer et al., 2005) and the concept of a general equilibrium to pay (GE-WTP) (Smith et al., 2004). A GE-WTP should be able to account for the value of non-marginal changes in a spatially explicit distribution of local public goods, thereby extending current hedonic pricing methodology. Commonly, such a GE-WTP is derived as a Hicksian WTP adjusted for endogenous prices. Endogenous prices are typically enforced by a market clearing condition, often a fixed supply, constraining the relocation of a population in response to the changes in local quality. This strongly resembles the set-up of the basic urban economics models in the Alonso tradition. For a closed city model in urban economics however, also the city size is endogenous. Therefore, a WTP that also allows for variation in the total amount of land used for residential purpose would be a suitable basis for further exploration of the combined welfare effects of quality and quantity aspects.

Existing locational sorting models, however, commonly assume that the total supply of housing is fixed. The population is thereby assumed to resort over the existing housing stock. This paper first develops a simplified theoretical equivalent to sorting models in section 3, which will be used to explore the implications of an endogenous total supply of residential area in section 5. This will facilitate an interpretation of a GE-WTP relative to an efficient supply of land.

3 Model

The main problem to be addressed in this paper will be stated in the next subsection. In subsection 3.2 the aggregate and the individual model will be developed for a Cobb-Douglas utility function, with an interpretation in subsection 3.3. Both will be used in the welfare analysis in section 4.

3.1 Problem background

Given a utilitarian social welfare function for a population of identical agents, by means of the sum of the indirect utilities for all agents over all locations, changes in supplied level of a single public good, q, can be valued using an aggregate willingness to pay (WTP) according to a compensation in income that is defined by (see e.g. Haab and McConnell (2002))

$$V\left(Y, P, q\right) = V\left(Y - WTP_{PE}, P, q^{*}\right). \tag{3.1}$$

Following Smith et al. (2004) this WTP is labelled *partial equilibrium*. The *price index* P contains the price levels for land or housing at all locations. If q is assumed to reflect the level of one non-spatial pure public good, the price index would not be affected by a change to level q^{*}. But if $\mathbf{q}$ is the vector of the levels of local public goods (one per location), the effect of capitalisation on the prices for land of changes in elements of q would be captured in

$$V\left(Y, P, \mathbf{q}\right) = V\left(Y - WTP_{GE}, P^{*}, \mathbf{q}^{*}\right). \tag{3.2}$$

The change of the price index for housing to changes in levels of local public goods could be interpreted as the response of the housing market. In a model with supply and demand for housing, these prices are endogenous. Hence, the specification of both supply and demand are necessary.

3.2 Cobb-Douglas case

The social welfare function (3.1) was defined as the sum of individual indirect utility levels. With N individuals and M locations, the number of individuals, n_j, at each location j can be expressed as the fraction times the size of the population, Nx_j. This results in the equivalence of the social welfare function and N times the *average* individual indirect utility level:

$$V \equiv \sum_{j}^{M} n_j v_j = N \sum_{j}^{M} x_j v_j = N\overline{v}. \tag{3.3}$$

Since it was assumed that all agents were identical and if the population is large enough, this average, $\overline{v}$, might be interpreted as the *expected* indirect utility level of the individual. This probabilistic interpretation typically adopted in the discrete choice literature (McFadden, 1984). Based on that literature, in this paper the individual indirect utility will be given by

$$\ln v_j = \ln y - \beta \ln p_j + \gamma \ln q_j + \varepsilon_{ij}. \tag{3.4}$$

Specification (3.4) is inspired by the locational sorting literature in Bayer et al. (2005), especially Timmins (2003). Here, ε is a i.i.d. error function that has a variance that is linearly dependent on μ. The probability of an individual choosing location location j is given by the conditional logit(McFadden, 1984), which is given for (3.4) by

$$x_j = \frac{\exp\left(\ln v_j/\mu\right)}{\sum_{k=1}^{M} \exp\left(\ln v_k/\mu\right)} = \frac{\left(q_j^{\gamma} \big/ p_j^{\beta}\right)^{1/\mu}}{\sum_{k=1}^{M} \left(q_k^{\gamma} \big/ p_k^{\beta}\right)^{1/\mu}}. \tag{3.5}$$

The deterministic part of (3.4) is consistent with a Cobb-Douglas specification for the direct utility:

$$v_j = \frac{y}{p_j^{\beta}} q_j^{\gamma}. \tag{3.6}$$

Expression (3.6) follows from the individual decision problem

$$\max_{s_j, z} u_j(s_j, z) \quad \text{s.t} \quad y = z + p_j s_j, \tag{3.7}$$

with

$$u_j(s_j, z) = \alpha s_j^{\beta} z^{1-\beta} q_j^{\gamma}, \tag{3.8}$$

where $\alpha \equiv \beta^{-\beta}(1-\beta)^{-(1-\beta)}$ for convenience. The individual demand for space at location j is therefore given by

$$s_j = \frac{\beta y}{p_j}. \tag{3.9}$$

Aggregate demand at location j follows from

$$S_j = n_j s_j = (N x_j)\left(\beta \frac{y}{p_j}\right). \tag{3.10}$$

A fixed supply per location that equals A and equating demand and supply (market clearing),

$$S_j = A, \tag{3.11}$$

or

$$(Nx_j)\left(\beta\frac{y}{p_j}\right) = A, \tag{3.12}$$

results in a relation between price and choice probability:

$$p_j = \frac{\beta Y}{A} x_j. \tag{3.13}$$

In appendix A it is shown that (3.13) together with (3.5) yield the following solution:

$$x_j = \frac{q_j^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}. \tag{3.14}$$

Finally, for simplicity, it will be assumed that $\mu \downarrow 0$ and thereby $\varepsilon_{ij} \downarrow 0$. This allows the social welfare function, substituting (3.6), (3.13) and (3.14) in (3.3) to be written as

$$\begin{aligned} N\overline{v} &= N\sum_{j=1}^{M} x_j v_j \\ &= Ny\left(\frac{A}{\beta Y}\right)^{\beta} \sum_{j=1}^{M} x_j \left(\frac{q_j^{\frac{\gamma}{\beta}}}{x_j}\right)^{\beta} \\ &= Y\left(\frac{A}{\beta Y}\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta}}\right)^{\beta} \sum_{j=1}^{M} x_j \\ &= YP^{-\beta}. \end{aligned} \tag{3.15}$$

Here,

$$P \equiv \left(\frac{A}{\beta Y}\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta}}\right)^{-1}. \tag{3.16}$$

Equation (3.15) is suitable for an interpretation in terms of (3.2), but first a broader range of interpretations concerning the various elements in this section will be explored in section 3.3.

3.3 Interpretation

The starting point for the interpretation of the model derived in section 3.2 is the logarithm of the indirect utility, plus random term, in (3.4). This expression is in principle suitable for econometric estimation, as demonstrated in Timmins (2003) and Bayer et al. (2005). The simplification to identical individuals is one step towards a theoretical interpretation, closer to Alonso (1964). The only individual element is the idiosyncratic component ε_{ij} defining an extra preference by individual i for location j, that cannot be related to the local quality level, q_j, by the observer. At the end of section 3.2 this component is assumed to be zero again, implying a population of truly identical agents.

The reason of resorting temporarily to this error term is twofold. First, it is used in the original econometric setting of conditional logit, leading to a choice probability (3.5), that in turn can be interpreted as a population density. The second reason is developed further in Grevers and van der Veen (2005). When relating the density/probability of (3.5) to local demand, as in (3.10), the demand function shows a constant elasticity of substitution (CES) between the various locations. This is a general property of a multinomial logit function, relating logit to product differentiation (Anderson et al., 1992). Therefore, an alternative way of deriving the social welfare function, (3.3), resembles the derivation of demand and supply for a Dixit-Stiglitz sub-utility function of product variety nested in a Cobb-Douglas utility function in Fujita et al. (1999, chapter 4). The utility function at this stage could also be thought of as the direct utility of a representative consumer and is written as

$$U = \alpha \hat{S}^{\beta} Z^{1-\beta}. \tag{3.17}$$

The utility function contains the following CES sub-utility function for *quality-*

adjusted total amount of land:

$$\hat{S} = \left[\sum_{j=1}^{M} \hat{S}_j^{\rho}\right]^{\frac{1}{\rho}} = \left[\sum_{j=1}^{M} \left(q_j^{\frac{\gamma}{\beta}} S_j\right)^{\rho}\right]^{\frac{1}{\rho}}. \tag{3.18}$$

With $\sigma \equiv \frac{1}{1-\rho}$ as the elasticity of substitution in (3.18), it can be shown that for the corresponding elasticity in (3.10): $\sigma = \frac{\beta}{\mu}$. Therefore, if $\mu \downarrow 0$, $\sigma \to \infty$, or $\rho \uparrow 1$.

This implies that if $\varepsilon_{ij} \downarrow 0$, all locations become perfect substitutes. The individuals will nevertheless sort themselves over the various locations, because of the supply constraint, $A_{ij} = A$, at each location. This supply constraint is essentially the same as in the discussion of the Alonso model in Fujita and Thisse (2002, p.82). The role of the amenity level q_j in (3.14) can be thought of as an analogy of the distance to the Central Business District (CBD) in the model by Alonso (1964). Depicted in fig. 1, it is shown how the population in equilibrium is distributed according to a positive relation between amenity level and population density.

Figure 1 about here

Also, related to (3.17), the price index P in (3.15) is a simplified version of the price index in the Dixit-Stiglitz model (Dixit and Stiglitz, 1977). Its main use in this paper, is to illustrate the dependence of social welfare on the number of locations, M. This parameter will be the main reference in the welfare analysis in section 4.

4 Welfare analysis

The basis type of welfare analysis concerns the application of (3.15) according to (3.2). With a change in quality level at location l, the price index—accounting for the price adjustments following a relocation of the individuals—, is given by

$$P^{*}=\left[\frac{A}{\beta Y}\left(q_{l}^{*\frac{\gamma}{\beta}}+\sum_{k\neq l}^{M}q_{k}^{\frac{\gamma}{\beta}}\right)\right]^{-1}. \tag{4.1}$$

The willingness to pay for this change—adjusting for changes in land prices according to (4.1)—is determined by the level of welfare in *spatial equilibrium*. From a public sector economics point of view, it is also important whether this spatial equilibrium reflects an optimal use of land.

Going back to the individual problem (3.8) in subsection 3.2, the distribution of the population in the equilibrium of (3.14) results from a two stage optimisation problem. First, the consumer optimises the amount of land at every location, s_j, relative to the numéraire, z. In the second stage, the optimal location is chosen. Since it was assumed that all individuals are identical, because of $\varepsilon_{ij}\downarrow 0$, the individual second is basically indifferent to the various locations. Stated differently, the real decentralised optimisation is reflected in the first stage. This conforms to the market equilibrium.

In order asses the efficiency of the land use, the market equilibrium will be compared with the optimal solution to the problem of a social planner. First, it will be assumed that the number of locations, M, is fixed. For the benevolent social planner, the problem would consist of

$$\max_{\mathbf{s},\mathbf{z}}\sum_{j=1}^{M}n_{j}u_{j}\left(s_{j},z\right)\quad\text{s.t}\quad Ny=Nz+\sum_{j=1}^{M}p_{j}A, \tag{4.2}$$

This problem is essentially the same as

$$\max_{\hat{S},Z} \hat{S}^{\beta} Z^{1-\beta} \quad \text{s.t} \quad Y = Z + \sum_{j=1}^{M} \left(\frac{p_j}{q_j^{\frac{\gamma}{\beta}}} \right) \left(A q_j^{\frac{\gamma}{\beta}} \right), \tag{4.3}$$

where

$$\hat{S} = \sum_{j=1}^{M} \hat{S}_j = \sum_{j=1}^{M} \left(A q_j^{\frac{\gamma}{\beta}} \right), \tag{4.4}$$

This follows from the fact—as was shown in section 3.3—that assuming $\varepsilon_{ij} \downarrow 0$ implies that the subutility function (3.18) reflects a choice between perfect substitutes (i.e., become linear). Alternatively, it might be interpreted as the social planner simply separates optimising the aggregate Nz from optimising $N(s_j q_j^{\frac{\gamma}{\beta}})$ for all j. Given the supply constraint (3.11), there is no degree of freedom left and the solution to the centralised problem (4.3) is the as the solution to the decentralised problem (see appendix B):

$$x_j = \frac{q_j^{\frac{\gamma}{\beta}}}{\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta}}}. \tag{4.5}$$

However, since the welfare level also depends on the number of locations, M, the question arrises how the welfare level is related to an *optimal* number of locations. In urban economics literature, the city border is usually defined by

$$p_M \geq p_A. \tag{4.6}$$

Condition (4.6) is inspired by the urban economics literature where a similar expression determines the city border, as the rent from agricultural use is assumed to higher beyond it (see e.g. Fujita and Thisse (2002, p.82)). For the land owner, this agricultural rent count as opportunity costs. Condition (4.6) appears in the second stage of the decentralised problem, as

$$\tilde{p}_l \equiv \max\{p_l, p_A\}. \tag{4.7}$$

This rent should be taken into account by the individual, altering (3.4):

$$\ln v_j = \ln y - \beta \ln \left[\max\{p_j, p_A\}\right] + \gamma \ln q_j + \varepsilon_{ij}, \tag{4.8}$$

Inserting (4.8) in (3.5) results in a problem that can be solved numerically. Given the fact, derived above, that the centralised problem is essentially the same as the decentralised problem, taking into account condition (4.7) will lead to an efficient allocation of land. This extends the results in the literature for the Alonso type of framework (Fujita and Thisse, 2002) to locational sorting models, conditional on the number of locations, M, being endogenous. Stated differently, an endogenous number of locations is necessary for deriving a GE-WTP based on two *efficient* spatial equilibria. Establishing a value for a GE-WTP on the current land use implicitly assumes that—in absence of agglomeration externalities—the current total amount of land used is optimal.

Locational sorting models are mainly applied in case of *endogenous amenities* (Timmins, 2003; Bayer et al., 2005). The type model can be illustrated by a simplified version of (3.4):

$$\ln v_j = \ln y - \beta \ln p_j + \gamma \ln q_j + \delta \ln x_j + \varepsilon_{ij}, \tag{4.9}$$

Here, x_j is the local population density, directly affecting the individual level of utility. Depending on the sign of δ it is an agglomeration or a congestion effect. It can be interpreted as an externality in terms of social (or non-market) interaction (Brock and Durlauf, 2003). Taking the third and the fourth term on the righthand side of (4.9) together as $\gamma \ln q_j x_j^\delta$ and inserting $N x_j s_j = A$ for the individual problem the direct utility (3.8) can be rewritten as

$$u_j\left(s_j, z\right) = \alpha s_j^\beta z^{1-\beta} q_j^\gamma(s_j). \tag{4.10}$$

Now a difference will appear between the market equilibrium and the social optimum. For the individual the amenity level, $q_j(s_j)$, will remain exogenous, not affecting her optimisation problem. This is by definition the case, since it concerns an externality. The social planner, however, could optimise the individual amount of land, also taking into account this externality. The optimisation problem (3.7) will yield

$$\begin{aligned}\frac{1}{\alpha}\frac{\partial u_j}{\partial s_j} &= \beta y - p_j s_j + \gamma s_j\left(y - p_j s_j\right)\frac{1}{q_j\left(s_j\right)}\frac{dq\left(s_j\right)}{ds_j} \\ &= \beta y - p_j s_j - e_j(s_j) = 0. \end{aligned} \tag{4.11}$$

Here, the sign of e is based on a positive externality ($\delta > 0$ in (4.9)). Because $\frac{dq(x_j)}{dx_j} > 0$, it follows that $\frac{dq(s_j)}{ds_j} < 0$. The individual demand that would correspond to the optimisation by a social planner will in case of a positive externality therefore be smaller than in the original problem without externalities (keeping the prices at the level of the original problem):

$$s_j = \frac{\beta y - e_j(s_j)}{p_j}. \tag{4.12}$$

Denoting the solution to (4.12) by s_j^*, keeping the supply of land per location at A, from (3.13) it follows that the optimal price p_j^* will be higher than the market price, because the local density will be higher. For a fixed population size, N, and allowing for the number of locations, M, to be endogenous—as above—, higher densities imply a lower M. This corresponds to the general notion that if positive externalities are internalised in the price of land an efficient allocation of land will yield a smaller agglomeration than in market equilibrium (Fujita and Thisse, 2002, p. 179-182).

5 Conclusions

This paper demonstrates how so-called locational sorting models can be interpreted in terms of land use models in traditional urban economics. Locational sorting models are recently introduced location choice models that help extending hedonic pricing methods for valuation of the benefits from non-marginal changes in local environmental quality. By adding the notion of total amount of consumed land from urban economics, quality and quantity aspects of land use are be combined in one consistent welfare measure.

For models that allow for agglomeration externalities—or *social interactions*—a distinction arises between the socially optimal and the market equilibrium. In the presence of externalities, the market allocation generally results in an oversupply of land.

Allowing for an endogenous total land consumption in locational sorting models will have direct the welfare implications, thereby affecting the measure of a WTP for non-marginal changes in local public good levels. An sorting model, as developed in this paper, that also reflects the optimality of the amount of land used in equilibrium will simultaneously address two aspects of a socially optimal allocation of land public policy is confronted with:

1. securing optimal allocation of land by markets,
2. supplying local public goods.

Given the original econometric context wherein locational sorting model were developed, an empirical implementation of the concepts developed here are expected to be feasible and an exciting route towards further applied economic land use research.

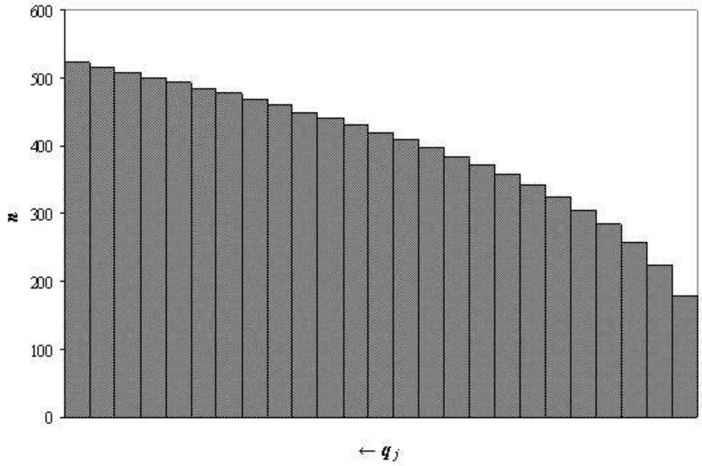

Figure 1: Example of a population distribution with $N = 10000$ cf. (3.4) with $\beta = 0.5$, $\gamma = 0.2$ and $\mu = 0$ (q decreases stepwise from 2.5 in steps of 0.1).

A Appendix

In this appendix, the analytical solution for the population frequency distribution of (3.14) will be derived.

$$x_j = \frac{\left(q_j^\gamma \big/ p_j^\beta\right)^{1/\mu}}{\sum_{k=1}^M \left(q_k^\gamma \big/ p_k^\beta\right)^{1/\mu}}. \tag{A.1}$$

Substituting (3.13):

$$x_j = \frac{\left(q_j^\gamma \big/ x_j^\beta\right)^{1/\mu}}{\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}}. \tag{A.2}$$

Reordering yields

$$x_j^{1+\beta/\mu} = \frac{q_j^{\gamma/\mu}}{\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}}, \tag{A.3}$$

or

$$x_j = \frac{q_j^{\gamma/(\beta+\mu)}}{\left[\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}\right]^{\mu/(\beta+\mu)}}. \tag{A.4}$$

Finally, using the identity

$$\sum_{j=1}^M x_j = 1, \tag{A.5}$$

allows the denominator of (A.4) to be written as

$$\left[\sum_{k=1}^M \left(q_k^\gamma \big/ x_k^\beta\right)^{1/\mu}\right]^{\mu/(\beta+\mu)} = \sum_{k=1}^M q_k^{\gamma/(\beta+\mu)}. \tag{A.6}$$

Substituting back in (A.4) solves for the population frequency:

$$x_j = \frac{q_j^{\gamma/(\beta+\mu)}}{\sum_{k=1}^{M} q_k^{\gamma/(\beta+\mu)}}. \tag{A.7}$$

B Appendix

In section 4 it was stated that the maximisation problem for the benevolent social planner,

$$\max_{\mathbf{s},\mathbf{z},M} \sum_{j=1}^{M} n_j u_j \left(s_j, z\right) \quad \text{s.t} \quad Ny = Nz + \sum_{j=1}^{M} p_j A, \tag{B.1}$$

is essentially the same as

$$\max_{\hat{S},Z} \hat{S}^{\beta} Z^{1-\beta} \quad \text{s.t} \quad Y = Z + \sum_{j=1}^{M} \left(\frac{p_j}{q_j^{\frac{\gamma}{\beta}}}\right) \left(A q_j^{\frac{\gamma}{\beta}}\right), \tag{B.2}$$

where

$$\hat{S} = \sum_{j=1}^{M} \hat{S}_j = \sum_{j=1}^{M} \left(A q_j^{\frac{\gamma}{\beta}}\right). \tag{B.3}$$

The equivalence follows from lacking a degree of freedom in maximising $\hat{S}$ because of the supply constraint per location. Therefore,

$$\begin{aligned} \sum_{j=1}^{M} n_j u_j &= N \sum_{j=1}^{M} x_j u_j \\ &= N \sum_{j=1}^{M} s_j^{\beta} z^{1-\beta} q_j^{\gamma} \\ &= N z^{1-\beta} \left(\frac{A}{N}\right)^{\beta} \sum_{j=1}^{M} x_j^{1-\beta} q_j^{\gamma} \\ &= Z^{1-\beta} A^{\beta} \sum_{j=1}^{M} x_j^{1-\beta} q_j^{\gamma}. \end{aligned} \tag{B.4}$$

Next, from

$$A^{\beta} \sum_{j=1}^{M} x_j^{1-\beta} \left(q_j^{\frac{\gamma}{\beta}}\right)^{\beta} = \left(\sum_{j=1}^{M} A q_j^{\frac{\gamma}{\beta}}\right)^{\beta}, \tag{B.5}$$

it follows that

$$\sum_{j=1}^{M} x_j^{1-\beta} \frac{\left(q_j^{\frac{\gamma}{\beta}}\right)^{\beta}}{\left(\sum_{j=1}^{M} q_j^{\frac{\gamma}{\beta}}\right)^{\beta}} = 1. \tag{B.6}$$

And finally, given $\sum_{j=1}^{M} x_j = 1$ the solution is given by

$$x_j = \frac{q_j^{\frac{\gamma}{\beta}}}{\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta}}}. \tag{B.7}$$

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