The IMAGE CGE Model: Understanding the Model Structure, Code and Solution Methods

Ronnie O'Toole
Trinity College Dublin.

Alan Matthews
Trinity College Dublin.

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Abstract

This working paper details the structure, code and solution methods for IMA*GE*, which is an acronym of "Irish Model of Agriculture, General Equilibrium". The IMAGE model is based on the widely known ORANI model (Dixon et al. 1982) of the Australian economy. The model has a theoretical structure that is typical of many CGE (Computable General Equilibrium) models. It is a static model, as it does not have any mechanism for the accumulation of capital. It is based entirely on the assumption of perfect competition, with no individual buyer or seller being able to influence price. Demand and supply equations are derived from the solution of optimisation problems (e.g. profit or utility maximization) for private sector agents. The model allows for multiple household types, export destinations, land types and labour occupations.

O'Toole: rpotoole@tcd.ie; Matthews: Alan.Matthews@tcd.ie. Model website: http://www.economics.tcd.ie/image.html

1 Introduction

This working paper details the construction, calibration and interpretation of IMA*GE*, which is an acronym of “Irish Model of Agriculture, General Equilibrium”. The IMAGE model is based on the widely known ORANI model (Dixon et al. 1982) of the Australian economy developed as part of the IMPACT project and which has been used extensively for policy analysis in Australia for nearly two decades. Adaptations of ORANI already exist for several countries, including Thailand, South Africa, Pakistan, Brazil, the Philippines, Japan, Indonesia, Venezuela, Taiwan and Denmark. The model also owes much to MONASH (Dixon and Rimmer, 2000), the dynamic successor to ORANI.

The model has a theoretical structure that is typical of many AGE (applied general equilibrium) models. It is a static model, as it does not have any mechanism for the accumulation of capital. It is based entirely on the assumption of perfect competition, with no individual buyer or seller being able to influence price. Demand and supply equations are derived from the solution of optimisation problems (e.g. profit or utility maximization) for private sector agents. The model allows for multiple household types, export destinations, land types and labour occupations. It also incorporates an explicit treatment of government revenue and expenditure. The core model consists of equations describing:

- Producers’ demands for produced inputs and primary factors;
- Producers’ supplies of commodities;
- Demands for inputs to capital formation;
- Household demands;
- Export demands;
- Government demands;
- The relationship of basic values to production costs and to purchasers’ prices;
- Market-clearing conditions for commodities and primary factors;
- Macroeconomic variables and price indices.

The discussion in this working paper is constructed on the basis of a chronological sequence of how a model might be constructed. An explanation as to how the model is implemented in the Tablo code is presented in Section 2. The functional forms, output linkages and price linkages underlying the model are discussed in section 3. The next step is to determine a closure for the model, and this is discussed in section 4. Section 5 discusses the approach used to solve the model, while section 6 concludes. The collection of the data required to solve the model is discussed in another working paper entitled *“The IMAGE CGE Model: Constructing the Base 1993 database”*. The full TABLO code of the model is available to download from www.economics.tcd.ie/image.html.

2 The Equation System

The model is implemented using the GEMPACK software (Harrison and Pearson, 1998). The equations file in GEMPACK is a so-called TABLO file which at first glance seems quite complex, though this not the case, as the actual ‘language’ is conventional algebra. Any difficulty lies in the (necessary) inclusion of a large number of coefficients, variables, formulae and equations etc. to adequately model a complex economic structure.

2.1 The TABLO Language

Care must be taken in interpreting the wording of TABLO, in that particular words have slightly more specific usage than in every day language. We will now briefly describe the important elements of the TABLO syntax.

Set

A collection of economic objects – e.g. commodities, industries, household types. In the line below we define a SET *commodities* to be called *COM*, specify that they are to be named C1, C2, C3, … , C34, and (as a useful comment) showed at the end of the line that the subscript *c* will be reserved for this set.

```
Set   COM         # Commodities #           (C1 - C34);         ! c !
```

All sets bar two are defined at the beginning of the program. The exceptions are only used in one short part of the program, so they are defined later on when they are needed rather than over-cluttering the opening section.

It is also possible that a SET is to be partitioned for some reason through the definition of (say) two new sets. To ensure that the program recognises that applying these two new SETs is equivalent to applying the original SET, we specify that they are subsets. For example, in the model the SET of industries is subdivided into agriculture and non-agriculture so as to calculate results such as Gross Agricultural Output. To ensure that the program recognises where these new SETs originate, we include the *"is subset of"* statement as below, where IND is defined as the set of industries in the model.

```
AGIND       is subset of IND;
NONAGIND    is subset of IND;
```

Variable

A variable in the mathematical sense is called one of two names in the ORANI coding convention, namely a COEFFICIENT (which is discussed below) or a VARIABLE. The word VARIABLE is retained for an economic variable that occurs in one or more equations. By convention, variables are written in upper case when they are in levels and in lower-case when in percentage-change form, with the distinction being made solely to ease interpretation. While typically a VARIABLE will be represented as a percentage-change, this can be altered. This is most often done when the denominator in the calculation of a percentage-change variable could realistically take the value of zero. For example, if the Balance of Trade happened by chance to be zero, then any percentage-change in this variable would be indeterminate, though an ordinary (Irish £) change would be perfectly admissible. Below we define the variable *x1(c,s,i)*, the percentage change in demands for intermediate commodities at basic prices, where SRC is defined as the set of origins of products to be used.

```
Variable
(All,c,COM)(All,s,SRC)(All,i,IND)  x1(c,s,i)  # Intermediate demand for commodities #
```

Coefficient

This is the current value of a levels variable. It can appear as the coefficient of a variable in an equation, a value of base data as read in from a data file, or some value derived from base data. The example below represents the second case where V1BAS, the flows of intermediate inputs at basic prices, is defined. COEFFICIENTs can either be parameter (constant throughout a simulation) or non-parameter (which is the default setting). There is no programming reason as such as to why a COEFFICIENT command is required. It is included in the code more as a presentational device to ease interpretation.

```
Coefficient  ! Basic Flows of Commodities!
(All,c,COM)(All,s,SRC)(All,i,IND)  V1BAS(c,s,i)
#Intermediate#;
```

Formula

The word FORMULA is reserved for those operations that involve the manipulation of one or more coefficients. To ease legibility, a FORMULA that calculates a coefficient frequently appears directly after the definition of the coefficient. In the case shown below, aggregate household consumption of each commodity by source is first of all ‘introduced’ by means of a COEFFICIENT statement, and then calculated by means of a FORMULA. If the qualifier ‘initial’ is used, then the FORMULA is only calculated during the first step of a multi-step simulation.

```
Coefficient
(All,c,COM)(All,s,SRC) V3BAS_H(c,s)# Households:Agg #;
Formula
(All,c,COM)(All,s,SRC) V3BAS_H(c,s) = Sum(h,HOU, V3BAS( c,s,h));
```

Update

An algebraic specification of how a given coefficient is to be updated after each step of a multi-step simulation. The example below shows a nice illustration of

COEFFICIENT, FORMULA and UPDATE. First, the domestic base price is defined for each good, which is initially set at 1 by use of a formula. Note that FORMULA is qualified by 'initial', otherwise the program would reset prices at 1 at the beginning of every step of the calculation. Finally, to specify how the domestic price is to change, we include an UPDATE statement.

Note the unusual syntax of the UPDATE statement. On the left hand side we have *P0DOM*, a levels coefficient, while on the right hand side we have *p0*, a percentage-change variable. Therefore the rule for update statements is to read the '=' as a 'by increasing it by'. So in the example below, it should be read as 'Update the levels coefficient *P0DOM* by increasing it by *p0*'.

```
Coefficient (All,c,COM) P0DOM(c) # levels domestic basic prices #;
Formula (Initial) (All,c,COM) P0DOM(c) = 1; !arbitrary initial
setting!

Update   (All,c,COM) P0DOM(c) = p0(c,"dom");
```

Equation

An algebraic specification of economic behaviour using both coefficients and variables. Note the name of the equation *E_x1lab*, given the purpose of the equation is to calculate *x1ab*. The use of this naming convention is used later on in TABmate to allow the program to 'suggest' a possible closure of the model. It does this by assuming that any VARIABLE that has an EQUATION named after it will be assumed endogenous.

```
Equation E_x1lab  # Demand for labour by industry and skill group #
(All,i,IND)(All,o,OCC)
x1lab(i,o) = x1lab_o(i) - SIGMA1LAB(i)*[p1lab(i,o) - p1lab_o(i)];
```

File, Read, Write & Display

The TABLO file contains the model relationships and interconnections, but must draw on external files to get much of the raw data, and ensure that all of this data is given an appropriate name within the model. FILE is the command used for naming files that contain data that will be used by the program. Below, the principle data set (named MDATA) is specified.

```
File  MDATA  # Data File #;
```

This file contains numerous vectors and matrices representing the different requirements of the model. For example, the vector with header (name) of 1ARM is the vector of Armington elasticities used in the bottom ‘source’ nest for intermediate production. Below the READ command specifies that the header “1BAS” will be known as the coefficient V1BAS in the model equations and formulae.

```
Read
 V1BAS From File MDATA Header "1BAS";
```

Finally, the WRITE command sends coefficient data to a specified data file, and has an identical syntax as shown for the READ command above.

2.2 Variables, Coefficients and Parameters of the model

While an effort has been made in constructing variable names to keep as close to the original ORANI naming scheme as possible, the fact that the listed equations include both levels and change variables results in a need to broaden the classification.

The adopted nomenclature will help the reader in understanding many of the equations. As far as possible, names for variables and coefficients conform to a system in which each name consists of 2 or more parts, as follows:

First, a letter indicating the type of variable:

a	technical change
Ä	ordinary (rather than percentage) change
f	shift variable
H	Indexing Parameter

p	price (IR£)
p$	price (foreign currency)
S	input Share
ó	elasticity of substitution
t	tax
V	Levels Value, IR£
w	% change value, IR£
x	input quantity

Secondly, one of the digits 0 to 6 indicating user:

1	current production
2	investment
3	consumption
4	export
5	government
6	inventories
0	all users, where user distinction is irrelevant

Thirdly, a variable might also have three or more letters if needed:

BAS	At basic prices
CAP	capital
CIF	imports (border prices)
IMP	imports (duty paid)
LAB	labour
LND	Land
LUX	Supernumerary part of consumption in LES
PRIM	all primary factors
PUR	At Purchasers' prices
SUB	Subsistence part of consumption in LES
TAR	Tariffs

Tax	indirect taxes
Tot	total or average over all inputs for some user

Finally, some variables will have an underscore (such as with VARIABLE_i) indicating in this case that VARIABLE has been summed over the set of industries.

2.3 Coding Conventions

Before explaining the various 'functions' contained in TABLO, it might be useful to specify some house keeping rules which will be obeyed when constructing the model.

Firstly, all names must be unique. For example, no two variables can be given the same name, and no equation can be given the same name as (say) a variable. TABLO does not differentiate between upper and lower case, so the variable *X_S(i)* and *x_s(i)* are indistinguishable as far as the program is concerned. For example, in Excerpt 21 (partially reproduced below), the variable *x3* appears in the TABLO file, which represents the rate of change of demand by each household type for each commodity by source. In reproducing the levels equation, *X3* is used to denote the level of demand by each household type for each commodity by source.

This results in commodity demands of:

$$X3(c,s,h) = A3(c,s) * X3\_S(c,h) * \left(A3(c,s) * \frac{P3(c,s)}{P3\_S(c)} \right)^{s_3(c)} \qquad (1)$$

which are written in TABLO form as:

```
! Excerpt 21 - 23 of TABLO input file: !
Coefficient (all,c,COM) SIGMA3(c) # Armington elasticities:
households #;
Read SIGMA3 from file H2DATA header "3ARM";

Equation E_x3  # Source-specific commodity demands #
```

```
(all,c,COM)(all,s,SRC)(all,h,HOU)
x3(c,s,h)-a3(c,s,h) = x3_s(c,h) - SIGMA3(c)*[
p3(c,s,h)+a3(c,s,h) - p3_s(c,h)];

Equation E_p3_s  # Effective price of commodity composite #
(all,c,COM)(all,h,HOU) p3_s(c,h) = sum{s,SRC,
S3(c,s,h)*[p3(c,s,h)+a3(c,s,h)]};
```

Secondly, as the TABLO file is quite long, we will retain the ORANI convention of breaking the code into numerous excerpts for ease of reference. Each excerpt is around 20 lines long, and will generally have a fairly well defined purpose.

Thirdly, as with any computer program, it is always useful to include detailed comments to aid the de-bugging process and ensure that others can easily interpret your work. Comments can be included in exclamation marks in TABLO. Alternatively, additional descriptions can be put in hashes. These are very similar to comments, though are taken 'more seriously' by TABLO. For example, if you use hashes to give a longer, more detailed name to a variable, then this longer name will be reproduced in the data files.

```
! A comment which will be completely ignored by the program !
# Usually used to give more information and will travel between various GEMPACK files #
```

3 Structure of the Model

3.1 Production

The model allows every industry to produce several commodities by using domestic or imported intermediates and a primary factor composite consisting of land, labour and capital. This would suggest a potentially very large and complex system that would be extremely difficult to calibrate. To keep the model to a manageable size, we assume, firstly, that each industry only produces one good and secondly, that input-output separability holds, which means the generalised production function for some industry:

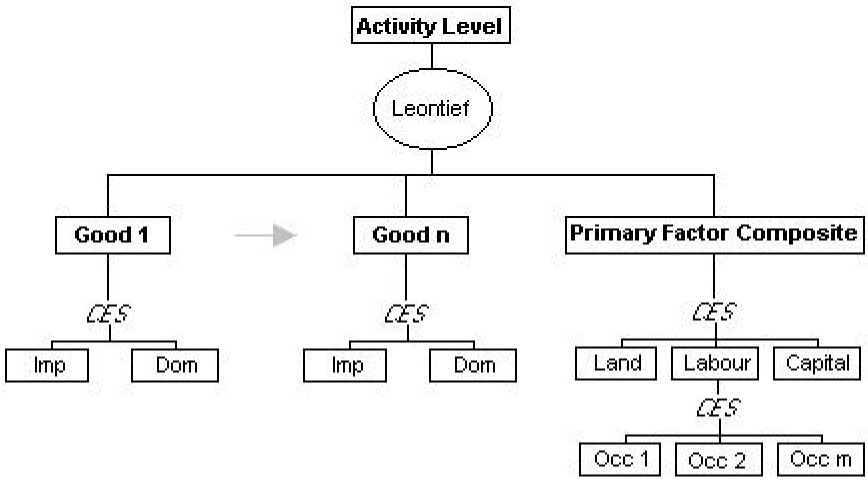
$$H(inputs) = G(outputs) \quad (2)$$

may be written as:

$$H(inputs) = Z = G(outputs) \quad (3)$$

where Z is an index of industry activity. The resulting structure for the H function is represented by Figure 1 below.

Figure 1
Form of Industrial Production



At the top level of the nest the volume employed of each of the n intermediate inputs and the primary factor composite by each firm is assumed to be in a constant proportion. Further, each of the intermediate goods is the product of a hypothesised 'mixing' industry which combines the (non-homothetic by the Armington assumption) imports and domestic production of good i. Each of these mixing industries are characterised by a CES function. The primary factor composite is formed through a combination of land, labour and capital, with labour itself being formed by a composite composed of a number of occupations.

3.1.1.1 Demand for Primary Factors

Demand for Labour by Occupation

As can be seen from Figure 1, labour employment in an industry is a CES composite of the various occupation types, namely agricultural workers, producers, labourers, transport/communication, clerical, finance, service, professional/technical and others. Labour demands for each industry are derived from the following optimisation problem:

Choose inputs of labour by profession:

$$X1LAB(i,o)$$

To minimise total labour cost:

$$\sum_{o}^{OCC} P1LAB(i,o) * X1LAB(i,o) \qquad (4)$$

This results in demands expressed in levels of:

$$X1LAB(i,o) = X1LAB\_O * \left(\frac{P1LAB(i,o)}{P1LAB\_O(i)} \right)^{s_{1LAB(i)}} \qquad (5)$$

The solution of this problem in percentage change form is shown in Excerpt 15. As can be seen from either representation, labour demand for each occupation and industry group is proportional to the total demand for labour in that industry. Changes in the relative price of the occupations induce substitution in favour of the cheaper occupations. The responsiveness of occupational employment in each industry depends on the magnitude of ó$_{1lab(i)}$ for each industry.

The second part of Excerpt 15 as shown in appendix 1 shows the calculation of the industry *i* labour price index. This is calculated in terms of percentage changes, and is a weighted average of the change in the price of each occupation employed in industry *i*, weighted by labour flows to each occupation. Take for example the situation whereby the cost of semi-skilled workers increased by 5%, and this occupation accounts for 50% of the wage bill of (say) the construction industry. In this case, the labour price index for the construction industry will rise by 2.5%, assuming that the cost of each of the other occupations remains unchanged.

A similar exercise calculates the optimal use of land of each class type assuming a CES function. The responsiveness of land use depends on the magnitude of $\sigma_{1lnd(i)}$ for each industry. The solution is:

$$X1LND(i,l) = X1LND\_L(i) * \left(\frac{P1LND(i,l)}{P1LND\_L(i)} \right)^{s_{1LND(i)}} \quad (6)$$

Demand for Primary Factor Composites

The next level up (as shown in figure 1) shows how the three factors of production (land, capital and labour) are combined to form the primary factor composite. While in other sections we have not included the technology/taste shift variables to ensure legibility, in this section they are shown by way of example.

The problem is to choose inputs of each primary factor:

$$X1LAB\_O(i) \qquad X1LND\_L(i) \qquad X1CAP(i)$$

to minimise total cost. This results in demands expressed in levels of:

$$\frac{X1LAB\_O(i)}{A1LAB\_O(i)} = X1PRIM(i) * \left(A1LAB\_O(i) * \frac{P1LAB\_O(i)}{P1PRIM(i)} \right)^{-s_{1PRIM}(i)} \quad (7)$$

$$\frac{X1LND\_L(i)}{A1LND\_L(i)} = X1PRIM(i) * \left(A1LND\_L(i) * \frac{P1LND\_L(i)}{P1PRIM(i)} \right)^{-s_{1PRIM}(i)} \quad (8)$$

$$\frac{X1CAP(i)}{A1CAP(i)} = X1PRIM(i) * \left(A1CAP(i) * \frac{P1CAP(i)}{P1PRIM(i)} \right)^{-s_{1PRIM}(i)} \qquad (9)$$

As can be seen, a number of technical change variables are incorporated, variables that are almost always set exogenously, and which allow an easy mechanism for incorporating a taste or technology shock. Excerpt 16 shows the equivalent percentage-change form as inputted into TABLO, which shows that demand for each factor is proportional to overall factor demand and a price term, assuming that the technical change variables are set exogenously at zero.

3.1.1.2 Demand for Intermediate Inputs

The level of demand for intermediate products is divided into two levels. The Armington (1969, 1970) assumption that imports are imperfect substitutes for domestic supplies is employed. In terms of the TABLO file, this requires the modeling of a hypothetical mixing industry (with a CES technology) that chooses combinations of:

$$X1(c,\text{"}domestic\text{"},i) \quad \text{and} \quad X1(c,\text{"}imported\text{"},i)$$

which are the domestic and imported varieties of the product, so as to minimise total costs. The resulting demand equations in levels are as follows:

$$X1(c,s,i) = X1\_S(c,i) * \left(\frac{P1(c,s)}{P1\_S(c)} \right)^{-s_1(i)} \qquad (10)$$

The interpretation is that (assuming no change in the taste/technology variables) the amount of each commodity demanded by industry *i* from, (say), domestic suppliers, for use in further processing is proportional to the amount of that commodity demanded by industry *i* and a price term. Again note how *X1* is appended by a *'_S'* to indicate that this is X1 summed over the various sources of the commodity.

The output of each of the hypothetical mixing industries is then combined in Leontief (fixed share) proportions with the primary factor aggregate, in what is the last stage of production. The Leontief production function is equivalent to a CES production function with the elasticity of substitution set at zero. The demand equations derived from the Leontief are as follows (including shift variables):

$$X1\_S(c,i) = A1PRIM(i) * A1TOT(i) * X1TOT(i) \qquad (11)$$

$$X1PRIM(i) = A1PRIM(i) * A1TOT(i) * X1TOT(i) \qquad (12)$$

Again, assuming that the taste/technology parameters are set exogenously, this implies that the amount of *X1PRIM* and each of *X1_S(c)* used depends only upon the level of industrial 'activity' in the industry in question, and not on the relative prices of these inputs. Note the *OCT* ("Other Cost Tickets") variables and equations that were used in the ORANI model for other costs, and is used in the IMAGE model as a mechanism to ensure the equality of supply and demand in the base data.

The final equation in Excerpt 18 (equation *E_p1tot*) incorporates the zero pure profits in production condition. Ignoring the taste/technology variables and the *OCT* component, it can be seen that the left hand side is the (change) in output price while the right hand side is a weighted average of input prices. Under the assumption of competitive pricing and a constant returns to scale technology, these must be equal. Again note the logic behind the naming of the zero pure profit equation. Finally, the model allows the facility for one industry to produce a number of commodities, though this is not utilised in the current version of the model.

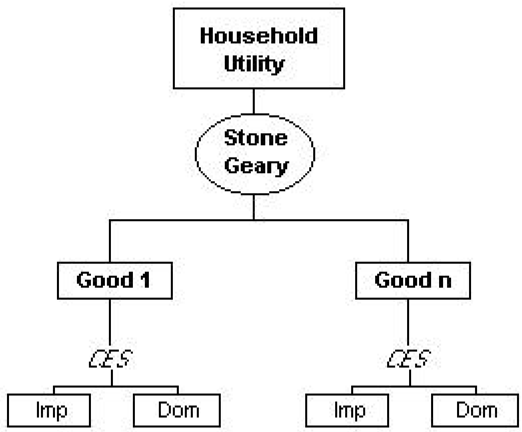
3.1.2 Household Demand

As regards household demand, the first decision is the choice of functional form to be used. Perhaps the most convenient would be demand functions derived by the maximisation of a Cobb-Douglas utility function. This results in demand functions with constant expenditure elasticities, unitary own-price elasticities and zero cross price elasticities. The alternative is to derive demands from a CES (Constant

Elasticity of Substitution) utility function, of which the Cobb-Douglas is a special case. The unitary substitution elasticity in the latter can now be replaced by any positive constant.

The third option, which is used, is to base household consumption on a Stone-Geary utility function, which will lead to a so-called LES (linear expenditure system). Household consumption is divided into two components, a minimum or subsistence amount and a luxury or supernumerary amount. This displacement of the origin allows a simple functional form for supernumerary demand, while also ensuring that the empirically observed changes in consumption patterns as income rises are not violated. Figure 2 shows the representation of the nesting structure for household demand.

Figure 2
Form of Household Demand



As can be seen from the diagram, at the lower nest, the equations are similar to the corresponding equations for intermediate and investment demands. Each of the three households (namely urban, farm and rural non-farm), *h*, chooses combinations of:

$$X3(c,"domestic",h) \quad \text{and} \quad X3(c,"imported",h)$$

to minimise a CES function. Ignoring the household subscript, this results in commodity demands of:

$$X3(c,s) = X3\_S(c) * \left(\frac{P3(c,s)}{P3\_S(c)} \right)^{-s_3(c)} \qquad (13)$$

The main nest in household demand is developed in Excerpts 22 and 23, the first of which reads in the data and calculates necessary coefficients, while the latter ‘does the work’ of calculating the commodity composition of household demand.

The demands are derived from the following (Stone-Geary) utility (per-household) function:

$$U(q) = \prod (X3\_S(c) - X3SUB(c))^{s} \qquad (14)$$

X3SUB(c) has the interpretation of a subsistence amount of consumption. The demand equations that arise from this utility function are:

$$X3\_S(c) = X3SUB(c) + S3LUX(c) * \frac{V3LUX\_C}{P3\_S(c)} \qquad (15)$$

Where:

$$V3LUX\_C = V3TOT - \sum X3SUB(c) * P3\_S(c) \qquad (16)$$

In other words, *V3LUX_C* is the (scalar) amount of income that is left over when all of the subsistence requirements have been purchased. The remaining income is divided into a share *S3LUX(c)* of total income to give *X3LUX(c).* This part is ‘luxury’ or ‘supernumerary’ expenditure. An additional feature of Excerpt 23 is the use of the coefficient *TINY* which neatly avoids uniqueness problems associated with the other element of equation *E_x3_h*, namely *V3BAS_H*, being equal to zero.

3.1.3 Investment Demand

Figure 3 shows the schematic representation of the production structure for units of fixed capital. As can be seen, the structure is very similar to that governing the production of goods, though no primary factors are used directly in the production of fixed capital. The discussion of the investment demand equations follows closely that of intermediate production. At the bottom level investors choose combinations of:

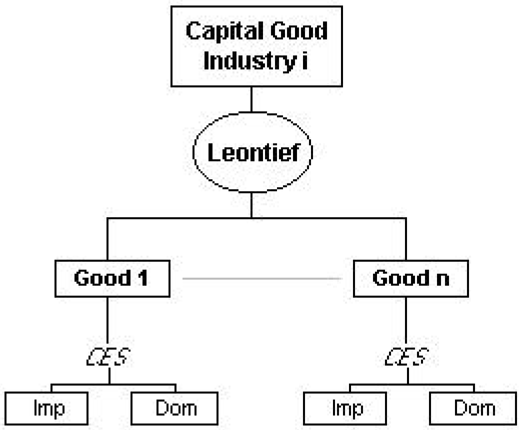
$$X2(c,"domestic",i) \quad \text{and} \quad X2(c,"imported",i)$$

the domestic and imported varieties of the product, so as to minimise total costs. The resulting demand equations in levels are as follows:

$$X2(c,s,i) = X2\_S(c,i) * \left(\frac{P2(c,s,i)}{P2\_S(c,i)} \right)^{-s_2(i)} \tag{17}$$

Figure 3

Form of Investment Demand



At the top level a Leontief technology is assumed, with each of the goods being used in fixed proportions. Hence *X2_S(i)* is directly proportional to *X2(c,s,i),* with no allowance for any substitution even if the relative price of inputs alters.

The final equation in Excerpt 20 (equation *E_p2tot*) incorporates the condition of zero pure profits in the production of fixed capital. The left hand side is the (change) in the total amount spent on capital goods, while the right hand side is a weighted average of input costs. Again, assuming competitive pricing and a constant returns to scale technology, these must be equal.

3.4 Treatment of Margin Commodities

Excerpt 25 contains all the equations specifying the margin demands[1] of the various categories of user.

One of the main problems with the published input-output tables as they stand is that margin commodities are treated as any other industry is treated. So, for example, wholesale/ retail margins are used as an intermediate input by other industries and are 'demanded' by consumers directly from the industry. Ideally we would like to be able to assign a margin flow to every underlying 'real' flow in the economy. To what extent is it a problem that we do not have this data, and to what lengths should we go to alleviate it?

By way of example, consider the impact of a doubling of oil prices due to the imposition of a green tax, the proceeds of which are then used to reduce tax on other consumer goods. Assume that this will reduce the quantity of oil demanded by the consumer by around 30% and that the quantity demanded of all other goods increases by around 5%. We will then have a rise in demand for margin commodities of 5%. It should be obvious that this margins figure takes no account of the reduced need for margin commodities because of the fall in use of oil products. In fact, the consumer is to a degree substituting away from using oil in favour of using the product 'transfer of oil', which is cheaper. This treatment of margins as substitutes rather than complements to the consumption of goods is clearly erroneous.

The problem for intermediate demand for oil is far less acute. If oil prices double, the model will predict that the oil industry will contract, releasing spare capacity to

[1] In IMA*GE* Transport and Trade Margins are generally assumed to be the margin commodities.

other sectors of the economy. As well as using (say) 10% less capital, 10% less labour, 10% less electricity etc., the oil industry will also use 10% less margin commodities. The fact that this margin flow is not directly linked to an underlying commodity flow is largely irrelevant.

It will only become relevant if, instead of shocking some commodity, or some industry, we change the underlying technology structure somehow. For example, if we wanted to implement a technologically driven decrease of oil use by farmers, then by changing the appropriate input-output coefficient, we have left total margin usage unchanged. So despite a reduction in oil demands in the economy, the exact same amount of resources is needed to transfer this reduced bulk from seller to buyer. Such types of shocks are relatively rare in CGE analyses and do not pose a problem. If a technological shock such as described above is to be implemented, a change in margin use will have to be calculated externally to the model. Therefore, a compromise can be reached. Where substitution possibilities are limited (e.g. Leontief technology), the treatment of margin commodities as a separate industry is adequate. Where substitution possibilities are significant (e.g. LES) this approach is inadequate. There seem to be significant advantages in allocating margin flows to their corresponding final demand flow, but no significant advantage in implementing a similar set of flows for intermediate production. In conclusion, we ignore margin commodities for intermediate flows and just incorporate them in the inter-industry structure, though we allocate margin commodities appropriately for final demand flows.

3.5 Exports

The export demand curve faced by Irish producers is assumed to be very slightly downward sloping. As can be seen from equation 18, the aggregate quantity of exports of commodity c (ignoring the quantity demand curve shifter *F4Q(c)* and the price demand curve shifter *F4P(c)*) is proportional to the export price divided by the exchange rate. The typical value chosen for the export elasticity is –20.

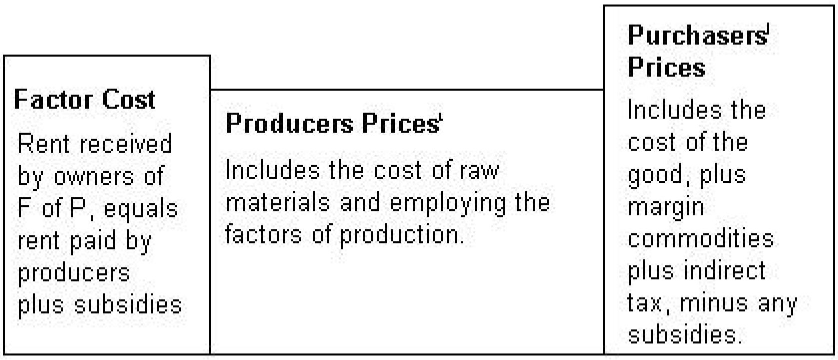
$$X4(c) = F4Q(c) * \left(\frac{P4(c)}{F4P(c) * PHI} \right)^{e} \quad (18)$$

3.6 The Price System

The price linkage system specifies the bridge between input and output prices, with the latter being formed as a function of the former, the use of margin commodities in getting the goods to market, indirect taxes and subsidies. Given that the model incorporates subsidies to factors of production, the system is somewhat more complex than typical CGE models. Its structure is represented in figure 4.

Figure 4:

The Price Linkages in the Model



As we can see from figure 4, there are three 'levels' of prices. When we talk of basic prices we mean the producers' prices represented by the middle block in figure 4. This consists of the cost of raw materials plus the cost of the (subsidised) factors or production. The purchasers' prices are represented in the right hand block, and consist of the basic cost of the good plus the cost of margin commodities in getting the goods to market plus the cost of indirect taxes minus any subsidies for selling to final demand markets. Finally, the rent received by the owners of factors of production is represented in the left hand block as the amount paid by users of the factors of production plus any subsidies. The price linkages in the commodity markets and factor markets are now dealt with in turn.

3.6.1 Zero Pure Profits in the Commodity Markets

This section deals with the relationship between the middle block in figure 4 and the right hand block. In effect, each equation says that output price must equal input prices to ensure zero pure profit. Excerpt 26 defines purchasers' prices for each of the five user groups. These are essentially zero-profit equations (as they are actually called for exporting and other) as the price a purchaser pays must equal the various costs associated with taking delivery of the goods. The purchasers' price is therefore equal to the basic price plus any sales tax due plus margin costs. Note again the use of the TINY variable to ensure that the right hand side variable is never indeterminate.

The first equation, *E_p1*, represents the main production price and says that the price received by producers is less than the price paid by purchasers' due to margins and tax. The power of the *ad valorem* tax is equal to the ratio of the tax and the market price, and the rate of change of the *ad valorem* tax is *t1*, which from excerpt 28 we can see can be shifted by:

- a rate independent of the market[2] which the good was being sold (*f1tax_csi*);
- a commodity specific, market independent rate (*f0tax_s(c)*).

An example of a use of the former might be if we wish to model the government raising VAT revenue by way of compensation for a loss of revenue/increase in expenditure elsewhere in the economy. We might use this shifter to indicate that this revenue is being raised from all commodities. An example of a use of the latter would be if we wished to raise the tax on beverage and tobacco, while leaving all other VAT rates unchanged.

Equations *E_p2*, *E_p3* and *E_p5* show the corresponding equations for investment, household expenditure and public expenditure respectively, with similar definitions for the rates of change of the power of the *ad valorem* taxes, *t2*, *t3* and *t5*. Note that *t3* is household independent, so the government must charge urban households the same rate of indirect taxation as rural households. This is likely to be violated in so far as a particular commodity is in fact a composite of a number of commodities that face a different tax rate depending on household. So, for example, increased expenditure on alcohol might be weighted towards spirits that have a very high

[2] By 'market' we mean the various components of final demand and intermediate sales to other companies.

marginal tax rate in (say) rural households while might be weighted towards (relatively) low taxed beer in urban areas.

Equation *E_p4* in a simplified version (excluding margin flows) is shown below. It shows that the purchasers' price (in IR£) for each commodity going to each destination[3] is equal to the basic price plus the export tax minus the export subsidy. Therefore the price wedge between the purchaser and the producer is an *ad valorem* tax and *ad valorem* subsidy, with associated rates of change, namely *t4*(c,d) and *t4exp*(c,d).

$$p4(c,d) = pe(c,d) + t4(c,d) - t4exp(c,d) \qquad (19)$$

The tax rate change variable has the facility to change by commodity, while also allowing a uniform tax shifter. The subsidy rate variable allows shifts that are commodity specific and independent of destination, destination specific and independent of commodity, both destination and commodity independent, and finally a shift variable that is both commodity and destination specific.

3.6.2 Zero Pure Profits in Factor Markets

This section deals with the relationship between the middle block in figure 4 and the left hand block. In effect, each equation says that the owners of each of the factors of production get the rent accruing to the factor plus any subsidy that accrues to it.

Equation *E_p1cap* says that the difference between the rate of change of *p1cap(i),* i.e. the actual rent that accrues to capital, is equal to the total rent minus the rate of subsidy payment. Therefore, where there are no tax rates and subsidies are strictly positive, the basic (owners) price will always be greater than the purchasers' (capital users) price. The equations *E_p1lab(i,o)* and *E_p1lnd(i,l)* have a similar definition but are extended to allow for the fact that there are nine different occupation types and three different land types.

[3] The destinations are the UK, Continental Europe and the Rest of the World.

As with indirect subsidies, factor subsidies are modeled to allow for a lot of flexibility. Consider land subsidies. They are allowed a land type specific change independent of industry, an industry specific change independent of land type, a shift which is both independent of land type and industry and finally a shift variable which is both specific in the land type and industry.

3.7 Other Equations

Excerpt 27 contains five equations, the first three of which are market-clearing conditions, while the last two define percentage changes for the aggregate demand for imports and labour. Equation *E_p0_B* ensures that demand equals supply for non-margin commodities. The right hand side of this equation includes a *Äx6* term, which measures any (by necessity, exogenous) change in the level of inventories.

Equation *E_x0imp* defines the variable x0imp, which is a measure of the (change in) aggregate import use. Similarly equation *E_x1lab* defines aggregate demand for occupation-specific labour. The final section incorporates numerous Excerpts that are included in the model. Many of these excerpts don't actually do any work, in the sense that they don't actually change any of the calculations. However they are very important to aid the interpretation of the results of any simulation and as a diagnostic tool to ensure the model is doing what it is expected to do.

GDP from Income and Expenditure Side

Excerpt 30 calculates the percentage change of the nominal aggregates, which make up GDP from the income side. The first three equations calculate returns to the three factors of production, namely land, labour and capital.

There are three further equations. The first relates to Other Cost Tickets that are all zero in this model. The change in indirect taxes must also be included as this is the government's share in value added, and thus forms part of GDP. Finally, pure profits exist for the returns to scale simulations and must be included. For the perfectly competitive core model, these will be zero.

Excerpt 31 calculates the percentage change of the nominal aggregates that make up GDP from the expenditure side. This will be calculated by reference to the familiar identity:

$$Y = C + I + G + In + (X - M) \qquad (20)$$

where *In* is inventory.

Trade Balance and Other Aggregates

The balance of trade is calculated in Excerpt 32 and is calculated as a percentage of GDP. It is not calculated as a percentage change as zero is a feasible value. The other equations of this excerpt define various volume, price and value indexes for imports, capital and labour.

Rates of Return and Investment

In the model, the creation of new units of capital is determined by the rate of return in each industry. The higher the rate of return the more capital that is created in that industry. Note that the rate of return is defined as twice the rate of change in the return on capital minus the change in the cost of a unit of capital. Therefore, if the cost of a unit of capital increases by as much as the increase in the return on capital, there will no increase in the return on capital. The reason for multiplying by a factor of 2 is in recognition of the fact that a lot of investment is needed simply to replace depreciated capital goods. The value 2.0 corresponds to the ratio Q (= ratio, gross to net rate of return) from Dixon et al (1982) and is a typical value of this ratio.

The equation *E_x2tot* relates the change in capital/investment ratio to the net rate of return minus the economy wide rate of return. The variable finv(i) allows for exogenous shifts in investment in each industry *i*.

4 Choosing the Model Closure

The model as specified will contain more variables (n) than equations (m), and will thus require a number of variables (n-m) to be set exogenously. From a purely mathematical perspective we must ensure that we choose the (n-m) variables appropriately to ensure that the coefficient matrix is invertible. Thankfully, economic intuition provides much guidance as to what constitutes a mathematically appropriate closure. For example, if we hold the price of (say) an imported good constant, chances are we will have to endogenise the quantity imported. If we were then to try and exogenise both variables, then elementary economics would tell us that we would need to allow demand or supply curves to shift appropriately – therefore we would need to endogenise taste and/or technology variables.

In each particular market the choice of closure depends primarily upon three not entirely mutually exclusive considerations.

Firstly, the choice depends upon the nature and availability of the data and the underlying phenomena that determine them. So, for example, in most traditional 'forward looking' modelling, the taste and technology parameters are held exogenous, as we as economists have little to say about the former, and are pretty vague at best in relation to the latter. Where these variables are not held exogenous, they are frequently used as a 'mop up' of any residual real movements left over after consideration has been made for relative price movements. Given that in most comparative-static experiments we are abstracting from time, it seems reasonable in these circumstances to hold technology and taste constant.[4]

Secondly, it depends on the assumed economy-wide responses. A classic example of this is that for many short-term simulations, the real (or perhaps nominal) wage rate is held constant in recognition of the assumption that the presence of such institutional rigidities such as unions prevents a quick response to a shock. Adjustment comes via the total numbers employed, which, it is argued, is much more sensitive to short run fluctuations. This sticky wage assumption can then be eased in a long run simulation, with real wages adjusting.

[4] A notable exception is the 'Transmission via Trade' school, worked examples of which in a GCE context can be found in Lee et al (2000) Lejour et al (2000) and van Meijl, H. and F. van Tongeren.

Finally, it depends on what you want to find out. So, if you were interested in the impact of a change in a tax rate, then this would be held exogenous and shocked by an appropriate amount. However, if the government wanted to target an employment level in a particular industry, we would allow the tax rate to vary endogenously so that the exogenous employment target is satisfied.

We can take a step towards the automation of the construction of an appropriate closure by use of a table of variables and equations as illustrated in Horridge *et al* (1998) (see table 1 below). There are five columns in this table. The first column indicates the dimension. The 'Macro' dimension might more properly be called the scalar dimension, as it refers to all scalar variables. The second column indicates the variable count in the entire model, while the third column indicates the equation count. So, for example, the version of the model presented below had 126 scalar variables and equations that explained 94 of these variables.

This leaves us with 32 variables that must be set exogenously. Given the convention of assuming that the equation named (for example) *E_delB* explains the variable *delB*, we can round up the 32 variables that have no matching equations and list them in column 5. In essence, this column suggests these variables as obvious candidates for setting as exogenous. Looking down the columns we can see that this procedure is done for each of the dimensions present in the model.

There is one final aspect of table 1 that requires explanation, and that is the numbers in brackets in the variable and equation counts. Looking at, for example, the COM variables, we see that the variable count is 19 + (1), while the equation count is 12 + (3). The variable in brackets - denoted by (1) - is in effect explained by three equations – denoted by (3). So, for example, the COM equation that defines industry use of commodities has three different forms depending on whether the purchasing industry is agriculture, manufacturing or services. This is to allow for a technology shift away from (say) electricity in all agricultural industries, in all manufacturing industries or in all service industries. Note that these variables and equations all cancel each other out. So, in the COM case, the exogenous count is 19 – 12 = 7.

The (1) in the variable count and the (3) in the equation count automatically cancel each other out. Through this method we can greatly simplify the process of identifying a mathematically adequate closure.

Table 1:
Tally of Variables and Equations

1 Dimension	2 Variable Count	3 Equation Count	4 Exog. Count	5 Unexplained Variables
MACRO	**126**	**94**	**32**	**Phi q_h omega f4pgen f5tot2 f3tot_h ffinv_I fgostax a1lnd_il f1lab_io f1prim_I f3tax_cs f5tax_cs f1tax_csi f2tax_csi f_t1cap_I f_t1lab_io f_t1lnd_il f_t4exp_cd x4_ntrad_d f4p_ntrad_d f4q_ntrad_d w0cif_adj_c f4tax_trad_d ff_subsidy_I f_twist_lk_I f4tax_ntrad_d ff_subsidy_ia ff_subsidy_im ff_subsidy_is twist_src_bar f_inctaxrate_h**
COM	**19 + (1)**	**12 + (3)**	**7**	**Ac f4p_d t0imp f0tax_s f_t4exp_d a1_si_agri ftwist_src**
COM*IND	**7 + (1)**	**5 + (5)**	**2**	**a1_s a2_s**
COM*HOU	**6**	**4**	**2**	**a3_s f3_s**
COM*SRC	**11 + (1)**	**9 + (3)**	**2**	**f5 fx6**
COM*SRC*IND	**9**	**7**	**2**	**a1 a2**
COM*SRC*IND*MAR	**4**	**2**	**2**	**a1mar a2mar**
COM*SRC*MAR	**5**	**3**	**2**	**a3mar a5mar**
COM*SRC*HOU	**1**	**0**	**1**	**a3**
IND	**37 + (1 + 1 + 1 + 1)**	**20 + (2 + 3 + 3 + 3)**	**17**	**a1oct f1lab_o a1cap a2tot f1oct f1tot ffinv p0cap a1prim a1lab_o a1lnd_I f_t1cap f_t1lnd_I f1tot_obs f_t1lab_o ff_subsidy f_twist_lk**
IND*OCC	**6**	**4**	**2**	**f_t11lab f1lab**
IND*LND	**5**	**3**	**2**	**f_t1lnd x1lnd**
OCC	**7**	**5**	**2**	**f1lab_I f_t1lab_i**
HOU	**18**	**15**	**3**	**f_inctaxrate a3_cs f3_cs**
HOU*HOU	**1**	**1**	**0**	
COM*DES	**6 + (1 + 1 + 1)**	**3 + (3 + 2 + 2)**	**3**	**f4p f4q f_t4exp**
OCC*HOU	**1**	**1**	**0**	
COM*FANCAT	**(1)**	**(4)**	**0**	
I1AGRI	**2**	**1**	**1**	**fitot_agri**
I2MANU	**6**	**3**	**3**	**f1tot_manu a1_manu_manu a1_manu_serv**
I3SERV	**2**	**1**	**1**	**f1tot_serv**
LND	**3**	**2**	**1**	**f_t1lnd_i**
COM*MAR*DES	**2**	**1**	**1**	**a4mar**
TOTAL	**284**	**196**	**88**	

5 Approach to Solving the Model

Because of the size and structure of CGE models, the limited processing capacity of computers until recently has presented modellers with a difficult computational task (Bautista, 1992). A major stimulus to early CGE modelling was provided by Scarf's (1967) algorithm, based on Brouwer's fixed-point theorem, for the computation of a Walrasian equilibrium system. According to Shoven and Whalley (1984), the recent advances, particularly in relation to the Scarf algorithm, mean 'that it is no longer the solution methods that constrain model applications, but the availability of data and the ability of modellers to specify key parameters'. By way of a worked example, a fixed-point algorithm is used in solving for equilibrium prices in Clarete's (1984) study.

One major drawback of the fixed-point algorithm, however, is that it does not take full advantage of the fact that our base data represents an initial general equilibrium solution. Using Johansen's (1960) procedure of taking total differentials of the logarithms of non-linear equations, the non-linear equation system is transformed into a linear one and a simple matrix calculation can be applied. This method is discussed in section 5.1. Of course, the solution obtained this way is an approximation, and the method gives no measure as to how large these approximation errors might be. These linearisation errors can be reduced or eliminated by using a procedure developed by Dixon et al. (1982) that combines Euler's method extended by a simple extrapolation exercise (see section 5.2). Finally we discuss Gragg's method, which is the default GEMPACK method, in section 5.3.

5.1 Johanson Solutions

Consider the general form of CGE models, in which an equilibrium is assumed to be observed at a vector, V, of length n. The vector satisfies the system of equations:

$$F(V) = 0 \qquad (21)$$

In the IMA*GE* model the function F is assumed to be differentiable, and is represented by m equations, the assumed functional forms of which are discussed in section 3. The number of variables n in the model will always be greater than or equal to the number of equations, otherwise the system would be over-determined. Hence we require n-m exogenous variables to close the model, and will solve the model for the remaining m endogenous variables.

To obtain the linearised form we must assume that the coefficient matrix is constant, so (21) above can be expressed as the product of a constant m * n matrix and the n * 1 vector of percentage-change in V:

$$Av = 0 \qquad (22)$$

Partitioning v into its exogenous (v_x) and its endogenous (v_n) components results in:

$$A_1 v_x + A_2 v_n = 0 \qquad (23)$$

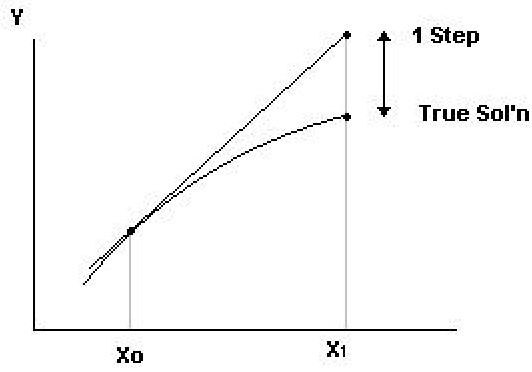
Finally, we can rearrange to give:

$$v_n = -A_2^{-1} A_1 v_x$$

The typical ij^{th} component, $[-A_2^{-1} A_1]_{ij}$, represents the elasticity of the i^{th} endogenous variable with respect to the j^{th} exogenous variable calculated at the initial base data set V^I. As we move away from this initial solution, if the model is to be accurate, we need to update the A matrix to reflect the new elasticities. For example, as the amount of labour in the economy rises relative to capital, then so will the values of the share parameters used in A (e.g. labour's share in total factor usage) which are used to derive the optimal use of factor resources given a CES technology. By employing Johanson's technique we choose not to update the matrix at all. The solution produced is as in figure 5.

Figure 5

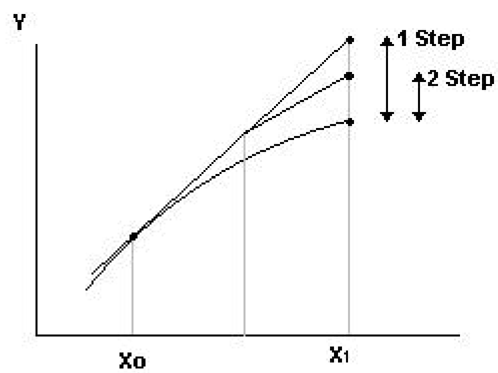
The Linear Johanson Solution



5.2 Euler's Method

Consider figure 5 again. If we bisect the exogenous shock (which is represented on the horizontal axis), apply half of the exogenous shock, recalculate the $[- A_2^{-1} A_1]$ matrix and then apply the remainder of the shock, we reduce the error in the simulation by ($ÄY_{n=1}$ - $ÄY_{n=2}$). We can then repeat this process by bisecting each of the new intervals again so that the entire shock has been subdivided into four sections (so n=4), with the result that the error is further reduced. In fact, as n , the number of intervals approaches infinity and therefore the width of the interval tends to zero, so we are in effect recalculating the coefficient matrix continuously, producing an exact solution. For a more rigorous mathematical discussion of Euler's method, see Dixon *et al*, 1980, pp 202-8.

Figure 6
Solution Using Euler's Method



Given all of the above, it is natural to ask, how large does n have to be to ensure that the linearisation errors are reduced satisfactorily? Using a simple extrapolation procedure (see Dixon *et al*, 1980, pp 206-7), the answer turns out to be very small, with n=2 generally providing a very satisfactory result, even for very large deviations in the exogenous variables.

This is achieved by way of an extrapolation procedure. With a value of n=1, namely a Johanson solution, the endogenous variables increase by Ä$Y_{n=1}$. With a value of n=2 the endogenous variables increase by Ä$Y_{n=2}$. Both of these are illustrated on figure 6. The improvement[5] in accuracy is (Ä$Y_{n=1}$ - Ä$Y_{n=2}$). Continuing and bisecting the 'leap' in exogenous values again gives us Ä$Y_{n=4}$, which is a further improvement in accuracy of (Ä$Y_{n=2}$ - Ä$Y_{n=4}$). Dixon *et al* comment that the simple rule, which works well in practice in the ORANI model, is that each bisection of the 'leap' in exogenous values implemented results in an accuracy improvement of half the improvement to the previous leap. So, for example:

$$\Delta Y_{n=1} - \Delta Y_{n=2} = 2 * \left(\Delta Y_{n=2} - \Delta Y_{n=4} \right) \tag{24}$$

We can take advantage of this shortcut and sum the series of errors, remembering that Ä$Y_{n=}$ is the true answer. The difference between the 1-step Johanson and the true solution is as follows:

$$\begin{aligned} \Delta Y_{n=1} - \Delta Y_{n=\infty} &= \left[\Delta Y_{n=1} - \Delta Y_{n=2} \right] + \left[\Delta Y_{n=2} - \Delta Y_{n=4} \right] + \ldots \\ &\quad \ldots + \left[\Delta Y_{n=2^k} - \Delta Y_{n=2^{k+1}} \right] + \ldots + \left[\ldots - \Delta Y_{n=\infty} \right] \end{aligned} \tag{25}$$

$$= \left[\Delta Y_{n=1} - \Delta Y_{n=2} \right] * \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \right) \tag{26}$$

[5] We must be careful – we cannot necessarily be sure that these are improvements in model accuracy at all. However, assuming that the functional forms could be closely approximated with a quadratic function is sufficient reassurance. The reason for this is that the derivative of a polynomial of degree (say) 3 will be of degree 2, while our approximation is linear so it will be at most of degree 1.

$$= [\Delta Y_{n=1} - \Delta Y_{n=2}] * 2 \qquad (27)$$

Finally, rearranging the above gives:

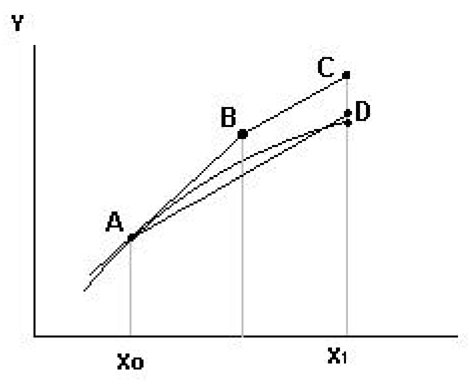
$$\Delta Y_{n=\infty} = 2 * \Delta Y_{n=2} - \Delta Y_{n=1} \qquad (28)$$

Hence by simply performing Euler up to *n*=2 and extrapolating, we can derive a very simple procedure for eliminating much of the linearisation error.

5.3 Gragg's Method

A further improvement in accuracy can be achieved by employing Gragg's method, which is in fact the default method assumed by GEMPACK. While both the Euler and Gragg method calculate the slope at B, the Euler method continues from B to C, while the Gragg method returns to the previous point. Generally, the Gragg method converges much more quickly than the Euler method, though for highly non-linear simulations, it has a tendency to diverge. In such a case, the Euler should be used, though this of course might also diverge.

Figure 7
Solution using Gragg's Method



Problems would only arise if more complex functional forms modelling more complex behavioural characteristics such as a backward bending labour supply curve were introduced. In essence, any function with significant 3rd order (or higher) derivatives may cause problems, though we can be fairly sure that even these would not cause problems in the fairly limited absolute deviations that are likely to be of concern to us in most simulations. Using this procedure with n=2 and employing the extrapolation procedure on a computer with a Pentium II 300MhZ processor on the full version of the model takes about three minutes. As such, model size or complexity is no barrier to us in deciding which simulations to run or what aspects of the economy to model.

6 Conclusion

This working paper has discussed the form of the model code, the technology assumptions, i.e. functional forms, underlying the model and the method used for solving the model.

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